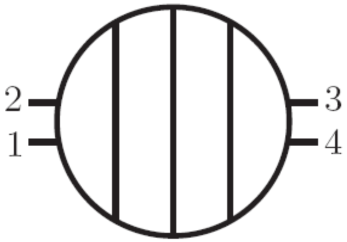
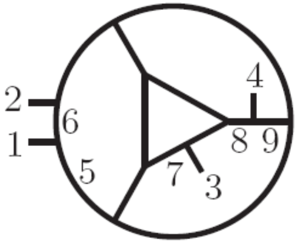
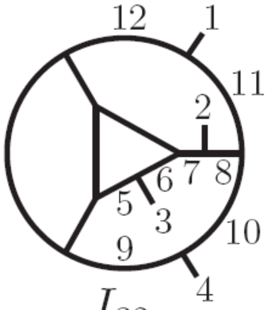
Perturbative Ultraviolet Behavior of $\mathcal{N} = 8$ Supergravity



I_1



I_{25}



I_{32}

I_{50}

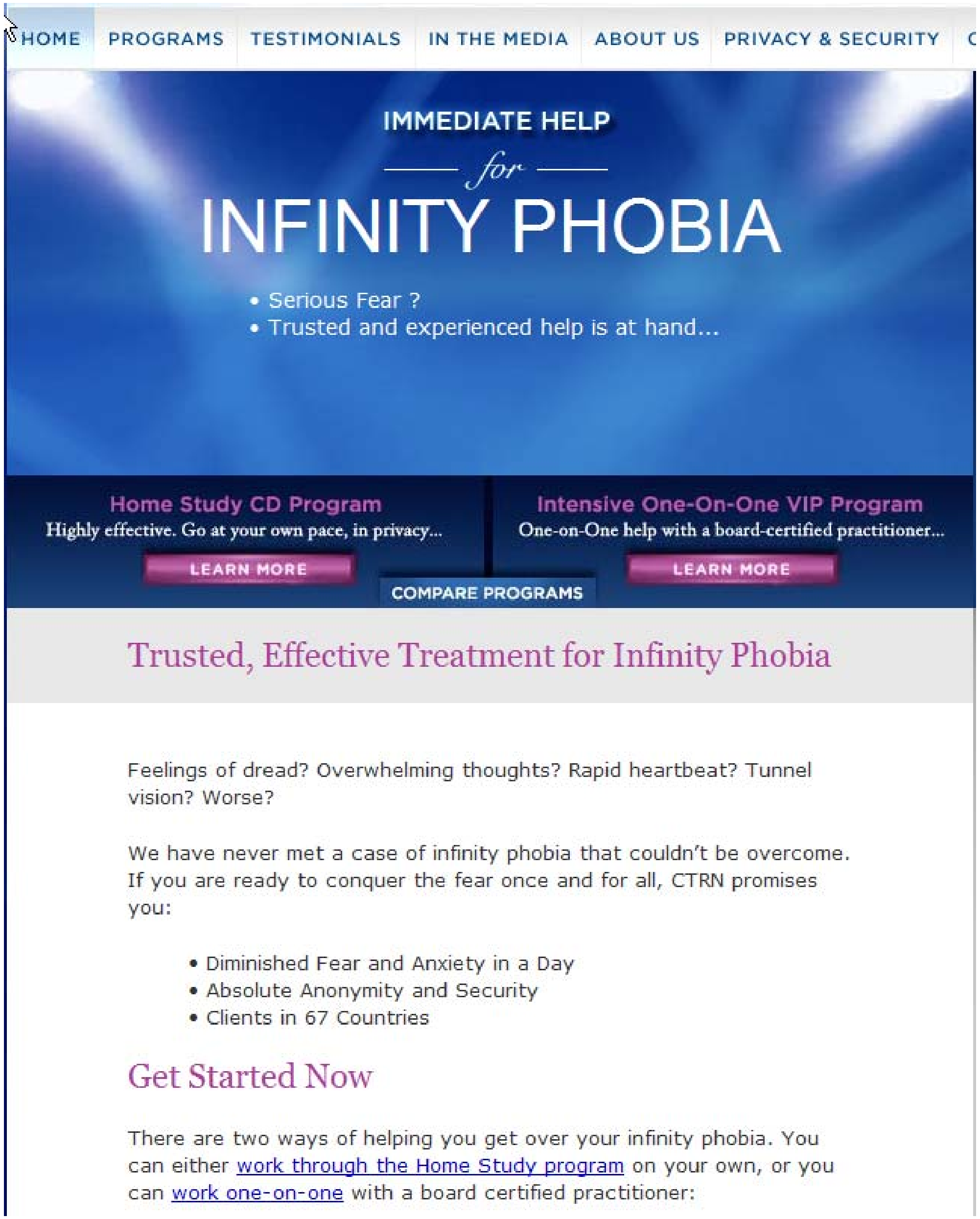
Z. Bern, J.J. Carrasco, LD, H. Johansson, R. Roiban

0905.2326 [PRL **103**, 08301 (2009)], 1003?.????

Lance Dixon (SLAC)
STRINGS 2010
Texas A & M
March 18, 2010

Introduction

- Quantum gravity is nonrenormalizable by power counting: the coupling, Newton's constant, $G_N = 1/M_{Pl}^2$ is dimensionful
- String theory cures the divergences of quantum gravity by introducing a new length scale, the string tension, at which particles are no longer pointlike.
- Is this necessary? Or could enough symmetry, e.g. **N=8** supersymmetry, allow a point particle theory of quantum gravity to be perturbatively ultraviolet finite?
- N=8 supergravity (ungauged) DeWit, Freedman (1977); Cremmer, Julia, Scherk (1978); Cremmer, Julia (1978,1979)
- Other point-like proposals include flow to (conjectured?) nontrivial fixed points:
  - asymptotic safety program Weinberg (1977); ...; Niedermaier, Reuter, Liv. Rev. Rel. **9**, 5 (2006), Weinberg talk
  - UV theory could be Lorentz asymmetric, but renormalizable Hořava, 0812.4287, 0901.3775, talk
- Here we will perturb around a (conjectured?) Gaussian fixed point



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Counterterm Basics

- Divergences associated with local counterterms

- On-shell counterterms are generally covariant, built out of products of Riemann tensor $R_{\mu\nu\sigma\rho}$ (& derivatives $\mathcal{D}_\mu$)

- Terms containing Ricci tensor $R_{\mu\nu}$ and scalar R removable by nonlinear field redefinition in Einstein action

$R^{\mu}_{\nu\sigma\rho} \sim \partial_\rho \Gamma^{\mu}_{\nu\sigma} \sim g^{\mu\kappa}\partial_\rho\partial_\nu g_{\kappa\sigma}$ has mass dimension 2

$G_N = 1/M_{Pl}^2$ has mass dimension -2

Each additional $R_{\mu\nu\sigma\rho}$ or $\mathcal{D}^2$ → 1 more loop

One-loop → $R_{\mu\nu\sigma\rho}R^{\mu\nu\sigma\rho}$

However, $R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\sigma\rho}R^{\mu\nu\sigma\rho}$

is Gauss-Bonnet term, total derivative in four dimensions.

So pure gravity is UV finite at one loop (but not with matter)

't Hooft, Veltman (1974)

Pure supergravity ($\mathcal{N} \geq 1$):
Divergences deferred to at least three loops

$R^3 \equiv R^{\lambda\rho}{}_{\mu\nu} R^{\mu\nu}{}_{\sigma\tau} R^{\sigma\tau}{}_{\lambda\rho}$ cannot be supersymmetrized

produces helicity amplitude (-+++) incompatible with SUSY Ward identitites Grisaru (1977); Tomboulis (1977)

However, at three loops, there is an N=8 supersymmetric counterterm, abbreviated R^4, plus (many) other terms containing other fields in N=8 multiplet.
Deser, Kay, Stelle (1977); Kallosh (1981); Howe, Stelle, Townsend (1981)

R^4 produces first subleading term in low-energy limit of 4-graviton scattering in type II string theory:

$$\alpha'^3 R^4 \Rightarrow \alpha'^3 stu \underbrace{M_4^{\text{tree}}(1,2,3,4)}_{\text{4-graviton amplitude in (super)gravity}}$$

Gross, Witten (1986)

Bose symmetric polynomial

$\mathcal{N}=8$ Constraints on Counterterms

Elvang, Freedman, Kiermaier, 100X.ijkl

- Use locality of on-shell amplitudes + powerful N=8 SUSY Ward identitites

Also related work by Kallosh

- N=8 SWI for maximally helicity violating (MHV) amplitudes:

$$\frac{M_n(++\cdots+-_i+\cdots+-_j+\cdots+)}{\langle i\,j\rangle^8} = \text{Bose symmetric}$$

- N=8 SWI for non-MHV amplitude – solved recently

  Elvang, Freedman, Kiermaier, 0911.3169

$\mathcal{D}^{2k}R^4$ → 4-point → MHV

→ amounts to classifying Bose-symmetric polynomials $P(s,t,u)$

$\mathcal{D}^{2k}R^5$ → still MHV → can still use Bose-symmetry

$\mathcal{D}^{2k}R^{6,7}$ → next-to-MHV analysis required

Chart of potential counterterms

Elvang, Freedman, Kiermaier, 100X.ijkl

Analytic proofs:

- $D^{2k} R^n$ MHV $\nexists$ for $n > 4$ and $k < 4$.
- $D^{2k} R^n$ NMHV $\nexists$ for $n > 5$ and $k < 2$.

Drummond, Heslop, Howe, Kerstan, th/0305202; Kallosh, 0906.3495

L							
3	R^4 MHV $\exists!$						
4	$D^2 R^4$ MHV $\nexists$	R^5 MHV $\nexists$					
5	$D^4 R^4$ MHV $\exists!$	$D^2 R^5$ MHV $\nexists$	R^6 (N)MHV $\nexists$				
6	$D^6 R^4$ MHV $\exists!$	$D^4 R^5$ MHV $\nexists$	$D^2 R^6$ (N)MHV $\nexists$	R^7 (N)MHV $\nexists$			
7	$D^8 R^4$ MHV $\exists!$	$D^6 R^5$ MHV $\nexists$	$D^4 R^6$ MHV $\nexists$ NMHV	$D^2 R^7$ (N)MHV $\nexists$	R^8 (N)MHV $\nexists$ N^2MHV?		
8	$D^{10} R^4$ MHV $\exists!$	$D^8 R^5$ MHV $\exists!$	$D^6 R^6$ MHV $\nexists$ NMHV?	$D^4 R^7$ MHV $\nexists$ NMHV?	$D^2 R^8$ (N)MHV $\nexists$ N^2MHV?	R^9 (N)MHV $\nexists$ N^2MHV?	
9	$D^{12} R^4$ 2×MHV	$D^{10} R^5$?×MHV	$D^8 R^6$ 2×MHV NMHV?	$D^6 R^7$ MHV $\nexists$ NMHV?	$D^4 R^8$ MHV $\nexists$ N or N^2MHV?	$D^2 R^9$ (N)MHV $\nexists$ N^2MHV?	R^{10} (N)MHV $\nexists$ N^2 or N^3MHV?

Until 7 loops, any divergences show up in 4-point amplitude!

- red: not excluded
- green: ?
- gray: excluded

$E_{7(7)}$ Constraints on Counterterms

- N=8 SUGRA has a continuous symmetry group, a noncompact form of E_7. Cremmer, Julia (1978,1979)
- 70 scalars parametrize coset space $E_{7(7)}/\mathrm{SU}(8)$, non-$\mathrm{SU}(8)$ part realized nonlinearly.
- $E_{7(7)}$ also implies amplitude Ward identitites, associated with limits as one or two scalars become soft

Bianchi, Elvang, Freedman, 0805.0757;
Arkani-Hamed, Cachazo, Kaplan, 0808.1446; Kallosh, Kugo, 0811.3414

- Single-soft limit of NMHV 6-point matrix element of R^4 doesn't vanish; indicates that R^4 violates $E_{7(7)}$

Broedel, LD, 0911.5704

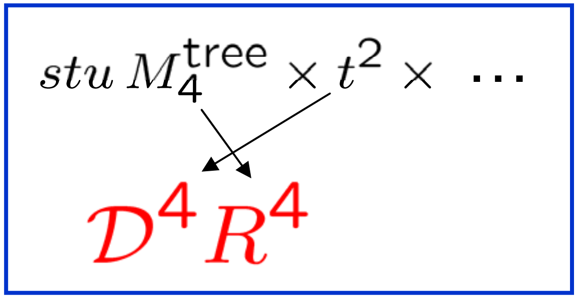
- R^4 non-invariance also suspected from superspace constructions

$E_{7(7)}$ Constraints (Cont.)

- A 7-loop N=8 supersymmetric counterterm was constructed long ago, but it was noted that the construction was not $E_{7(7)}$ invariant   Howe, Lindstrom (1981)
- More recent discussion of whether there is a (non-vanishing) 7-loop $E_{7(7)}$ invariant counterterm from the volume of the on-shell N=8 superspace   Brossard, Howe, Stelle 0908.3883
- 8-loop $E_{7(7)}$ invariant counterterm definitely nonvanishing, coincides with $\mathcal{D}^{10}R^4$ Howe, Lindstrom (1981) ; Kallosh (1981)

Other early hints that $\mathcal{N}=8$ is very special

- Found at two loops, and suggested for $L \geq 2$ that R^4 does **not** appear without **four** extra derivatives



$stu\, M_4^{\text{tree}} \times t^2 \times \cdots$

$\mathcal{D}^4 R^4$

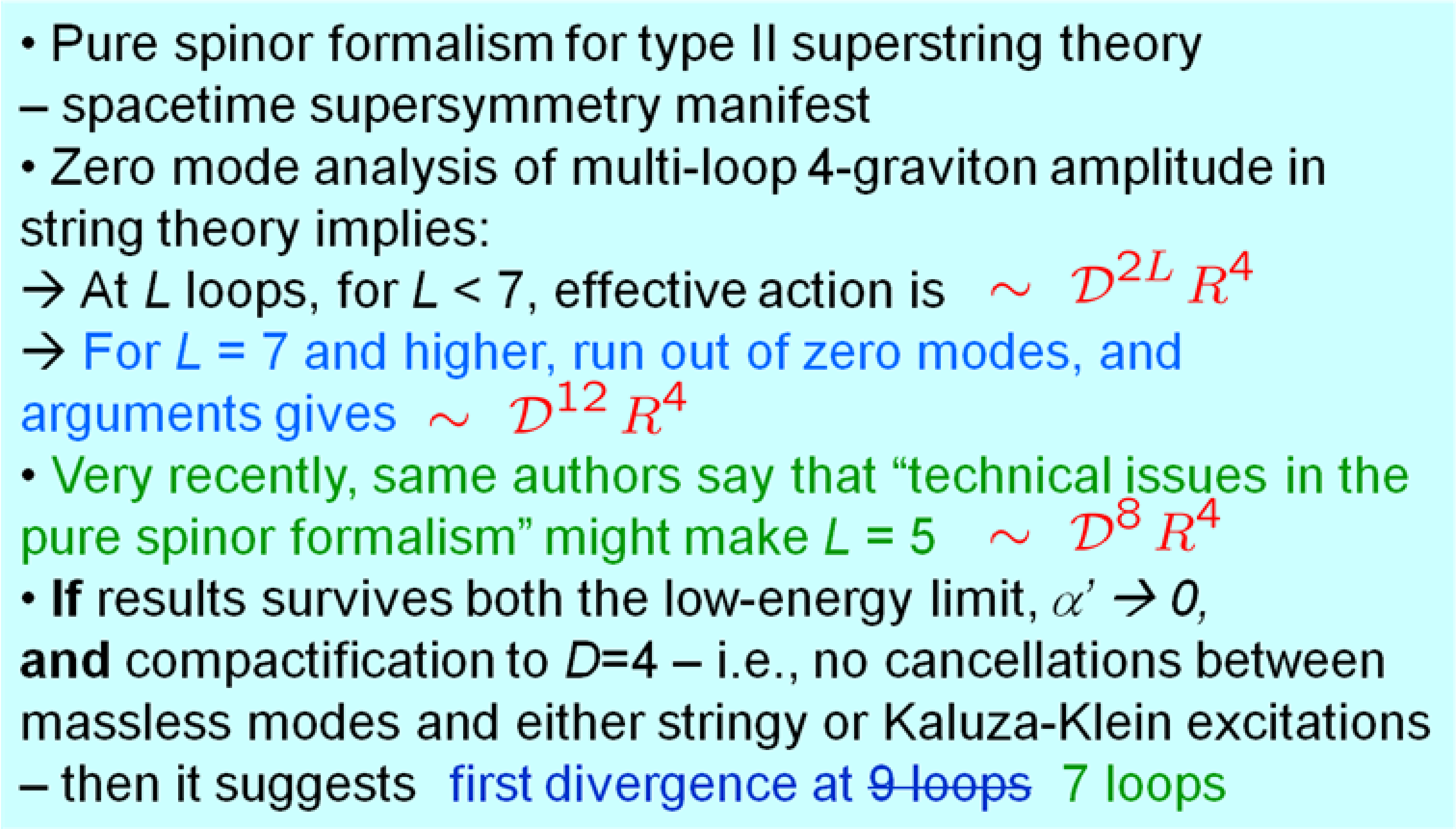
$\Leftrightarrow$ divergence at five loops?

Bern, LD, Dunbar, Perelstein, Rozowsky (1998)

- Superspace-based speculation that $D=4$ case diverges only at $L=6$, not $L=5$ — Howe, Stelle, hep-th/0211279
- However, more recent analysis predicts $D=4$ case diverges at $L=5$, [and $D=5$ case diverges at $L=4$] unless additional cancellation mechanisms are present — Bossard, Howe, Stelle, 0901.4661
- Multi-loop string results seem not to allow even $\mathcal{D}^4 R^4$ past $L=2$ — Berkovits, hep-th/0609006; Green, Russo, Vanhove, hep-th/0611273
- String/M duality arguments with similar conclusions, suggesting possible finiteness for all L. — Green, Russo, Vanhove, hep-th/0610299
- Light-cone superspace suggests finiteness until $L=7$ — Kallosh, 0903.4630
- “No triangle” cancellations for 1-loop amplitudes

Zero-mode counting in string theory

Berkovits, hep-th/0609006; Green, Russo, Vanhove, hep-th/0611273, 1002.3805, Green talk



- Pure spinor formalism for type II superstring theory – spacetime supersymmetry manifest
- Zero mode analysis of multi-loop 4-graviton amplitude in string theory implies:

→ At *L* loops, for *L* < 7, effective action is $\sim \mathcal{D}^{2L} R^4$

→ For *L* = 7 and higher, run out of zero modes, and arguments gives $\sim \mathcal{D}^{12} R^4$

- Very recently, same authors say that "technical issues in the pure spinor formalism" might make *L* = 5 $\sim \mathcal{D}^{8} R^4$
- **If** results survives both the low-energy limit, α' → 0, **and** compactification to *D*=4 – i.e., no cancellations between massless modes and either stringy or Kaluza-Klein excitations – then it suggests first divergence at ~~9 loops~~ 7 loops

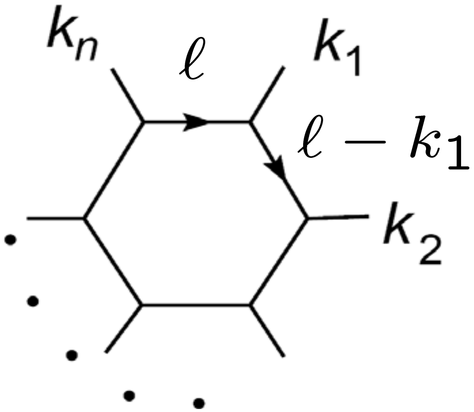
“No triangle” property

Bjerrum-Bohr et al., hep-th/0610043; Bern, Carrasco, Forde, Ita, Johansson, 0707.1035 (pure gravity) ; Kallosh, 0711.2108; Bjerrum-Bohr, Vanhove, 0802.0868

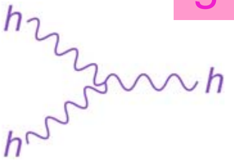
Proofs: Bjerrum-Bohr, Vanhove, 0805.3682; Arkani-Hamed, Cachazo, Kaplan, 0808.1446

- Statement about UV behavior of N=8 SUGRA amplitudes at **one loop** but with **arbitrarily many external legs:**

  “N=8 UV behavior no worse than N=4 SYM at one loop”
- Samples arbitrarily many powers of loop momenta
- Necessary but not sufficient for excellent **multi-loop** behavior
- Implies specific **multi-loop** cancellations Bern, LD, Roiban, th/0611086

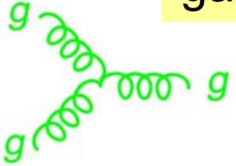


gravity (spin 2)



$$\supset \ \ell^{\mu_1}\,\ell^{\mu_2}\,\eta^{\nu_1\rho_1}\,\eta^{\nu_2\rho_2} + \cdots$$

gauge theory (spin 1)



$$\supset \ \ell^{\mu}\,\eta^{\nu\rho} + \cdots$$

UV info from scattering amplitudes:

$\mathcal{N}=8$ vs. $\mathcal{N}=4$ SYM

- Study 4-graviton amplitudes in higher-dimensional versions of N=8 supergravity to see what critical dimension D_c they begin to diverge in, as a function of loop number L
- Compare with analogous results for N=4 super-Yang-Mills theory (a finite theory in $D = 4$).

Key technical ideas: Bern, LD, Dunbar, Perelstein, Rozowsky (1998)

- Kawai-Lewellen-Tye (KLT) (1986) relations to express N=8 supergravity tree amplitudes in terms of simpler N=4 super-Yang-Mills tree amplitudes
- Unitarity to reduce multi-loop amplitudes to products of trees

Bern, LD, Dunbar, Kosower (1994)

Results now available through four loops BCDJR, 0905.2326

$\mathcal{N}=8$ vs. $\mathcal{N}=4$ SYM

DeWit, Freedman (1977); Cremmer, Julia, Scherk (1978); Cremmer, Julia (1978,1979)

$2^8 = 256$ massless states, ~ expansion of $(x+y)^8$

$\mathcal{N}=8$: $1 \leftrightarrow 8 \leftrightarrow 28 \leftrightarrow 56 \leftrightarrow 70 \leftrightarrow 56 \leftrightarrow 28 \leftrightarrow 8 \leftrightarrow 1$

helicity : $-2 \quad -\frac{3}{2} \quad -1 \quad -\frac{1}{2} \quad 0 \quad \frac{1}{2} \quad 1 \quad \frac{3}{2} \quad 2$

SUSY $\longleftrightarrow$

$h^- \quad \psi_i^- \quad v_{ij}^- \quad \chi_{ijk}^- \quad s_{ijkl} \quad \chi_{ijk}^+ \quad v_{ij}^+ \quad \psi_i^+ \quad h^+$

$\mathcal{N}=4$ SYM : $1 \quad 4 \quad 6 \quad 4 \quad 1$

$g^- \quad \lambda_A^- \quad \phi_{AB} \quad \lambda_A^+ \quad g^+$

all in adjoint representation

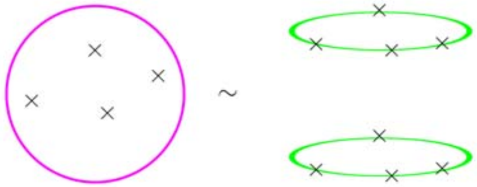
$2^4 = 16$ states ~ expansion of $(x+y)^4$

$$\Rightarrow \quad [\mathcal{N}=8] \ = \ [\mathcal{N}=4] \otimes [\mathcal{N}=4]$$

Kawai-Lewellen-Tye relations

KLT, 1986

Derive from relation between open & closed string amplitudes.



Low-energy limit gives N=8 supergravity amplitudes $M_n^{\rm tree}$ as quadratic combinations of N=4 SYM amplitudes $A_n^{\rm tree}$, consistent with product structure of Fock space,

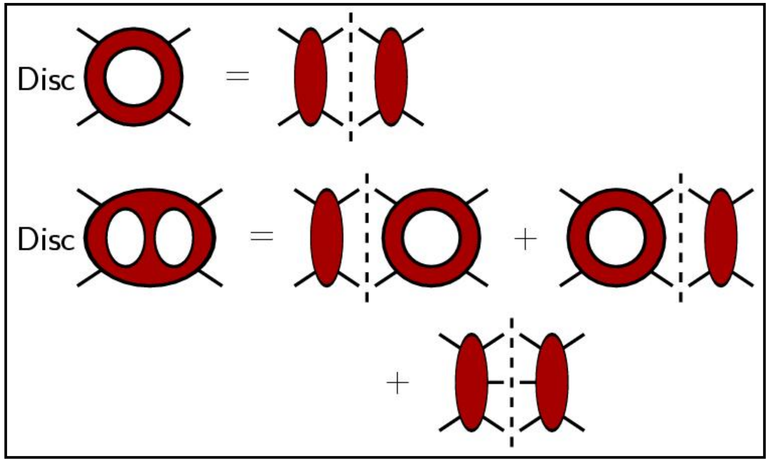
$$[\mathcal{N}=8] \;=\; [\mathcal{N}=4] \otimes [\mathcal{N}=4]$$

$$\begin{aligned}
M_3^{\rm tree}(1,2,3) &= [A_3^{\rm tree}(1,2,3)]^2 \\
M_4^{\rm tree}(1,2,3,4) &= -i\, s_{12}\, A_4^{\rm tree}(1,2,3,4)\, A_4^{\rm tree}(1,2,4,3) \\
M_5^{\rm tree}(1,2,3,4,5) &= i\, s_{12} s_{23}\, A_5^{\rm tree}(1,2,3,4,5)\, A_5^{\rm tree}(2,1,4,3,5) \\
&\quad + (2 \leftrightarrow 3) \\
M_6^{\rm tree}(1,2,3,4,5,6) &= \cdots
\end{aligned}$$

Amplitudes via perturbative unitarity

- *S*-matrix a unitary operator between in and out states

→ unitarity relations (cutting rules) for amplitudes



- Reconstruction of full amplitudes from cuts very efficient, due to simple structure of tree and lower-loop helicity amplitudes

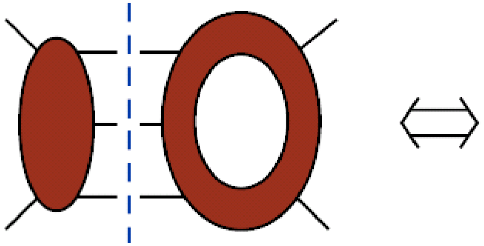
- Generalized unitarity (more propagators open) necessary to reduce everything to trees (in order to apply KLT relations)

Multi-loop generalized unitarity

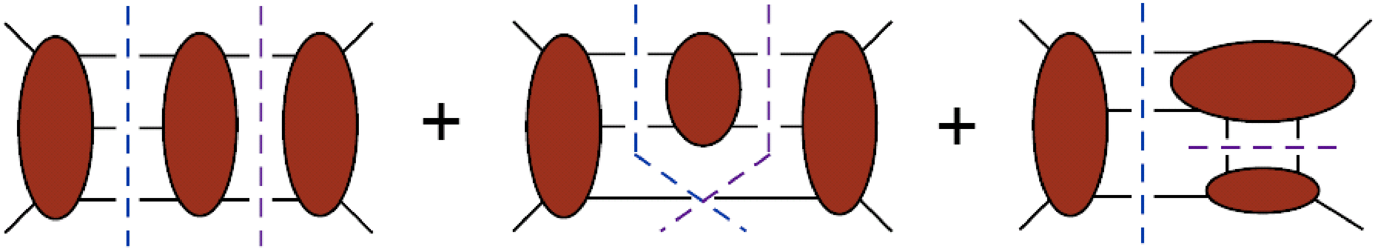
Bern, LD, Kosower, hep-ph/0001001; Bern, Czakon, LD, Kosower, Smirnov hep-th/0610248; Bern, Carrasco, LD, Johansson, Kosower, Roiban, hep-th/0702112; BCJK, 0705.1864; Cachazo, Skinner, 0801.4574; Cachazo, 0803.1988; Cachazo, Spradlin, Volovich, 0805.4832

Ordinary cuts of multi-loop amplitudes contain loop amplitudes.

For example, at 3 loops, one encounters the product of a 5-point tree and a 5-point one-loop amplitude:



Cut 5-point loop amplitude further, into (4-point tree) x (5-point tree), in all 3 inequivalent ways:

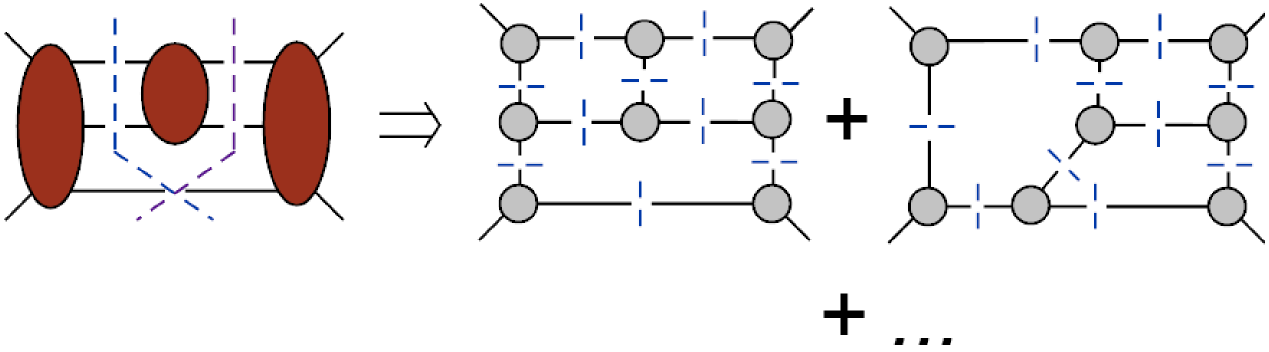


cut conditions satisfied by real momenta

Method of maximal cuts

Complex cut momenta make sense out of all-massless 3-point kinematics – can chop an amplitude entirely into 3-point trees

→ **maximal cuts**

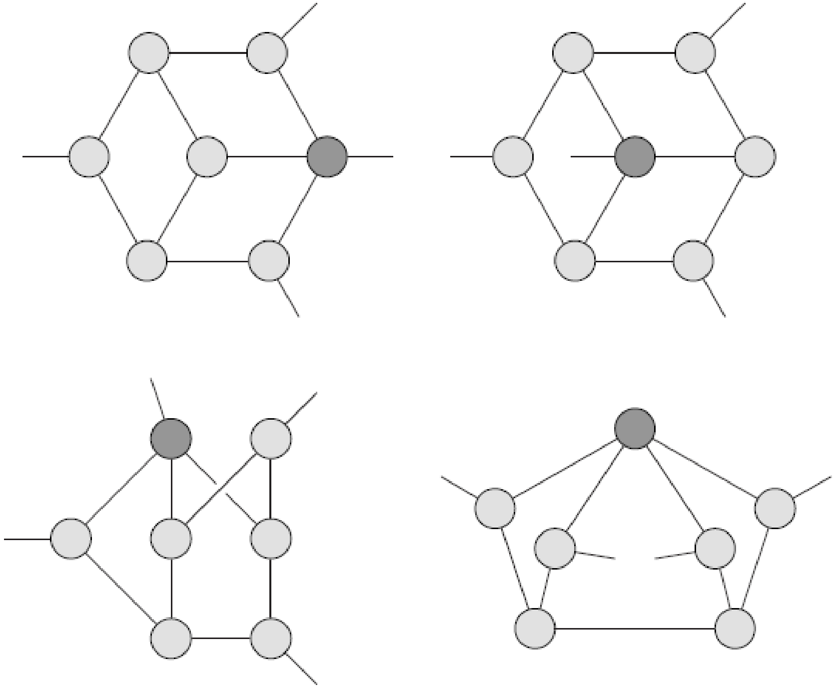


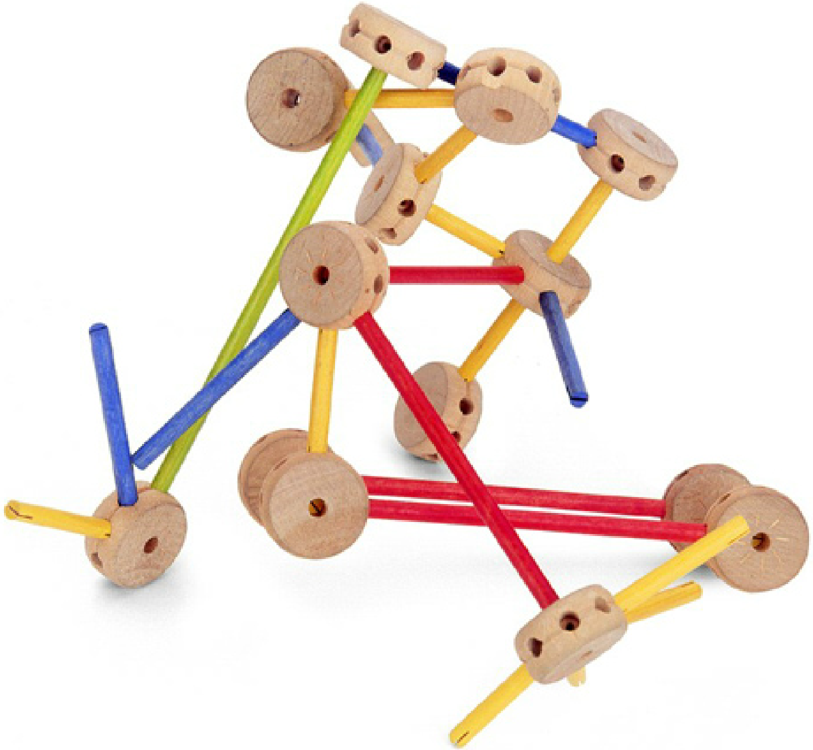
Maximal cuts are maximally simple,
yet give excellent starting point for constructing full answer

For example, in planar (leading in N_c) N=4 SYM
they find all terms in the complete answer for 1, 2 and 3 loops

Remaining terms found **systematically**: Let 1 or 2 propagators collapse from each maximal cut → **near-maximal cuts**

Amplitude assembly from near-maximal cuts is child's play





Multi-loop "KLT copying"

Bern, LD, Dunbar, Perelstein, Rozowsky (1998)

- N=8 SUGRA cuts are products of N=8 SUGRA trees, summed over all internal states.
- KLT relations let us write N=8 cuts very simply as:

sums of products of two copies of N=4 SYM cuts

$$[\mathcal{N}=8] = [\mathcal{N}=4] \otimes [\mathcal{N}=4] \implies \boxed{\sum_{\mathcal{N}=8} = \sum_{\mathcal{N}=4} \sum_{\mathcal{N}=4}}$$

- Need both **planar** (large N_c) and **non-planar** terms in corresponding multi-loop N=4 SYM amplitude

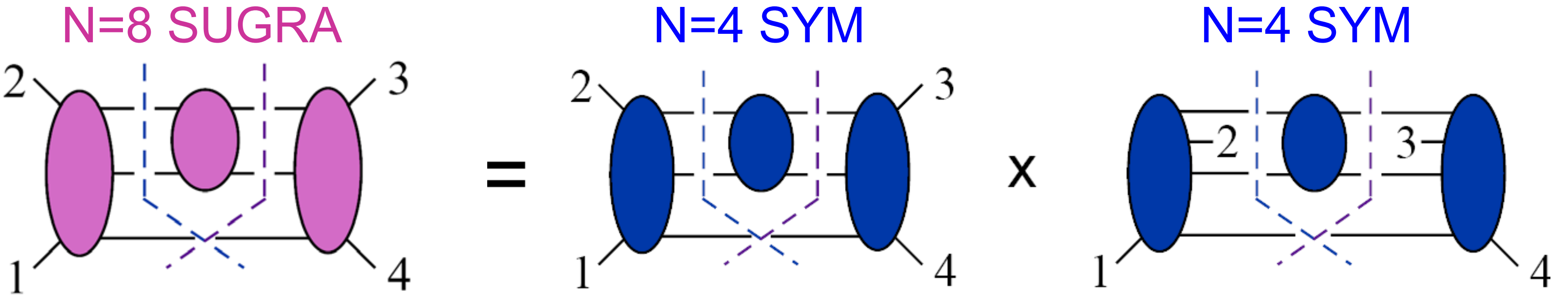
KLT copying at 3 loops

Using

$$M_4^{\text{tree}}(1,2,3,4) = -i\frac{st}{u}[A_4^{\text{tree}}(1,2,3,4)]^2$$

$$M_5^{\text{tree}}(1,2,3,4,5) = -i\, s_{51}s_{23}\, A_5^{\text{tree}}(1,2,3,4,5)\, A_5^{\text{tree}}(1,4,2,3,5) + (1 \leftrightarrow 2)$$

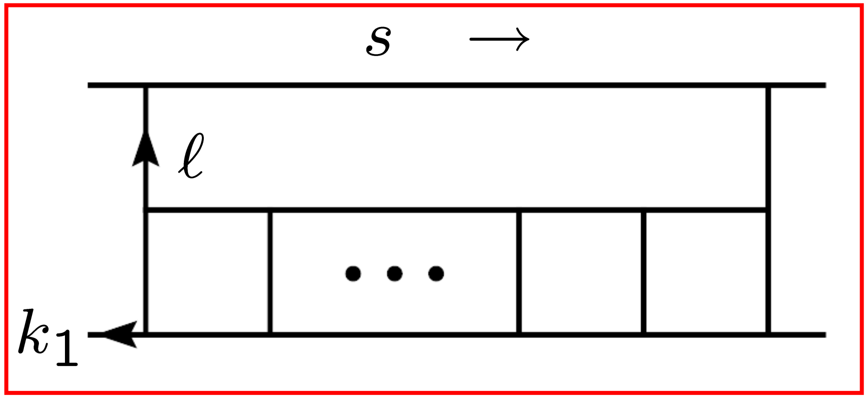
it is easy to see that



rational function of Lorentz products of external and cut momenta;
all state sums already performed

+ permutations $(1 \leftrightarrow 2,\ 3 \leftrightarrow 4)$

For $L>2$, UV behavior of generic integrals looks worse in N=8



N=4 SYM

$$st\,A_4^{\rm tree} \times t \times [(\ell+k_1)^2]^{L-2}$$

N=8 supergravity

$$stu\,M_4^{\rm tree} \times t^2 \times [(\ell+k_1)^2]^{2(L-2)}$$

← 2 from HE behavior of gravity

Integral in *D* dimensions scales as

$$\mathcal{I} \sim \int d^{DL}\ell \, \frac{(\ell^2)^{2(L-2)}}{(\ell^2)^{3L+1}}$$

→ Critical dimension D_c for log divergence (if no cancellations) obeys

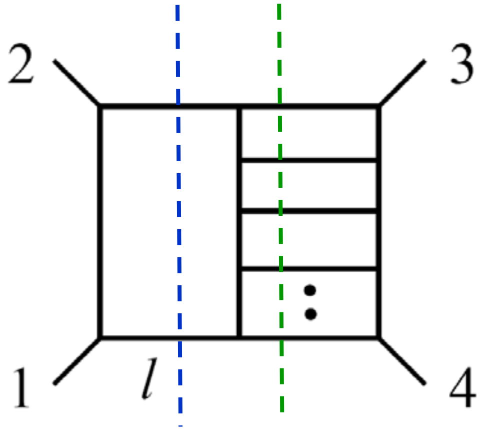
$$\frac{D_c L}{2} + 2(L-2) = 3L+1 \quad \Rightarrow$$

$$D_c = 2 + \frac{10}{L}$$ N=8

$$D_c = 4 + \frac{6}{L}$$ N=4 SYM

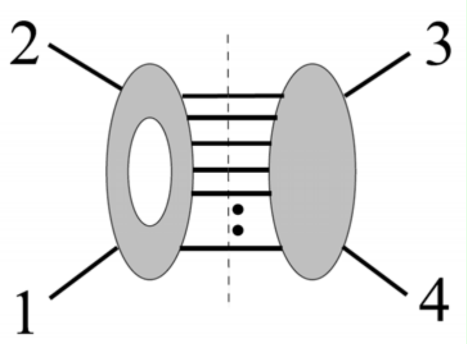
BDDPR (1998)

But: No-triangle → better behavior



2-particle cut exposes Regge-like ladder topology, containing numerator factor of $[(l+k_4)^2]^{2(L-2)}$

L-particle cut exposes one-loop (*L*+2)-point amplitude – but $[(l+k_4)^2]^{2(L-2)}$ would (heavily) violate the no-triangle property



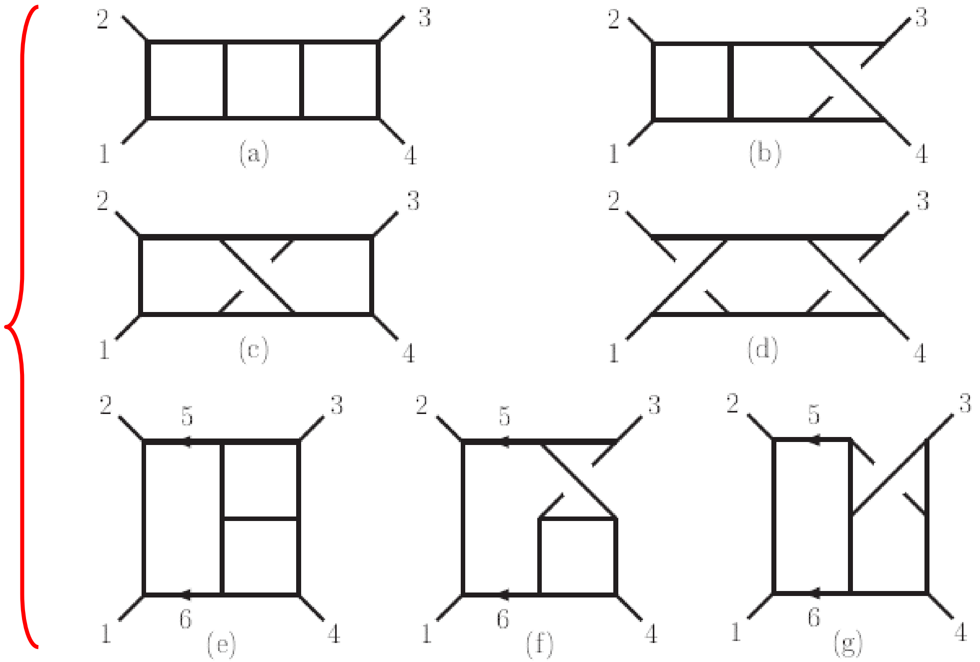
- Implies additional cancellations in the left loop BDR hep-th/0611086
- Inspired computation of full 4-graviton amplitude at 3 & 4 loops

3 loop amplitude

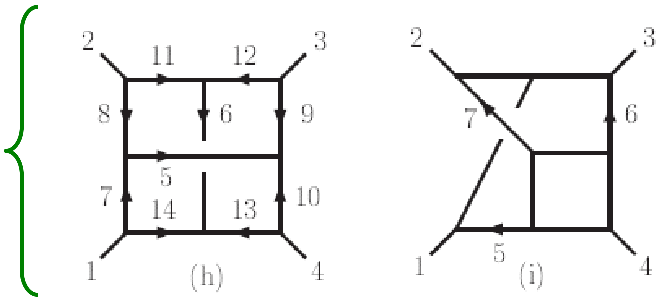
Bern, Carrasco, LD, Johansson, Kosower, Roiban, th/0702112
Bern, Carrasco, LD, Johansson, Roiban, 0808.4112

Nine basic integral topologies

Seven (a)-(g) long known (2-particle cuts → easily determine using "rung rule") BDDPR (1998)



Two new ones (h), (i) have no 2-particle cuts

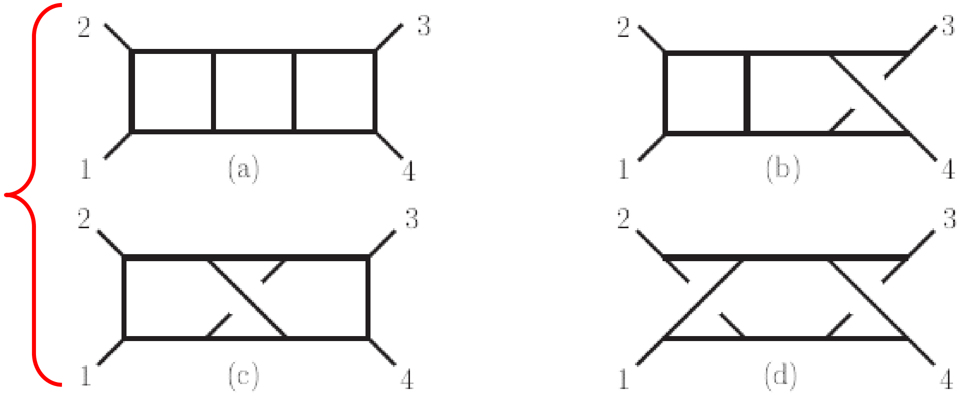


N=4 numerators at 3 loops

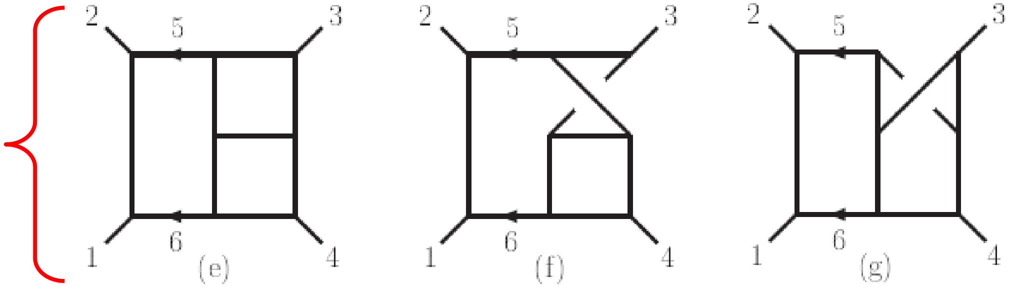
Overall
$st\,A_4^{\text{tree}}$

$$s_{iM} = (k_i + \ell_M)^2$$
$$\tau_{iM} = 2k_i \cdot \ell_M$$

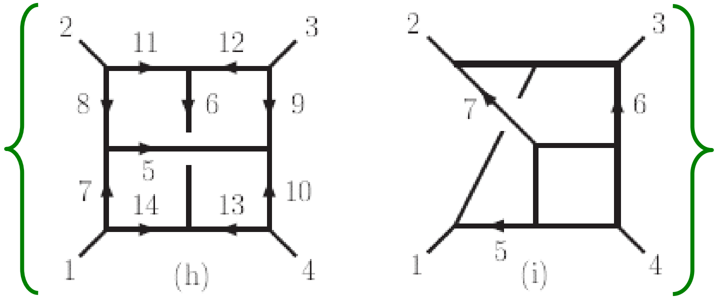
$$s_{12}^2 \Big\{$$



$$s_{12}\, s_{46} \Big\{$$



$$s_{12}(\tau_{26}+\tau_{36}) + s_{14}(\tau_{15}+\tau_{25}) + s_{12}s_{14} \Big\{$$



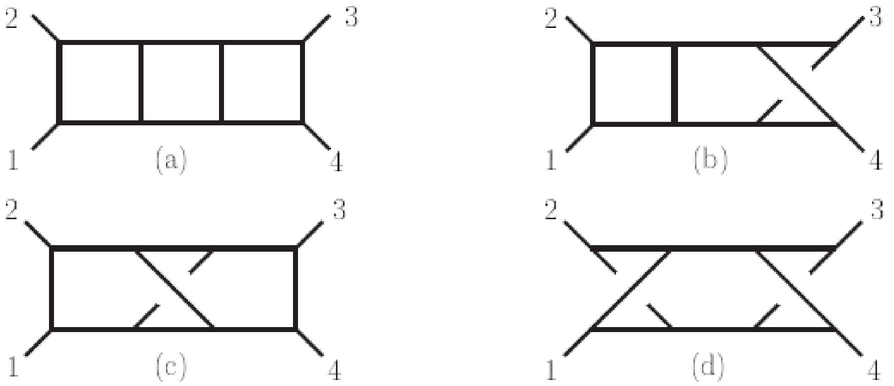
$$\Big\}\; s_{12}\, s_{45} - s_{14}\, s_{46} - \frac{1}{3}(s_{12} - s_{14})\,\ell_7^2$$

manifestly quadratic in loop momentum ℓ_M

N=8 numerators at 3 loops

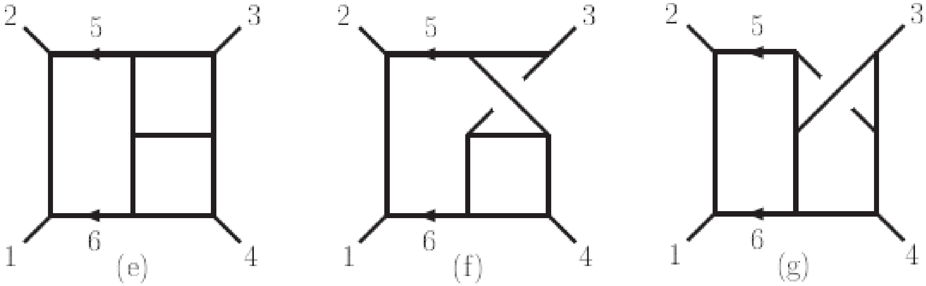
Overall
$(st A_4^{\rm tree})^2 = stu\, M_4^{\rm tree}$

s_{12}^4

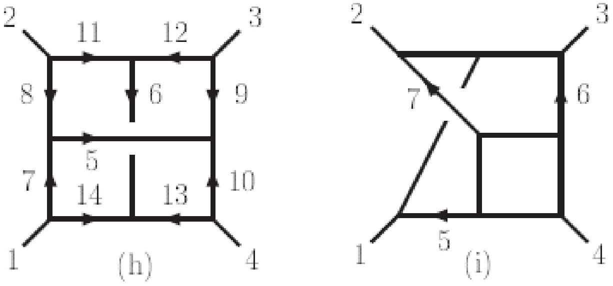


$$s_{iM} = (k_i + \ell_M)^2$$
$$\tau_{iM} = 2k_i \cdot \ell_M$$

$s_{12}^2\, \tau_{35}\, \tau_{46}$



$$\begin{aligned}&(s_{12}(\tau_{26}+\tau_{36})+s_{14}(\tau_{15}+\tau_{25})+s_{12}s_{14})^2\\ &+(s_{12}^2(\tau_{26}+\tau_{36})-s_{14}^2(\tau_{15}+\tau_{25}))\\ &\times(\tau_{17}+\tau_{28}+\tau_{39}+\tau_{4,10})\\ &+s_{12}^2(\tau_{17}\,\tau_{28}+\tau_{39}\,\tau_{4,10})\\ &+s_{14}^2(\tau_{28}\,\tau_{39}+\tau_{17}\,\tau_{4,10})\\ &+s_{13}^2(\tau_{17}\,\tau_{39}+\tau_{28}\,\tau_{4,10})\end{aligned}$$



$$\begin{aligned}&(s_{12}\,\tau_{45}-s_{14}\,\tau_{46})^2\\ &-\tau_{27}(s_{12}^2\,\tau_{45}+s_{14}^2\,\tau_{46})\\ &-\tau_{15}(s_{12}^2\,\tau_{47}+s_{13}^2\,\tau_{46})\\ &-\tau_{36}(s_{14}^2\,\tau_{47}+s_{13}^2\,\tau_{45})\\ &+l_5^2\,s_{12}^2\,s_{14}+l_6^2\,s_{12}\,s_{14}^2\\ &-\frac{1}{3}l_7^2\,s_{12}\,s_{13}\,s_{14}\end{aligned}$$

also manifestly quadratic in loop momentum ℓ_M

BCDJR (2008)

N=8 no worse than N=4 SYM in UV

Manifest quadratic representation at 3 loops – same behavior as N=4 SYM – implies same critical dimension still for $L = 3$: $\boxed{D_c \leq 4 + \frac{6}{L} = 6}$

- Evaluate UV poles in integrals

→ no further cancellation

- At 3 loops, $D_c = 6$ for N=8 SUGRA as well as N=4 SYM:

$$M_4^{(3),D=6-2\epsilon}\Big|_{\text{pole}} = \frac{1}{\epsilon}\frac{5\zeta_3}{(4\pi)^9}\left(\frac{\kappa}{2}\right)^8 (s_{12}s_{13}s_{14})^2 M_4^{\text{tree}}$$

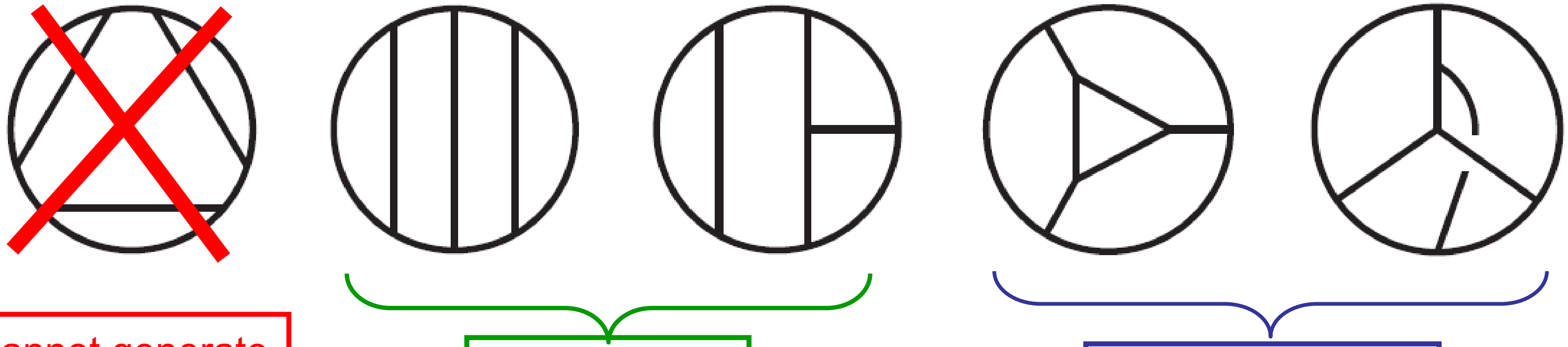
$\mathcal{D}^6 R^4$ counterterm

Recently recovered via string theory (up to factor of 9?)

Green, Russo, Vanhove, 1002.3805, Green talk

4 loops

- Begin with 4-loop cubic vacuum graphs
- Decorate them with 4 external legs to generate 50 nonvanishing cubic 4-point graphs
- Determine the 50 numerator factors, first for N=4 SYM, then, using KLT, for N=8 supergravity



cannot generate a nonvanishing (no-triangle) cubic 4-point graph

only generate rung rule topologies

the most complex

4 loop graphs

Number of cubic 4-point graphs with nonvanishing coefficients and various topological properties:

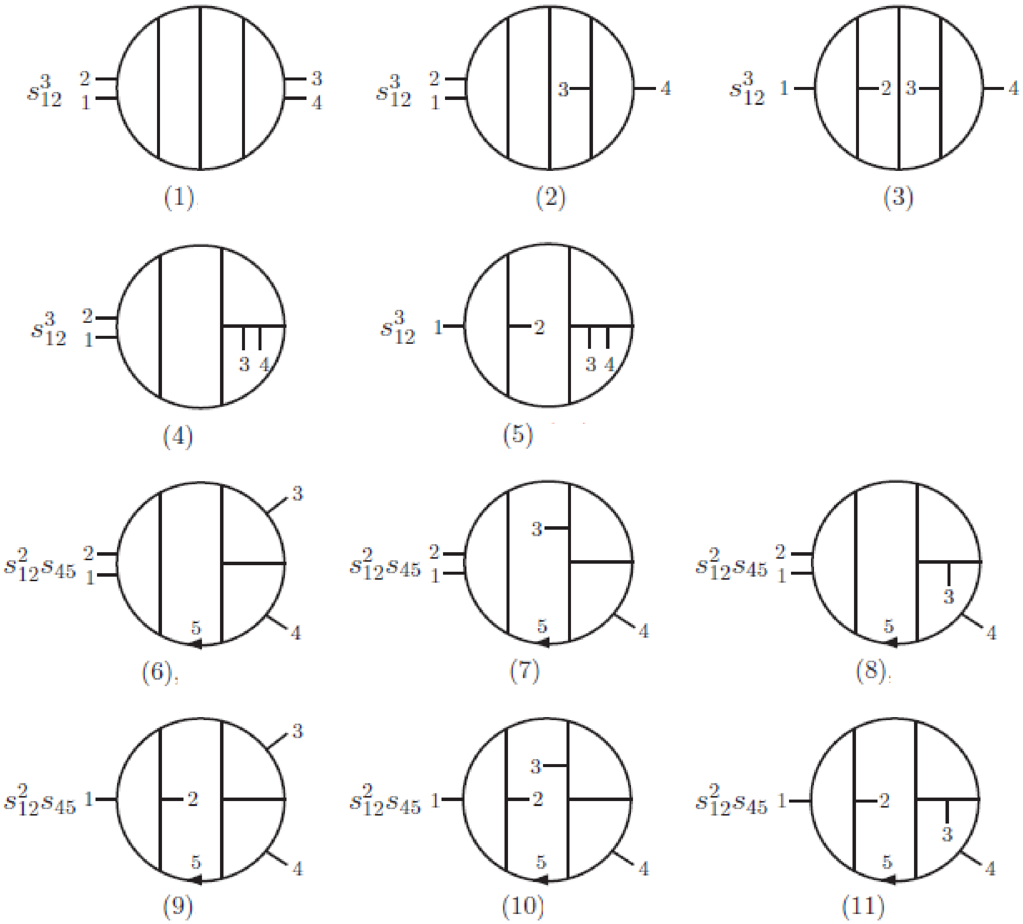
L	vacuum cubic	4-pt cubic	planar	non-planar	non-rung-rule	non-box-cut
1	1	1	1	0	0	0
2	1	2	1	1	0	0
3	2	9	2	7	2	1
4	5	50	6	44	18	5

worked out for N=4 in 2006 (BCDKS)

simple method to get numerator fails

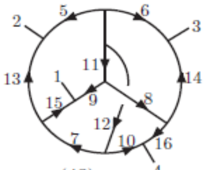
another method fails

Simplest (rung rule) graphs N=4 SYM numerators shown



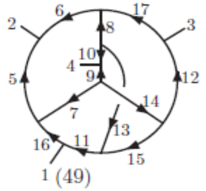
Most complex graphs
N=4 SYM numerators shown
[N=8 SUGRA numerators much larger]

$$\begin{aligned}
&s_{12}(s_{2,10}s_{39} - s_{47}s_{18} + s_{2,10}s_{59} + s_{39}s_{6,10} + s_{23}s_{6,11}) - s_{23}s_{57}s_{68} - s_{13}s_{59}s_{6,10}\\
&+l_6^2(s_{12}s_{35} + s_{12}s_{4,\overline{12}} - s_{23}s_{59}) + l_5^2(s_{12}s_{26} + s_{12}s_{1,\overline{11}} - s_{23}s_{6,10})\\
&+l_9^2(s_{12}s_{12,\overline{13}} - s_{13}s_{10,11}) + l_{10}^2(s_{12}s_{11,\overline{14}} - s_{13}s_{9,12})\\
&-l_{13}^2s_{12}s_{11,\overline{14}} - l_{14}^2s_{12}s_{12,\overline{13}} + (s_{13} - 2s_{12})l_9^2l_{10}^2\\
&+s_{23}(l_5^2l_6^2 - l_7^2l_8^2 + l_6^2l_7^2 + l_5^2l_8^2) + s_{12}l_{13}^2l_{14}^2 + s_{12}l_5^2l_6^2\\
&+s_{12}(-l_5^2l_8^2 + l_5^2l_9^2 - l_5^2l_{11}^2 - l_5^2l_{15}^2 - l_9^2l_{15}^2)\\
&+s_{12}(-l_6^2l_7^2 + l_6^2l_{10}^2 - l_6^2l_{12}^2 - l_6^2l_{16}^2 - l_{10}^2l_{16}^2)\\
&+s_{23}(l_9^2l_{12}^2 + l_{10}^2l_{11}^2 - l_7^2l_9^2 - l_8^2l_{10}^2) + s_{13}(l_9^2l_{11}^2 + l_{10}^2l_{12}^2)
\end{aligned}$$



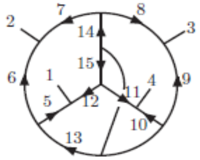
(48)

$$\begin{aligned}
&s_{12}(s_{47}s_{5,12} - s_{19}s_{36} - s_{48}s_{36}) + s_{23}(s_{48}s_{6,11} - s_{15}s_{3,10} - s_{15}s_{47}) - s_{12}s_{23}s_{11,12}\\
&+l_5^2(s_{23}s_{7,12} - s_{23}s_{4,15} - s_{13}s_{10,11}) + l_6^2(s_{12}s_{8,11} - s_{12}s_{4,\overline{15}} - s_{13}s_{9,12})\\
&+l_9^2(s_{23}s_{3,15} - s_{12}s_{3\overline{8}} + s_{23}s_{6,10}) + l_{10}^2(s_{12}s_{1,\overline{15}} - s_{23}s_{1\overline{7}} + s_{12}s_{59})\\
&+l_{13}^2(s_{12}s_{23} + s_{12}s_{38} - s_{23}s_{6,11}) + l_{14}^2(s_{23}s_{12} + s_{23}s_{17} - s_{12}s_{5,12})\\
&+l_{11}^2s_{23}(s_{4,12} - s_{6,10}) + l_{12}^2s_{12}(s_{4,11} - s_{59})\\
&+s_{13}(l_7^2l_8^2 + l_5^2l_8^2 + l_6^2l_7^2 + l_{11}^2l_{12}^2 + l_{10}^2l_{16}^2 + l_9^2l_{17}^2 - l_9^2l_{12}^2 - l_{10}^2l_{11}^2)\\
&+s_{12}(-l_5^2l_{10}^2 + l_6^2(l_{14}^2 + l_{13}^2 - l_{10}^2) + l_{12}^2(l_9^2 - l_5^2 - l_7^2 + l_{14}^2) + l_8^2(l_9^2 + l_{16}^2))\\
&+s_{23}(-l_6^2l_9^2 + l_5^2(l_{13}^2 + l_{14}^2 - l_9^2) + l_{11}^2(l_{10}^2 - l_6^2 - l_8^2 + l_{13}^2) + l_7^2(l_{10}^2 + l_{17}^2))\\
&+s_{12}(l_{12}^2l_{13}^2 - l_8^2l_{13}^2 - l_{10}^2l_{13}^2 - l_{10}^2l_{14}^2 - l_{13}^2l_{17}^2) + s_{23}(l_{11}^2l_{14}^2 - l_7^2l_{14}^2 - l_9^2l_{14}^2 - l_9^2l_{13}^2 - l_{14}^2l_{16}^2)
\end{aligned}$$



(49)

$$\begin{aligned}
&s_{12}s_{28}s_{4,12} - s_{12}s_{37}s_{1,11} - s_{23}s_{16}s_{3,10}\\
&+s_{23}s_{25}s_{49} + \tfrac{1}{2}s_{12}s_{23}(s_{13,15} - s_{13,14})\\
&+s_{12}(l_6^2l_{10}^2 - l_5^2l_9^2) + s_{23}(l_7^2l_{11}^2 - l_8^2l_{12}^2)
\end{aligned}$$

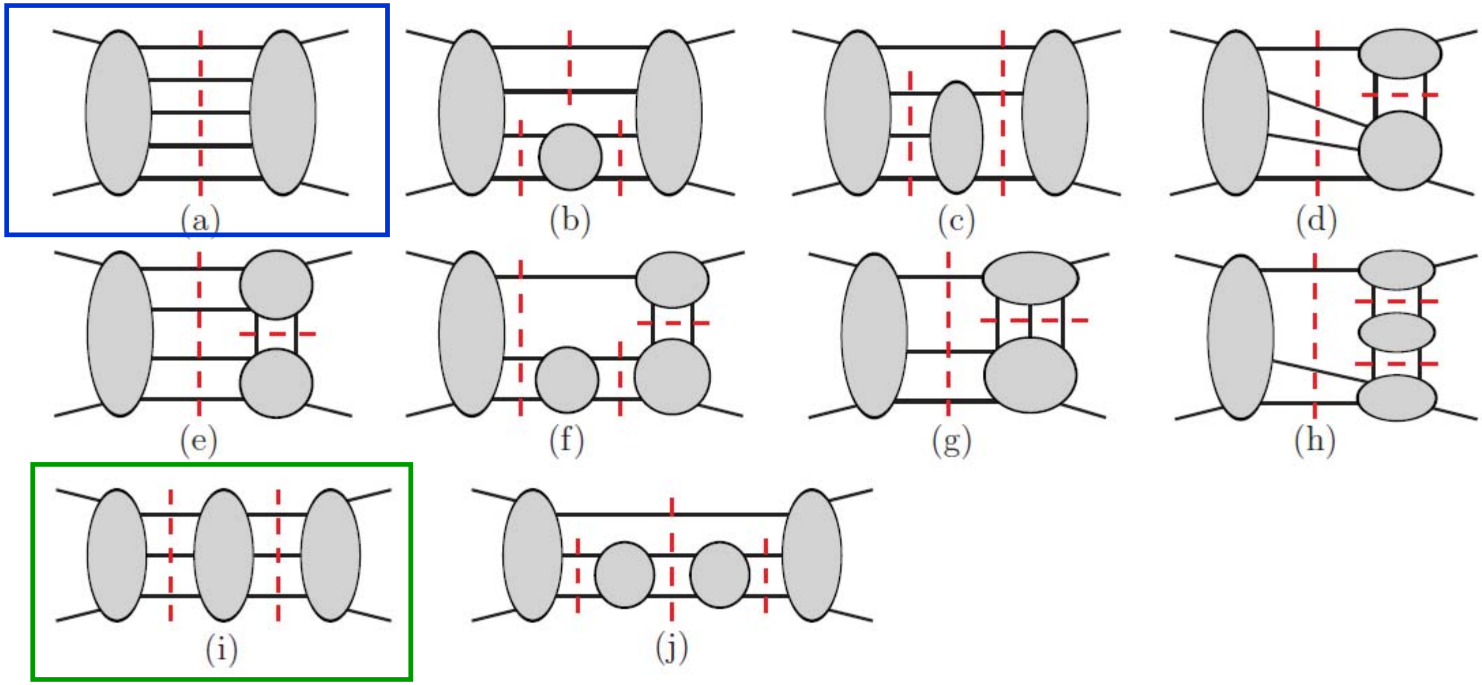


(50)

Checks on N=4 result

- Lots of different products of MHV tree amplitudes.
- NMHV_7 * anti-NMHV_7 and MHV_5 * NMHV_6 * anti-MHV_5

  – evaluated by Elvang, Freedman, Kiermaier, 0808.1720



UV behavior of N=8 at 4 loops

- All 50 cubic graphs have numerator factors composed of terms

$$\sim \quad k^{12-m}\, l^m$$

loop momenta l
external momenta k

- Maximum value of m turns out to be 8 in every integral

- Integrals all have 13 propagators, so

$$\boxed{\mathcal{I} \ \sim \int d^{4D}l \, l^{8-26}}$$

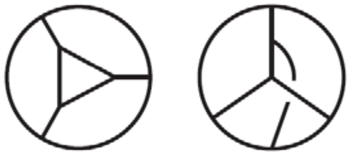
- Manifestly finite in $D=4$: 4 x 4 + 8 – 26 = - 2 < 0

- Not manifestly finite in $D=5$: 4 x 5 + 8 – 26 = + 2 > 0

In order to show that $D_c = 4 + \frac{6}{L} = 5.5$ need to show that $l^8, \ l^7, \ l^6, \ l^5$ all cancel

Cancellations between integrals

- Cancellation of $k^4\ l^8$ terms [vanishing of coefficient of $\mathcal{D}^4 R^4$] simple: just set external momenta $k_i \rightarrow 0$, collect coefficients of 2 resulting vacuum diagrams, observe that the 2 coefficients cancel.



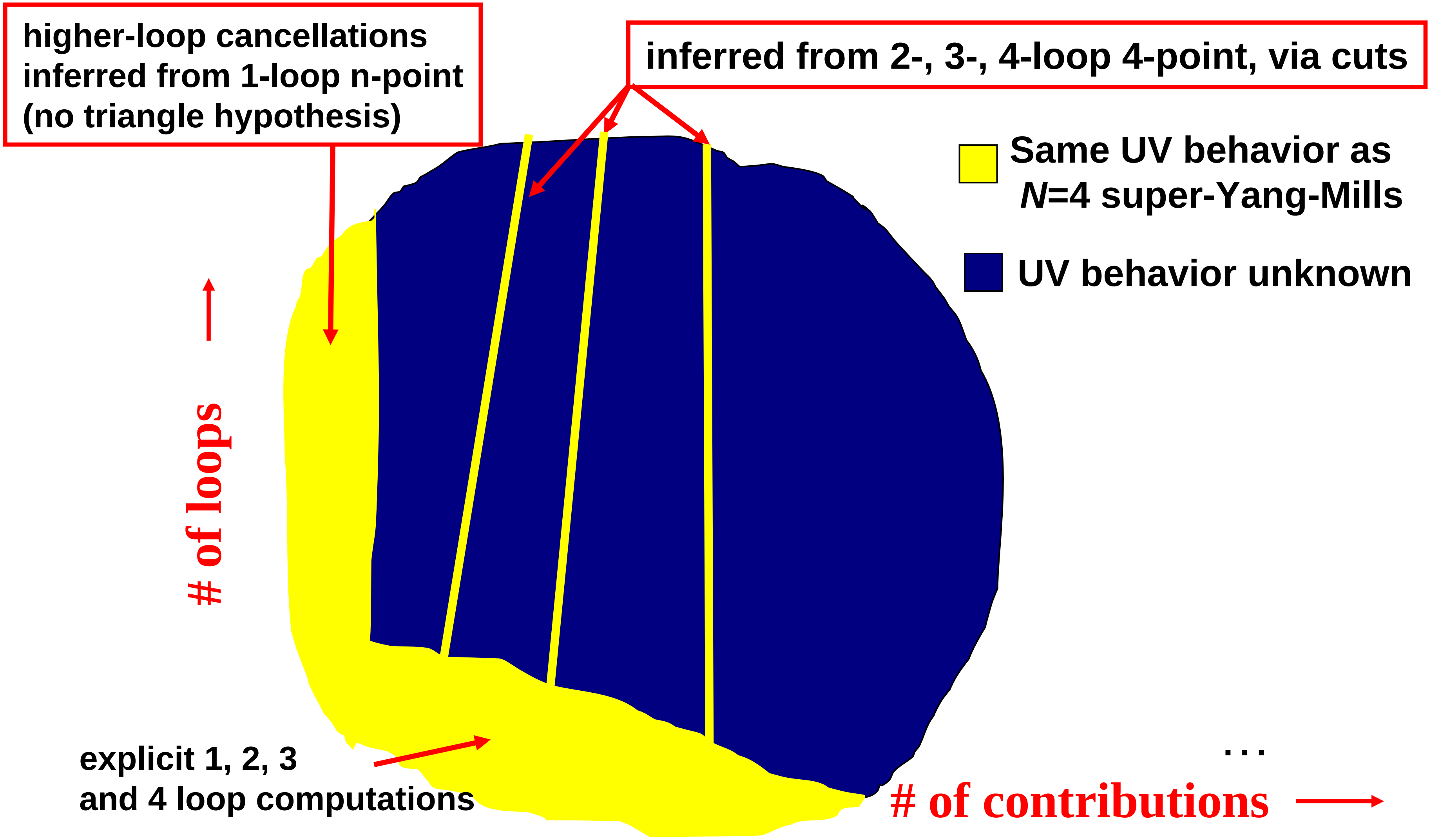
- Cancellation of $k^5\ l^7$ [and $k^7\ l^5$] terms is trivial: Lorentz invariance does not allow an odd-power divergence.

- Cancellation of $k^6\ l^6$ terms [vanishing of coefficient of $\mathcal{D}^6 R^4$] more intricate: Expand to second subleading order in limit $k_i \rightarrow 0$, generating 30 different vacuum integrals.
- Evaluating UV poles for all 30 integrals (or alternatively deriving consistency relations between them), we find that

UV pole cancels in $D=5\text{-}2\varepsilon$

N=8 SUGRA still no worse than N=4 SYM in UV at 4 loops!

Peeking beyond four loops



higher-loop cancellations inferred from 1-loop n-point (no triangle hypothesis)

inferred from 2-, 3-, 4-loop 4-point, via cuts

🟨 **Same UV behavior as *N*=4 super-Yang-Mills**

🟦 **UV behavior unknown**

# of loops →

explicit 1, 2, 3 and 4 loop computations

. . .

# of contributions →

5 loops?

A bit daunting but more enticing now that GRV 1002.3805 speculate L=5 behavior might be worse than $\mathcal{D}^{2L} R^4$ ←→ N=8 worse than N=4 at **only** L=5

Number of cubic 4-point graphs with nonvanishing coefficients

L	vacuum cubic	4-pt cubic	planar	non-planar	non-rung-rule	non-box-cut
1	1	1	1	0	0	0
2	1	2	1	1	0	0
3	2	9	2	7	2	1
4	5	50	6	44	18	5
5	16	439	19	420	?	?

worked out for N=4 in 2007 (BCJK)

What might it all mean?

- Suppose N=8 SUGRA is finite to all loop orders.
- Does this mean it is a nonperturbatively consistent theory of quantum gravity?
- No!
- At least two reasons it might need a nonperturbative completion:
  - Likely L! or worse growth of the order L coefficients,

$$\sim \; L! \; (s/M_{Pl}^2)^L$$

  - Different $E_{7(7)}$ behavior of the perturbative series (invariant!) compared with the $E_{7(7)}$ behavior of the mass spectrum of black holes (non-invariant!)

Is N=8 SUGRA "only" as good as QED?

- QED is renormalizable, but its perturbation series has zero radius of convergence in α: $\sim L!\ \alpha^L$
- UV renormalons associated with UV Landau pole
- But for small α it works pretty well:
g_e - 2 agrees with experiment to 10 digits
- Also, tree-level (super)gravity works well for $s << M_{Pl}^2$
- Many pointlike nonperturbative UV completions for QED: asymptotically free GUTs
- What is/are nonperturbative UV completion(s) for N=8 SUGRA? Could some be pointlike too?
- Some say N=8 SUGRA is in the "Swampland" – not connected to string theory beyond p.t. Green, Ooguri, Schwarz
- If so, then maybe UV completion has to be pointlike?!

Conclusions and open questions

- Through 4 loops, 4-graviton scattering amplitude of N=8 supergravity has UV behavior no worse than the corresponding 4-gluon amplitude of N=4 SYM.
- Will same continue to happen at higher loops? Partial evidence from generalized unitarity supports this, but 5 loops is the (next) acid test.
- If so, N=8 supergravity would be a perturbatively finite, pointlike theory of quantum gravity.
- Is there a nonperturbative UV completion?
- Although it may not be of direct phenomenological relevance, could N=8 cancellations point the way to other, more realistic, finite theories with less supersymmetry?
- Swampland restoration project?

Extra slides

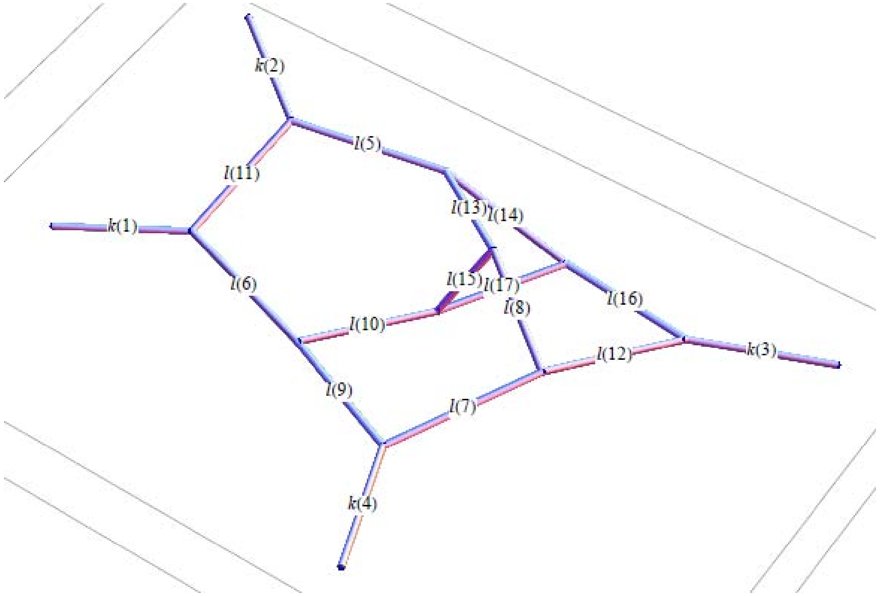
All the N=8 SUGRA details you could ever want … and more

In two locations we provide all 50 numerator factors for the 4-loop N=8 SUGRA amplitude, in Mathematica readable files:

- aux/* in the source of the arXiv version of 0905.2326 [hep-th]
- EPAPS Document No. E-PRLTAO-103-025932 (Windows-compatible)

Plus:

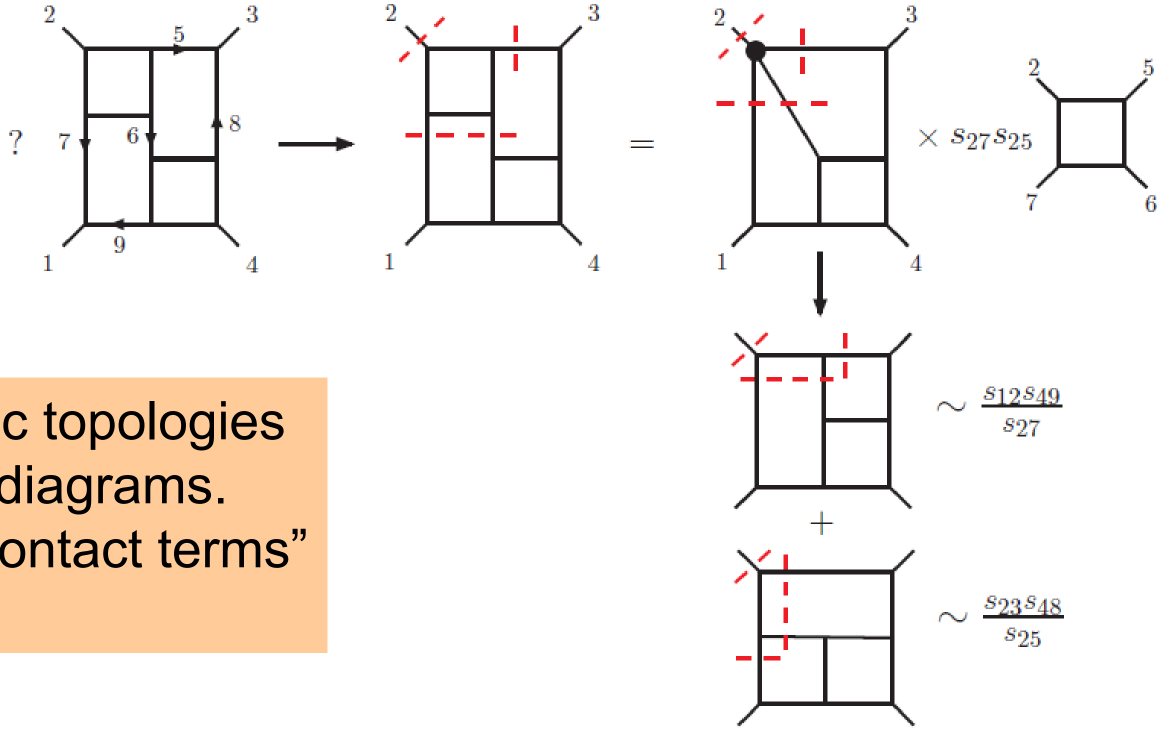
- use Mathematica's free Player to rotate 3D views of all 50 graphs
- Mathematica tools for extracting information about the 50 numerators, etc.



Box cut

Bern, Carrasco, Johansson, Kosower, 0705.1864

- If the diagram contains a box subdiagram, can use the simplicity of the 1-loop 4-point amplitude to compute the numerator very simply
- Planar example:



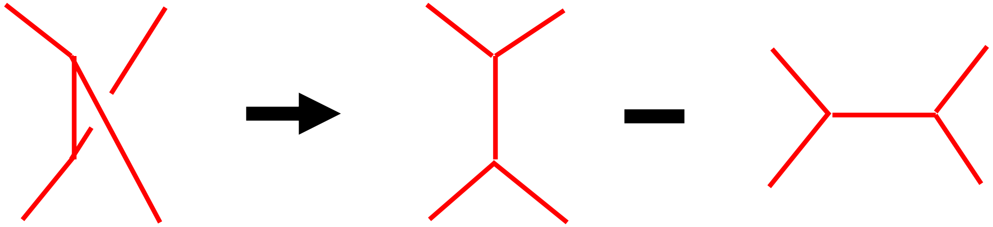
- Only five 4-loop cubic topologies do not have box subdiagrams.
- But there are also "contact terms" to determine.

$$\longrightarrow \quad ? = s_{27}s_{25}\left(\frac{s_{12}s_{49}}{s_{27}} + \frac{s_{23}s_{48}}{s_{25}}\right) = s_{12}s_{25}s_{49} + s_{23}s_{27}s_{48}$$

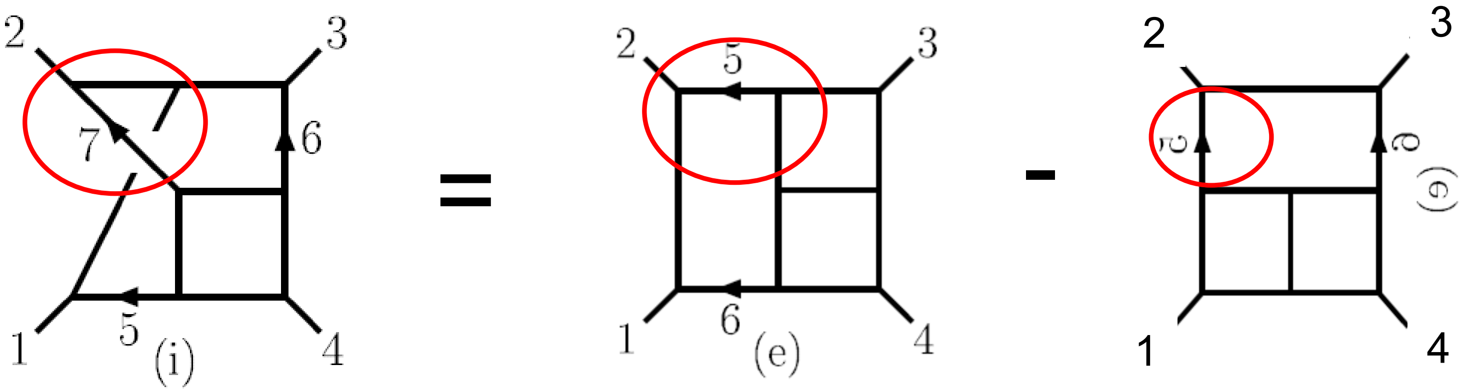
Twist identity

- If the diagram contains a four-point tree subdiagram, can use a Jacobi-like identity to relate it to other diagrams.

Bern, Carrasco, Johansson, 0805.3993



- Relate non-planar topologies to planar, etc.
- For example, at 3 loops, (i) = (e) – $(e)^T$ [+ contact terms]



N=4 SYM in UV at three loops

- UV poles of integrals in $D_c = 6 - 2\varepsilon$ [BCDJR 2008] dressed with color factors

→
$$\boxed{\begin{aligned} \mathcal{A}_4^{(3)}(1,2,3,4)|_{\text{pole}} &= -\frac{g^8\,\mathcal{K}}{3\,(4\pi)^9\,\epsilon}(N_c^3 + 36\,\zeta(3)\,N_c)\,\Big[s_{12}\,(\text{Tr}_{1324} + \text{Tr}_{1423}) \\ &\qquad + s_{23}\,(\text{Tr}_{1243} + \text{Tr}_{1342}) + s_{13}\,(\text{Tr}_{1234} + \text{Tr}_{1432})\Big] \end{aligned}}$$

$$\begin{aligned} \text{Tr}_{ijkl} &\equiv \text{Tr}(T^{a_i}T^{a_j}T^{a_k}T^{a_l}) \\ \text{Tr}_{ij} &\equiv \text{Tr}(T^{a_i}T^{a_j}) = \delta^{a_i a_j} \end{aligned}$$

- Corresponds to $\mathcal{D}^2\,\text{Tr}F^4$ type counterterms.
- Absence of double-trace terms of form $\mathcal{D}^2[\,\text{Tr}F^2]^2$ at $L = 3$.

N=4 SYM in UV at four loops

- Combining UV poles of integrals in $D_c = 5.5 - 2\varepsilon$ with color factors

$$\rightarrow \boxed{\begin{aligned} \mathcal{A}_4^{(4)}(1,2,3,4)|_{\text{pole}} \;=\; & -6\, g^{10}\, \mathcal{K}\, N_c^2 \big[N_c^2\, V_1 + 12\,(V_1 + 2\,V_2 + V_8) \big] \\ & \times \big[s_{12}\,(\mathrm{Tr}_{1324} + \mathrm{Tr}_{1423}) + s_{23}\,(\mathrm{Tr}_{1243} + \mathrm{Tr}_{1342}) \\ & + s_{13}\,(\mathrm{Tr}_{1234} + \mathrm{Tr}_{1432}) \big] \end{aligned}}$$

- V_i are 4-loop vacuum integrals
- Again corresponds to $\mathcal{D}^2\,\mathrm{Tr}\,F^4$ type counterterms.
- Later arguments for absence of double-trace terms at L = 3,4
  - – string theory Berkovits, Green, Russo, Vanhove, 0908.1923
  - – ¼ BPS invariant protected Bossard, Howe, Stelle 0908.3883