Charges of Monopole Operators in SYM-CS Theories

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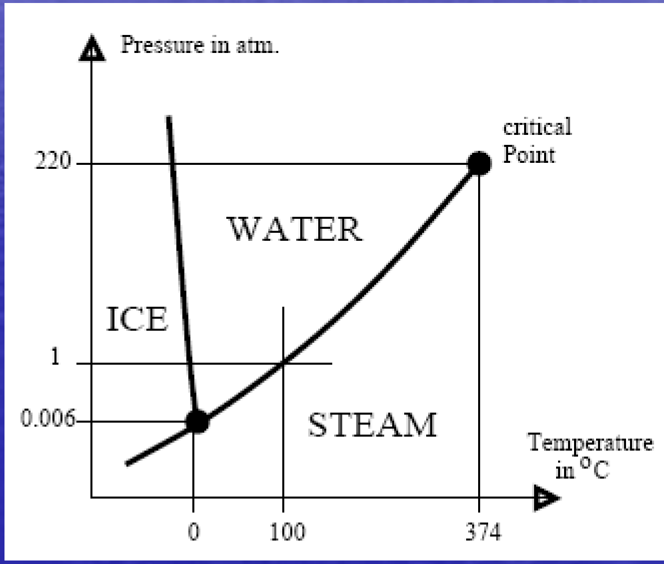
The Promise of AdS_4/CFT_3

- Besides describing all of known particle physics, Quantum Field Theory is important for understanding the vicinity of certain phase transitions, such as the all-important water/vapor transition.
- Here we are interested in a 3-d (Euclidean) QFT.



- This transition is in the 3-d Ising Model Universality Class.
- Other common transitions are described by 3-d QFT with O(N) symmetry.
- 3-d theories are also very important in describing 2-d quantum systems, such as those in the Quantum Hall effect, high-Tc superconductors, etc. Tuning various parameters can lead to fixed points (`Quantum Criticality') in 2+1 dimensions.

A Challenge

- Can we find a set of strongly coupled 3-d conformal theories that have tractable AdS_4 duals?
- One of the motivations is that they could serve as toy models, or `Hyperbolic Cow' approximations, to some physical systems, much like the $\mathcal{N}$=4 SYM in 4-d seems to approximate some aspects of QCD.
- See an article by I.K., J. Maldacena in January 2009 issue of Physics Today

M2 Brane Theory

- The theory on N coincident M2-branes has $\mathcal{N}$=8 superconformal symmetry, the maximum possible in 3 dimensions.
- When N is large, its dual description is provided by the weakly curved AdS_4 x S^7 background in 11-dimensional M-theory.
- This dual description is tractable and makes many non-trivial predictions.

What is the M2 Brane Theory?

- It is the Infrared limit of the D2-brane theory, the $\mathcal{N}$=8 supersymmetric Yang-Mills theory in 2+1 dimensions, i.e. it describes the degrees of freedom at energy much lower than $(g_{YM})^2$
- The number of such degrees of freedom $\sim N^{3/2}$ is much lower than the number of UV degrees of freedom $\sim N^2$. I.K., A. Tseytlin
- Is there a more direct way to characterize the Infrared Superconformal Theory? Schwarz suggested that it should be a Chern-Simons gauge theory.

The BLG Theory

- In a remarkable development, Bagger and Lambert, and Gustavsson formulated an SO(4) Chern-Simons Gauge Theory with manifest $\mathcal{N}$=8 superconformal gauge symmetry. In Van Raamsdonk's SU(2)xSU(2) formulation,

$$X^* = -\varepsilon X \varepsilon$$

$$\epsilon = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$$\begin{aligned} \mathcal{S} = \int d^3x \, \mathrm{tr}\Big[& -(\mathcal{D}^\mu X^I)^\dagger \mathcal{D}_\mu X^I + i\bar{\Psi}^\dagger \Gamma^\mu \mathcal{D}_\mu \Psi \\ & - \frac{2if}{3} \bar{\Psi}^\dagger \Gamma^{IJ} (X^I X^{J\dagger} \Psi + X^J \Psi^\dagger X^I + \Psi X^{I\dagger} X^J) - \frac{8f^2}{3} \mathrm{tr}\, X^{[I} X^{\dagger J} X^{K]} X^{\dagger [K} X^J X^{\dagger I]} \\ & + \frac{1}{2f} \epsilon^{\mu\nu\lambda} (A_\mu \partial_\nu A_\lambda + \frac{2i}{3} A_\mu A_\nu A_\lambda) - \frac{1}{2f} \epsilon^{\mu\nu\lambda} (\hat{A}_\mu \partial_\nu \hat{A}_\lambda + \frac{2i}{3} \hat{A}_\mu \hat{A}_\nu \hat{A}_\lambda) \Big] \end{aligned}$$

- X^I are the 8 fields transforming in (2,2), which is the 4 of SO(4)

$$X^I = \frac{1}{2}(x_4^I \mathbb{1} + i x_i^I \sigma^i)$$

$\mathcal{N}$=2 Superspace Formulation

- Define bi-fundamental superfields rotated by SU(4)$_{\text{flavor}}$ symmetry

$$\mathcal{Z} = Z(x_L) + \sqrt{2}\theta\zeta(x_L) + \theta^2\, F(x_L)\ ,$$
$$\bar{\mathcal{Z}} = Z^\dagger(x_R) - \sqrt{2}\bar{\theta}\zeta^\dagger(x_R) - \bar{\theta}^2\, F^\dagger(x_R)$$

$$Z^A = X^A + iX^{A+4}$$

$$Z^{\ddagger A} := -\varepsilon(Z^A)^{\mathrm{T}}\varepsilon = X^{\dagger A} + iX^{\dagger A+4}$$

- The superpotential is Benna, IK, Klose, Smedback,

$$\mathrm{W} = \frac{1}{4!}\epsilon_{ABCD}\,\mathrm{tr}\,\mathcal{Z}^A\mathcal{Z}^{\ddagger B}\mathcal{Z}^C\mathcal{Z}^{\ddagger D}$$

- Using SO(4) gauge group notation,

$$\mathrm{W} = -\frac{1}{8\cdot 4!}\,\epsilon_{ABCD}\epsilon^{abcd}\,\mathcal{Z}_a^A\mathcal{Z}_b^B\mathcal{Z}_c^C\mathcal{Z}_d^D$$

The ABJM Theory

- Aharony, Bergman, Jafferis and Maldacena argued that the correct description of a pair of M2-branes is slightly different. It involves U(2) x U(2) gauge theory.
- The SU(4) flavor symmetry is not manifest because of the choice of complex combinations

$$Z^1 = X^1 + iX^5 \,, \qquad W_1 = X^{3\dagger} + iX^{7\dagger}$$
$$Z^2 = X^2 + iX^6 \,, \qquad W_2 = X^{4\dagger} + iX^{8\dagger}$$

- The manifest flavor symmetry is SU(2)xSU(2)

$$\mathrm{W} = \frac{1}{4}\epsilon_{AC}\epsilon^{BD}\,\mathrm{tr}\,\mathcal{Z}^A\mathcal{W}_B\mathcal{Z}^C\mathcal{W}_D$$

- For N M2-branes ABJM theory easily generalizes to U(N) x U(N). The theory with Chern-Simons coefficient k is then conjectured to be dual to $AdS_4 \times S_7/Z_k$ supported by N units of flux.
- For k>2 this theory has $\mathcal{N}$=6 super-conformal symmetry, in agreement with this conjecture. Bandres, Lipstein, Schwarz

$SU(4)_R$ Symmetry

- The global symmetry rotating the 6 supercharges is SO(6)~SU(4). The classical action of this theory indeed has this symmetry. Benna, IK, Klose, Smedback

$$V^{\rm bos} = -\frac{L^2}{48}\,{\rm tr}\Big[Y^A Y_A^\dagger Y^B Y_B^\dagger Y^C Y_C^\dagger + Y_A^\dagger Y^A Y_B^\dagger Y^B Y_C^\dagger Y^C + 4 Y^A Y_B^\dagger Y^C Y_A^\dagger Y^B Y_C^\dagger - 6 Y^A Y_B^\dagger Y^B Y_A^\dagger Y^C Y_C^\dagger\Big]$$

$$V^{\rm ferm} = \frac{iL}{4}\,{\rm tr}\Big[Y_A^\dagger Y^A \psi^{B\dagger}\psi_B - Y^A Y_A^\dagger \psi_B \psi^{B\dagger} + 2 Y^A Y_B^\dagger \psi_A \psi^{B\dagger} - 2 Y_A^\dagger Y^B \psi^{A\dagger}\psi_B - \epsilon^{ABCD} Y_A^\dagger \psi_B Y_C^\dagger \psi_D + \epsilon_{ABCD} Y^A \psi^{B\dagger} Y^C \psi^{D\dagger}\Big]\,.$$

Y^A, A=1,…4, are complex N x N matrices.

$$Y^A = \{Z^1, Z^2, W^{1\dagger}, W^{2\dagger}\}$$

- The SU(4) symmetry currents have the standard structure.
- In fact, there is also a $U(1)_B$ current which enhances the symmetry to U(4).
- For k=1 or 2 the global symmetry should enhance to SO(8) according to the ABJM conjecture. In order to write the additional currents we have to employ the \`monopole operators' such as $(\mathcal{M}^{-2})^{\hat{a}\hat{b}}_{ab}$

$$J_\mu^{AB} = \mathcal{M}^{-2}\Big[Y^A \mathcal{D}_\mu Y^B - \mathcal{D}_\mu Y^A Y^B + i\psi^{\dagger A}\gamma^\mu \psi^{\dagger B}\Big]$$

Monopole Operators

- They modify the behavior of fields near an insertion point to create a U(1) magnetic flux through a 2-sphere surrounding the point
  Borokhov, Kapustin, Wu
  Kapustin, Witten

$$A = \frac{H}{2} \frac{\pm 1 - \cos\theta}{r} \mathrm{d}\varphi$$

- For a U(N) gauge theory, the generator H describes the U(1) embedding. Due to Dirac quantization, it is labeled by a set of integers:

$$H = \mathrm{diag}(q_1, \ldots, q_N) \qquad q_1 \geq q_2 \ldots \geq q_N$$

- With Chern-Simons level k, these operators transform under U(N) as a representation with a Young tableaux with $kq_1, kq_2, \ldots, kq_N$ rows.

- In a recent paper, Benna, Klose and I studied the monopole operators in the U(N) x U(N) ABJM theory with $H = \hat{H}$
- For example, the operators like $(\mathcal{M}^{-2})^{\hat{a}\hat{b}}_{ab}$ appearing in the SO(8) R-symmetry currents correspond to $kq_1 = k\hat{q}_1 = 2$
- It is important to prove that the monopole insertion does not alter the `naïve' global charges and dimensions of these currents. This is difficult because the ABJM theory is strongly coupled for small k.

A Strategy

- We will consider the weakly coupled UV completion of the ABJM theory obtained through adding the SYM terms and adjoint kinetic terms that maintain $\mathcal{N}=3$ SUSY. J. Maldacena
- This theory has SU(2) R-symmetry, and we will calculate the representations of monopole operators using semi-classical methods.
- Since the SU(2) spin cannot change along the RG flow, we will be able to show that the ABJM theory has singlet monopole operators.

$\mathcal{N}$=3 Yang-Mills Chern-Simons

- We add the U(N) x U(N) super-Yang-Mills and the corresponding chiral adjoint kinetic terms required by $\mathcal{N}$=3 SUSY

$$\mathcal{S}_{\mathrm{YM}} = \frac{1}{4g^2} \int d^3x\, d^2\theta \ \mathrm{tr}\Big[\mathcal{U}^\alpha \mathcal{U}_\alpha + \hat{\mathcal{U}}^\alpha \hat{\mathcal{U}}_\alpha\Big]$$

$$\mathcal{S}_{\mathrm{adj}} = \frac{1}{g^2} \int d^3x\, d^4\theta \ \mathrm{tr}\Big[-\bar{\Phi} e^{-\mathcal{V}} \Phi e^{\mathcal{V}} - \bar{\hat{\Phi}} e^{-\hat{\mathcal{V}}} \hat{\Phi} e^{\hat{\mathcal{V}}}\Big]$$

- These fields are coupled to N_f bi-fundamental hypermultiplets

$$\mathcal{S}_{\mathrm{mat}} = \int d^3x\, d^4\theta \ \mathrm{tr}\Big[-\bar{\mathcal{Z}}_A e^{-\mathcal{V}} \mathcal{Z}^A e^{\hat{\mathcal{V}}} - \bar{\mathcal{W}}^A e^{-\hat{\mathcal{V}}} \mathcal{W}_A e^{\mathcal{V}}\Big]$$

- The superpotential contains quadratic terms related to the C-S levels

$$W = \mathrm{tr}\big(\Phi \mathcal{Z}^A \mathcal{W}_A + \hat{\Phi} \mathcal{W}_A \mathcal{Z}^A\big) + \frac{k}{8\pi} \mathrm{tr}\big(\Phi\Phi - \hat{\Phi}\hat{\Phi}\big)$$

- Far in the IR, the adjoint chirals become non-dynamical. Integrating them out makes W quartic. For N_f=2 the theory reduces to ABJM. We will also consider other values of N_f and show how the dimensions of monopole operators depend on N_f.

- The bi-fundamental scalars, and their fermion superpartners combine into doublets of $SU(2)_R$.

$$X^{Aa} = \begin{pmatrix} Z^A \\ W^{\dagger A} \end{pmatrix} \quad , \quad X^\dagger_{Aa} = \begin{pmatrix} Z^\dagger_A \\ W_A \end{pmatrix}$$

$$\xi^{Aa} = \begin{pmatrix} \omega^{\dagger A}\, e^{i\pi/4} \\ \zeta^A\, e^{-i\pi/4} \end{pmatrix} \quad , \quad \xi^\dagger_{Aa} = \begin{pmatrix} \omega_A\, e^{-i\pi/4} \\ \zeta^\dagger_A\, e^{i\pi/4} \end{pmatrix}$$

- For each U(N), two adjoint scalar fields from the chiral multiplet combine with one from the vector multiplets to form a triplet of $SU(2)_R$.

$$\phi^a_b = \phi_i(\sigma_i)^a_b = \begin{pmatrix} -\sigma & \phi^\dagger \\ \phi & \sigma \end{pmatrix} \quad , \quad \hat{\phi}^a_b = \hat{\phi}_i(\sigma_i)^a_b = \begin{pmatrix} \hat{\sigma} & \hat{\phi}^\dagger \\ \hat{\phi} & -\hat{\sigma} \end{pmatrix}$$

- Here is the complete field content of the theory and how it assembles into SU(2) reps

manifest symmetry	Super fields	Component fields								
		dynamical in the IR				auxiliary in the IR				aux.
$\mathrm{U}(1)_R \times \mathrm{SU}(N_f)_{\mathrm{fl}}$	$\mathcal{V}$	A				σ		$\chi_\sigma,\ \chi_\sigma^\dagger$		D
	$\hat{\mathcal{V}}$	$\hat{A}$					$\hat{\sigma}$		$\hat{\chi}_\sigma,\ \hat{\chi}_\sigma^\dagger$	$\hat{\mathrm{D}}$
	$\mathit{\Phi}$					ϕ		χ_ϕ		F_ϕ
	$\bar{\mathit{\Phi}}$					$\phi^\dagger$		$\chi_\phi^\dagger$		$F_\phi^\dagger$
	$\hat{\mathit{\Phi}}$						$\hat{\phi}$		$\hat{\chi}_\phi$	$\hat{F}_\phi$
	$\bar{\hat{\mathit{\Phi}}}$						$\hat{\phi}^\dagger$		$\hat{\chi}_\phi^\dagger$	$\hat{F}_\phi^\dagger$
	$\mathcal{Z}^A$	Z^A		ζ^A						F^A
	$\bar{\mathcal{Z}}_A$		$Z_A^\dagger$		$\zeta_A^\dagger$					$F_A^\dagger$
	$\mathcal{W}_A$		W_A		ω_A					G_A
	$\bar{\mathcal{W}}^A$	$W^{\dagger A}$		$\omega^{\dagger A}$						$G^{\dagger A}$
$\mathrm{SU}(2)_R \times \mathrm{SU}(N_f)_{\mathrm{fl}}$		X^{Aa}	$X_{Aa}^\dagger$	ξ^{Aa}	$\xi_{Aa}^\dagger$	ϕ_i	$\hat{\phi}_i$	λ^{ab}	$\hat{\lambda}^{ab}$	

- We carry out Euclidean continuation, and then map from R^3 to $R \times S^2$ by setting $r = e^\tau$ and rescaling the fields according to

$$\mathcal{A} = e^{-\dim(\mathcal{A})\tau} \tilde{\mathcal{A}}$$

- The effective interaction strength acquires dependence on τ

$$\tilde{g} = e^{\tau/2} g$$

- We can study the monopoles semiclassically for $\tau \to -\infty$ where the theory is weakly coupled.

Semi-classical monopoles

- In the weak coupling limit, there are classical (anti-)BPS monopole solutions $\eta = \pm 1$

$$A = \hat{A} = \frac{H}{2}(\pm 1 - \cos\theta)\mathrm{d}\varphi \quad , \quad \phi_i = -\hat{\phi}_i = -\eta \frac{H}{2}\delta_{i3}$$

- The choice of scalar field breaks the SU(2) global symmetry to a U(1). To restore the SU(2) symmetry we introduce a SU(2)/U(1) collective coordinate $\phi_i = -\hat{\phi}_i = -\frac{H}{2} n_i(\tau)$
- Its quantum mechanics is analogous to that of a particle on a 2-d sphere.

- The collective coordinate effective action has the general form

$$\Gamma(\vec{n}) = \int d\tau \Big[-V_{\text{eff}}(\vec{n}) + i\,\dot{n}_i A_i(\vec{n}) + \tfrac{1}{2}\dot{n}_i \dot{n}_j B_{ij}(\vec{n}) + \dots \Big]$$

- The Wess-Zumino term, which is linear in the time derivative arises due to the interaction with the fermions:

$$\begin{aligned} \mathcal{S} = \int d\tau\, d\Omega\ \mathrm{tr}\Big[& -i\xi^{\dagger}_{Aa} \not{D} \xi^{Aa} - \tfrac{i}{2}\, n_i\, \xi^{\dagger}_{Aa} (\sigma_i)^a{}_b [H, \xi^{Ab}] \\ & - i\lambda^{\dagger}_{1a} \not{D} \lambda^{1a} + \tfrac{i}{2}\, n_i\, \lambda^{\dagger}_{1a} (\sigma_i)^a{}_b [H, \lambda^{1b}] \\ & - i\hat{\lambda}^{\dagger}_{1a} \not{D} \hat{\lambda}^{1a} + \tfrac{i}{2}\, n_i\, \hat{\lambda}^{\dagger}_{1a} (\sigma_i)^a{}_b [H, \hat{\lambda}^{1b}] \Big] \end{aligned}$$

- Note the sign difference in the interaction term between the hypers and vectors.

Monopole of Monopoles

- Remarkably, the induced gauge potential looks like that of a magnetic monopole at the center of the 2-sphere.
- This `effective monopole' is not to be confused with the original monopole we are quantizing.
- A useful building block is a single fermion coupling to the collective coordinate with charge q: $\mathcal{S} = \int d\tau \, d\Omega \left[-i\psi_a^\dagger \not{D} \psi^a - \frac{i}{2} q n_i(\tau) \, \psi_a^\dagger (\sigma_i)^a{}_b \psi^b \right]$

- The induced gauge potential is then

$$\partial_i A_j(\vec{n}) - \partial_j A_i(\vec{n}) = \frac{|q|}{2}\,\epsilon_{ijk}\frac{n_k}{|\vec{n}|^3}$$

- The full contribution of the hyper-multiplet fermions is then

$$\partial_i A_j(\vec{n}) - \partial_j A_i(\vec{n}) = N_f\,\frac{q_{\rm tot}}{2}\,\epsilon_{ijk}\frac{n_k}{|\vec{n}|^3}$$

$$q_{\rm tot} = \sum_{r,s=1}^{N} |q_{rs}| = 2\sum_{r>s} |q_r - q_s|$$

- The vector multiplet fermions contribute with a crucial opposite sign

$$\partial_i A_j(\vec{n}) - \partial_j A_i(\vec{n}) = -2\times\frac{q_{\rm tot}}{2}\,\epsilon_{ijk}\frac{n_k}{|\vec{n}|^3}$$

- The total effective magnetic monopole charge is $h = (N_f - 2)\,q_{\rm tot} = (N_f - 2)\sum_{r,s} |q_r - q_s|$

Collective Quantization

- The effective action is

$$\Gamma(\vec{n}) = \int d\tau \left[\tfrac{1}{2} M \dot{\vec{n}}^2 + i\,\vec{A}(\vec{n}) \cdot \dot{\vec{n}} + \lambda(\vec{n}^2 - 1)\right]$$

with the mass M depending on τ. The angular momentum, i.e. the $SU(2)_R$ charge is still conserved:

$$\vec{L} = M\,\vec{n} \times \dot{\vec{n}} - \frac{h}{2}\,\vec{n}$$

- The smallest representation has

$$l = \frac{|h|}{2} = \left|\frac{N_f}{2} - 1\right| \sum_{r,s} |q_r - q_s|$$

- Only for N_f=2, i.e. the ABJM case, do we find singlet monopoles.

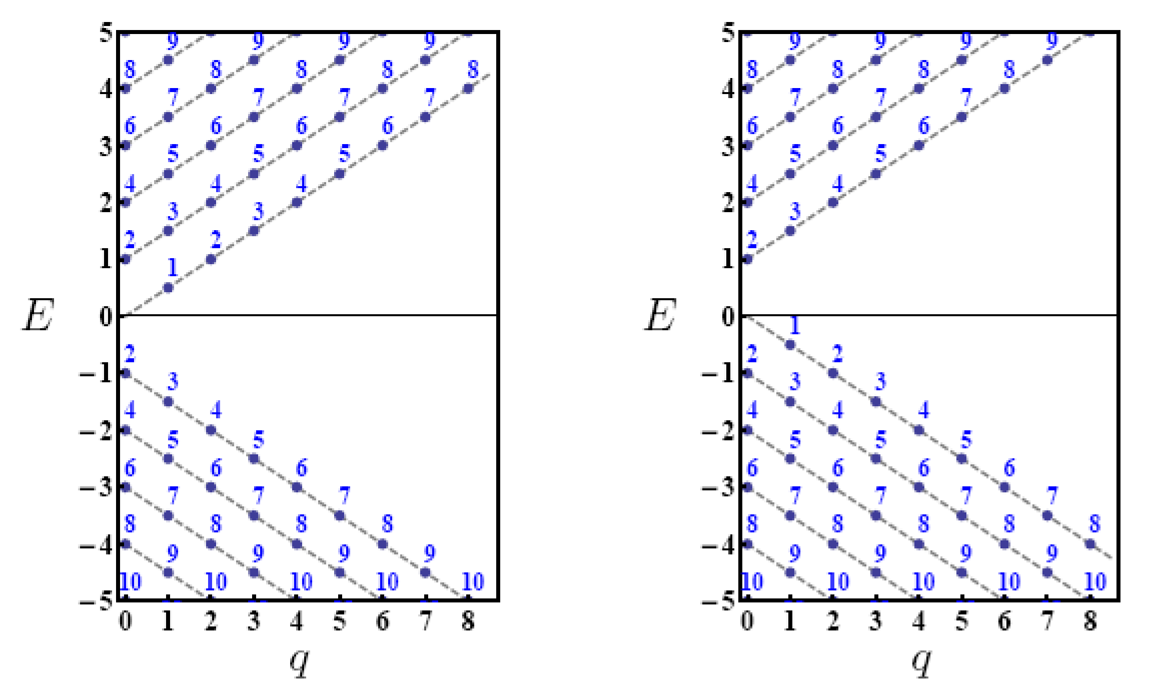
Induced R-charge

- Another way to fix the effective monopole charge h is to calculate the U(1) R-charge induced by the fermions for a monopole with a fixed orientation, $n_i = \delta_{i3}$
- The R-charge operator is

$$Q = -i \int d\Omega \, \mathrm{tr}\Big[-\tfrac{1}{2}\,\zeta_A^\dagger \gamma^\tau \zeta^A - \tfrac{1}{2}\,\omega^{\dagger A} \gamma^\tau \omega_A + \chi_\sigma^\dagger \gamma^\tau \chi_\sigma + \hat{\chi}_\sigma^\dagger \gamma^\tau \hat{\chi}_\sigma\Big]$$

where we recognize -1/2 and 1 as the R-charges of the bi-fundamentals and the adjoints, respectively.

- In the presence of a BPS or anti-BPS monopole, the induced charge is due to the unpaired modes in the fermion energy spectrum. Borokhov, Kapustin, Wu



(b) BPS (c) anti-BPS

- This leads to the value of induced charge

$$Q_R^{\text{mon}} = \eta \left(\frac{N_f}{2} - 1 \right) \sum_{r,s=1}^{N} |q_r - q_s|$$

  which up to a sign coincides with h/2. This is because the $U(1)_R$ charge is the 3[rd] component of `angular momentum' for a fixed collective coordinate.

$$\vec{L} = M\,\vec{n} \times \dot{\vec{n}} - \frac{h}{2}\,\vec{n}$$

- The relative sign between the vector and hyper contributions is obviously dictated by their $U(1)_R$ charges.
- Our result for $U(1)_R$ charge agrees with a calculation of the ABJM partition function done by Seok Kim who used a deformation that produces a weak coupling limit. It is also consistent with earlier work by Gaiotto, Witten; Imamura; Gaiotto, Jafferis.

- Similarly, it is not hard to see that the monopoles do not acquire any SU(N_f) charge. Therefore, in the ABJM case, they do not carry any global charges. Due to the BPS relation between dimension and R-charge, the monopoles do not contribute to dimension of composite operators. Thus, the AdS/CFT duality conjectured by ABJM appears to have passed a non-trivial test.
- See also the related work by Gustavsson, Rey; Berenstein, Park that just appeared.

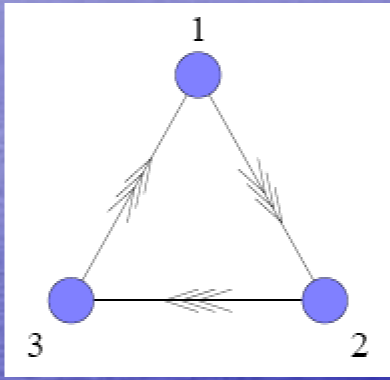
Outlook

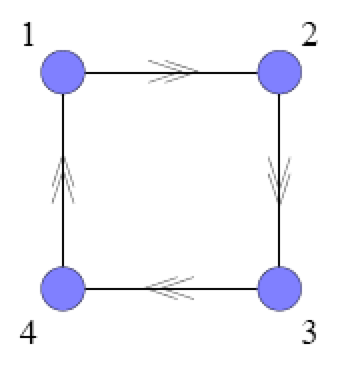
- We can use similar methods to study any SCFT that has a weakly coupled $\mathcal{N}$=3 YM-CS description. Imamura; Jafferis and Tomasiello
- For example, there is an interesting class of quiver Chern-Simons gauge theories whose AdS duals have been proposed. These theories possess weakly coupled UV completions where the monopoles can be studied semiclassically.

- $\mathcal{N}$=2 gauge theories are more challenging, since the abelian R-charge may be renormalized along the flow. If we assume that it does not, then we observe a curious relation between the induced monopole R-charge in a 3-d gauge theory and the chiral anomaly coefficient in its 4-d parent.
- In some models, both are proportional to the sum over the fermionic R-charges. For example, in the 4-d parent theories of the 3-d models we have studied, the $U(1)_R$ anomaly coefficient is $(1 - N_f/2)N$
- The consistent theory is the `conifold quiver.'

- Various other familiar 4-d quivers assume interesting new identitites in 3-d:

M^{111}, Q^{222}, etc.





- We conjecture that, in $\mathcal{N}$=2 CS quiver theories that have consistent 4-d `parents' there are monopoles that contribute vanishing R-charge to composite operators.
- Ultimate Hope: to find a `simple' dual of a 3-d fixed point realized in Nature.