The Higher-Spin Challenge

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Some related reviews:

- N. Bouatta, G. Compere, A.S., hep-th/0409068
- D. Francia and A.S., hep-th/0601199
- A.S., Sezgin, Sundell, hep-th/0501156
- X. Bekaert, S. Cnockaert, C. Iazeolla, M.A. Vasiliev, hep-th/0503128
- D. Sorokin, hep-th/0405069
- A.S., D. Sorokin, P. Sundell, M.A. Vasiliev, Phys. Reports (Book?)

D. Francia
M. Tsulaia, J. Mourad , A. Campoleoni

Rome - STRINGS 2009

Higher Spins : Field Theory

In 4D : symmetric tensors (spinor-tensors) $\varphi_{\mu_1 \ldots \mu_s}$ or $\psi_{\alpha;\mu_1 \ldots \mu_s}$

Dirac - Fierz - Pauli conditions

$$\begin{aligned} (\Box - M^2)\varphi_{\mu_1 \ldots \mu_s} &= 0 \\ \partial^{\mu_1}\varphi_{\mu_1 \ldots \mu_s} &= 0 \\ \varphi^{\mu_1}{}_{\mu_1 \ldots \mu_s} &= 0 \end{aligned}$$

$$\begin{aligned} (i\not{\partial} - M)\Psi_{\mu_1 \ldots \mu_s} &= 0 \\ \partial^{\mu_1}\Psi_{\mu_1 \ldots \mu_s} &= 0 \\ \gamma^{\mu_1}\Psi_{\mu_1 \ldots \mu_s} &= 0 \end{aligned}$$

Lagrangians: many additional fields

Higher Spins : Field Theory

Long search for free action principles:

- Metric-like formulation:

  - *Singh ,Hagen (1974):* action for free MASSIVE higher spins
  - *Fronsdal (1978):* MASSLESS higher spins with constrained gauge symmetry

$$\delta\varphi_{\mu_1\ldots\mu_s} = \partial_{\mu_1}\Lambda_{\mu_2\ldots\mu_s} + \ldots$$
$$\Lambda\,' \equiv \Lambda^{\rho}{}_{\rho\mu_4\ldots\mu_s} = 0$$
$$\varphi\,'' \equiv \varphi^{\rho\sigma}{}_{\rho\sigma\mu_5\ldots\mu_s} = 0$$

  - *Buchbinder, Pashnev, Tsulaia et al (1998-):* remove constraints by BRST
  - *Francia, AS, Mourad (2002-):* minimal (non)local form, NO trace constraints
  - *Mixed symmetry :* …, *Labastida, [Bekaert, Boulanger; De Medeiros, Hull],…*
    Campoleoni, Francia, Mourad, AS, 2008 and 2009

- Frame-like formulation: (Vasiliev et al, 1980's-, Skvortsov, 2008, Boulanger, Iazeolla, Sundell, 2008)

$$e_\mu{}^a \longrightarrow \varphi_\mu{}^{a_1\ldots a_{s-1}}$$
$$\omega_\mu{}^{a,b} \longrightarrow \left\{\varphi_\mu{}^{a_1\ldots a_{s-1},b_1\ldots b_t}\right\}_{t=1,\ldots s-1}$$

Plan

- Free Higher Spins [Bose for brevity] :
  - constrained (Fronsdal) and unconstrained LOCAL formulations
  - minimal NON-LOCAL (geometric) formulation
  - triplets and String Field Theory
  - unconstrained mixed symmetry from Bianchi identities
- External Currents and applications :
  - vDVZ (dis)continuities
- Interacting (symmetric) Higher Spins :
  - the "classic" problems
  - currents and interactions
  - the Vasiliev equations
  - link with unconstrained formulation

Free Symmetric Higher Spins

Fronsdal (1978): natural extension of s=1,2 cases (BUT : with CONSTRAINTS)

$$\mathcal{F}_\mu \equiv \Box A_\mu - \partial_\mu \partial \cdot A = 0$$

$$\mathcal{F}_{\mu\nu} \equiv \Box h_{\mu\nu} - (\partial_\mu \partial \cdot h_\nu + (\mu \leftrightarrow \nu)) + \partial_\mu \partial_\nu h' = 0$$

$$\ldots$$

$$\mathcal{F}_{\mu_1 \ldots \mu_s} \equiv \Box \varphi_{\mu_1 \ldots \mu_s} - (\partial_{\mu_1} \partial \cdot \varphi_{\mu_2 \ldots \mu_s} + \ldots) + (\partial_{\mu_1} \partial_{\mu_2} \varphi'_{\mu_3 \ldots \mu_s} + \ldots) = 0$$

$$\delta \mathcal{F}_\mu = 0 \ , \quad \delta \mathcal{F}_{\mu\nu} = 0$$

but

$$\delta \mathcal{F}_{\mu_1 \ldots \mu_s} = 3 \partial_{(\mu_1} \partial_{\mu_2} \partial_{\mu_3} \Lambda'_{\mu_4 \ldots \mu_s)}$$

("primes" = traces)

Can simplify notation (and algebra) **hiding** space-time indices:

$$\varphi_{\mu_1 \ldots \mu_s} \to \varphi \ , \quad \varphi'_{\mu_3 \ldots \mu_s} \to \varphi'$$

$$\partial_{\mu_1} \varphi_{\mu_2 \ldots \mu_{s+1}} + \ldots \to \partial \varphi$$

$$\mathcal{F}_{\mu_1 \ldots \mu_s} \to \mathcal{F} \equiv \Box \varphi - \partial \partial \cdot \varphi + \partial^2 \varphi'$$

$$\delta \mathcal{F} = 3 \partial^3 \Lambda'$$

$\Longrightarrow$

1st Fronsdal constraint

$$\Lambda' = 0$$

Free Symmetric Higher Spins

Bianchi identity → "classical anomaly" :

$$\partial \cdot \mathcal{F} - \frac{1}{2}\partial \mathcal{F}' = -\frac{3}{2}\partial^3 \varphi''$$

$$\mathcal{L} = \varphi \left(\mathcal{F} - \frac{1}{2}\eta \mathcal{F}' \right)$$

$$\delta \mathcal{L} = -s\Lambda \left(\partial \cdot \mathcal{F} - \frac{1}{2}\partial \mathcal{F}' \right)$$

$$\varphi'' = 0$$

2nd Fronsdal constraint

Natural to try and forego these "trace" constraints:

- BRST (non minimal) *(Buchbinder, Pashnev, Tsulaia, …, 1998-)*
- Minimal compensator form *(Francia, AS, Mourad, 2002 -)*

$$\mathcal{F} = 0 \rightarrow \mathcal{A} \equiv \mathcal{F} - 3\partial^3 \alpha = 0$$

$$\partial \cdot \mathcal{A} - \frac{1}{2}\partial \mathcal{A}' = -\frac{3}{2}\partial^3 \mathcal{C}$$

$$\delta \alpha = \Lambda'$$

$$\mathcal{C} = \varphi'' - 4\partial \cdot \alpha + \partial \alpha'$$

S=3 (Schwinger)

$$\mathcal{L} = \varphi \left(\mathcal{A} - \frac{1}{2}\eta \mathcal{A}' \right) - \frac{3}{4}\binom{s}{3}\alpha \partial \cdot \mathcal{A}' + 3\binom{s}{4}\beta \mathcal{C}$$

Unconstrained Lagrangian
[2-derivative : *(Francia, 2007)*]

Free Higher-Spin Geometry

What are we gaining ?

$$\mathcal{F}^{(n+1)} = \mathcal{F}^{(n)} + \frac{1}{(n+1)(2n+1)} \frac{\partial^2}{\Box} \mathcal{F}'^{(n)} - \frac{1}{n+1} \frac{\partial}{\Box} \partial \cdot \mathcal{F}^{(n)}$$

$$\mathcal{F}^{(k)} = (2k+1) \frac{\partial^{2k+1}}{\Box^{k-1}} \alpha^{[k-1]}$$

After some iterations: NON-LOCAL gauge invariant equation for φ ONLY

s = 2: $$\delta h_{\mu\nu} = \partial_\mu \Lambda_\nu + \partial_\nu \Lambda_\mu \rightarrow \delta \Gamma^{\mu}_{\nu\rho} = \partial_\nu \partial_\rho \Lambda^\mu \rightarrow \delta R^{\alpha}_{\mu\nu\rho} = 0$$

s > 2: Hierarchy of connections and curvatures *(de Wit and Freedman, 1980)*

$$\Gamma_{\mu;\nu_1\ldots\nu_s}, \ldots, \; \Gamma_{\mu_1\ldots\mu_{s-1};\nu_1\ldots\nu_s} \; ; \; \mathcal{R}_{\mu_1\ldots\mu_s;\nu_1\ldots\nu_s}$$

"Irreducible" NON LOCAL form of the equations :

$$s = 2n+1 : \qquad \frac{1}{\Box^n} \partial_\mu \mathcal{R}^{\mu[n];\nu_1\ldots\nu_s} = 0$$

$$s = 2n : \qquad \frac{1}{\Box^{n-1}} \mathcal{R}^{[n];\nu_1\ldots\nu_s} = 0$$

String Theory & Free Higher Spins

(Kato and Ogawa, 1982; Witten; Neveu, West et al, 1985,….)

$$\mathcal{Q}\,|\Psi\rangle = 0$$
$$\delta\,|\Psi\rangle = \mathcal{Q}\,|\Lambda\rangle$$

BRST equations for "contracted" Virasoro:

$$L_k = \frac{1}{2}\sum_{l=-\infty}^{+\infty} \alpha^{\mu}_{k-l}\,\alpha_{\mu l}$$
$$[L_k, L_l] = (k-l)\,L_{k+l} + \frac{\mathcal{D}}{12}\,m\,(m^2-1)\,\delta_{k+l,0}$$

$\alpha' \rightarrow \infty$

$$\ell_0 = p^2$$
$$\ell_k = p\cdot\alpha_k$$
$$[\ell_m, \ell_n] = m\,\ell_0\,\delta_{m+n,0}$$

First open bosonic Regge trajectory → TRIPLETS

Propagate: s,s-2,s-4, …

(A. Bengtsson, 1986; Henneaux, Teitelboim, 1987)
(Pashnev, Tsulaia, 1998; Francia, AS, 2002; Bonelli, 2003; AS, Tsulaia, 2003)

$$\Box\,\varphi = \partial\,C\,,$$
$$\partial\cdot\varphi - \partial\,D = C$$
$$\Box\,D = \partial\cdot C$$

Frame formulation : (Sorokin, Vasiliev, 2008)

On-shell truncation : (Francia, AS, 2002)

$$\varphi' - 2\,D = \partial\,\alpha$$

$$\mathcal{F} = 3\,\partial^{\,3}\alpha$$
$$\varphi'' = 4\partial\cdot\alpha + \partial\alpha'$$
$$\delta\varphi = \partial\Lambda\,,\qquad \delta\alpha = \Lambda'$$

Off-shell truncation : (Buchbinder, Krykhtin, Reshetnyak 2007)

Mixed Symmetry

- Independent fields for D > 5
- Index "families"
- Non-Abelian gl(N) algebra mixing them

$$\varphi_{\mu_1 \ldots \mu_{s_1}} \rightarrow \varphi_{\mu_1 \ldots \mu_{s_1}; \ldots ; \rho_1, \ldots, \rho_{s_N}}$$
$$\delta\varphi = \partial\,\Lambda \;\rightarrow \delta\varphi = \partial^i\,\Lambda_i$$

- Labastida constraints (1987):

$$T_{(ij}\,\Lambda_{k)} = 0$$
$$T_{(ij}\,T_{kl)}\,\varphi = 0$$

  - NOT all (double) traces vanish → Higher traces in Lagrangians !
  - Constrained bosonic Lagrangians and fermionic field equations
  - Key tool: self-adjointness of kinetic operator

Recently: *(Campoleoni, Francia, Mourad, AS, 2008, and 2009)*

  - Unconstrained bosonic Lagrangians
  - Weyl-like symmetries in sporadic cases determined by gl(N) algebra
  - (Un)constrained fermionic Lagrangians [NOT here for brevity]
  - Key tool: Bianchi identities

Mixed Symmetry

(Campoleoni, Francia, Mourad, AS, 2008, and 2009)

$$\delta\varphi = \partial\,\Lambda \;\longrightarrow\; \delta\varphi = \partial^i\,\Lambda_i$$

$$\mathcal{F} = \Box\varphi - \partial^i\,\partial_i\varphi + \frac{1}{2}\,\partial^i\,\partial^j\,T_{ij}\,\varphi$$

$$\delta\,\mathcal{F} = \frac{1}{6}\,\partial^i\partial^j\partial^k\,T_{(\,ij}\,\Lambda_{\,k\,)}$$

$$\partial_i\,\mathcal{F} - \frac{1}{2}\,\partial^j\,T_{ij}\,\mathcal{F} = -\,\frac{1}{12}\,\partial^j\partial^k\partial^l\,T_{(\,ij}\,T_{kl\,)}\,\varphi$$

Multi-symmetric tensors :

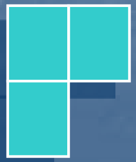
Bianchi identities :

- neatly encode the (not independent) Labastida constraints
- [compensators $\alpha_{i|jk}(\Phi_i)$, Lagrange multipliers $\beta_{i|jkl}$]

$$\partial_i\,\mathcal{A} - \frac{1}{2}\,\partial^j\,T_{ij}\,\mathcal{A} = -\,\frac{1}{4}\,\partial^j\partial^k\partial^l\,\mathcal{C}_{\,ijkl}$$

(2,1) projection

- **new features** from traces of Bianchi's (not simply constraints)



$$\partial_i\,T_{jk}\,\mathcal{A} - \frac{1}{2}\,\partial_{(\,j}\,T_{k\,)\,i}\,\mathcal{A} = \frac{1}{6}\,\partial^l\left(2\,T_{il}\,T_{jk} - T_{i\,(\,j}\,T_{k\,)\,l}\right)\mathcal{A}$$
$$+\,\frac{1}{4}\,\partial^l\partial^m\left(2\,\partial_i\,\mathcal{C}_{jklm} - \partial_{(\,j}\,\mathcal{C}_{\,k\,)\,ilm}\right) - \frac{1}{12}\,\partial^l\partial^m\partial^n\left(2\,T_{jk}\,\mathcal{C}_{ilmn} - T_{i\,(\,j}\,\mathcal{C}_{\,k\,)\,lmn}\right)\,.$$

Mixed Symmetry

(Campoleoni, Francia, Mourad, AS, 2008, and 2009)

Two-family Lagrangians:

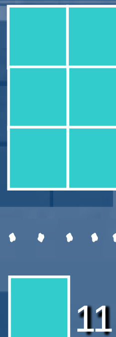
$$
\begin{aligned}
\mathcal{L}\,(\varphi, \Phi_i, \beta_{ijkl}\,) \;=\;& \frac{1}{2}\,\langle\,\varphi\,,\,\mathcal{A}\,-\,\frac{1}{2}\,\eta^{ij}\,T_{ij}\,\mathcal{A} + \frac{1}{36}\,\eta^{ij}\,\eta^{kl}\,\big(\,2\,T_{ij}\,T_{kl}\,-\,T_{i\,(\,k}\,T_{l\,)\,j}\,\big)\,\mathcal{A}\,\rangle \\
&-\,\frac{1}{24}\,\langle\,\alpha_{\,ijk}(\Phi)\,,\,\partial_{\,(\,i}\,T_{jk\,)}\,\mathcal{A} - \frac{1}{12}\,\eta^{lm}\,\big(\,2\,\partial_{\,(\,i}\,T_{jk\,)}\,T_{lm}\,-\,\partial_{\,(\,i\,|}\,T_{l\,|\,j}\,T_{k\,)\,m}\,\big)\,\mathcal{A}\,\rangle \\
&+\,\frac{1}{8}\,\langle\,\beta_{\,ijkl}\,,\,\mathcal{C}_{\,ijkl}\,\rangle\,,
\end{aligned}
$$

- New terms determined by the (2,1) trace of the Bianchi identity
- **More families :** higher traces
- (2,…,2,1) trace of Bianchi's:
- **Also:** 2-derivative form

$$
\begin{aligned}
&(\,p+2\,)\;Y_{\{2^p,1\}}\,\partial_{\,k}\,\mathcal{A}^{\,[\,p\,]}{}_{i_1 j_1,\,\dots\,,\,i_p j_p}\,-\,\partial^{\,l}\,\mathcal{A}^{\,[\,p+1\,]}{}_{i_1 j_1,\,\dots\,,\,i_p j_p\,,\,k\,l} \\
&=\,-\,\frac{1}{2}\;Y_{\{2^p,1\}}\,T_{i_1 j_1}\,\dots\,T_{i_p j_p}\,\partial^{\,l}\partial^{\,m}\partial^{\,n}\,\mathcal{C}_{\,klmn}\,.
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{L}\;=\;& \frac{1}{2}\,\langle\,\varphi\;,\;\sum_{p\,=\,0}^{N}\,\frac{(-1)^{\,p}}{p\,!\,(\,p+1\,)\,!}\;\eta^{i_1 j_1}\,\dots\,\eta^{i_{\,p} j_{\,p}}\,\mathcal{A}^{\,[\,p\,]}{}_{i_1 j_1,\,\dots\,,\,i_p j_p}\,\rangle \\
&-\,\frac{1}{4}\,\langle\,\alpha_{\,ijk}\;,\;\sum_{p\,=\,0}^{N-1}\,\frac{(-1)^{\,p}}{p\,!\,(\,p+3\,)\,!}\;\eta^{i_1 j_1}\,\dots\,\eta^{i_p j_p}\,\partial_{\,(\,i}\,\mathcal{A}^{\,[\,p+1\,]}{}_{jk\,),\,i_1 j_1,\,\dots\,,\,i_p j_p}\,\rangle \\
&+\,\frac{1}{8}\,\langle\,\beta_{\,ijkl}\;,\;\mathcal{C}_{\,ijkl}\,\rangle\,.
\end{aligned}
$$

(2,…,2,1)
projection



Mixed Symmetry

(Campoleoni, Francia, Mourad, AS, 2008, and 2009)

Reduction: e.g. passing from $R_{\mu\nu} - 1/2\, g_{\mu\nu} R = 0$ to $R_{\mu\nu} = 0$

- One family (symmetric tensors) :

$$\boxed{\mathcal{A} - \frac{1}{2}\,\eta\,\mathcal{A}' + \eta^2\,\mathcal{B} = 0} \Rightarrow \boxed{\mathcal{A} = 0}$$

  - Always possible via traces, with one exception: 2D gravity

- Two families :

$$\boxed{\mathcal{A} - \frac{1}{2}\,\eta^{ij}\,\mathcal{A}'_{ij} + \frac{1}{12}\,\eta^{ij}\,\eta^{kl}\,\mathcal{A}''_{ij\,,\,kl} + \frac{1}{2}\,\eta^{ij}\,\eta^{kl}\,\mathcal{B}_{ijkl} = 0}$$

  - looks simple, but …

$$\boxed{[\,(D-2)\,\delta_i^k\,\delta_j^l + \delta^k_{(i}\,S^l{}_{j)}\,]\left(\mathcal{A}'_{ij} - \frac{1}{3}\,\eta^{mn}\,\mathcal{A}''_{ij\,,\,mn} - 2\,\eta^{mn}\,\mathcal{B}^{(T)}{}_{ijmn}\right) = 0}$$

  - $S^i{}_j$: displace indices → gl(2) algebra

$$\boxed{[\,S^i{}_j\,,\,S^k{}_l\,] = \delta^k_j\,S^i{}_l - \delta^i_l\,S^k{}_j}$$

Mixed Symmetry

(Campoleoni, Francia, Mourad, AS, 2008, and to appear)

- Spectrum of : $\boxed{[\,(D-2)\,\delta_i^k\,\delta_j^l\,+\,\delta_{(i}^k\,S^l{}_{j)}\,]}$
- Zero modes according to first column of table:
- What happens ? **Weyl-like symmetries**
- e.g. 2D gravity: $\boxed{\delta\, h_{\mu\nu} = \eta_{\mu\nu}\,\Omega}$
- **N families :** Weyl-like symmetries:

$$\boxed{\delta\,\varphi = \eta\,\Omega\ ,\ \delta\,\varphi = \eta\,\eta\,\Omega\ ,\dots}$$

	$\mathcal{A}'$		$\mathcal{A}''$	
D	s_1	s_2	s_1	s_2
2	2	0		
2	2	1		
2	3	1		
~~2~~	~~s~~	~~3~~		
3	2	1	2	2
4	s	2		

Weyl-like symmetries
(irred. fields)

$$\delta\,\varphi = \eta^{ij}\,\Omega_{ij}$$

$$\delta\,\mathcal{E} = \sum_{p=1}^{N}\frac{(-1)^p}{p!\,(p+1)!}\,\eta^{i_1j_1}\dots\eta^{i_pj_p}\,Y_{\{2^p\}}\sum_{n=1}^{p}\prod_{r\neq n}T_{i_rj_r}\,\{\,(D-2)\,\Omega_{\,i_nj_n}+\,S^k{}_{(\,i_n}\Omega_{\,j_n\,)\,k}\,\}$$

$$+\sum_{p=2}^{N+1}\frac{(-1)^p}{p\,(p-1)!\,p!}\,\eta^{i_1j_1}\dots\eta^{i_pj_p}\,Y_{\{4,2^{p-2}\}}\sum_{n=1}^{p}\Big(Y_{\{2^{p-1}\}}\prod_{r\neq n}T_{i_rj_r}\Big)\,\Omega_{\,i_nj_n}$$

Multiplier terms

External currents

(Francia, Mourad, AS, 2007, 2008)

$$\boxed{J \text{ ——— } J}$$

- Residues of current exchanges reflect the **degrees of freedom**

- s=1 :

$$p^2 A_\mu - p_\mu p \cdot A = J_\mu$$

$$p^2 J^\mu A_\mu = J^\mu J_\mu \Rightarrow J_i J_i$$

- All s :

$$\mathcal{A} - \frac{1}{2}\eta\,\mathcal{A}' + \eta^2 \mathcal{B} = J$$

$$\partial \cdot \mathcal{A}' - (2\partial + \eta\,\partial\cdot)\,\mathcal{B} = 0$$

$$\varphi'' - 4\,\partial \cdot \alpha - \partial\,\alpha' = 0$$

Unique non-local Lagrangian :
(e.g. for s=3)

$$\mathcal{A} = \frac{1}{\Box}\,\partial \cdot \mathcal{R}' + \frac{1}{2}\,\frac{\partial^2}{\Box^2}\,\partial \cdot \mathcal{R}''$$

vD-V-Z (Dis)continuity for HS

J ———— J

$$m = 0 \; : \quad T_{\mu\nu}T^{\mu\nu} \; - \; \frac{1}{2}(T')^2$$

$$m \neq 0 \; : \quad T_{\mu\nu}T^{\mu\nu} \; - \; \frac{1}{3}(T')^2$$

(van Dam, Veltman; Zakharov, 1970)

- VDVZ discontinuity : comparing of D and (D+1) massless exchanges
- First present for s=2
- All s: can describe irreducibly a massive field à la Scherk-Schwarz from (D+1) dimensions :
  [e.g. for s=2 : h_{MN} → (h_{mn} cos(my) , A_m sin(my), φ cos(my))]

$$\langle J, \Box\, \varphi \rangle = \sum_n \frac{1}{n!\, 2^{2n} \left(3 - s - \frac{D}{2}\right)_n} \; \langle J^{[n]} \; , \; J^{[n]} \rangle$$

(A)dS extension (still free level): [radial reduction] *(Fronsdal, 1979; Biswas, Siegel, 2002)*

◇∀s : Discontinuity → smooth interpolation in $(mL)^2$ *(∀s: Francia, Mourad, AS, 2008)*

(s=2 : Higuchi, 1987; Porrati, 2001; Kogan, Mouslopoulos, Papazoglou,2001; Duff, Liu, Sati, 2004)

◇[Liouville's Theorem !]

Poles at (even) "partly massless" states
(Deser, Nepomechie, Waldron, 1983 -)

⟷

VDVZ discontinuity!

HS Interactions

Problems :

- Usual coupling with gravity → "naked" Weyl tensors *(Aragone, Deser,1979)*
- Weinberg's 1964 argument , Coleman – Mandula
- Velo-Zwanziger inconsistencies
- Weinberg – Witten *(see Porrati, 2008)*
- ....

But:

Berends, Burgers, van Dam, 1982)
(Bengtsson[2], Brink, 1983)
(Boulanger et al, 2001 -)
(Metsaev, 2005,2007)
(Buchbinder,Fotopoulos, Irges, Petkou, Tsulaia, 2006)
(Boulanger, Leclerc, Sundell, 2008)
(Zinoviev, 2008)
(Manvelyan, Mkrtchyan, Ruhl, 2009)
(Bekaert, Mourad, Joung, 2009)

- (Light-cone or covariant) 3-vertices

  [higher derivatives]

- Scattering via current exchanges
- Contact terms can resolve Velo-Zwanziger *(Porrati, Rahman, 2009)*

Vasiliev eqs: *(Vasiliev, 1990, 2003)* *(Sezgin, Sundell, 2001)*

- Deformed low-derivative with $\Lambda \neq 0$
- Infinitely many fields

Scattering via Current Exchanges

(Bekaert, Mourad, Joung, 2009)

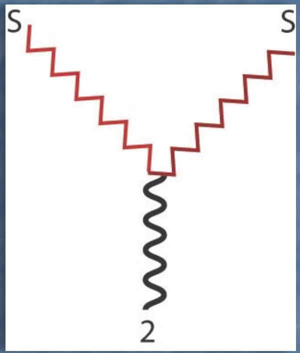
Key ingredients:

- (0,0,s) couplings via conserved currents: $$\left|\phi(x+i\,y)\right|^2$$
- current exchange formula: $$\langle J, \Box\,\varphi\rangle = \sum_n \frac{1}{n!\,2^{2n}\left(3-s-\frac{D}{2}\right)_n}\,\langle J^{[n]}\,,\,J^{[n]}\rangle$$
- dimensionful coupling constant λ
- dimensionless coupling function g(z): $$g(z) = \sum_{n=0}^{\infty} g_n\,\frac{z^n}{n!}$$

  [String Theory: exponential g]
- In 4D: $$\mathcal{A}(s,t,u) \sim \frac{\lambda^2}{t}\left\{g\left[-\frac{\lambda^2}{8}\left(\sqrt{s}+\sqrt{-u}\right)^2\right]+g\left[-\frac{\lambda^2}{8}\left(\sqrt{s}-\sqrt{-u}\right)^2\right]-g_0\right\} + \text{crossing}$$
- **In principle:** good high-energy behavior possible with exotic g

Cubic HS Interactions

- Closer look at old difficulties (for definiteness s-s-2 case)



  - **Aragone-Deser:** **NO** "standard" gravity coupling for **massless** HS around **flat** space;

  - **Fradkin-Vasiliev :** higher-derivative terms around (A)dS;
  - **Boulanger et al:** ALMOST UNIQUE highest non-Abelian CUBIC coupling (seed) for massless HS around flat space;

e.g. 3-3-2 :

$$\boxed{\begin{aligned}\mathcal{L}_3 \quad \sim \quad & \tilde{w}_{\alpha\beta\gamma\delta}\Big[2\ \widetilde{\phi}'_{\mu}\partial^{\beta}\partial^{\delta}\widetilde{\phi}^{\alpha\gamma\mu} + \widetilde{\phi}^{\alpha\gamma}{}_{\mu}\partial^{\delta}\partial^{\mu}\widetilde{\phi}'^{\beta} - 3\ \widetilde{\phi}'^{\alpha}\partial^{\delta}\partial^{\mu}\widetilde{\phi}^{\beta\gamma}{}_{\mu} \\ & +2\ \widetilde{\phi}^{\alpha}{}_{\mu\nu}\partial^{(\delta}\partial^{\nu)}\widetilde{\phi}^{\beta\gamma\mu} + \partial_{\mu}\widetilde{\phi}^{\alpha\gamma\mu}\partial_{\nu}\widetilde{\phi}^{\beta\delta\nu} - \widetilde{\phi}^{\alpha\gamma\mu}\partial_{\mu}\partial_{\nu}\widetilde{\phi}^{\beta\delta\nu} \\ & -2\ \partial^{(\mu}\widetilde{\phi}^{\nu)\alpha\gamma}\partial_{\mu}\widetilde{\phi}^{\beta\delta}{}_{\nu} - 2\ \widetilde{\phi}^{\alpha\gamma}{}_{\mu}\partial^{\delta}\partial^{\nu}\widetilde{\phi}^{\beta\mu}{}_{\nu} + \widetilde{\phi}'^{\alpha}\partial^{\beta}\partial^{\delta}\widetilde{\phi}'^{\gamma} - \widetilde{\phi}^{\alpha}{}_{\mu\nu}\partial^{\beta}\partial^{\delta}\widetilde{\phi}^{\gamma\mu\nu}\Big]\end{aligned}}$$

(Boulanger, Leclerc, Sundell, 2008)

- DEFORMING highest vertex to (A)dS one can recover consistent "standard" couplings WITH tails, singular limit as $\Lambda \rightarrow 0$

Closed-form HS Interactions

Vasiliev' s setting:

1. **Extend** frame formulation of gravity :
2. Symmetric tensors of arbitrary spin

- All (even) spin-s fields:

3. ∞ - dim. HS-algebra via **oscillators** (coordinates and momenta):

$$g_{\mu\nu} \longrightarrow e_\mu^a \, , \, \omega_\mu^{ab} \longrightarrow \omega_\mu^{A;B}$$

$$\omega_\mu^{A;B} : \begin{array}{|c|}\hline A \\ \hline B \\ \hline\end{array}$$

$$\varphi_{\mu_1 \dots \mu_s} \longrightarrow \omega_\mu^{A_1 \dots A_{s-1};B_1 \dots B_{s-1}}$$

$$\omega_\mu^{A_1 \dots A_{s-1};B_1 \dots B_{s-1}} : \begin{array}{|c|c|c|}\hline A_1 & \cdots & A_{s-1} \\ \hline B_1 & \cdots & B_{s-1} \\ \hline\end{array}$$

$$M_{A;B} \rightarrow M_{A_1 \dots A_s;B_1 \dots B_s}$$

$$M_{A;B} = x_A p_B - x_B p_A$$

$$M_{A_1 \dots A_s;B_1 \dots B_s} = x_{A_1} \dots x_{A_s} p_{B_1} \dots p_{B_s} \pm \dots$$

(Twisted) Adjoint

3. [Weyl ordered (symmetric) polynomials in (x,p) or *- products]

4. One-form A in adjoint of HS algebra :
   (all ω's : HS vielbeins and connections)
   - One field for any (even) s

$$\mathcal{A}(x^\mu|x_A, p_A)$$
$$F = dA + A \wedge \star A$$

5. Zero-form Φ in "twisted adjoint":
   - ∞ fields for any (even) s
   - " Weyl " (+ derivatives) :

$$\Phi^{A_1 \ldots A_{s+k}; B_1 \ldots B_s} : \quad \begin{array}{|c|c|c|c|c|c|} \hline & \cdots & & s & \cdots & s+k \\ \hline & \cdots & s \\ \cline{1-3} \end{array}$$

Scalar : φ

- Spin-2 equation in the form:
  - trace recovers " Ricci=0"

" Riemann = Weyl "

The Vasiliev equations

$$\widehat{F} = \frac{i}{2}\, dZ^i \wedge dZ_i\ \widehat{\Phi} \star k$$
$$\widehat{D}\,\widehat{\Phi} = 0$$

$$\widehat{A}\,(x\,|\,Y,Z))$$
$$\widehat{\Phi}\,(x\,|\,Y,Z))$$

- Background independent (non-Lagrangian)
- [Extra Z variables] ; Φ*κ: "twisted adjoint" Φ → adjoint
- [Chan-Paton extension to all (even and odd) ranks]
- Consistent : Bianchi for F implies second eq .
  - Cartan Integrable System (but with 0-form)
  - Unfolding (One-derivative equations)
- This form (not spinor 4D form) needs further specifications

$$dW^i = f^i(W)$$
$$\frac{\partial f^i(W)}{\partial W^j} \wedge f^j(W) = 0$$

(Sullivan, 1977; D'Auria, Fre', 1982)

Linearization

Free flat limit (with traces in A → Strong Proj.)

$$\partial_{[\mu}\,\omega_{\nu],a(s-1),b(k)}^{(s-1,k)} + \omega_{[\mu|,a(s-1),|\nu]b(k)}^{(s-1,k+1)} = \delta_{k,s-1}\,\Phi_{[\mu|a(s-1),|\nu]b(s-1)}$$

- s = 2 :

$$\partial_{[\mu}\,e_{\nu]}^{a} - \omega_{[\mu\nu]}{}^{a} = 0 \rightarrow \omega(e)$$
$$R_{\mu\nu}{}^{ab} = \partial_{[\mu}\,\omega_{\nu]}{}^{ab} = \Phi_{\mu a,\nu b} \rightarrow R_{\nu b} = 0$$

- s = 3 :
  - a first equation, analogous of the vielbein postulate giving ω(e)
  - a second equation, defining a second-order kinetic operator

$$\omega_{\mu,ab,c}^{(2,2)\ c} = \Box\,\varphi_{\mu ab} - \partial_a\,\partial\cdot\varphi_{\mu b} - \partial_b\,\partial\cdot\varphi_{\mu a} + \partial_a\,\partial_b\,\varphi'_{\mu}$$

  - a third equation providing the constraint

$$\partial_{[\mu}\omega_{\nu],ab,c}^{(2,2)\ c} = 0 \rightarrow \omega_{\mu,ab,c}^{(2,2)\,c} = \partial_\mu\beta_{ab}$$

Recovering the compensator

(AS, Sezgin, Sundell, 2004)

$$\Box\,\varphi_{\mu ab} - \partial_a\,\partial\cdot\varphi_{\mu b} - \partial_b\,\partial\cdot\varphi_{\mu a} + \partial_a\,\partial_b\,\varphi'{}_{\mu} \;=\; \partial_\mu\,\beta_{ab}$$

$$\partial_{[\mu}(\partial\cdot\varphi_{a]b} - \partial_{a]}\varphi'_b) \;=\; \partial_{[\mu}\beta_{b]a}$$

(Dubois-Violette, Henneaux, 1999)

$$\beta_{ab} = (\partial\cdot\varphi_{ab} - \partial_a\varphi'_b - \partial_b\varphi'_a) + 3\,\partial_a\partial_b\alpha$$

$$\mathcal{F} \;=\; 3\,\partial^3\,\alpha$$

Outlook

Today :

- **Systematics** of free higher-spin dynamics
- (Essentially) the **extension** to (A)dS backgrounds
- **Link** with free String Theory via (generalized) triplets

- Two paradigmatic examples of "interacting higher spins":
  - String Theory (mixed symmetry, massive, mostly flat)
  - Vasiliev system (symmetric tensors, massless, curved)

Key objectives :

- Systematics of HS Interactions
- Geometrical framework for HS interactions
- Lessons from (and for) String Theory

Internal expansion

$$\widehat{F} = \frac{i}{2}\, dZ^i \wedge dZ_i\ \widehat{\Phi} \star k$$
$$\widehat{D}\,\widehat{\Phi} = 0$$

- Gauge field A in (x,z) space: $\widehat{A}(x^\mu|Y^{iA}, Z^{iA}) = \widehat{A}_\mu dx^\mu + \widehat{A}_{ia} dZ^{ia} + \widehat{A}_i dZ^i$

$$\widehat{A}_{ia} = 0\ ,\ \partial_{ia}\widehat{\Phi} = 0$$
$$\partial_{ia}\widehat{A}_i = 0\ ,\ \partial_{ia}\widehat{A}_\mu = 0$$

$$\Rightarrow$$

$$\widehat{F}_{ij} = -\, i\ \epsilon_{ij}\ \widehat{\Phi} \star k$$
$$\widehat{F}_{\mu\nu} = 0\ ,\qquad \widehat{F}_{\mu i} = 0$$
$$D_\mu \widehat{\Phi} = 0\ ,\ D_i \widehat{\Phi} = 0$$

Internal equations: power series in Φ by successive iterations

Unfolding

- Linearized Φ – equation :

$$D_\mu \Phi + \frac{1}{2i}\{P_a, \Phi\} = 0$$

- Unfolding :

$$\begin{aligned}
&\varphi_\mu = \partial_\mu \varphi \\
&\varphi_{\mu\nu} = \partial_\mu \varphi_\nu \\
&\ldots \\
&\varphi_{\mu(n+1)} = \partial_\mu \varphi_{\mu(n)}
\end{aligned}
\Rightarrow
\eta^{\mu\nu}\varphi_{\mu\nu} = 0 \rightarrow \Box\varphi = 0$$

Uniform description of HS interactions

Cartan Integrable Systems

$$\widehat{F} = \frac{i}{2} dZ^i \wedge dZ_i \; \widehat{\Phi} \star k$$
$$D\widehat{\Phi} = 0$$

Cartan Integrable System :

(Sullivan, 1977; D'Auria, Fre', 1982)

- e.g. Chern-Simons theory
- manifestly consistent eqs
- gauge covariance
- manifest diff covariance
- non-Lagrangian

$$R^i \equiv dW^i + f^i(W) = 0 \quad \text{with} \quad f^i(W) \frac{\partial f^i(W)}{\partial W^j} = 0$$
$$\delta W^i = d\Lambda^i - \Lambda^j \frac{\partial f^i(W)}{\partial W^j}$$
$$\delta R^i = (-)^j \Lambda^j R^k \frac{\partial^2 f^i(W)}{\partial W^k \partial W^j}$$

- NEW INGREDIENT : 0-form Φ