N=2 Gauge Theories

S-duality, holography and a surprise

DG, G. Moore, A. Neitzke  to appear

DG:  arXiv:0904.2715


Overview

M5 branes on Riemann surfaces.
• 6d worldvolume field theory on Riemann surface C
• Flow to IR to engineer N=2 4d SCFTs

Outline:
• Motivations
• How do the 4d theories look like?
• Can they be built directly in 4d?
• A final surprise: Liouville theory from 4d gauge theory
Motivations.

Maldacena, Nunez: hep-th/0007018.

$\text{AdS}_5 \times M_6$ M-theory solutions

- RG flow to an IR regular geometry \text{MN}
- UV: M5 branes wrapping a Riemann surface $C$
- Part of a larger family of $M_6$ \text{LLM}
  - Hard to build new solutions

Dual to unknown $N=2$ four dimensional SCFTs
Motivations.

Kontsevich, Soibelman: 0811.2435.
DG, Moore, Neitzke: 0807.4723

Mathematical progress: KS Wall Crossing Formula
• About “motivic Donaldson-Thomas invariants”
• Predicts jumps of BPS degeneracies in N=2 theories!

Physical explanation? GMN
• Consider the N=2 theory on a circle.
• BPS instantons correct moduli space metric
• Metric is continuous! Wall crossing formula follows
Motivations

Witten: hep-th/9703166.
Cherkis, Kapustin: hep-th/0006050

We needed examples of BPS spectra, moduli spaces.
• A lot of N=2 quivers have a IIA brane realization. Witten
• Lift to a system of M5 branes.
• Some M5 wrap a cylinder or torus C
• Some intersect transversally

Focus on (2,0) theory on C
• Transversal M5s as defects
• Allows computation of BPS spectra, 3d moduli space
Motivations

Now we have useful defects available!

- Consider M5 on punctured Riemann surface C
- Place defects at punctures.
- A very large class of 4d N=2 SCFTs.
- It includes many well known gauge theories
- Most theories are unknown

We identified the holographic dual defects in AdS$_5 \times$ M$_6$

GM
Motivations

Argyres. Seiberg: 0711.0054.

Generalization of S-duality.
- Consider $N_f=6$ SU(3) gauge theory at strong coupling
- A dual weakly coupled SU(2) gauge theory emerges
- Coupled to interacting $E_6$ SCFT

Can we produce other new, interesting $N=2$ theories?

S-duality will be crucial to understand M5 branes on $C!$
(2,0) theory on $C$

How to wrap M5 branes on $C$?
- Worldvolume theory of $N$ M5s is $(2,0)\ A_{N-1}$ SCFT
- Needs twisting to preserve SUSY on $C$
- $N=2$ SUSY in 4 dimensions

Only complex structure of $C$ matters in IR
- Seiberg-Witten curve only depends on complex structure
- BPS spectrum only depends on complex structure
- Gravity RG flow to metric of constant negative curvature
- $3g-3+n$ moduli of $C$ are couplings of 4d IR theory
(2,0) theory on C

What sort of couplings are those?

- Consider some degeneration limit of C
- Long, thin tubes develop.
- Moduli $\tau_i$ of tubes parametrize C
Degeneration of C

(2,0) theory on a circle is is 5d SU(N) Yang Mills

- Long thin tubes give 5d SYM on a long segment
Degeneration of C

5d SYM on segments may give 4d SYM, coupling $\tau_i$
- Hint of $(3g-3)$ weakly coupled SU(N) gauge groups
- What are they coupled to?
- Tubes connected to $(2g-2)$ “pair of pants” theories.

We expect some sort of “generalized quiver”
Degeneration of C

Riemann surfaces can degenerate in many ways

There must be a network of S-dualities
S-duality in SU(2) gauge theories

Nf=4 SU(2) gauge theory is a SCFT

- Exactly marginal gauge coupling $\tau$
- Usual SL(2,Z) S-duality action $\tau \rightarrow (a \tau + b)/(c \tau + d)$
Nf=4 SU(2) has SO(8) flavor symmetry
• SL(2,Z) acts through triality on SO(8)
• Exchanges electrons in $8_v$, monopoles in $8_s$, dyons in $8_c$
S-duality in SU(2) gauge theories

Reformulating triality

- Consider subgroup SO(4)xSO(4) in SO(8)
- Rewrite it as \([SU(2)_a \times SU(2)_b] \times [SU(2)_c \times SU(2)_d]\)
- \(SL(2,Z)\) permutes \((a,b,c,d)\)
  - \(8_v=(2a \times 2b) + (2c \times 2d)\)
  - \(8_s=(2a \times 2c) + (2b \times 2d)\)
  - \(8_c=(2a \times 2d) + (2c \times 2b)\)
2 M5 on Riemann Surface

Brane construction for SU(2) Nf=4: A_1 (2,0) theory on a sphere with four defects.
The SU(2) pants

Two SU(2) doublets have SO(4) = SU(2)_2 × SU(2)_3 flavor symmetry.

• All three SU(2)_{1,2,3} can be gauged, play symmetric role
• A lot in common with 2 M5 on a pair of pants...
• Can S-duality of Nf=4 SU(2) insure consistency of gluing?
• Let’s glue, and see...
S-duality in SU(2) quiver gauge theories

Consider a superconformal quiver with two SU(2) nodes

S-duality at first node permutes SU(2)_a, SU(2)_b, SU(2)_c

At second node permutes SU(2)_c, SU(2)_d, SU(2)_e

They do not commute!

Full duality group permutes five SU(2) groups
S-duality in SU(2) quiver gauge theories

Now three nodes

\[
\begin{array}{c}
\text{SU(2)}_a \\
\text{SU(2)}_b \\
\text{SU(2)} \\
\text{SU(2)}_c \\
\text{SU(2)} \\
\text{SU(2)}_d \\
\text{SU(2)}_e \\
\text{SU(2)}_f \\
\text{SU(2)} \\
\text{SU(2)} \\
\text{SU(2)}_c \\
\end{array}
\]
2 M5 on Riemann Surface

\[
\begin{array}{c}
\text{SU(2)}_a \quad \text{SU(2)}_b \quad \text{SU(2)}_c \quad \text{SU(2)}_d \quad \text{SU(2)}_e \quad \text{SU(2)}_f \\
\end{array}
\]

\[
\begin{array}{c}
b \quad q_1 \quad c \quad q_2 \quad d \quad q_3 \quad e \quad f \\
a
\end{array}
\]
2 M5 on Riemann Surface
The SU(3) pants

What’s pair of pants for three M5s?
• SU(3) x SU(3) x SU(3) subgroup in E₆
• Argyres Seiberg duality + brane construction:
  • E₆ SCFT from A₂ (2,0) theory on sphere with three defects
The SU(N) pants

What’s pair of pants for N M5s?

• Needs SU(N) x SU(N) x SU(N) flavor symmetry
• Generalized Argyres Seiberg duality + brane construction:
  • Start from quiver of N-2 SU(N) gauge groups
  • Careful strong coupling limit decouples SU(N-1) dual group
  • Theory splits into desired pair of pants, plus extra stuff
Conclusions.. but not yet the end

We can build N=2 theories labeled by Riemann surface and choice of defects

Gauge coupling moduli space has several distinct weak coupling regions, where theory is a ``generalized quiver'' possibly involving non-trivial interacting SCFTs as matter. Labelled by decompositions of the surface.

Class includes most known N=2 theories, we can produce holographic dual, determine BPS spectra, treat in an unified manner.
A final surprise

N=2 gauge theory and the theory of Riemann surfaces

• How deep does the connection run?

Nekrasov instanton partition function of SU(2) quivers computes 2d CFT conformal blocks! AGT

• SU(2) Nf=4 gives four points on a sphere
• SU(2) N=2* gives one point on torus
• etc. etc.

Full partition function coincides with Liouville theory
The basic idea

$A_1$ Theories are labelled by Riemann surface. Lagrangian descriptions by sewing: build the surface from pairs of pants.

Virasoro conformal blocks for a Riemann surface depend on a choice of sewing. They are defined as a sum over Virasoro descendants of highest weight vectors in the intermediate channels.
Extensions

Anything you can compute from a Lagrangian description, and depends on gauge couplings, should provide a "modular functor", i.e. an object attached to a pant decomposition of a Riemann surface, with interesting properties under channel dualities, fusion, braiding, factorization, etc.

If you are strong enough to deal with non-Lagrangian pair of pants theories, $A_N$ theories should also be fun.