R Symmetries, Supersymmetry Breaking, and A Bound on The Superpotential

Strings2010. Texas A&M University

Michael Dine

Department of Physics
University of California, Santa Cruz
Work with G. Festuccia, Z. Komargodski.

March, 2010

String Theory at the Dawn of the LHC Era

As we meet, the LHC program is finally beginning. Can string theory have any impact on our understanding of phenomena which we may observe?

- Supersymmetry?
- Warping?
- Technicolor?
- Just one lonely higgs?

Supersymmetry

Virtues well known (hierarchy, presence in many classical string vacua, unification, dark matter). But reasons for skepticism:

1. Little hierarchy
2. Unification: why generic in string theory?
3. Hierarchy: landscape (light higgs anthropic?)

Reasons for (renewed) optimism:

1. The study of metastable susy breaking (ISS) has opened rich possibilities for model building; no longer the complexity of earlier models for dynamical supersymmetry breaking.
2. Supersymmetry, even in a landscape, can account for hierarchies, as in traditional naturalness ($e^{-\frac{8\pi^2}{g^2}}$) (Banks, Gorbatov, Thomas, M.D.).
3. Supersymmetry, in a landscape, accounts for stability – i.e. the very existence of (metastable) states. (Festuccia, Morisse, van den Broek, M.D.)

All of this motivates revisiting issues low energy supersymmetry. While I won't consider string constructions per se, I will focus on an important connection with gravity: the cosmological constant. I will not be attempting to provide a new explanation, but rather simply asking about the features of the low energy lagrangian in a world with approximate SUSY and small Λ.

Outline

- The Cosmological Constant and the Superpotential
- Possible roles for Discrete R Symmetries
  1. supersymmetry breaking
  2. cosmological constant
  3. μ problem (and small masses generally)in gauge mediation
- A Theorem About the Superpotential in Theories with Continuous R symmetries

With supersymmetry, an inevitable connection of *low* energy physics and gravity

$$\langle |W|^2 \rangle = 3 \langle |F|^2 \rangle M_p^2 + \text{ tiny}. \quad (1)$$

So not just F small, but also W. Why?

1. Some sort of accident? E.g. KKLT assume tuning of W relative to F (presumably anthropically).
2. R symmetries can account for small W. (Banks) $< W >$ can be correlated naturally with the scale of supersymmetry breaking. *Scale of R symmetry breaking: set by cosmological constant.*

Suggests a role for R symmetries. In string theory (gravity theory): discrete symmetries. Such symmetries are interesting from several points of view:

1. Cosmological constant
2. Give rise to approximate continuous R symmetries at low energies which can account for supersymmetry breaking (Nelson-Seiberg).
3. Account for small, dimensionful parameters.
4. Suppression of proton decay and other rare processes.

Continuous R Symmetries from Discrete Symmetries

OR model:

$$W = X_2(A_0^2 - f) + mA_0 Y_2 \tag{2}$$

(subscripts denote R charges). If, e.g., $|m^2| > |f|$, $F_X = f$.

Can arise as low energy limit of a model with a discrete R symmetry:

$$X_2 \to e^{\frac{2\pi i}{N}} X_2;\ \ Y_2 \to e^{\frac{2\pi i}{N}} Y_2;\ \ A_0 \to A_0. \tag{3}$$

Allows $\delta W = \frac{X^{N-n} Y^{n+1}}{M_p^{N-2}}$. N susy vacua far away. Approximate, accidental R symmetry. SUSY breaking metastable.

$\langle W \rangle$: Gaugino Condensation

W transforms under any R symmetry; an order parameter for R breaking.

Gaugino condensation: $\langle \lambda\lambda \rangle \equiv \langle W \rangle$ breaks discrete R without breaking supersymmetry.

Readily generalized (J. Kehayias, M.D.) to include order parameters of dimension one.

E.g. N_f flavors, N colors, $N_f < N$:

$$W = S_{ff'} \bar{Q}_f Q'_f + \mathrm{Tr} S^3 \tag{4}$$

exhibits a $Z_{2(3N-N_f)}$ symmetry, spontaneously broken by $\langle S \rangle$; $\langle \bar{Q} Q \rangle$; $\langle W \rangle$.

Gauge Mediation/Retrofitting

Gauge mediation: traditional objection: c.c. requires large constant in W, unrelated to anything else.

Retrofitted Models (Feng, Silverstein, M.D.): OR parameter f from

$$\frac{XW_\alpha^2}{M_p} \to X\frac{\langle\lambda\lambda\rangle}{M_p}. \tag{5}$$

Need $\langle W \rangle = fM_p$, consistent with our requirements for canceling c.c. Makes retrofitting, or something like it, inevitable in gauge mediation.

Other small mass parameters: m, μ-term, arise from dynamical breaking of discrete R symmetry. E.g.

$$W_\mu = \frac{S^2}{M_p} H_U H_D. \tag{6}$$

Readily build realistic models of gauge mediation/dynamical supersymmetry breaking with all scales dynamical, no μ problem, and prediction of a large tan β.

R Symmetry Breaking in Supergravity

Supergravity (moduli): $W = f\, M_p\, g(X/M_p)$. $X \ll M_p$ could give approximate R, along lines of Nelson Seiberg. But unclear how one can get a large enough W under these circumstances (suppressed by both R breaking and susy breaking?).

Alternatively, again, retrofit scales.

These sorts of questions motivate study of W itself as an order parameter for R-symmetry breaking.

A Bound on W

Theorem: In any theory with spontaneous breaking of a continuous R-symmetry and SUSY:

$$|\langle W \rangle| \leq \frac{1}{2}|F|f_a$$

where F is the Goldstino decay constant and f_a is the R-axion decay constant.
I will illustrate first in broad classes of models, and then prove generally.

The bound in O'Raifeartaigh models

Consider a generic renormalizable OR model with an R-symmetry $\Phi_i \to e^{iq_i\xi}\Phi_i$.

$$K = \sum_i \Phi_i \bar{\Phi}_i, \quad W(\Phi_i) = f_i \Phi_i + m_{ij}\Phi_i\Phi_j + \lambda_{ijk}\Phi_i\Phi_j\Phi_k$$

If the theory breaks SUSY at $\phi_i^{(0)}$ then classically it has a pseudomoduli space parameterized by the goldstino superpartner[Rey; Shih, Komargodski].

$$G = \sum_i \left(\frac{\partial W}{\partial \phi_i}\right)\psi_i, \qquad \Phi = \sum_i \left(\frac{\partial W}{\partial \phi_i}\right)\delta\phi_i,$$

Wherever the R-symmetry is broken there is also a flat direction corresponding to the R-axion.

Define two complex vectors $w_i = q_i \phi_i$ and $v_i^\dagger = \frac{\partial W}{\partial \phi_i}$
Since the superpotential has R-charge 2,

$$2\langle W \rangle = \sum_j q_j \phi_j \frac{\partial W(\phi_i)}{\partial \phi_j} = \langle v, w \rangle$$

On the pseudomoduli space we can write

$$|F|^2 = \sum_i \left(\frac{\partial W}{\partial \phi_i}\right) \left(\frac{\partial W}{\partial \phi_i}\right)^* = \langle v, v \rangle$$

Parameterizing $\phi_i(x) = \langle \phi_i(x) \rangle e^{i q_i a(x)}$ we obtain for the R-axion kinetic term:

$$\left(\sum_i |\phi_i(x)|^2 q_i^2\right) (\partial a)^2 \Rightarrow f_a^2 = \langle w, w \rangle$$

Then by the Cauchy-Schwarz inequality:

$$4|\langle W\rangle|^2 = |\langle v, w\rangle|^2 \leq \langle v, v\rangle\langle w, w\rangle = |F|^2 f_a^2$$

which is the bound to be established.

- The bound is saturated if $v \propto w$, in which case the R-axion is the Goldstino superpartner.
- Adding gauge interactions strengthens the bound because the D terms contribution to the potential makes $|F|^2$ larger.

Simple Example

$$W = \sum_{i=1}^{N_2} X_i f_i(\phi_a).$$

Here there are N_2 fields, X_i, of R charge 2, and N_0 fields, ϕ_a, of R charge zero. If $N_2 > N_0$, supersymmetry is broken, and there is a classical moduli space ($\frac{\partial W}{\partial \phi_a} = 0$ are N_0 equations for N_2 unknowns). On this moduli space

$$f_a = 2\sum |X_i|^2 \quad F = \left|\sum_i W_i\right|^2 \quad |\langle W\rangle|^2 = \left|\sum X_i W_i\right|^2$$

More intricate R assignments are more interesting (also needed (Shih) to obtain spontaneous R breaking on the pseudomoduli space).

The bound in general Sigma Models

Consider a theory with arbitrary Kähler potential and W.

$$K(\phi_i, \bar{\phi}_j)\ , \qquad W(\phi_i)\ .$$

The Kähler metric g is Hermitian and positive definite so it can be decomposed as

$$g = LL^\dagger\ , \qquad g^{-1} = (L^\dagger)^{-1} L^{-1}\ .$$

We can modify the definitions of the two vectors

$$w_j = \sum_i R_i \phi_i L_{ij}, \qquad v_j^\dagger = \sum_k L^{-1}_{jk} \frac{\partial W}{\partial \phi_k}$$

.

Then as before we have:

$$
\begin{aligned}
f_a^2 &= \sum_{i,j} q_i \phi_i g_{ij} q_j \phi_j^* = \langle w, w \rangle, \\
|F|^2 &= \sum_{i,j} \left(\frac{\partial W}{\partial \phi_i} \right) g_{ij}^{-1} \left(\frac{\partial W}{\partial \phi_j} \right)^* = \langle v, v \rangle, \\
2W &= \sum_i q_i \phi_i \frac{\partial W}{\partial \phi_i} = \sum_{i,j,k} q_i \phi_i L_{ij} L_{jk}^{-1} \frac{\partial W}{\partial \phi_k} = \langle v, w \rangle
\end{aligned}
$$

Again, the bound follows by using the Cauchy-Schwarz inequality.

Notice that we assumed the positivity of g. This is a hint that causality is an underpinning of the would-be bound.

Proving the Bound in Full Generality: The Low Energy Non-Linear Lagrangian (Komargodski, Seiberg)

In weakly coupled theories, if the bound is saturated at the level of two-derivative terms, it might be spoiled by higher derivative operators. A more interesting question is provided by strongly coupled theories (e.g. $SU(5)$ with a $\bar{5}$ and 10) which break supersymmetry and don't admit a supersymmetric (linear) sigma model description. But the weakly coupled theories exhibit the main points.

Higher Derivative Terms in the Linear Models

Consider a theory with a single chiral field, X, and a superpotential $W = fX$ (and arbitrary Kähler potential). Such a theory saturates the bound at the level of terms with two derivatives. A term such as

$$-\int d^4\theta \frac{\epsilon}{M^4} X^\dagger X \partial_\mu X^\dagger \partial^\mu X. \tag{7}$$

changes f_a without affecting the Goldstino decay constant (it changes the normalization of the kinetic terms for X). Whether the bound is satisfied seems to depend on the sign of ϵ. Calculating loop corrections in simple models one finds that the sign is such that the bound is satisfied. Why?

The sign of ϵ is constrained by general field theoretic principles (causality, unitarity). In particular, there is an interaction between the axion and the Goldstino of the form:

$$-i\, C\, \partial^{\mu} \tilde{G} \sigma^{\lambda} \bar{\tilde{G}} \partial_{\mu} a \partial_{\lambda} a \tag{8}$$

$C \propto \epsilon$. The sign of this interaction is fixed, by an argument due to Adams, Arkani-Hamed, Dubovsky, Nicolis, Rattazzi.

Causality Constraints

Consider the propagation of the Goldstino in a background for the R-axion field such that $\partial_\mu a(x) = V_\mu$. To first order in V_μ:

$$(k_\mu \bar{\sigma}^\mu + C(V \cdot k) V_\mu \bar{\sigma}^\mu)\, G = 0 \; .$$

Multiplying by $V^\rho \sigma_\rho$:

$$V_\rho k_\mu \sigma^\rho \bar{\sigma}^\mu G = 0$$

Multiplying by $k_\mu \sigma^\mu$ we get the for the dispersion relation

$$-(k^2 - C(V \cdot k) k_\rho V_\mu \, (\sigma^\rho \bar{\sigma}^\mu)) G = -\left(k^2 + 2C(V \cdot k)^2\right) G = 0$$

$k^2 = -2C(V \cdot k)^2$

Goldstino propagation is superluminal unless $C \geq 0$. I.e. the sign is constrained (in such a way that the bound is satisfied). The argument can be formulated in a more general way, making the connection to unitarity more direct (Adams et al).

The general low energy theory for the Goldstino $G_\alpha(x)$ and the R-axion $a(x)$

Now we are prepared to consider the problem in a model independent way. Following Komargodski and Seiberg, write a low energy effective lagrangian containing the Goldstino and R axion (and possibly other light fields) in which these symmetries are non-linearly realized.
Under the R-symmetry

$$a(x) \rightarrow a(x) + \xi, \qquad G_\alpha(x) \rightarrow e^{i\xi} G_\alpha(x), \qquad \theta_\alpha \rightarrow e^{i\xi}\theta_\alpha$$

The action of SUSY is more complicated and can be described in terms of constrained superfields.

The Goldstino

The Goldstino is described by a chiral superfield X_{NL} satisfying the constraint $(X_{NL})^2 = 0$.

$$X_{NL} = \frac{G^2(y)}{2F(y)} + \sqrt{2}\theta G(y) + \theta^2 F(y), \qquad y^\mu = x^\mu + i\theta\sigma^\mu\bar{\theta}$$

The R-axion, $a(x)$, is introduced via a chiral superfield $\mathcal{R}_{NL} = e^{i\mathcal{A}}$ subject to the constraint

$$X_{NL}\left(\mathcal{A}_{NL} - \mathcal{A}_{NL}^\dagger\right) = 0$$

In components the chiral superfield $\mathcal{A}$ is:

$$\mathcal{A} = a + \text{quadratic and higher order in Golstino, axion.} \tag{9}$$

Under R $\quad a(x) \to a(x) + \xi, \quad \mathcal{A}_{NL} \to \mathcal{A}_{NL} + \xi, \quad \mathcal{R}_{NL} \to e^{i\xi}\mathcal{R}_{NL}$

Non-Linear Lagrangian

Up to field redefinitions the R-invariant Lagrangian is necessarily:

$$\int d^4\theta \left(|X_{NL}|^2 + f_a^2 |\mathcal{R}_{NL}|^2\right) + \int d^2\theta \left(f\, X_{NL} + \tilde{f}\, \mathcal{R}_{NL}^2\right) + c.c$$

There are three parameters:

- The Goldstino decay constant f
- The R-axion decay constant f_a
- $\langle W \rangle = \tilde{f}$

They seem completely independent.

Goldstone-Axion Interactions

Expanding in components, and after a suitable field redefinition, include

$$2i\left(\frac{f_a^2}{f^2} - 4\frac{|\tilde{f}|^2}{f^4}\right)\partial^\mu \tilde{G}\sigma^\lambda \bar{\tilde{G}}\partial_\mu a \partial_\lambda a \qquad (10)$$

The coefficient of this operator is controlled by the same causality considerations we encountered in the linear sigma model, and it is proportional to the quantity which enters in our proposed theorem:

$$C = 2\left(\frac{f_a^2}{f^2} - 4\frac{|\tilde{f}|^2}{f^4}\right) \geq 0$$

Implications

- We expect that any *continuous* R symmetry is approximate and accidental. We also require that $|\langle W\rangle| = \sqrt{3}|F|M_p$. So if $\langle W\rangle$ is controlled by R-symmetric dynamics, the smallness of the c.c. implies

$$f_a > 2\sqrt{3}M_p$$

- Supergravity: Large breaking of any approximate continuous symmetry likely means large breaking of any discrete symmetry as well. Such symmetries then can't be relevant to understanding supersymmetry breaking, suppression of flavor changing process.

- Nelson Seiberg theorem: The bound provides new insight into the Nelson Seiberg theorem. For example, R symmetry breaking, generically, implies supersymmetry breaking.
- Gauge Mediation: Cosmological constant requires new interactions which explicitly break any continuous approximate R symmetries. Gaugino condensation and its generalizations *at this scale* readily allow understanding of other required small scales, including f, μ and B_μ.