Non-perturbative effects and wall crossing in 4d N=1,2 string vacua

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Based on I. Garcia-Etxebarria, A.U., arXiv:0711.1430
I. Garcia-Etxebarria, F. Marchesano, A.U. arXiv:0805.0713
A.U. arXiv:0808.2918
A. Collinucci, P. Soler, A.U., arXiv:09041133

D-brane instanton effects

[Becker's, Strominger; Witten; Harvey, Moore; ...]

Violate certain perturbatively exact U(1) global symmetries

- Ex: Take one complex structure modulus in IIA CY orientifold

$$T = t + i\,a = \int_C \mathrm{Re}\,\Omega + i \int_C C_3$$

PQ symmetry $a \to a + \lambda$ violated by D2-brane instanton $\simeq e^{-T}$

⇒ Stabilization of moduli perturbatively protected by PQ

[Kachru, Kallosh, Linde, Trivedi]

- With 4d gauge D6-branes, gauging of PQ by U(1) in U(N)

From D6-branes on C'

$$\int_{C' \times M_4} C_5 \wedge \mathrm{tr}\, F \to \int_{M_4} B_2 \wedge \mathrm{tr}\, F \to \int d^4x\, (\,\partial_\mu a + A_\mu\,)^2$$

⇒ Instanton on C violates $U(1)_{C'}$ by $n=I_{CC'}$ units

⇒ Fermion zero modes at $I_{CC'}$ intersections $\quad \int d\lambda\, d\tilde{\lambda}\, e^{-T+\lambda\Phi\tilde{\lambda}} = e^{-T}\,\Phi$

⇒ Role in generating perturbatively forbidden couplings

$$e^{-T}\,\Phi_1 \ldots \Phi_n$$

[Blumenhagen, Cvetic, Weigand; Ibanez, AU; Florea, Kachru, McGreevy, Saulina]

D-brane instantons and wall crossing

- Need to understand physics of multi-instantons

  - Sum over instanton sector, multiply wrapped instantons

  - Single instanton contributions not well defined over moduli space

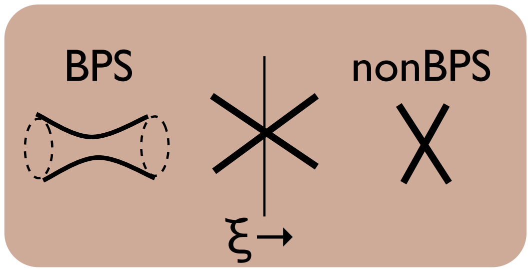
- List of BPS D-brane instantons can jump discontinuously in real codimension one in closed string modulus: Walls of BPS stability

  Closed string modulus couples as FI-term
  Fayet model on worldvolume of D-brane instanton

Marginal stability:

BPS ⇒ split BPS ⇒ non-BPS

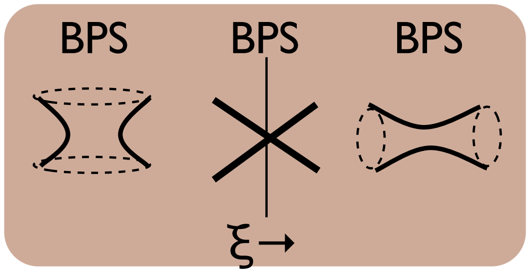
$$V_D = (|\phi|^2 + \xi)^2$$



Threshold stability:

BPS ⇒ split BPS ⇒BPS

$$V_D = (|\phi_1|^2 - |\phi_2|^2 + \xi)^2$$



- Fate of non-perturbative terms upon wall crossing?

Focus on 4d N=1

- Clue to the resolution:
  Number of (exact) fermion zero modes

  - Determines the kind of 4d superspace interaction

  - Is topological (continuous upon change of parameters)

  - Must include Goldstinos, at least 2 for BPS, 4 for non-BPS

Application to non-perturbative superpotentials

- Generated by BPS instantons with exactly 2 fermion zero modes

- Instantons contributing to superpotential can never become non-BPS

Not enough fermion zero modes to account for 4 required goldstinos

$\Rightarrow$ Powerful criterion: Any instanton which can become non-BPS cannot contribute to superpotential

$\Rightarrow$ Safe against marginal wall crossing

- What about threshold stability?

Theshold wall crossing, instantons, and N=1 superpotentials

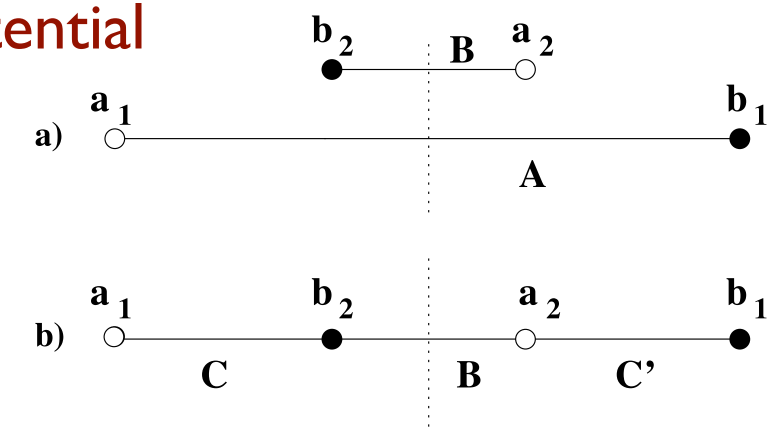
Threshold walls are "harmless" for BPS D-brane particles in 4d N=2

Index counting BPS particle states is continuous, see later

But for BPS D-brane instantons, potentially very dramatic effect !

For instance naive jump in superpotential inconsistent with holomorphy

Ex: Orientifold of double C* fibration



a) Away from wall, two instanton contributions from D2's on A, B

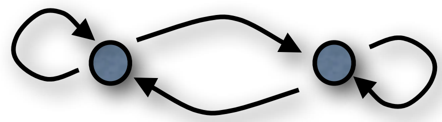
$$W = f_1 e^{-T_B} + f_2 e^{-T_A}$$

b) At threshold wall, instanton A disappears.
Naively only contribution from D2's on B (D2 on C/C' has 4 fzm)

How is $\exp(-T_A)$ generated?

Resolution of puzzle involves <u>2-instanton process</u>

- BPS instantons splits into two BPS instantons



2-instanton process: 0-dim quiver theory

Ex: Translational Goldstones x_1, x_2; "Goldstinos" $\theta_1, \tilde{\theta}_1, \theta_2$;
bi-fundamental hyperm. Φ_{12}, Φ_{21} ie $\varphi_{12}, \varphi_{21}, \chi_{12}, \chi_{21}$

Contribution to superpotential localize onto $x_1=x_2$, couplings are

$$(\chi_{12}\,(\theta_1-\theta_2))\varphi_{12}^* - (\chi_{21}\,(\theta_1-\theta_2))\varphi_{21}^* + (\bar{\chi}_{12}\tilde{\theta})\varphi_{12} - (\bar{\chi}_{21}\tilde{\theta})\varphi_{21}$$

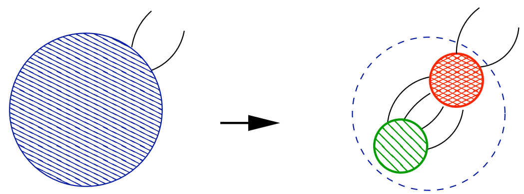
$$\chi_{12}\varphi_{21}\chi_{12}\varphi_{21} + 2\chi_{12}\chi_{21}\varphi_{12}\varphi_{21} + \varphi_{12}\chi_{21}\varphi_{12}\chi_{21} \quad + \text{h.c.}$$

All fermions couple except for the overall Goldstinos $\theta_1+\theta_2$

Pull down interactions in exp($-S_{inst}$) and soak up zero modes

We recover $$S_{4d} \simeq \int d^4\, d^2\theta\, e^{-(T_1+T_2)} = \int d^4\, d^2\theta\, e^{-T}$$

- Decay products combine into 2-instanton process reconstructing amplitude before decay



Marginal stability and 4d N=1 higher F-terms

- Global picture for instantons generating higher F-terms

- Non-perturbative higher F-terms are continuous across general lines of marginal stability (BPS ⇒ non-BPS)

- Consistent w/ standard wisdom of BPS=F-term, non-BPS=D-term

Inst. amplitude as 4d operator in non-trivial Beasley-Witten cohomology:
- Locally in moduli space, can be written as a D-term
- Obstruction (localized on BPS locus) to write as global D-term

$$\int d^4x\, d^4\theta\, e^{-V(T_i,\bar{T}_i}\, f(\Sigma,\bar{\Sigma}) \leftrightarrow \int d^4x\, d^2\theta\, e^{-T}\, \bar{D}\bar{\Sigma}\, \bar{D}\bar{\Sigma} + (\mathrm{Dterm})$$

locally, D-term
(generic non-BPS)

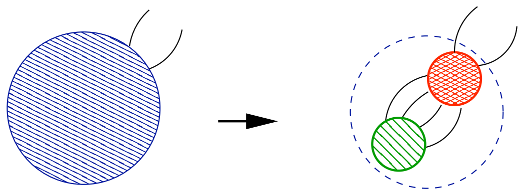
Globally, core F-term (BPS)
cannot be written as Dterm

Non-holomorphic dependences all go into ‘exact’ D-term piece

- Open question: For 4d N=2 relate to physical interpretation of Kontsevich, Soibelmann wall crossing formula by Gaiotto, Moore, Neitzke

Topological string connection?

- 4d non-perturbative terms insensitive to the stability of D-brane instantons



- Reminiscent of D-branes in topological strings

Non-perturbative F-terms as ‘universal’ functions, defined in terms of the category of holomorphic branes

- Suggests the topological string should be able to compute them

Naive objection:

Topological strings depend on ‘wrong’ moduli (‘wrong’ D-branes)
e.g. A-model depend on Kahler, instantons in IIA correct complex structure

Solution:

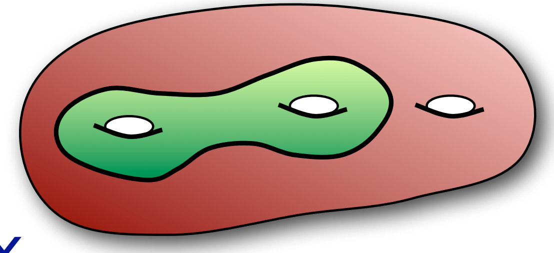
- A-model encodes non-perturbative D-brane particle effects in IIA on X
- Apply ‘c-map’: compactification on S^1 to 3d and T-duality [Ooguri, Vafa ‘96]
- A-model encodes D-brane instanton effects on T-dual IIB on same X

Obs: Related to S-duality of A- and B-models [Nekrasov, Ooguri, Vafa]
Compactification to 3d reminiscent of Gaiotto, Moore, Neitzke

Focus on 4d N=2

GV interpretation of A-model

[Gopakumar, Vafa]



- Partition function $Z_{\rm top} = \sum_g \lambda^{2g-2} F_g(t)$
- Computes F-terms for v.m. in 4d N=2 IIA on CY

[Antoniadis, Gava, Narain, Taylor]

$$S_{4d} = \int d^4x\, d^4\theta \sum_g F_g(t)\, \mathcal{W}^{2g} = \int d^4x\, F_g(t)\, R_+^2\, F_+^{2g-2} + \ldots$$

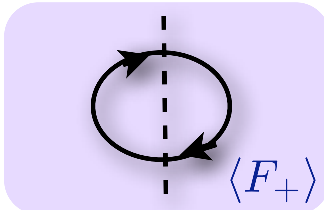
- Turn on graviphoton background $\langle F_+ \rangle = \lambda \to S_{4d} = \int d^4x\, Z_{\rm top}(\lambda)\, R_+^2$

⇒ Schwinger diagram for electric charges: D2/D0-branes

Perturbative F_g from integrating out genus g D2/D0 particles

⇒ Non-pert. creation of 4d D2/D0-particles by Schwinger effect

- Lift to M-theory on CY x S^1 with wrapped genus g M2 particles with momentum on S^1



$$Z_{\rm top,n.p.} = \sum_{r,\, m,\, (n^r_{\mathbf{k}})} n^r_{\mathbf{k}} \int_\epsilon^\infty \frac{ds}{s} \left(2 \sin \frac{s}{2}\right)^{2r-2} \exp\left[-2\pi \frac{s}{\lambda}(k_i t_i + i\, m)\right]$$

D-brane instantons from M-theory on T2

- GV give non perturbative effects for v.m. from 4d D2/D0-particles
- Relate to D-brane instantons by S^1 compactification and T-duality

[Ooguri, Vafa ' 96]

Non-pert. effects on hyperm. moduli space of dual 4d IIB on CY

- IIA on CY x S^1 computed as M-theory on CY x T^2 (genus g M2 particles with momentum on T^2), then shrink T^2 for IIB

  Manifest SL(2,Z) on type IIB side
  Includes D1/D(-1)-brane instantons (general (p,q) string instantons)

- Higher F-terms for hm's are difficult [Michel, Pioline], focus on genus 0

Corrections to hm metric

$$Z_{\mathrm{M2}} = \frac{1}{4\pi} \sum_{\mathbf{k}} n^{(0)}_{k_a} \sum_{(m,n)\neq(0,0)} \frac{\tau_2^{3/2}}{|m+\tau n|^{3/2}} \left(1 + 2\pi\, |m+\tau n|\, k_a\, t^a\right) e^{-S_{k_1,k_2}}$$

$\downarrow$

$$Z_{\mathrm{D-inst}} = \frac{1}{4\pi} \sum_{k_a} n_{k_a} \sum_{m\neq 0, n\in\mathbf{Z}} \frac{|z+q|^{1/2}}{|p|^{3/2}} \left[1 + \sum_{k=1}^{\infty} \frac{\Gamma(3/2+k)}{k!\,\Gamma(3/2-k)} \left(4\pi|p\tau_2||z+q|\right)^{-k}\right] e^{-S_{(p,q)}}$$

Reproduces [Robles-Llana, Rocek, Saueressig, Theis, Vandoren, '06]

Any wall crossing?

- GV includes only instantons with zero mutual "intersection number"
(e.g. D1, D(-1) but no D3, D5) i.e. no magnetic charges in Schwinger
  - ⇒ Only threshold stability walls
  - ⇒ Enough to discuss continuity of superpotential (in 4d N=1 setup)

- <u>Continuity across threshold walls from continuity of GV invariants</u>

  Well-known, but useful to understand microscopically to apply in 4d N=1

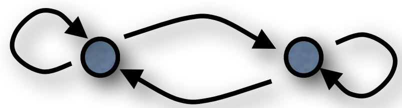
- Can construct examples of splitting of genus 0 D-brane particles

  Classically a BPS D-brane particle splits into two BPS ones

  Quantum level: 1-dim quiver quantum mechanics [Denef '02]

  2-particle system has one bound state at threshold

  [Denef, deBoer, El-Showk, Messamah, Van den Bleeken]



  Bound state in Schwinger loop ensures continuity

  Particle bound state in Schwinger loop maps to 2-instanton process

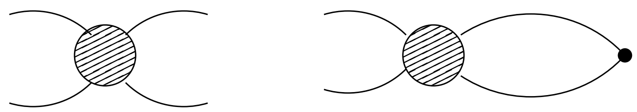
- Non-trivial microscopic miracles, trivially encoded by topological string

Implications and tools for N=1

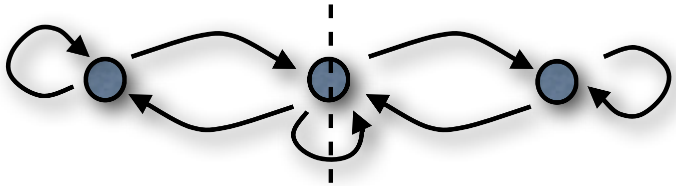
Focus on 4d N=1

Need to reduce to N=1 to really discuss superpotentials

- Turn on N=2 → N=1 fluxes
  - Flux lifts instanton fermion zero modes so N=2 instantons contribute to superpotential [Bergshoeff, Kallosh, Kashani-Poor, Sorkin, Tomasiello; ...; Billo, Ferro, Frau, Fucito, Lerda, Morales]
  - Effective 4d theory description [A.U.]
    Full N=2 hm metric + flux superpotential = non.pert superpotential of N=1 flux model



- Introduce orientifold planes
  - Unoriented topological string may produce useful tools [Walcher, Krefl, ...]
  - Partial lesson to study threshold walls: particle-instanton dictionary
    2 -particle system: orientifolded 1-dim quiver quantum mechanics
    Can check existence of bound state



Matrix Model instantons

Back to 4d N=2

- Matrix models describe topological string B-model on certain local CY's [Dijkgraaf, Vafa]
- "Non-perturbative definition": Matrix model instantons e^{-N} [Mariño; Mariño, Schiappa, Weiss;...]

  Jump of eigenvalues between branch cuts

  Strength of effect matches D-brane on 3-cycle

- We interpret them as effects of particles from D3-branes on 3-cycle

  in contrast with usually proposed as domain walls from D5's on 3-cycle

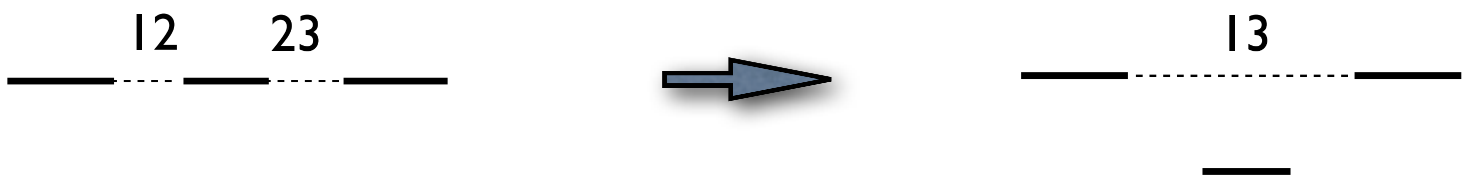
- Fits nicely with Schwinger interpretation

  Only effects of D3's on A-type cycles

- Provides simple alternative explanation for continuity across walls

  Effect of instanton 13, independent of whether 13 splits as 12+23

  Matrix model has conservative force field (work independent of path)



D6's, DT invariants and marginal wall crossing

4d N=2

- Can the topological string describe true marginal wall crossing?

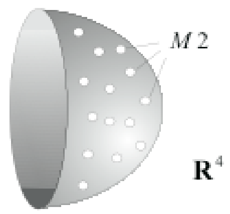
  How to include charges beyond D2-D0 e.g. D6-branes?

- Tantalizing suggestion to exploit the 4d-5d connection

[Dijkgraaf, Vafa, Verlinde]

M2's on CYx S^1xTN → D2/D0 on CYxTN = Ztop on CY x R^4

M2's on CYx S^1xTN → D6/D2/D0 on CYx S^1



- Relates GV invariants to D6/D2/D0-particle running on S^1

  Computes D5/D1/D(-1) instantons in T-dual IIB

  No wall crossing for GV ⇒ continuity of instanton effects a la KS/GMN??

  Trick limited to one D6, equiv. linear approximation in twistor description of hm metric in Alexandrov, Pioline, Saueressig, Vandoren

- Tantalizing, but full inclusion of all charges still far from clear

Conclusions

D-brane instantons wall crossing in 4d N=1

- Holomorphy of 4d N=1 terms across stability walls
- Criteria for the generation of superpotentials

  Instantons which can become non-BPS cannot generate superp.
  (can iff misalignment breaks spacetime susy)

Non-perturbative effects in 4d N=2 from topological string

- GV interpretation yields effects from 'electric' D-brane instantons
- Compactification and T-duality are naturally required
- Lesson of particle-instanton T-duality carries over to 4d N=1

  Multi-instantons ensuring holomorphy of superpotential relate to threshold bound states, morally unorientable GV invariants

Open questions:

- N=2: General charges and KS/GMN
- N=1: One loop D-particle diagram leading to D-instanton sum