(Non)geometric Aspects of Black Holes Microstates

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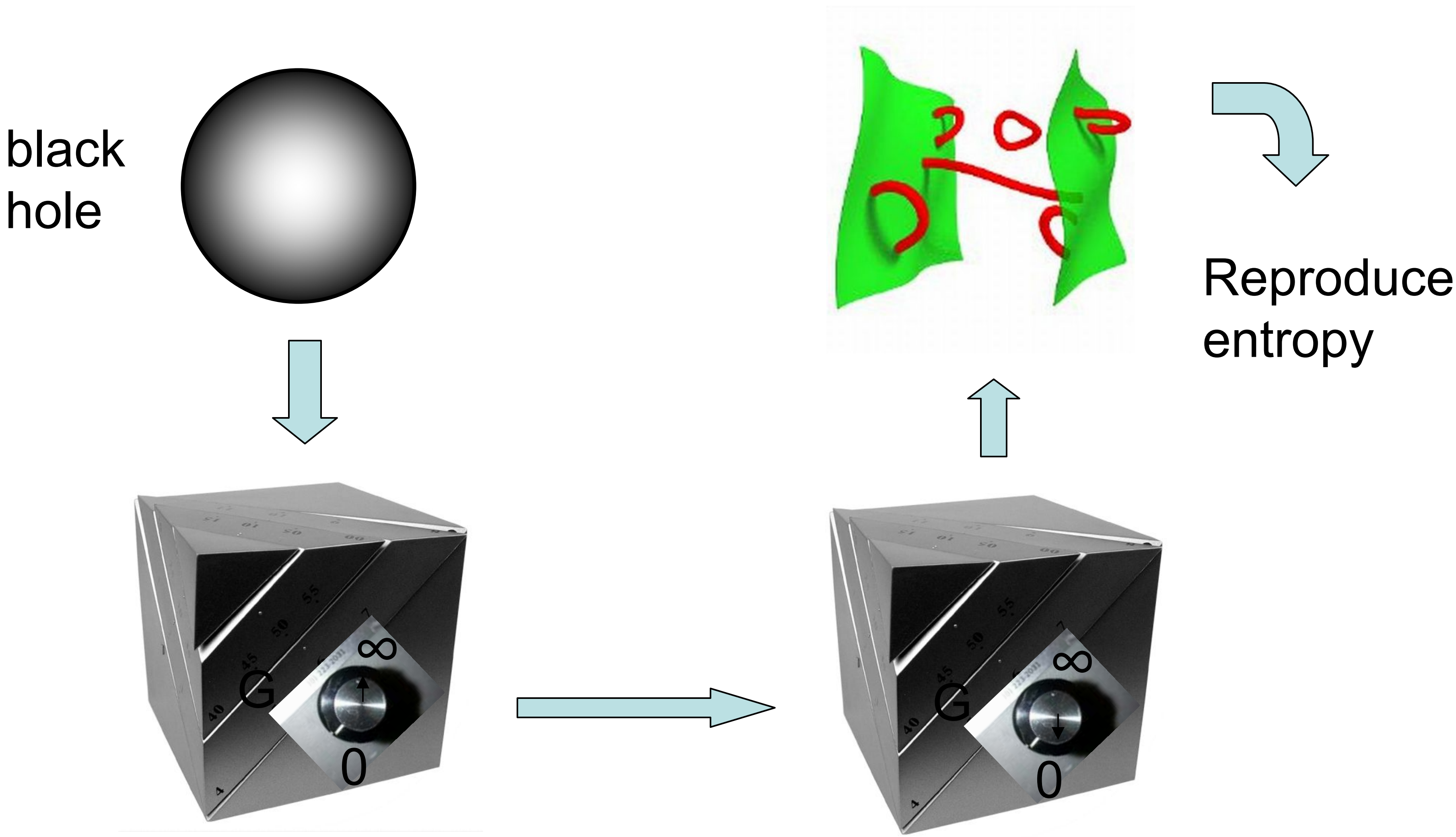
Based on:

arXiv:0807.4556 - JdB, Sheer El-Showk, Ilies Messamah, Dieter van den Bleeken
arXiv:0906.0011 - JdB, Sheer El-Showk, Ilies Messamah, Dieter van den Bleeken.
Work in progress - Bena, Berkooz, JdB, El-Showk
Work in progress - JdB, Shigemori

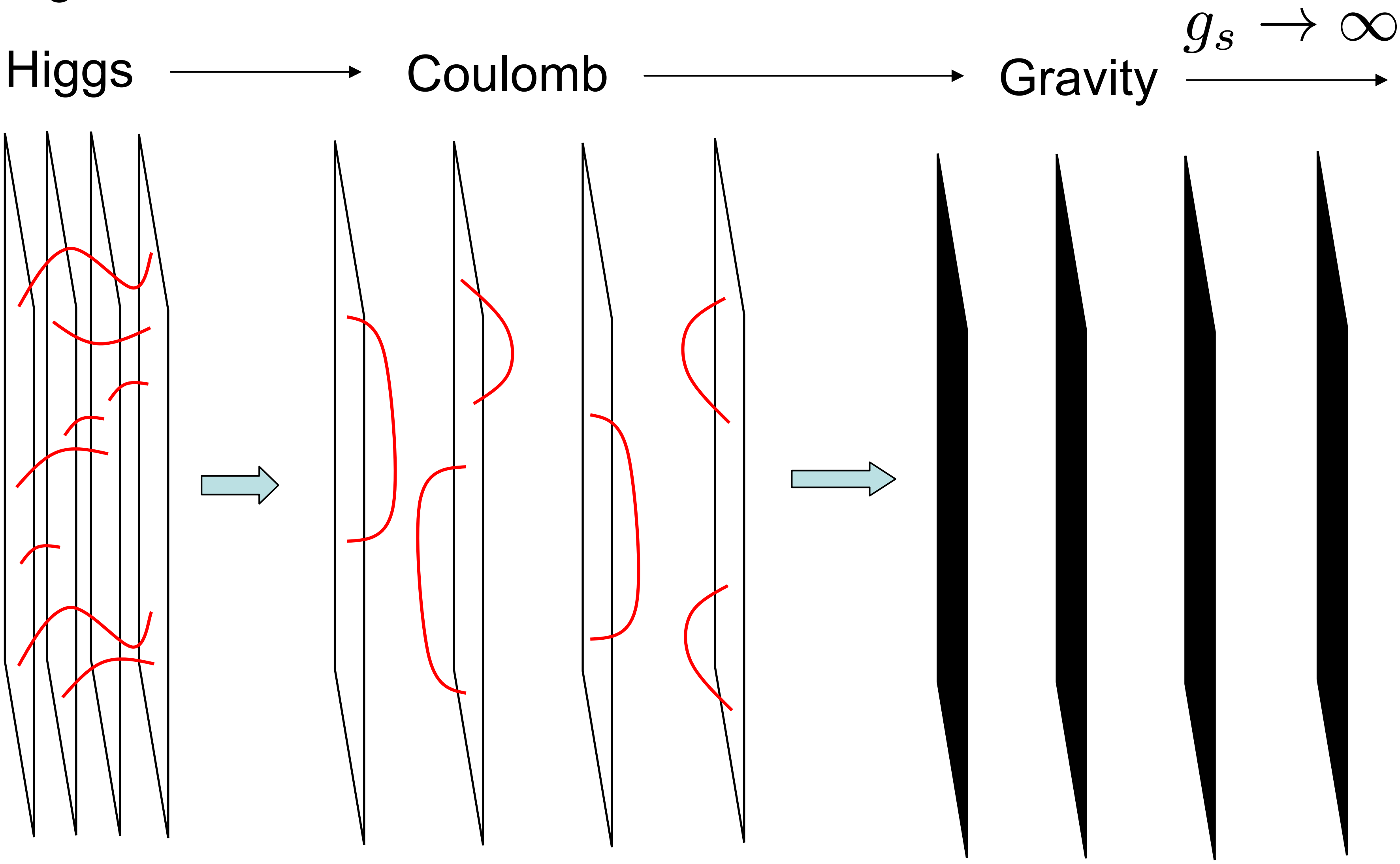
Outline

1. Higgs → Coulomb → Geometry
2. Microstates for large supersymmetric black holes
3. Quantum effects in deep throats
4. The number of smooth supergravity solutions
5. Beyond Geometry
6. Conclusions

Conventional approach to black holes in string theory:



It would be very interesting could reverse the arrows in this picture and find a geometrical description of the individual degrees of freedom.



Difficult to do: look at examples. For 4d black holes in e.g. IIA/CY, relevant gauge theory is N=4 SUSY quantum mechanics (ignoring internal features of wrapped branes).

Denef; Balasubramanian, Gimon, Levi; Denef, Moore

Simplest case: $U(1) \times U(1)$ gauge theory with N bifundamentals.

	Fields	Constraints	Moduli space	Ground states
H	Q_a	$\sum \lvert Q_a \rvert^2 = \theta$	$\mathbb{CP}^{N-1}$	$H^*(\mathbb{CP}^{N-1})$
C	$\vec{X}$	$\lvert \vec{X} \rvert = \frac{N}{\theta}$	$\mathbb{CP}^1$	$H^0(\mathbb{CP}^1, \mathcal{O}(N-1))$

The number of degrees of freedom remains the same! Can see how the two are related by taking $g_s \to \infty$ in the classical theory. Coulomb branch fields become non-dynamical:

Berkooz, Verlinde

$$\vec{X} = \frac{\sum_a \bar{\psi}^a \vec{\sigma} \psi_a}{\sum_a |Q_a|^2}.$$

Act on Higgs branch as the Lefschetz SU(2). Form a finite dimensional representation of SU(2). Commutation relations agree with symplectic structure on Coulomb branch. Form what is commonly referred to as a fuzzy two-sphere.

$$\mathcal{M}_C(\text{fuzzy}) \subset H^*(\mathcal{M}_H).$$

In general, need to restrict the fermionic states to be flavor singlets at half-filling to recover the Coulomb branch.

In general, the Coulomb branch is realized as a fuzzy manifold inside the Higgs branch, but does not see the entire Higgs branch. Precise geometric criterion which states disappear is unclear at present.

It appears this typically happens whenever the gauge theory has a “scaling” regime, where on the Coulomb branch one can take some $|\vec{X}_i - \vec{X}_j| \to 0$. More on this later.

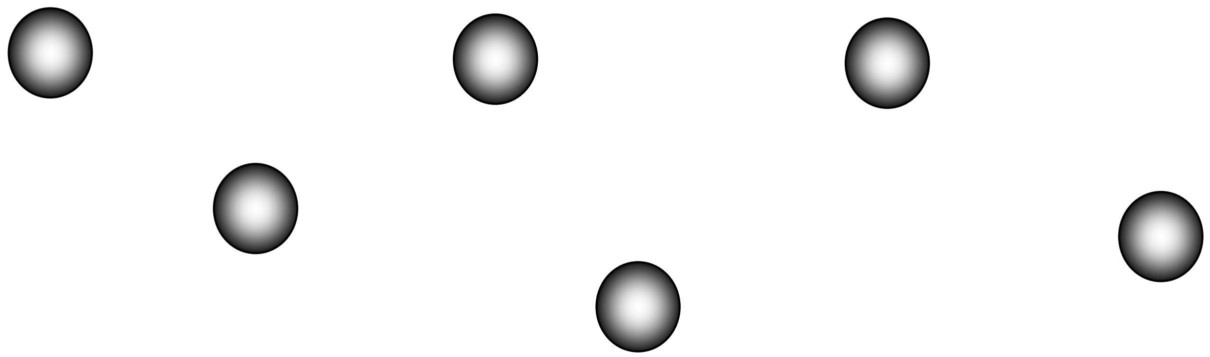
The next step is to map the Coulomb branch to spacetime geometries. The relevant geometries are known special cases of multi-centered 4d black hole geometries.

Large supersymmetric black holes carrying electric charge Q and magnetic charge P exist in four dimensions. (P and Q can be vectors with many components).

There exists however a much larger set of solutions of the gravitational field equations, which includes bound states of black holes, and also many smooth solutions.

Lopes Cardoso, de Wit, Kappeli, Mohaupt; Denef; Bates, Denef; Balasubramanian, Gimon, Levi

Put black holes with charges $\Gamma_i = (P_i, Q_i)$ at locations $\vec{x}_i \in \mathbb{R}^3$



There are corresponding solutions of the field equations only if (necessary, not sufficient)

$$\langle h, \Gamma_i \rangle + \sum_{j \neq i} \frac{\langle \Gamma_j, \Gamma_i \rangle}{|\vec{x}_j - \vec{x}_i|} = 0$$

Here, $\langle \Gamma_1, \Gamma_2 \rangle = P_1 \cdot Q_2 - P_2 \cdot Q_1$ is the electric-magnetic duality invariant pairing between charge vectors. The constant vector h determines the asymptotics of the solution.

Solutions are stationary with angular momentum

$$\vec{J} = \frac{1}{4} \sum_{i \neq j} \langle \Gamma_i, \Gamma_j \rangle \frac{\vec{x}_i - \vec{x}_j}{|\vec{x}_i - \vec{x}_j|}$$

These describe precisely backreacted Coulomb branch solutions. The (one-loop) D-term equations of the gauge theory

$$\theta_i + \sum_{j\neq i} \frac{N_{ij}}{|\vec{x}_j - \vec{x}_i|} = 0$$

N_{ij} =#bifundamentals

are the same as the supergravity equation

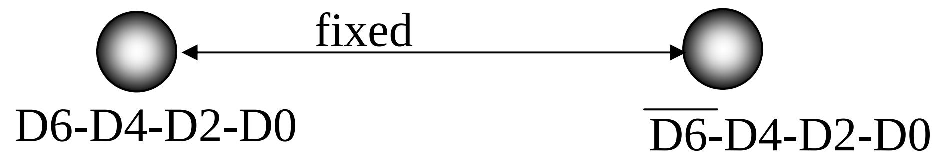
$$\langle h, \Gamma_i \rangle + \sum_{j\neq i} \frac{\langle \Gamma_j, \Gamma_i \rangle}{|\vec{x}_j - \vec{x}_i|} = 0$$

Typical setup: type IIA on CY

Magnetic charges: D6,D4

Electric charges: D0,D2

Previous gauge theory example:



- ❑ Whenever the total D6-brane charge of a solution vanishes, one can take a decoupling limit so that the geometry (after uplifting to d=5) becomes asymptotic to $AdS_3 x S^2 x CY$. (dual=MSW (0,4) CFT) Maldacena, Strominger, Witten
- ❑ When the centers correspond to D6 branes with only a world-volume gauge field, or D0 branes, the 5d uplift is a smooth geometry.
- ❑Uplift of a D4-D2-D0 black hole yields the BTZ black hole, and can apply Cardy.

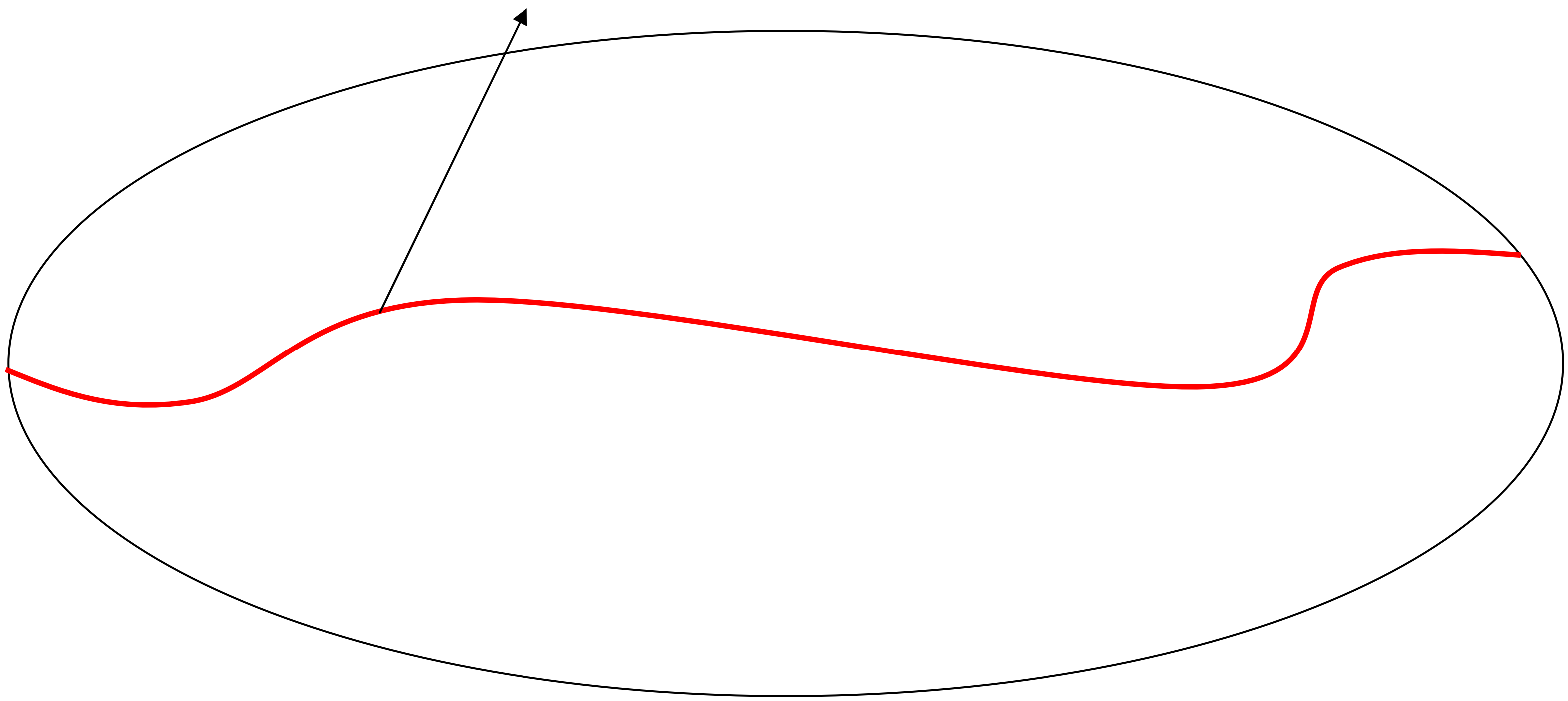
Only when the supergravity solution is smooth it is honestly geometrical. Singularities and sources require extra input.

These smooth solutions are then candidate microstates for the black hole we started out with (Mathur….) More precisely, they form a classical phase space which after quantization provides the required microscopic degrees of freedom.

Note that it is easy to arrange a situation so that the charges carried by the smooth geometries are identical to those of a large supersymmetric black hole.

To quantize need the symplectic form.

Set of smooth solutions



Full phase space=set of all solutions of the equations of motion.

$$\omega \sim \int d\Sigma^\mu \left(\delta \frac{\delta \mathcal{L}}{\delta(\partial^\mu \phi)} \wedge \delta\phi \right)$$

Result:

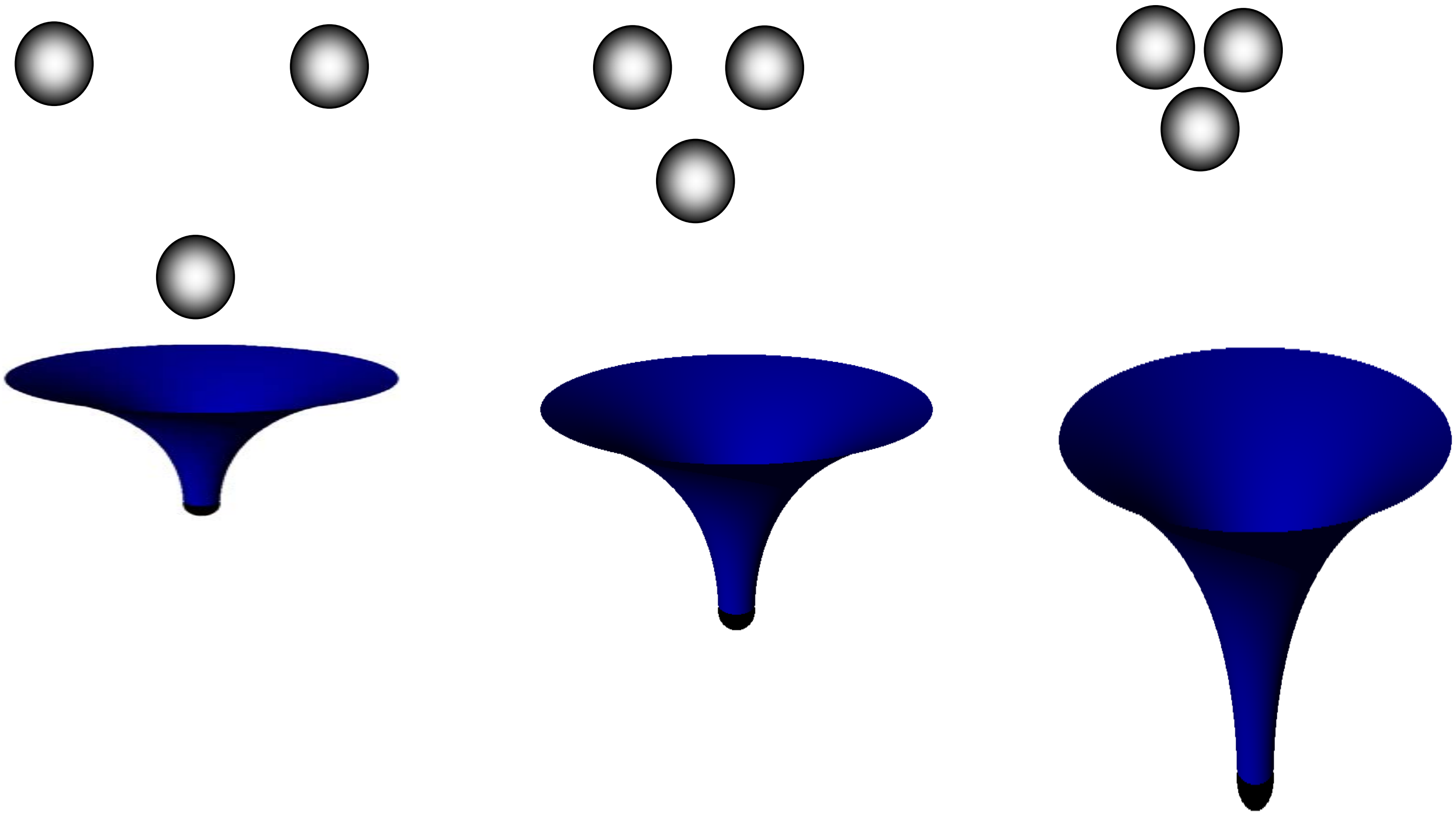
$$\omega = \frac{1}{4} \sum_{p \neq q} \langle \Gamma_i, \Gamma_j \rangle \frac{\epsilon_{ijk} (\delta(x_p - x_q)^i \wedge \delta(x_p - x_q)^j) \, (x_p - x_q)^k}{|\mathbf{x}_p - \mathbf{x}_q|^3}$$

Can now use various methods to quantize the phase space, e.g. geometric quantization. Can explicitly find wavefunctions for various cases.

In this way, find purely geometric duals for some components of the Coulomb branch, and number of states agrees with $H^0(\mathcal{M}_C, \mathcal{L})$ in those cases.

Bena, Wang, Warner; Denef, Moore

Of particular interest: scaling solutions: solutions where the constituents can approach each other arbitrarily closely.



In space-time, a very deep throat develops, which approximates the geometry outside a black hole ever more closely.

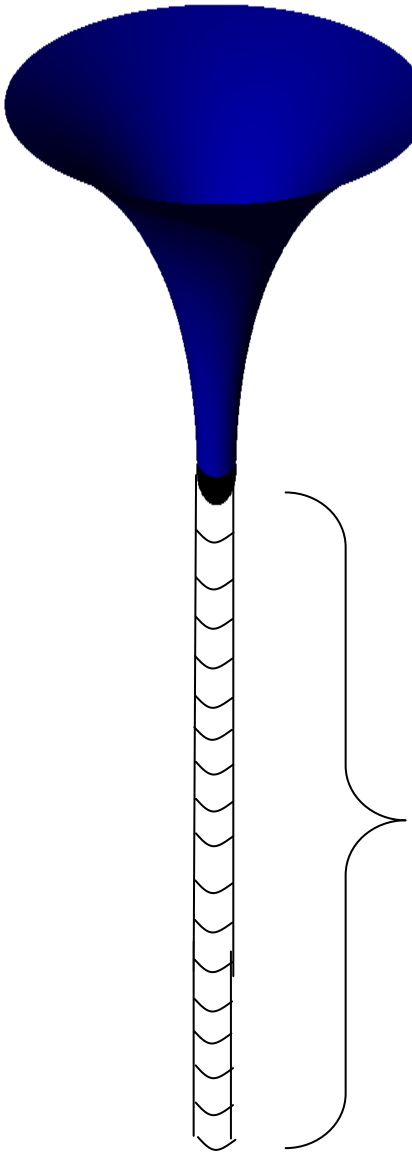
None of these geometries has large curvature: they should all be reliably described by general relativity.

However, this conclusion is incorrect!

The symplectic volume of this set of solutions is finite. Throats that are deeper than a certain critical depth are all part of the same ħ-size cell in phase space: wave-functions cannot be localized on such geometries.

Quantum effects become highly macroscopic and make the physics of very deep throats nonlocal.

This is an entirely new breakdown of effective field theory.



Wave functions have support on all these geometries

As a further consistency check of this picture, it also resolves an apparent inconsistency that emerges when embedding these geometries in AdS/CFT.

This is related to the fact that very deep throats seem to support a continuum of states as seen by an observer at infinity, while the field theories dual to AdS usually have a gap in the spectrum.

Bena, Wang, Warner

The gap one obtains agrees with the expected gap 1/c in the dual field theory (the dual 2d field theory appears after lifting the solutions to five dimensions and taking a decoupling limit).

This non-local breakdown of effective field theory near the horizon is perhaps exactly the sort of thing one needs in order to reconcile the information paradox with effective field theory?

(It has been argued that the information paradox cannot be resolved in perturbation theory)

Notice that the scale that appears is 1/c, which seems to be a scale that appears often in this context. Evidence for a universal underlying long string picture?

The breakdown is somewhat reminiscent of the breakdown of the statistical description of near-extremal black holes when $\left|T\left(\frac{\partial T}{\partial M}\right)\right| \ll T$.

Preskill, Schwarz, Shapere, Trivedi, Wilzcek

As has been argued by e.g. by Sen, the full partition function has the following structure

$$Z = Z_{\rm hair} + Z_{\rm hair} Z_{\rm BH} + Z_{\rm hair} Z_{\rm BH} Z_{\rm BH} + \ldots$$

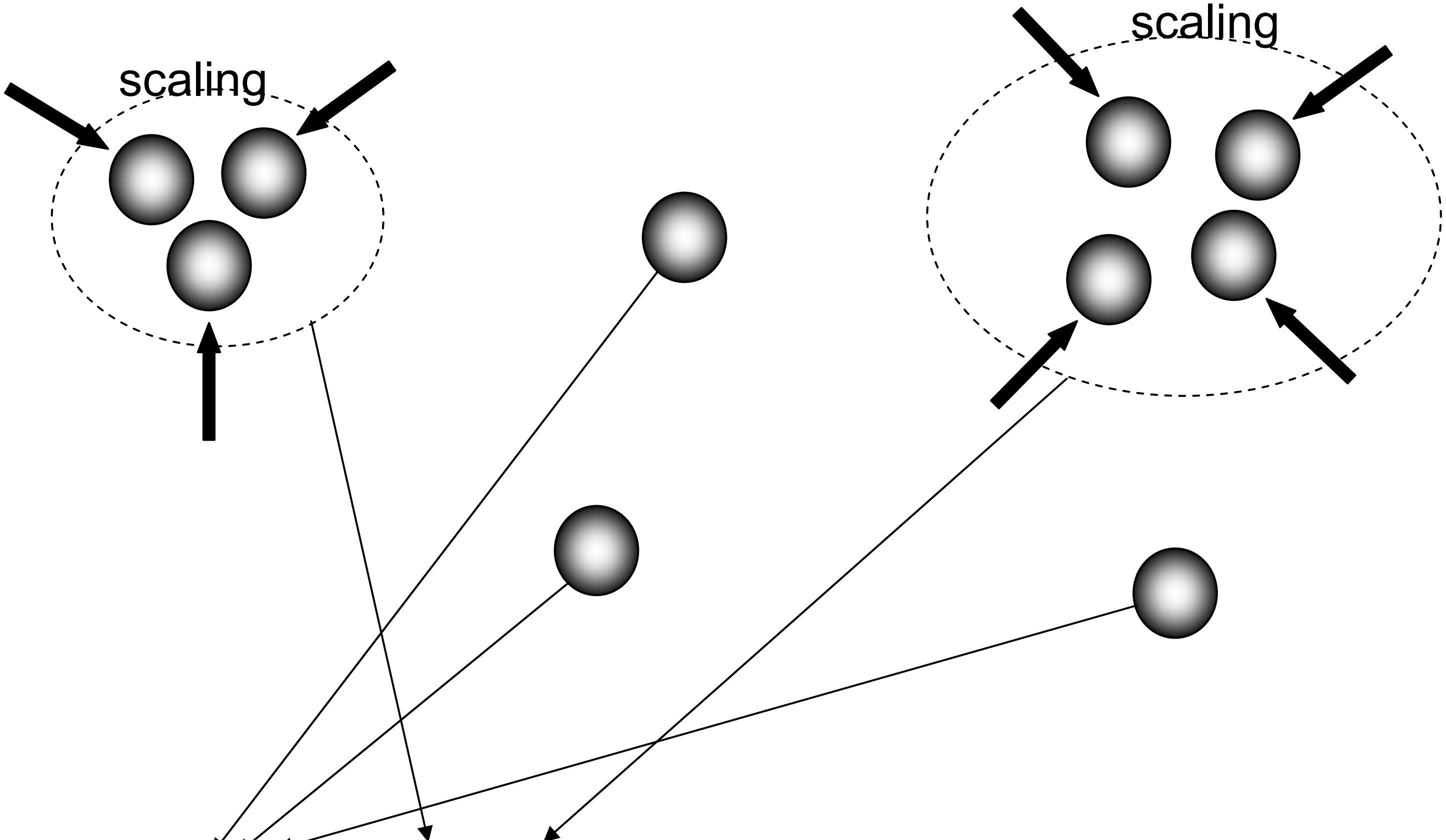
and one may wonder to which terms the smooth solutions we are talking about contribute. One might be inclined to view the smooth solutions as hair, but that is not obvious:

•This is not a sum over Euclidean saddle points, smooth solutions can typically not be Wick rotated.

•Smooth solutions are also not obviously related to one-loop determinants.

•Reminiscent of Farey tail expansion, except there $Z_{\rm hair}$ only contains polar states which never coexist with black holes.

For the ½-BPS black holes we considered, there is natural split into “hair” and “black holes”.



$$Z_{\rm hair} Z_{\rm BH} Z_{\rm BH} + \dots$$

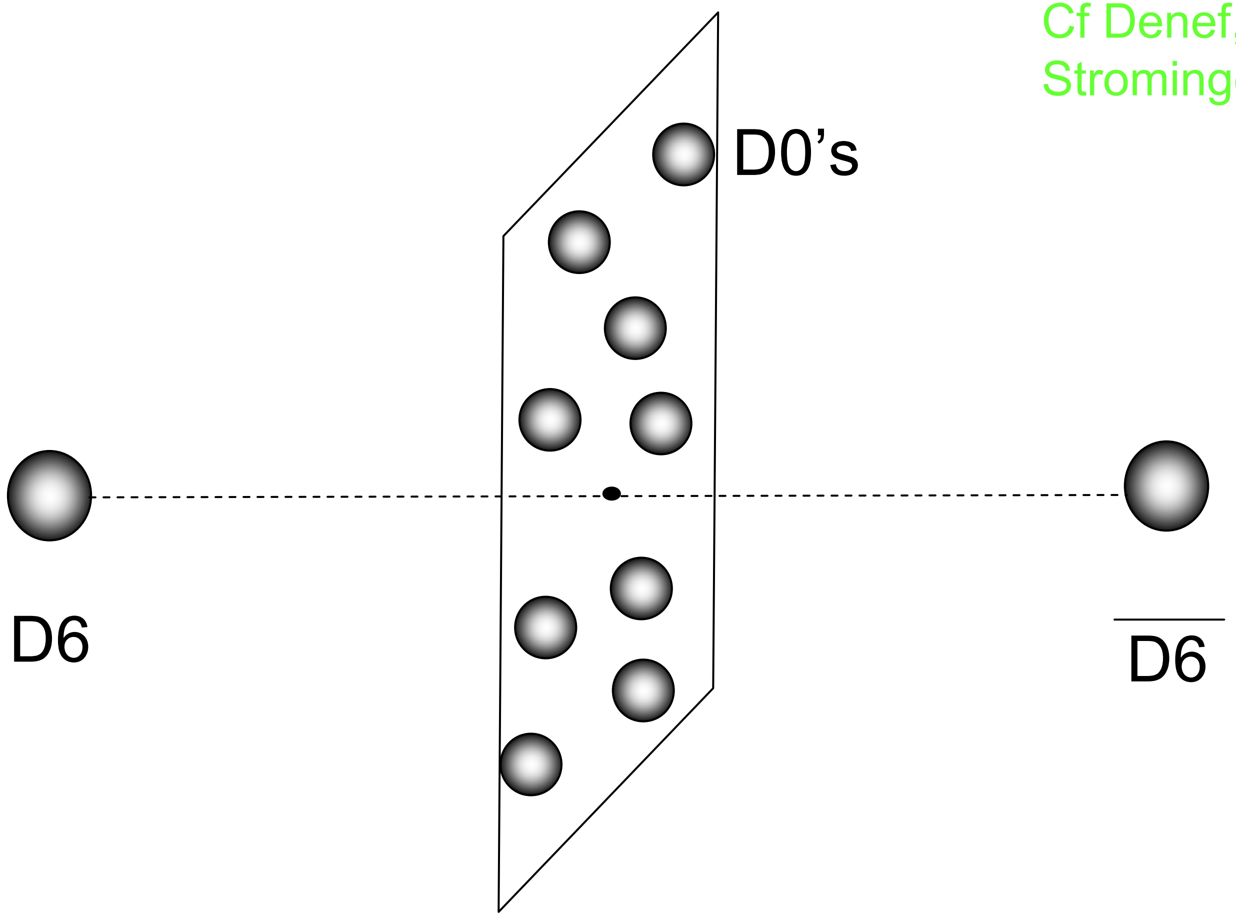
This is also suggested by considering split attractor flows.

Are there sufficiently many smooth supergravity solutions to account for the black hole entropy?

We lost some states along the way, and this is not a prediction of AdS/CFT.

Largest set we have been able to find:

Cf Denef, Gaiotto, Strominger, vdBleeken, Yin



In terms of standard 2d CFT quantum numbers we find the following number of states:

$$\left(\tfrac{3}{16}\zeta(3)L_0^2\right)^{1/3} \qquad L_0 \leq c/6$$

$$\left(\tfrac{3}{2}c\zeta(3)(L_0 - \tfrac{c}{12})\right)^{1/3} \qquad L_0 \geq c/6$$

This is less than the black hole entropy, which scales as

$$S \sim 2\pi \left(\tfrac{c}{6}L_0\right)^{1/2}$$

Perhaps we are simply missing many solutions?

Try to find upper bound: count the number of states in a gas of BPS supergravitons. Idea is that all smooth BPS solutions are obtained by taking a superposition of free BPS supergravitons and letting the system backreact. Because of the BPS bound, the energy of the system cannot become be lowered.

After all, classical solutions can be thought of as coherent superpositions of gravitons…

Thus we compute the partition function of a gas of BPS supergravitons; spectrum can be read off from the the KK modes of M-theory on CYxS^2. Result:

$$\mathcal{Z} = \mathrm{Tr}_{\mathrm{NS,BPS}}(-1)^F q^{L_0} y^{\tilde{L}_0 - 1/2}$$

equals

$$\mathcal{Z} = Z_{\{\frac{1}{2},\frac{1}{2}\}}^{2h^{1,2}+2}\, Z_{\{0,1\}}^{h^{1,1}-1}\, Z_{\{1,0\}}^{h^{1,1}-1}\, Z_{\{-1,2\}}\, Z_{\{0,2\}}\, Z_{\{1,1\}}\, Z_{\{2,1\}}$$

where

$$Z_{\{s,\tilde{h}_{\min}\}} = \prod_{n\geq 0} \prod_{m\geq 0} \left(1 - y^{m+\tilde{h}_{\min}-1/2} q^{n+m+\tilde{h}_{\min}+s}\right)^{(-1)^{2s+1}}$$

We put y=1 and compute the asymptotics of this partition function. Result:

$$S \sim \left(\frac{3}{16}\zeta(3)L_0^2\right)^{1/3}$$

Clearly backreaction will be important. Difficult to deal with, but can impose one dynamical feature: stringy exclusion principle.

Maldacena, Strominger

The stringy exclusion principle is related to the fact that the spins of primaries in a level k SU(2) WZW cannot exceed k/2. Thus we reinstate y and keep only the terms where the power of y is at most c/6.

Now we find precisely the same result as before:

$$S \sim \left(\tfrac{3}{16}\zeta(3)L_0^2\right)^{1/3} \qquad L_0 \leq c/6$$

$$S \sim \left(\tfrac{3}{2}c\zeta(3)(L_0 - \tfrac{c}{12})\right)^{1/3} \quad L_0 \geq c/6$$

- Strongly suggests supergravity is not sufficient to account for the entropy.
- Stringy exclusion principle is visible in classical supergravity (and not so stringy).

In particular, this suggests that all attempts to quantize gravity on its own are futile and will never lead to a consistent unitary theory with black holes.

This is of course perfectly fine: string theory was invented to yield a consistent quantum theory of gravity, so it would have been somewhat disappointing if we could get away with gravity alone.

This statement is also supported by the N=4 case, where one can show that multicentered configurations can never contribute to the index.

Dabholkar, Guica, Murthy, Nampuri

Caveat:

Aminneborg, Begtsson, Brill, Holst, Peldan

In d=3 there are many solutions which are identical to a black hole outside the horizon but have structure behind it. There may even be enough solutions of this type to account for the black hole entropy (Maloney). Not clear whether these solutions should be viewed as pure states though.

Such solutions cannot obviously be made by throwing gravitons in global AdS.

Another possibility:

Recall that the F1-D0 system can puff up into a supertube, a D2-brane whose cross section can be an arbitrary curve.

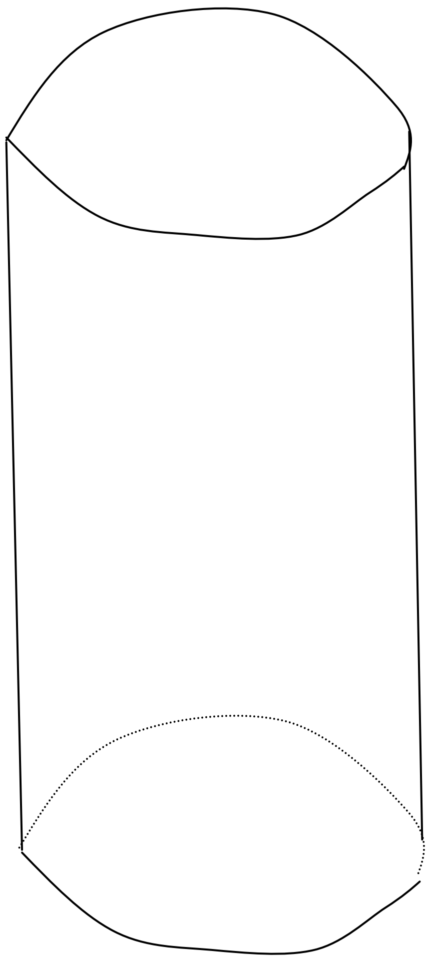
Mateos, Townsend

One can reproduce the number of F1-D0 bound states by quantizing supertubes.

Marold, Palmer

In a suitable duality frame, they can be described by smooth supergravity solutions.

Lunin, Mathur



If we T-dualize the F1-D0 system, many other systems can be shown to puff up into supertubes.

In IIA on T^6, with D4 branes wrapping the 6789 and D4 branes wrapping the 4589 directions, the resulting supertube is made of an extended object which one gets by T-dualizing an NS5-brane in two transversal directions: a 5_2^2-brane.

These exotic objects also appear when U-dualizing conventional branes in three dimensions. Their tensions can involve strange powers of g_s such as g_s^{-3}, and strange powers of the radii. Elitzur, Giveon, Kutasov, Rabinovici; Obers, Pioline

Above reasoning suggests such exotic branes may play an important role in understanding microstates.

JdB, Shigemori

General picture in d=3, e.g. in M-theory compactified on T^8

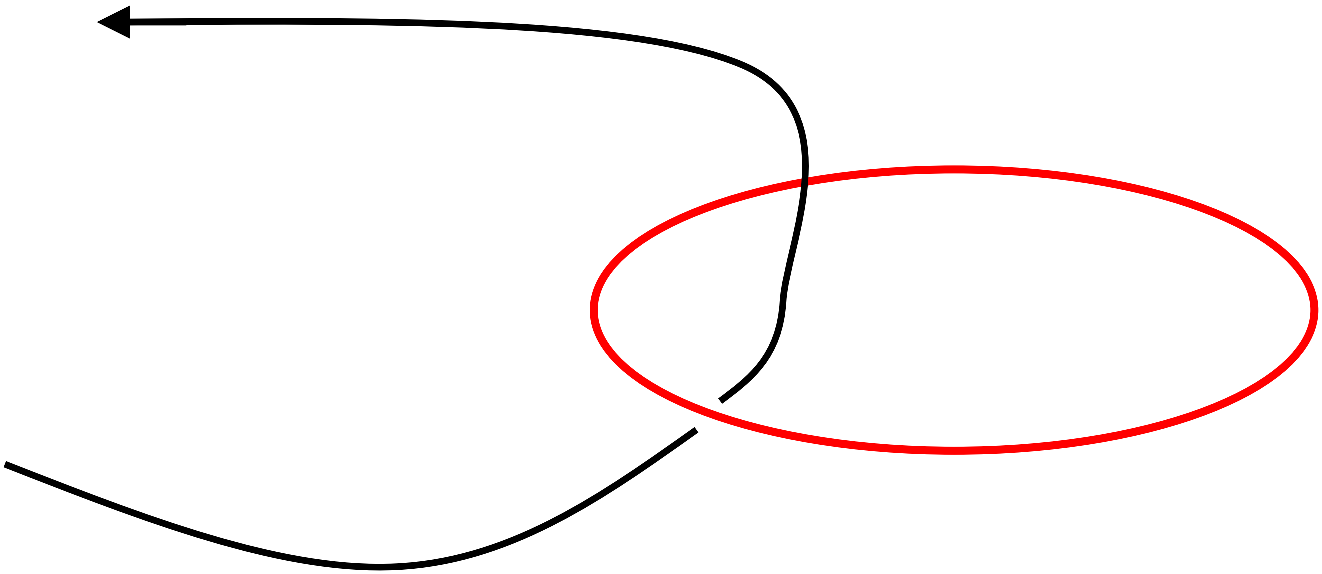
Scalar moduli space: $SO(16)\backslash E_8(\mathbb{R})/E_8(\mathbb{Z})$

Point particles carry "charges" given by an element of $E_8(\mathbb{Z})$
In general these are non-geometric in higher dimensions, just like in non-geometric compactifications.

Can describe explicit 3d gravity solutions by embedding the standard complex upper half plane in the scalar moduli space: $SO(2)\backslash SL_2(\mathbb{R}) \hookrightarrow SO(16)\backslash E_8(\mathbb{R})$

(cf cosmic strings)

Can explicitly construct the metric for the 5^2_2 supertube.



As one moves through the loop, one picks up a non-geometric twist. Not visible at infinity.

OUTLOOK:

- Described progress towards understanding which microscopic degrees of freedom of black holes may be visible in gravity and which ones may not. More work needs to be done.

- Extend breakdown of effective field theory and discussion of quantum effects to generic Schwarzschild black holes: AdS/CFT may allow us to make some progress in this direction.

- It appears that supersymmetric black holes cannot be described in terms of gravitational degrees of freedom only, but perhaps non-geometric solutions of gravity may allow one to improve the situation. What does smoothness mean? To be continued.