2+1 Topics

1) Cosmology

2) Insightful D-branes
w/ Horowitz, Lawrence

} Executive Summary (papers online)

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3) $\boxed{\text{F is for Four} \quad \text{w/ Polchinski}}$ } New (in progress)

1) String theory as Rocket Science

CMB observables

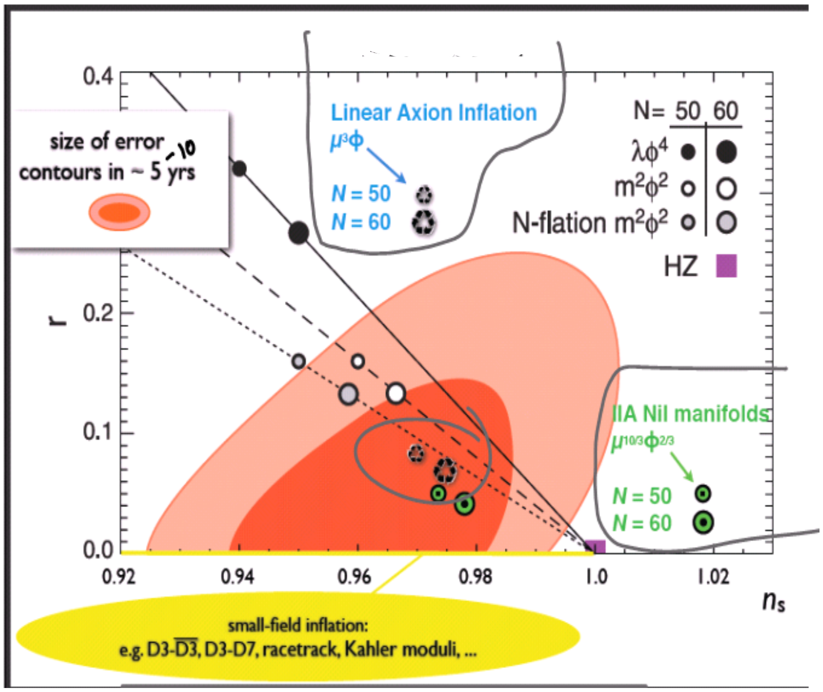
- gravity waves (B mode polarization)
- non-Gaussianity

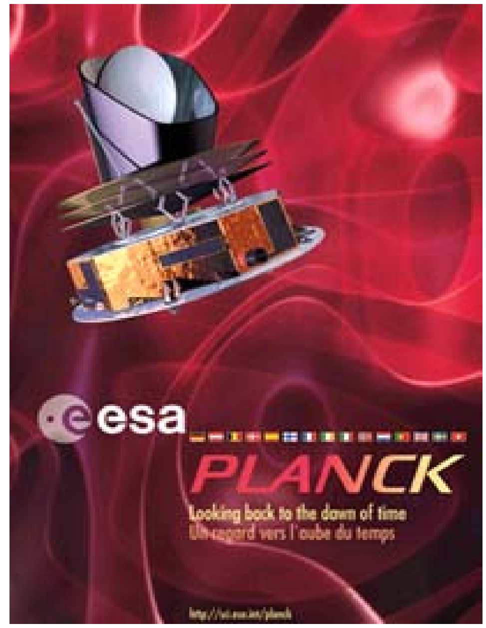
are UV sensitive

$$\text{e.g.} \quad \frac{\Delta\Phi}{M_p} \sim \left(\frac{r}{0.01}\right)^{1/2} \qquad r = \frac{\text{tensor}}{\text{scalar}}$$

Must control contributions to inflaton effective action at $\geq$ dimension 6

- D3 - $\overline{\text{D3}}$ warping, tuning of $\frac{V''}{V} M_p^2$
- Monodromy symmetry $\rightarrow r \sim .07$
- trapping $\rightarrow$ new source of $\delta\rho$, non-Gaussian
- DBI $\rightarrow$ N.G. · Nflation · Kähler ...

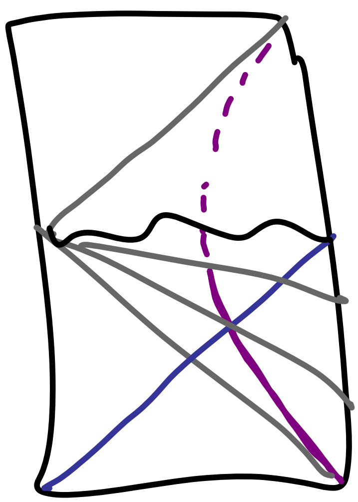




- Refs : • Review papers
  - CMB Pol Inflation working group report '08
- Planck launch + 3 years = 2012
  KITP workshop on Primordial Cosmology   ES, M. Zaldarriaga

Insightful D-branes

G. Horowitz
A. Lawrence
ES



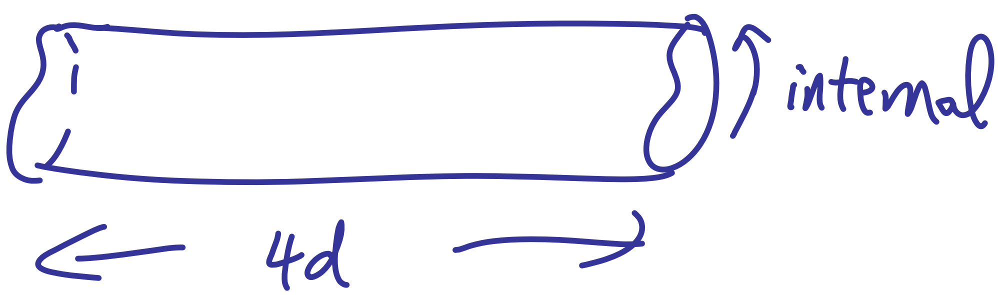
- Poincare Patch

AdS/CFT duality on compact hyperbolic spatial slices governs physics inside the horizon of a black hole.

F is for Four

J. Polchinski
E.S.

We'd like a non-perturbative formulation of 4d physics:



with a hierarchy of energy scales

$$m_{\left\{\substack{KK \\ string \\ D}\right.} \gg \frac{1}{L_{4d}}$$

Outline

- The problem & previous attempts
- Our strategy
- Consistency conditions (cf singularities)
- Candidate Examples

$$AdS_{\substack{5\\4}} \times \text{Small}_{\substack{5\\6}} \Leftrightarrow CFT_{\substack{4\\3}}$$

stabilized compactifications $\Leftrightarrow$ IR limit of concrete brane systems (w/ SUSY)

- Generalizations toward dS

AdS/CFT (and previously, BFSS matrix theory) formulate some backgrounds non-perturbatively, but did not (yet) get down to 4d :

BFSS : 11d $\leftrightarrow$ N D0-brane Q.M.

4d (max susy) $\leftrightarrow$ $\underbrace{\text{D7-branes}}$ on T^7

↳ codim 2 → log potential → $C_{N \leq 24}$

AdS/CFT: ① $\underbrace{AdS_2 \times S^2}_{\text{want } L_{AdS} \to \infty} \times \underbrace{CY}_{\text{small } \checkmark}$

↳ IR divergences in AdS_2

AdS/CFT ②

$$AdS_4 \times \underbrace{\begin{cases} S^7 & (M) \\ S^7/\Gamma & \\ \mathbb{CP}^3 & (IIA) \end{cases}}_{L_{internal} \sim L_{AdS}}$$

No hierarchy of scales in Freund-Rubin compactifications.

Basic reason: In 11/10 d Einstein equations

$$\underbrace{R_{MN} - \frac{1}{2} R G_{MN}}_{\text{Internal} + 4d} = 8\pi G \underbrace{T_{MN}}_{\text{flux}}$$

all three contributions are of the same order in the solution

Or in terms of the 4d effective potential:

$$S = \int d^{10}x \frac{\sqrt{G}}{\alpha'^4}\left(\underbrace{\frac{\mathcal{R}}{g_s^2}}_{\text{4d potential energy}} + F_p^2 + \cdots\right)$$

$$U_{\mathcal{R}} \sim -M_4^2 \frac{1}{R^2 \alpha'} + \cdots$$

R = curvature radius in string units.

$$U_{\mathcal{R}} \sim M_4^2 \Lambda \Rightarrow R_{AdS} \sim R$$

in Freund-Rubin

On the other hand, we can construct

$$(A)dS_4 \times X_{small}$$

in an apparently large number of ways B^2S DRS BP MSS GKP+KKLT...

suggesting a rich set of dual CFT_3 s.

- Not a priori realized as near-horizon limit of brane system
- Can read off interesting properties:

$$N_{d.o.f.} \sim L_{(A)dS}^2 M_p^2 \leq N^b$$

b ← betti #

N ← flux #

ES BvdB AAB

This talk:

SUSY Brane construction

Near Horizon

AdS × Small

Low energy:

QFT

Using 7-branes to nearly cancel Curvature energy

⟹ Electric + Magnetic Flavors

cf Aharony
Fayazzudin
Maldacena

Strategy: Start from known, Freund-Rubin dual pair:

$$\underbrace{AdS \times S} \leftrightarrow QFT$$

(at least brane construction)

add ingredients which cancel or nearly cancel $U_{\mathcal{R}}$ $\leftrightarrow$ additional field content, couplings of QFT

stabilize the moduli $\rightarrow AdS_{\substack{5\\4}}$ $\leftrightarrow$ $CFT_{\substack{4\\3}}$

7- branes compete with curvature energy:

$$\mathcal{U}_7 \sim \underbrace{\gamma_7}_{\frac{1}{g_s^2}} \cdot \underbrace{R^4}_{R^6 \cdot \frac{1}{R^2}} \alpha'^2 \sim M_4^2 \frac{1}{R^2 \alpha'}$$

(p, q)

Of course 7Bs, being codimension 2, have large IR back reaction ...

The interplay between curvature & 7-brane energy is accurately captured using the techniques of F-theory :

Vafa '96

$$T^2 \to X$$
$$\downarrow$$
$$B$$

$$\tau_{T^2} = C_0 + \frac{i}{g_s} \quad \text{in IIB}$$

Plan: Start from
Freund-Rubin dual pair:

AdS x S $\leftrightarrow$ QFT
(brane construction)

add 7-branes $\leftrightarrow$ additional flavors*
which cancel or couplings of
nearly cancel QFT
U_R

stabilize the moduli $\rightarrow$ AdS4 $\leftrightarrow$ CFT_3

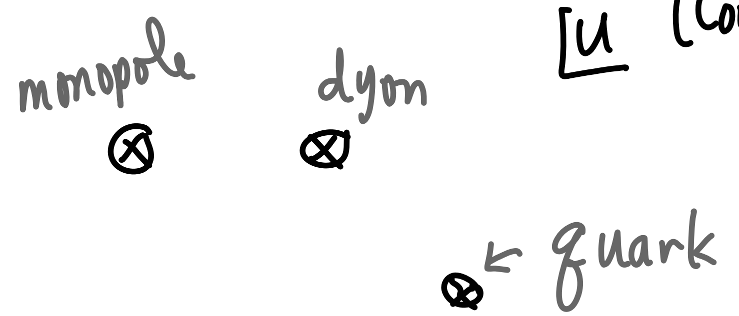
* in general, both electric & magnetic cf Douglas/Shenker, Argyres/Douglas, Argyres-Plesser-Seiberg-Witten

D3, D7 and Electric/Magnetic Matter

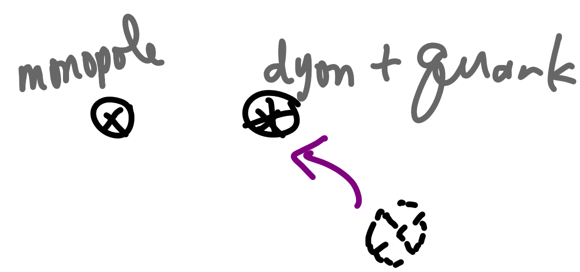
- 4d $\mathcal{N}=2$ $SU(2)$ SYM w/ N_f hypermultiplets

Seiberg-Witten solution

$\boxed{u}$ (Coulomb branch)



AD/APSW: can change mass matrix M such that



mutually nonlocal matter is light.

- In brane constructions (Sen, Banks Douglas Seiberg ...)

$u \leftrightarrow$ D3 position

$\otimes \leftrightarrow$ 7B position $\qquad \tau_{YM} \leftrightarrow \tau_{IIB}$

The T^2 varying over B can be described as

Vafa
Morrison-Vafa
Kachru Intriligator
Morrison Vafa
...

$$y^2 - x^3 - x f(u) z^4 - g(u) z^6 = 0$$

coordinates on B

i.e. as a degree 6 hypersurface in $WP^2(2,3,1)$.

For a Kähler base B, one can formulate the T^2 fibration

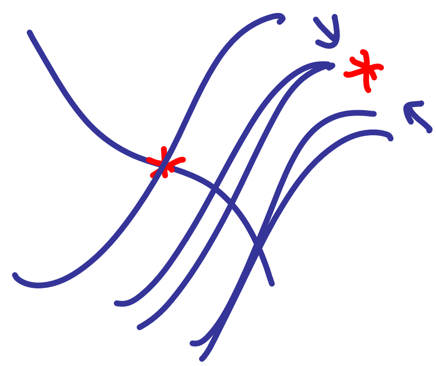
$$\begin{array}{ccc} T^2 & \to & X \\ & & \downarrow \\ & & B \end{array}$$

as a hypersurface in $B \times WP^2(2,3,1)$, and as the target space of a (2,2) gauged linear σ-model (GLSM) Witten

7-branes live at the locus

$$\Delta = 27 g^2 + 4 f^3 = 0$$

Singularities



- Some allowed (e.g. enhanced gauge symmetries)
- Some not allowed

A criterion for allowed singularities:

cf W, SW '90s

In the GLSM, singularities arise from extra non-compact branches of scalar field space

bulk of geometry

Compute (using GLSM) $\hat{c}_{throat}$:

If $\hat{c}_{throat} \geq \hat{c}_{bulk}$ then truly singular. (Otherwise linear dilaton $\rightarrow$ mass gap in throat)

This agrees with known cases ...

e.g.

$$\int d^2\theta \; P\left(y^2 - x^3 - x \overset{\sim 0}{f(u)} z^4 - \overset{\sim u^n}{g(u)} z^6 \right)$$

branch with $\underbrace{\langle p \rangle \overset{\text{(D-term)}}{\sim} \langle z \rangle}_{\hat{c} = 1}$; $Y = X = 0 = U$

$\hat{c}_Y = 0$ (massive)

$\hat{c}_X = \frac{1}{3}$

$\hat{c}_U = 1 - \frac{2}{n}$

$\Rightarrow$ singular if $n \geq 6$ ✓

... and can be applied widely

Let us start with IIB on

$Y_5 \times S^1$ with $\int_Y F_5 = N_c$

where Y_5 is an S^1 (Hopf) fibration over a Kähler base

(7-branes will be extended along fiber)

$$\begin{array}{ccc} S^1_f & \to & Y \\ & & \downarrow \\ & & B \end{array}$$

First study 7-branes on B

Examples: $Y = S^5$, $B = \mathbb{CP}^2$ or $\mathbb{WP}^2$

Topologically $S^2 \times S^3$

$Y = T^{1,1}$ $B = \mathbb{CP}^1 \times \mathbb{CP}^1$ Klebanov/Witten

$Y = Y^{pq}$ $B = \ldots$ Gauntlett Martelli Sparks Waldram

$Y = L^{abc}$ $B = \ldots$ Cvetic Liu Pope

can have many 3-cycles

base of toric CY_3 cones Hanany...

" " non-toric " Martelli/Sparks/Yau

$$\begin{array}{ccc} S^1_f & \to & Y \\ & & \downarrow \\ & & B \end{array} \cong B \text{ with "metric flux" } F_{met} = J_B$$

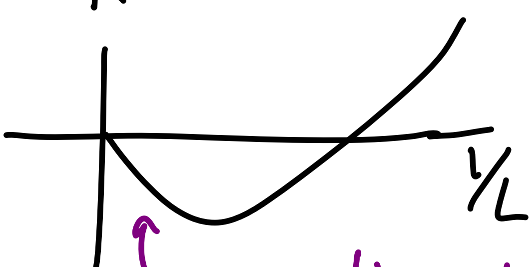
size R_f size R

(useful description for small fiber)

Two classes of candidate examples:

① 7-branes fully cancel curvature energy: e.g. 36 7-branes on $\mathbb{CP}^2$

$$U_R + U_7 = 0 \quad \text{(F-theory on CY)}$$



negative term from O-planes

$$R_{AdS} \gg R \gg R_f$$

② 7-branes nearly cancel $U_R^{(B)}$

$Y \times S^1_\perp \times AdS_4$

$$\frac{Q_3^2}{R_\perp^2 R^4}$$

$$U\Big|_{g_s \sim 1} \sim M_p^4 \left(\frac{1}{R^4 R_6 R_f}\right)\left(\frac{R_f^2}{R^4} - \frac{\varepsilon}{R^2} + \frac{N_c^2}{R^8 R_f^2} + \frac{Q_1^2}{R_\perp^2}\right)$$

enforce with e.g. E_n 7-branes

↳ stable minimum with $R_f \ll R \ll R_{AdS}$

Related (simpler) AdS_5 model:

$$U\Big|_{g_s \sim 1} \sim \frac{M_5^5}{(R^5 R_f)^{2/3}}\left(\frac{R_f^2}{R^4} - \frac{\varepsilon}{R^2} + \frac{N_c^2}{R^8 R_f^2}\right)$$

To get started, consider

$$Y = S^5 \quad \text{(topologically)}$$

Start from the $\mathbb{CP}^2$ model:

(2,2) chiral multiplets u_1, u_2, u_3

$U(1)$ Gauge symmetry

$$(u_1, u_2, u_3) \simeq e^{2\pi i \alpha}(u_1, u_2, u_3)$$

$$\Rightarrow D^2 = \left(|u_1|^2 + |u_2|^2 + |u_3|^2 - R^2\right)^2$$

$\hookrightarrow$ $D = 0$ alone gives S^5

$\rightarrow$ α parameterizes S^1_{fiber}

$$\begin{array}{ccc} S^1_f & \rightarrow & S^5 \\ & & \downarrow \\ & & \mathbb{CP}^2 \end{array}$$

$$ds^2_{S^5} = d\sigma^2_{\mathbb{CP}^2} + R_f^2 (d\alpha + A)^2$$

$$dA = J$$

Gibbons, Pope

To add the 7-branes, want a T^2 fibration over $B = \mathbb{CP}^2$

Gauged Linear σ-model becomes

	u_1	u_2	u_3	x	y	z	P
$T^2\{U(1)$	0	0	0	2	3	1	-6
$\times$							
$\mathbb{CP}^2\{U(1)$	1	1	1	0	0	q_z	0

$$S_W = \int d^2\sigma\, d^2\theta\, P\left(y^2 - x^3 - f(u)\,x z^4 - g(u)\,z^6\right)$$

Now, $\displaystyle\sum_{\text{fields } I} q_I = 0$ is the Calabi-Yau condition (ensuring anomaly-free $U(1)\times U(1)$ R-symmetries appropriate to (2,2) SCFT)

Witten

The running of R^2 in

$$D^2 = \left(|u_1|^2 + |u_2|^2 + |u_3|^2 + q_x |x|^2 + \tfrac{3}{2} q_x |y|^2 - |p|^2 - R^2\right)^2$$

is $\quad M \frac{\partial R^2}{\partial M} \sim \sum_I q_I$

Now $\quad \sum q_I = 3 + q_z$

and the degree of $G = y^2 - x^3 - f x z^4 - g z^6$

is $\quad \deg_G = \deg_g = 6 q_z = 18 - 6 \sum q_I$

Fully canceling curvature energy

means $\quad \sum q_I = 0 \Rightarrow \deg_g = 18$

$\Rightarrow \deg \Delta = 36 \Rightarrow$ 36 7-branes

(This agrees with naive result from $\mathcal{U}_7 \sim \cdots (\varphi_7 \times \mathrm{vol})$ ✓)

- The 7Bs are extended along U(1) fiber

As next step, generalize to cases where we do <u>not</u> fully cancel the curvature energy. Consider

T^2 fibration over $B = \underline{W}\mathbb{P}^2$

Gauged Linear σ-model

	u_1	u_2	u_3	x	y	z	P
$T^2\{U(1)$	0	0	0	2	3	1	-6
$\times$							
$W\mathbb{P}^2\{U(1)$	$\underline{\underline{w_1}}$	$\underline{\underline{w_2}}$	$\underline{\underline{w_3}}$	0	0	$w_0 - \Sigma w$	0

$$S_W = \int d^2\sigma\, d^2\theta\; P\left(y^2 - x^3 - f(u)\, x z^4 - g(u)\, z^6\right)$$

Again $\beta_{R^2,\,\text{full}} \sim \sum q = w_0$

in the full system including the 7Bs.

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in the full system including the 7Bs.

For WP^2 <u>alone</u>,

$$\beta_{R^2,\,wp^2} = W_1 + W_2 + W_3$$

$\Rightarrow$ If $\quad W_0 << W_1 + W_2 + W_3$

then we <u>almost</u> cancel the curvature.

$$\to \quad \mathcal{U}_{R,\,\text{full}} \sim M_p^4 \left(\frac{g_s^2}{Vol}\right)^2 \cdot Vol \cdot \left(-\frac{\varepsilon}{R^2}\right)$$

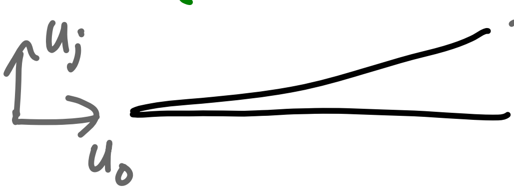
with $\boxed{\varepsilon \sim \dfrac{W_0}{W_1 + W_2 + W_3}}$

(using the NLSM result $\beta \sim R_{MN}$)

We can describe the full brane system (whose low-energy limit is the QFT) as F-theory on the following noncompact CY_4:

u_0	u_1	u_2	u_3	x	y	z	p
0	0	0	0	2	3	1	-6
$-w_0$	w_1	w_2	w_3	0	0	$w_0 - \Sigma w$	0

- $\Sigma w = 0$ overall; cross-section geometry as above

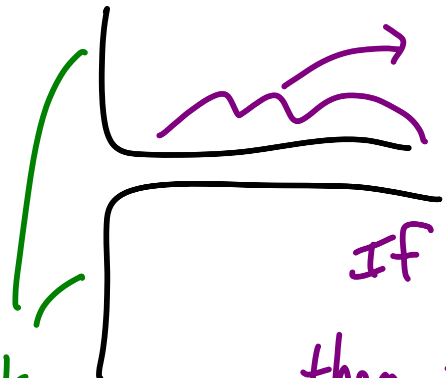


} small if $w_0 \ll w_j$: $w_0 |u_0|^2 \sim w_j |u_j|^2$

- This + N_c D3-branes at tip ($u_0 = u_j = 0$) is the brane construction.

Altogether preserves 4 supercharges.

* Singularities of 7-branes:



bulk of geometry

compute (using GLSM) $\hat{c}_{\text{throat}}$:

If $\hat{c}_{\text{throat}} \geq \hat{c}_{\text{bulk}}$ then truly singular. (Otherwise linear dilaton $\to$ mass gap in throat)

w_0	W_1	W_2	W_3	z
-1	1	n	n	$-2n$

$\to$ nonsingular but anisotropic

w_0	W_1	W_2	W_3	z
$-w_0$	$w-\delta$	w	$w+\delta$	w_0-3w

isotropic but singular $g \sim \sum_I \underbrace{u_1^I \, u_2^{18+\frac{3w_0}{\delta}-2I}}_{} \, u_3^{\frac{I-3w_0}{\delta}}$

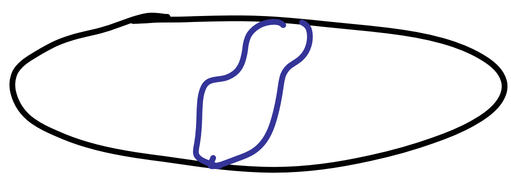
high degree $\to$ high $\hat{c}$

Generalizations (≈ isotropic, nonsingular)

-

u_0	u_1	u_2	u_3	u_4	$\cdot\cdot z \cdot\cdot$	$\tilde{p}$
-1	1	n	n	n	$-2n$	$-n$

$\int d^2\theta \, \tilde{P} \, \tilde{G}$



F term restricts to isotropic subspace.

-

u_0	u_1	$\cdots$	u_D	z
$-w_0$	w_1	$\cdots$	w_D	$w_0 - \Sigma w$
0	Q^a_1	$\cdots$	Q^a_D	$-\Sigma Q^a$

$a = 1 \ldots D-3$ extra $U(1)$ conditions

So choose nonsingular f, g & then w, Q under which that f & g are invariant.

Remarks

- 7-brane moduli approximately flat ($\beta = 0$ in sigma model)

- Sen (orientifold) limit may provide purely electric description

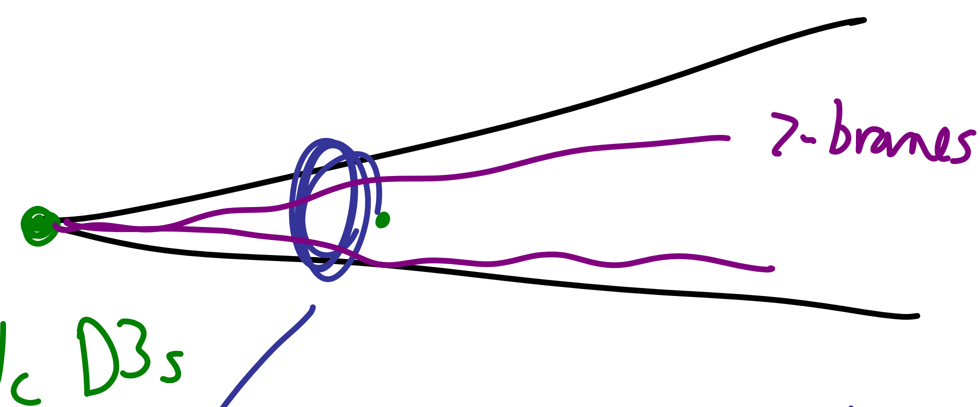
- Dasgupta/Mukhi constant coupling is enhanced symmetry point ($\Rightarrow$ extremum)

Number of degrees of freedom:

AdS_5 :

$$N_{d.o.f.}^{(7Bs)} \sim M_5^3 R_{AdS}^3 \sim \frac{1}{\varepsilon^3} \cdot N_{d.o.f.}^{(no\ 7Bs)}$$

$$R_f \sim \varepsilon R_{AdS} \,, \quad R^2 \sim \varepsilon R_{AdS}^2$$

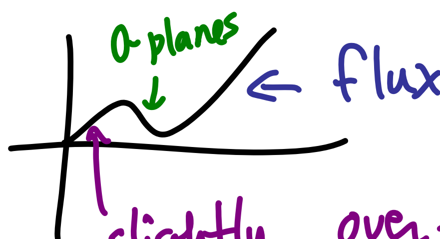


$\rightarrow$ # of light states

$$\sim \frac{1}{(\varepsilon^{\frac{1}{2}})^4} \cdot \frac{1}{\varepsilon} \sim \frac{1}{\varepsilon^3}$$

Future directions

- QFT content & couplings from brane system
- dS_4 :



slightly over-cancel curvature energy

Now no tachyons are allowed.

- GKP + KKLT use 7-branes (F-theory) and have 5-form flux. Interpret as above?
- Similar generalization of Gaiotto/Maldacena?