N=2 Gauge Theories

S-duality, holography and a surprise

DG, G. Moore, A. Neitzke to appear

DG: arXiv:0904.2715

DG, J. Maldacena: arXiv:0904.4466

L.F.Alday, DG, Y.Tachikawa: arXiv:0906.3219

Overview

M5 branes on Riemann surfaces.

- 6d worldvolume field theory on Riemann surface C
- Flow to IR to engineer N=2 4d SCFTs

Outline:

- Motivations
- How do the 4d theories look like?
- Can they be built directly in 4d?
- A final surprise: Liouville theory from 4d gauge theory

Motivations.

Maldacena, Nunez: hep-th/0007018.

Lin, Lunin, Maldacena: hep-th/0409174.

$AdS_5 \times M_6$ M-theory solutions

- RG flow to an IR regular geometry MN
- UV: M5 branes wrapping a Riemann surface C
- Part of a larger family of M_6 LLM
  - Hard to build new solutions

Dual to unknown N=2 four dimensional SCFTs

Motivations.

Kontsevich, Soibelman: 0811.2435.

DG, Moore, Neitzke: 0807.4723

Mathematical progress: KS Wall Crossing Formula

- About “motivic Donaldson-Thomas invariants”
- Predicts jumps of BPS degeneracies in N=2 theories!

Physical explanation? GMN

- Consider the N=2 theory on a circle.
- BPS instantons correct moduli space metric
- Metric is continuous! Wall crossing formula follows

Motivations

Witten: hep-th/9703166.

Cherkis, Kapustin:hep-th/0006050

We needed examples of BPS spectra, moduli spaces.

- A lot of N=2 quivers have a IIA brane realization. Witten
- Lift to a system of M5 branes.
- Some M5 wrap a cylinder or torus C
- Some intersect transversally

Focus on (2,0) theory on C

- Transversal M5s as defects
- Allows computation of BPS spectra, 3d moduli space

Motivations

Now we have useful defects available!

- Consider M5 on punctured Riemann surface C
- Place defects at punctures.
- A very large class of 4d N=2 SCFTs.
- It includes many well known gauge theories
- Most theories are unknown

We identified the holographic dual defects in $AdS_5 \times M_6$ GM

Motivations

Argyres. Seiberg: 0711.0054.

Generalization of S-duality.

- Consider Nf=6 SU(3) gauge theory at strong coupling
- A dual weakly coupled SU(2) gauge theory emerges
- Coupled to interacting E_6 SCFT

Can we produce other new, interesting N=2 theories?

S-duality will be crucial to understand M5 branes on C!

(2,0) theory on C

How to wrap M5 branes on C?

- Worldvolume theory of N M5s is (2,0) A_{N-1} SCFT
- Needs twisting to preserve SUSY on C
- N=2 SUSY in 4 dimensions

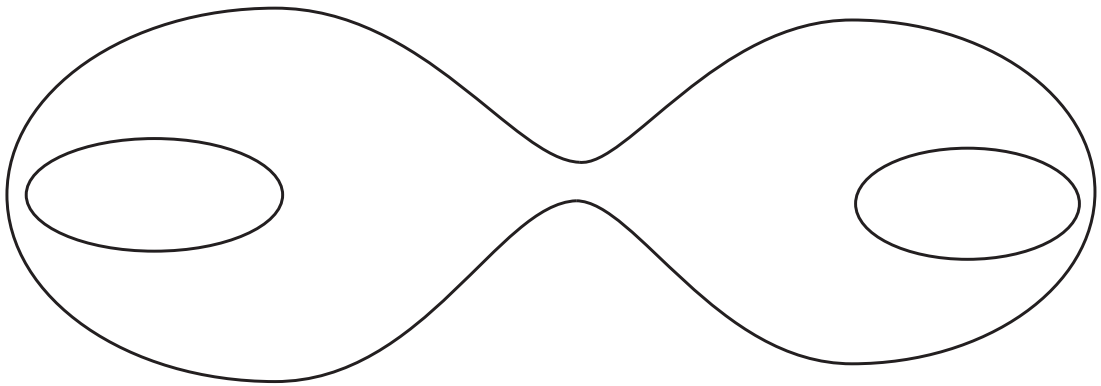
Only complex structure of C matters in IR

- Seiberg-Witten curve only depends on complex structure
- BPS spectrum only depends on complex structure
- Gravity RG flow to metric of constant negative curvature
- 3g-3+n moduli of C are couplings of 4d IR theory

(2,0) theory on C

What sort of couplings are those?

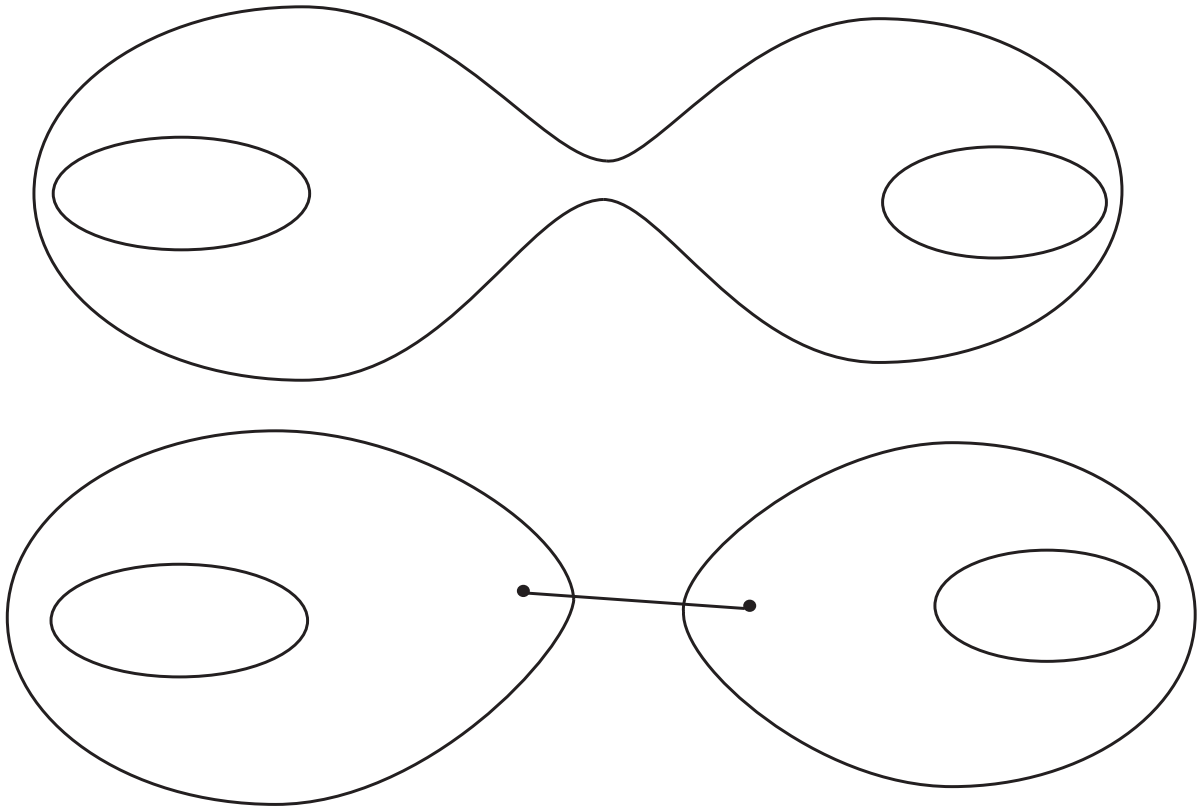
- Consider some degeneration limit of C
- Long, thin tubes develop.
- Moduli τ_i of tubes parametrize C



Degeneration of C

(2,0) theory on a circle is is 5d SU(N) Yang Mills

- Long thin tubes give 5d SYM on a long segment

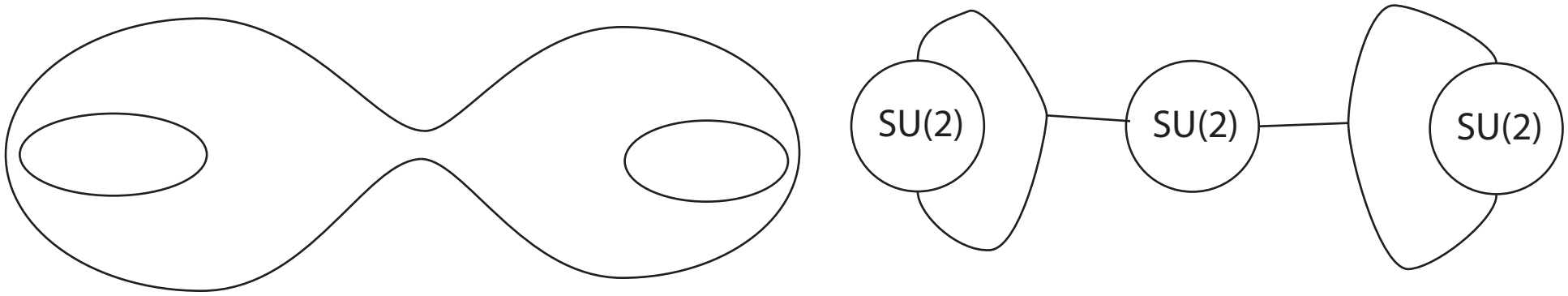


Degeneration of C

5d SYM on segments may give 4d SYM, coupling τ_i

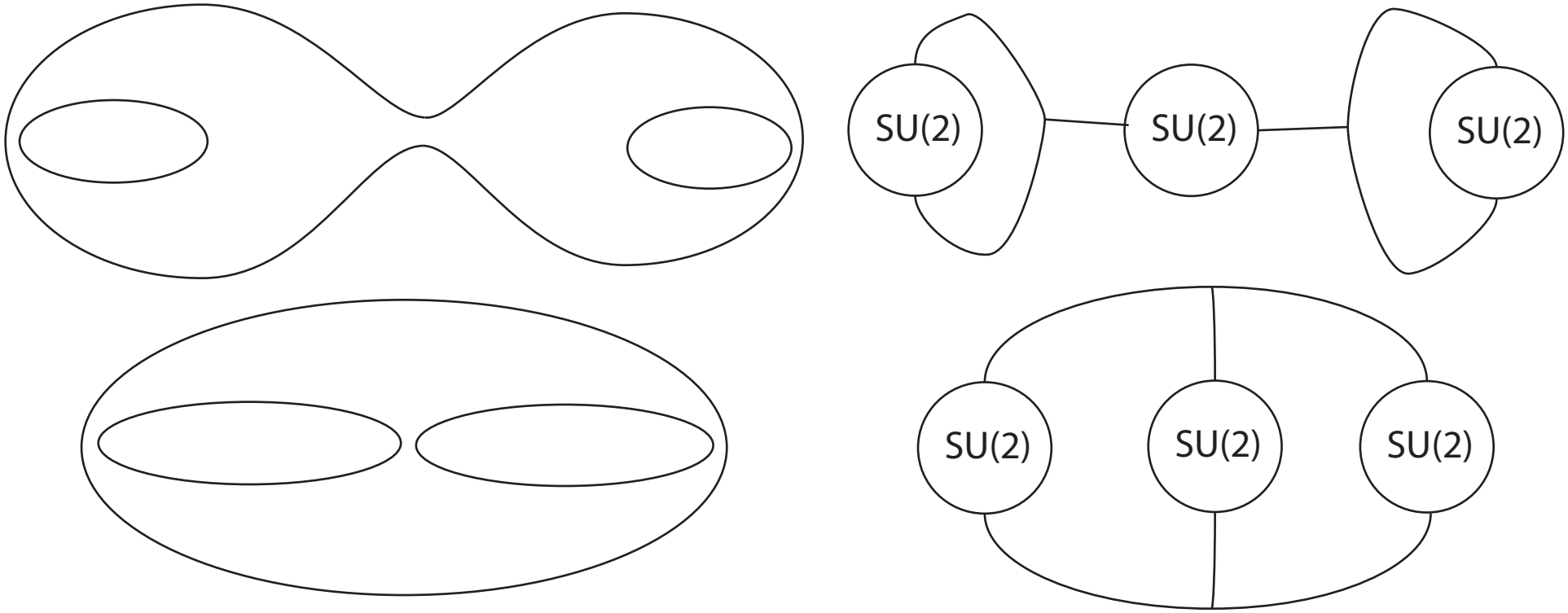
- Hint of (3g-3) weakly coupled SU(N) gauge groups
- What are they coupled to?
- Tubes connected to (2g-2) “pair of pants” theories.

We expect some sort of “generalized quiver”



Degeneration of C

Riemann surfaces can degenerate in many ways

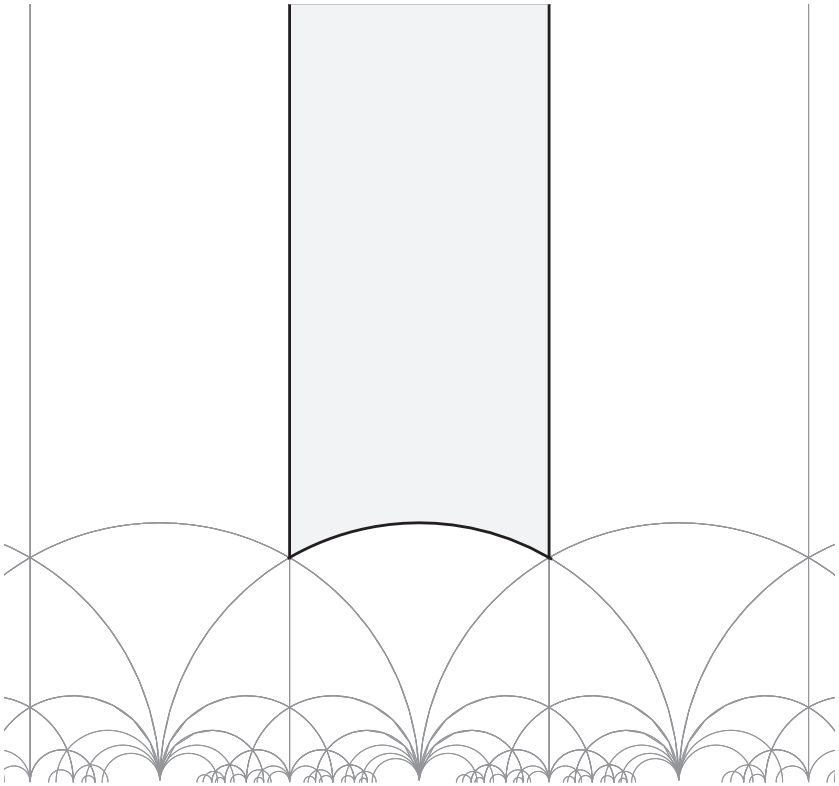


There must be a network of S-dualities

S-duality in SU(2) gauge theories

Nf=4 SU(2) gauge theory is a SCFT

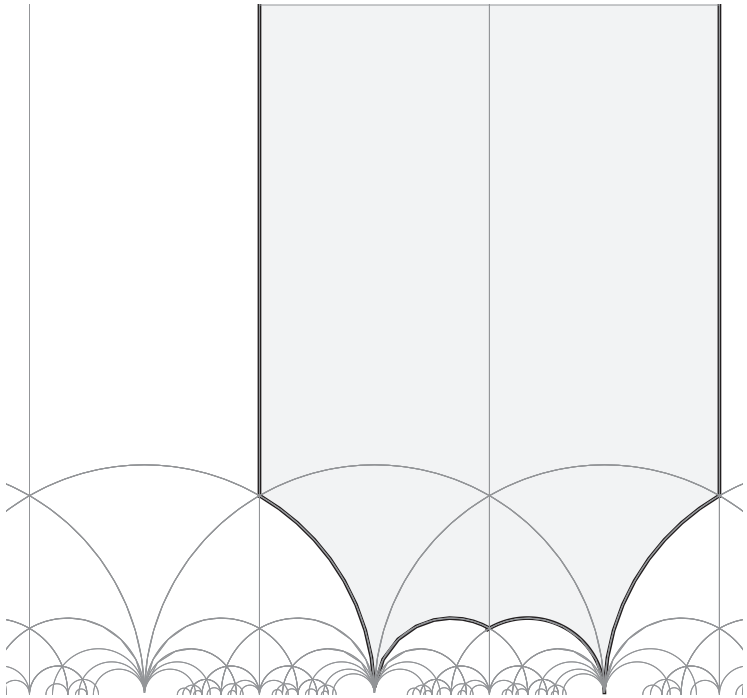
- Exactly marginal gauge coupling τ
- Usual SL(2,Z) S-duality action $\tau \rightarrow (a\,\tau + b)/(c\,\tau + d)$



S-duality in SU(2) gauge theories

Nf=4 SU(2) has SO(8) flavor symmetry

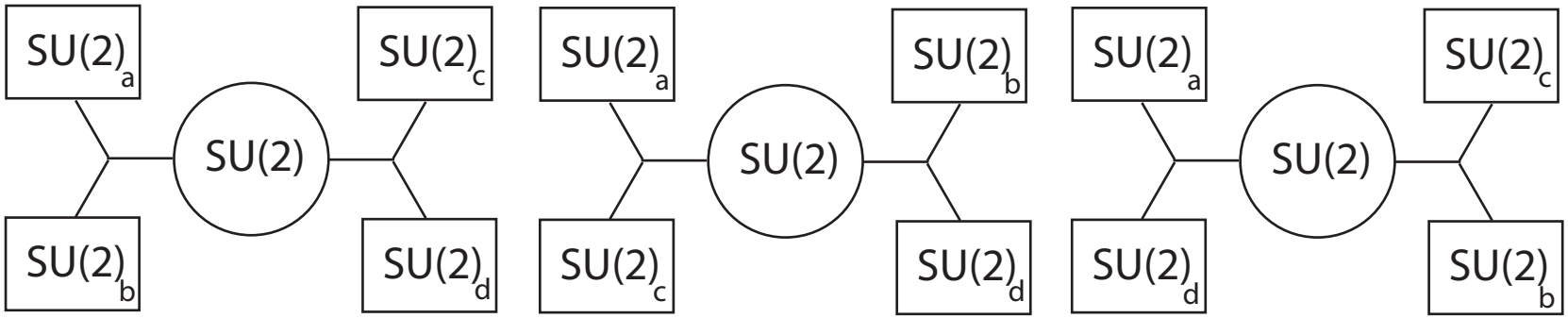
- SL(2,Z) acts through triality on SO(8)
- Exchanges electrons in 8_v, monopoles in 8_s,dyons in 8_c



S-duality in SU(2) gauge theories

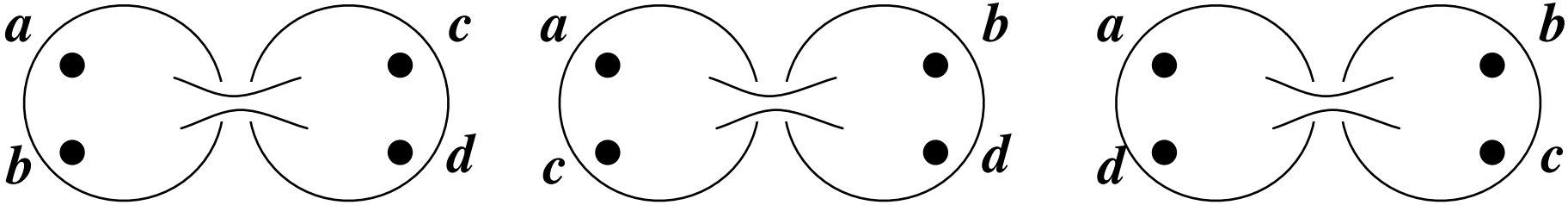
Reformulating triality

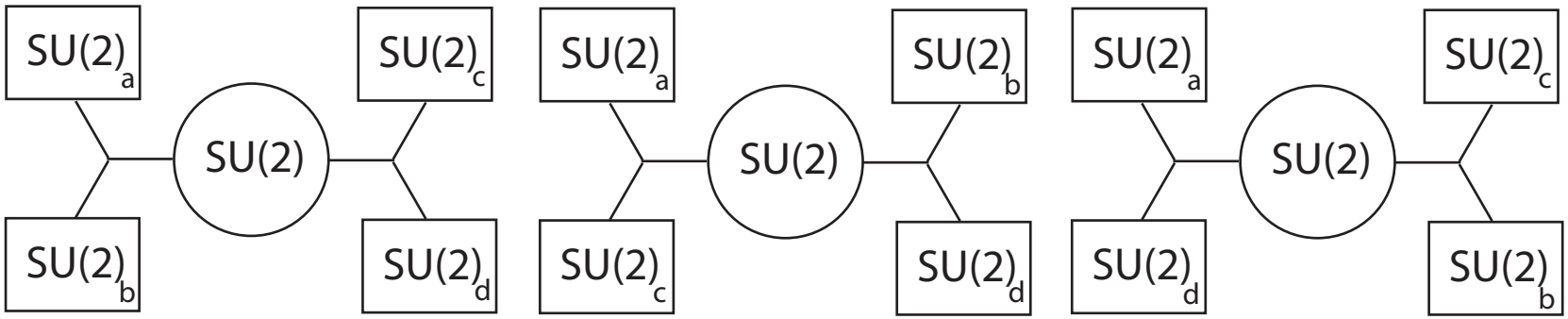
- Consider subgroup SO(4)xSO(4) in SO(8)
- Rewrite it as $[SU(2)_a \times SU(2)_b] \times [SU(2)_c \times SU(2)_d]$
- SL(2,Z) permutes (a,b,c,d)
  - $8_v=(2_a \times 2_b) + (2_c \times 2_d)$
  - $8_s=(2_a \times 2_c) + (2_b \times 2_d)$
  - $8_c=(2_a \times 2_d) + (2_c \times 2_b)$



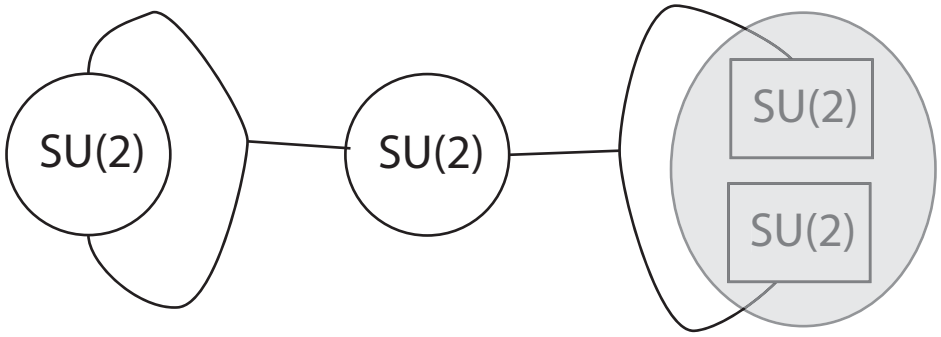
2 M5 on Riemann Surface

Brane construction for SU(2) Nf=4: A_1 (2,0) theory on a sphere with four defects.





The SU(2) pants

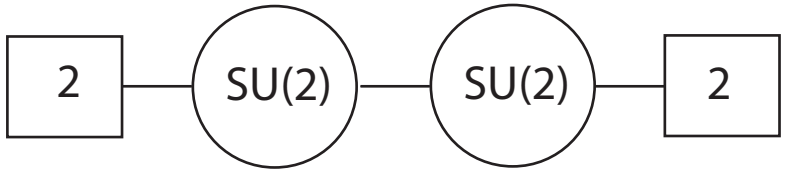


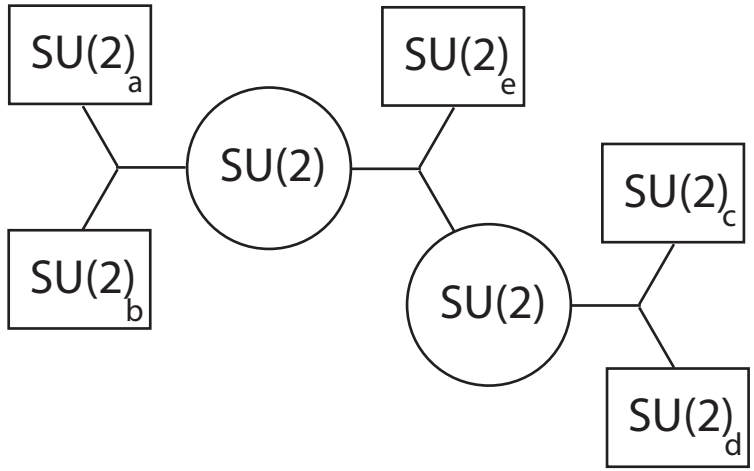
Two $SU(2)_1$ doublets have $SO(4) = SU(2)_2 \times SU(2)_3$ flavor symmetry.

- All three $SU(2)_{1,2,3}$ can be gauged, play symmetric role
- A lot in common with 2 M5 on a pair of pants...
- Can S-duality of Nf=4 SU(2) insure consistency of gluing?
- Let's glue, and see...

S-duality in SU(2) quiver gauge theories

Consider a superconformal quiver with two SU(2) nodes





S-duality at first node permutes $SU(2)_a$, $SU(2)_b$, $SU(2)_c$

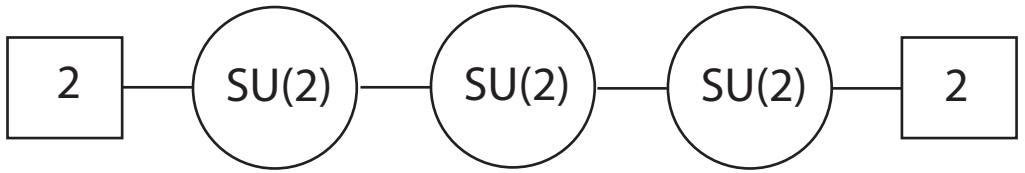
At second node permutes $SU(2)_c$, $SU(2)_d$, $SU(2)_c$

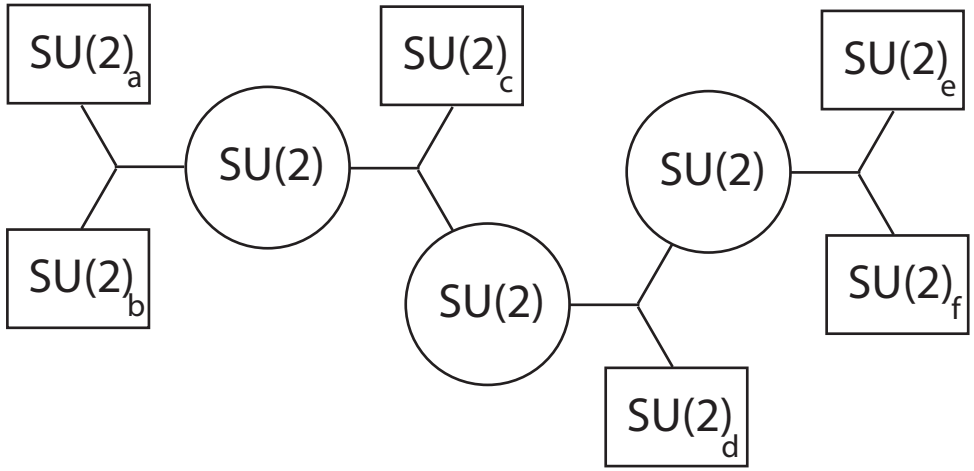
They do not commute!

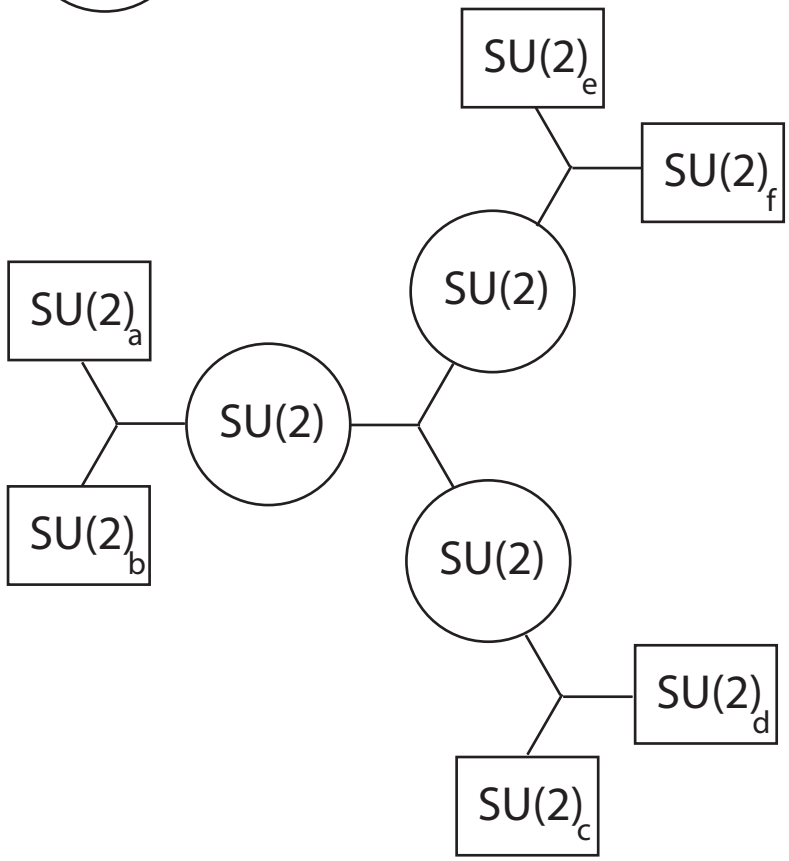
Full duality group permutes five SU(2) groups

S-duality in SU(2) quiver gauge theories

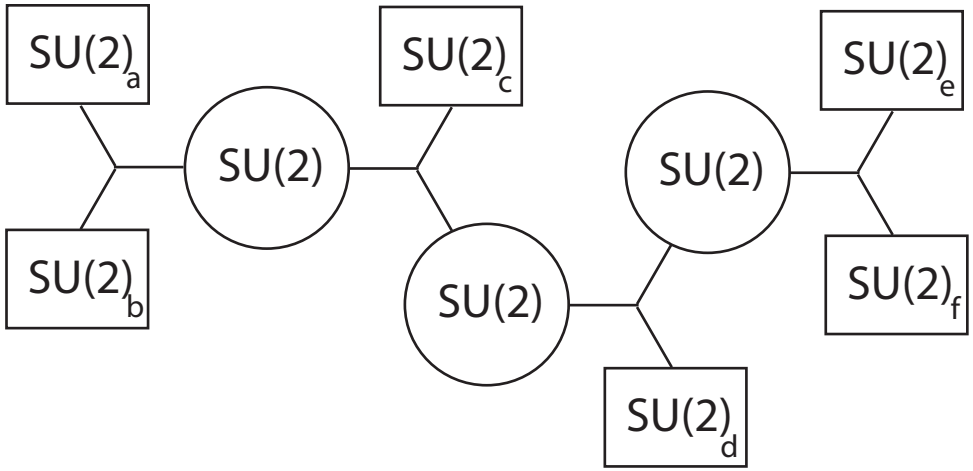
Now three nodes

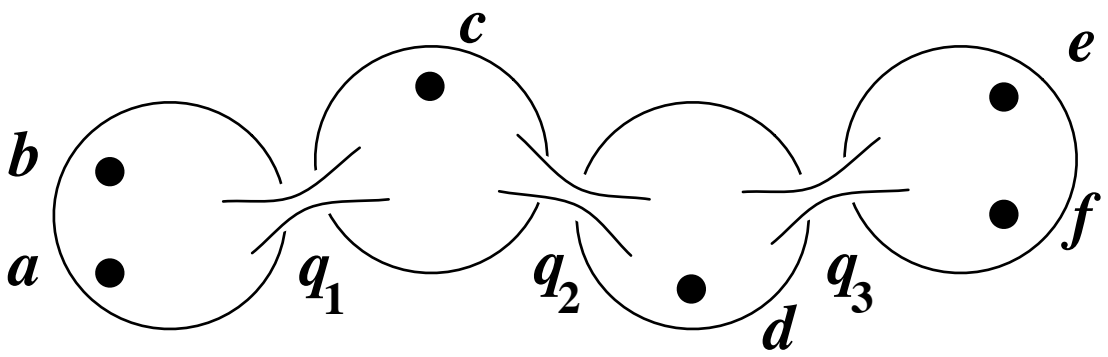




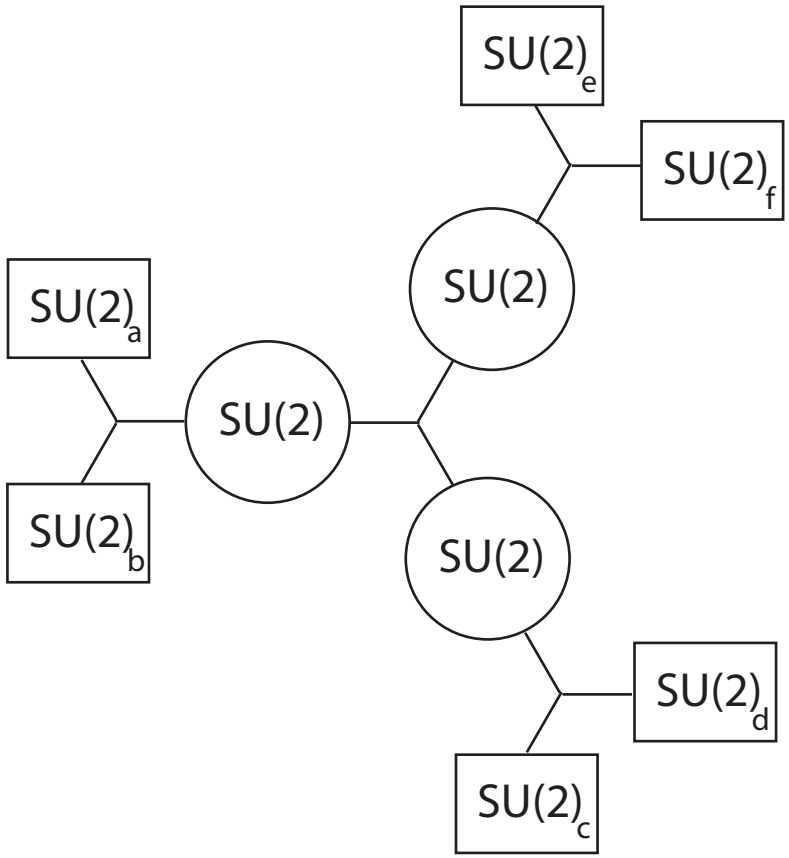


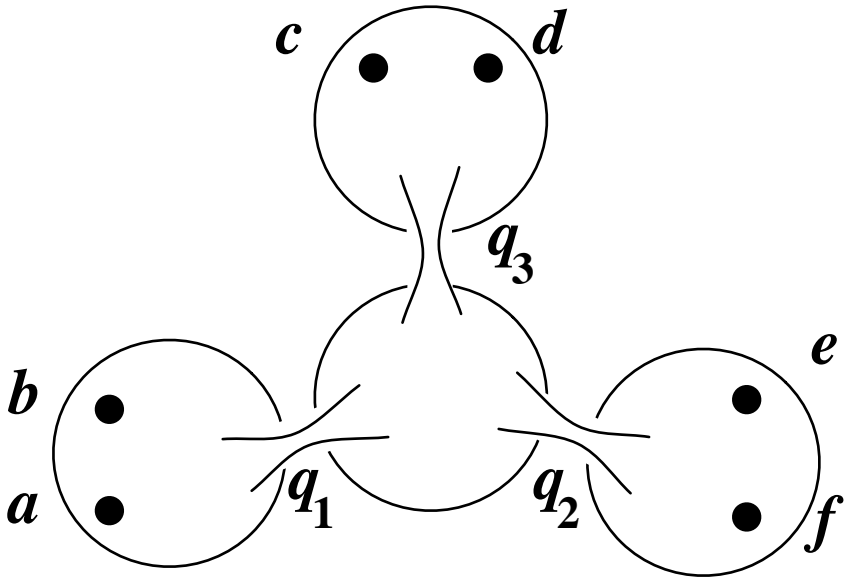
2 M5 on Riemann Surface





2 M5 on Riemann Surface

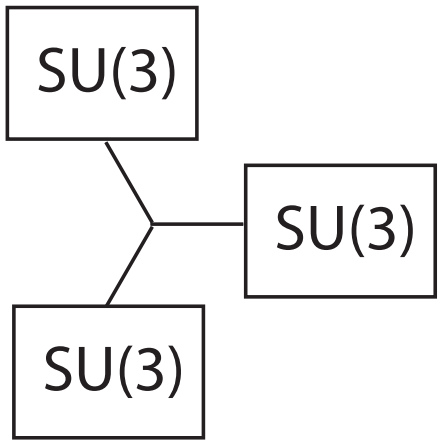




The SU(3) pants

What's pair of pants for three M5s?

- SU(3) x SU(3) x SU(3) subgroup in E_6
- Argyres Seiberg duality + brane construction:
  - E_6 SCFT from A_2 (2,0) theory on sphere with three defects



The SU(N) pants

What's pair of pants for N M5s?

- Needs SU(N) x SU(N) x SU(N) flavor symmetry
- Generalized Argyres Seiberg duality + brane construction:
  - Start from quiver of N-2 SU(N) gauge groups
  - Careful strong coupling limit decouples SU(N-1) dual group
  - Theory splits into desired pair of pants, plus extra stuff

Conclusions.. but not yet the end

We can build N=2 theories labeled by Riemann surface and choice of defects

Gauge coupling moduli space has several distinct weak coupling regions, where theory is a ``generalized quiver'' possibly involving non-trivial interacting SCFTs as matter. Labelled by decompositions of the surface.

Class includes most known N=2 theories, we can produce holographic dual, determine BPS spectra, treat in an unified manner.

A final surprise

N=2 gauge theory and the theory of Riemann surfaces

- How deep does the connection run?

Nekrasov instanton partition function of SU(2) quivers computes 2d CFT conformal blocks! AGT

- SU(2) Nf=4 gives four points on a sphere
- SU(2) N=2* gives one point on torus
- etc. etc.

Full partition function coincides with Liouville theory

The basic idea

A_1 Theories are labelled by Riemann surface.

Lagrangian descriptions by sewing:

build the surface from pairs of pants.

Virasoro conformal blocks for a Riemann surface depend on a choice of sewing.

They are defined as a sum over Virasoro descendants of highest weight vectors in the intermediate channels.

Extensions

Anything you can compute from a Lagrangian description, and depends on gauge couplings, should provide a ``modular functor'', i.e. an object attached to a pant decomposition of a Riemann surface, with interesting properties under channel dualities, fusion, braiding, factorization, etc.

If you are strong enough to deal with non-Lagrangian pair of pants theories, A_N theories should also be fun.