Holographic Superconductors in M-Theory

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Applied AdS/CFT

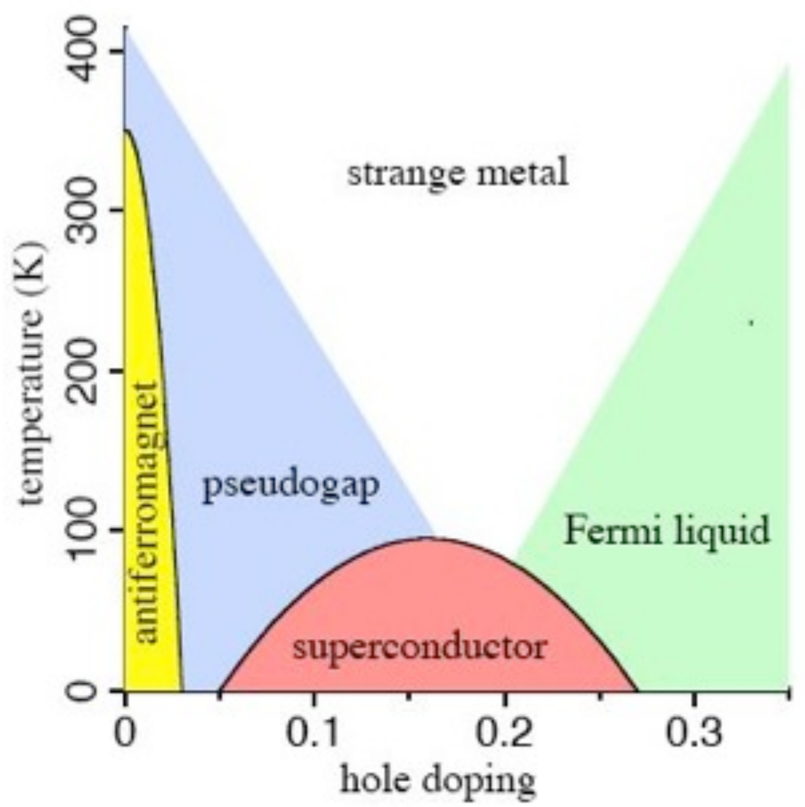
- The AdS/CFT correspondence is a powerful tool to study strongly coupled quantum field theories.

Can holography be applied to condensed matter systems?

- One focus: systems with strongly coupled "quantum critical points" (phase transitions at zero temperature).

- Some superconductors ("heavy fermions", high Tc cuprates) are associated with quantum critical points.

Phase diagram: of High Tc cuprates e.g. $La_{2-x}Sr_xCuO_4$



Quantum critical point under the superconducting dome?

Holographic Superconductors

Ingredients [Gubser][Hartnoll,Herzog,Horowitz]

- Strongly coupled CFT in d=3 with a holographic AdS dual
- Superconductivity (superfluidity): CFT should have a $U(1)$ global symmetry that can be spontaneously broken by an operator $\mathcal{O}$ acquiring a vev $< \mathcal{O} > \neq 0$ for $T \leq T_c$

  Gravity theory should have a $U(1)$ gauge symmetry and a charged field χ which can spontaneously break the symmetry.
- Finite temperature: consider black holes in the AdS space
- Critical temperature: consider the CFT to be held at finite chemical potential μ (finite charge density) which corresponds to considering electrically charged black holes

Want:

- High temp electrically charged black holes preserving $U(1)$

  Charged field $\chi = 0$ dual to $< \mathcal{O} >= 0$

- Low temp electrically charged black holes that spontaneously break the $U(1)$

  Charged field $\chi \neq 0$ - "charged hair"- dual to $< \mathcal{O} >\neq 0$

Almost all work has been in phenomenological or "bottom up" models. Focus on a few degrees of freedom.

The most popular model consists of metric, g gauge field, A single charged scalar field χ with a potential given by a simple mass term $V(\chi) = -m^2|\chi|^2$
[HHH]

Many insights, but:

- Is there any well defined underlying CFT?
- If phenomenological model is viewed just as an approximation to a model that can be embedded in string theory it might not capture e.g. interesting low temperature behaviour (as we will see)

Outline of Talk

Goal: Construct holographic superconductors in M-theory

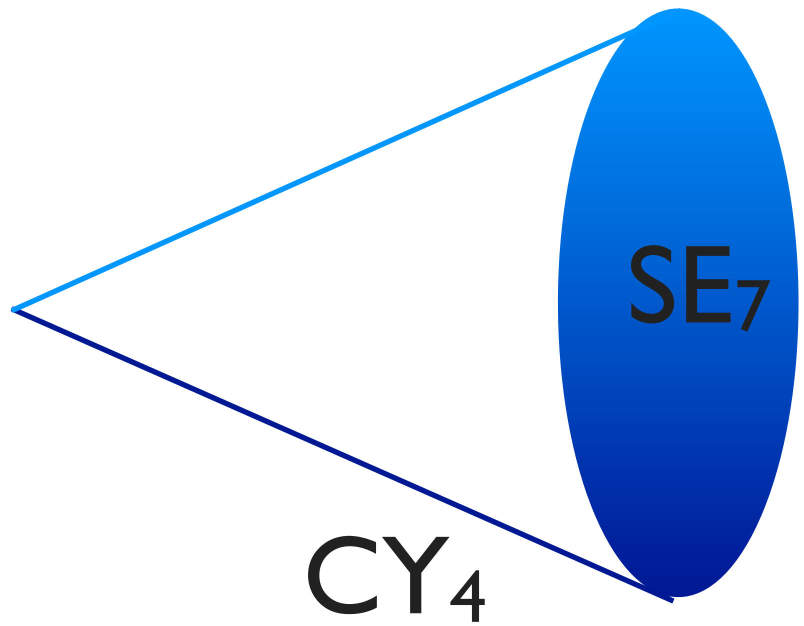
- Consistent KK truncations using Sasaki-Einstein spaces to give a non-linear extension of [HHH] model

  Includes a number of interesting features:

- Extra neutral scalar field - dual to a relevant operator
- Superconducting dome that "masks" finite entropy density of unbroken phase black holes at zero temperature
- Zero temperature limit of superconducting black holes have AdS_4 factors in the IR: dual to emergent quantum criticality with three dimensional conformal invariance.

  (bottom up: [Gubser,Rocha])

D=7 Sasaki-Einstein spaces

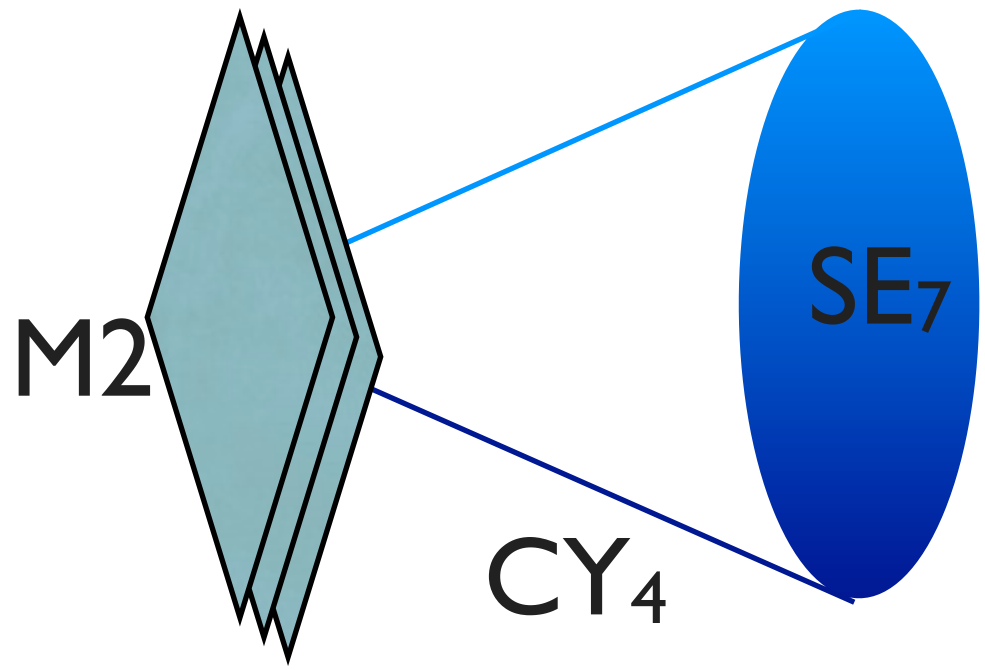


- $ds^2(SE_7)$ is a D=7 SE metric iff the cone metric is CY_4

- Locally SE_7 is a U(1) fibration over a D=6 Kahler-Einstein space:

$$ds^2(SE_7) = ds^2(KE_6) + \eta \otimes \eta \qquad \text{with} \qquad d\eta = 2J_{KE_6}$$

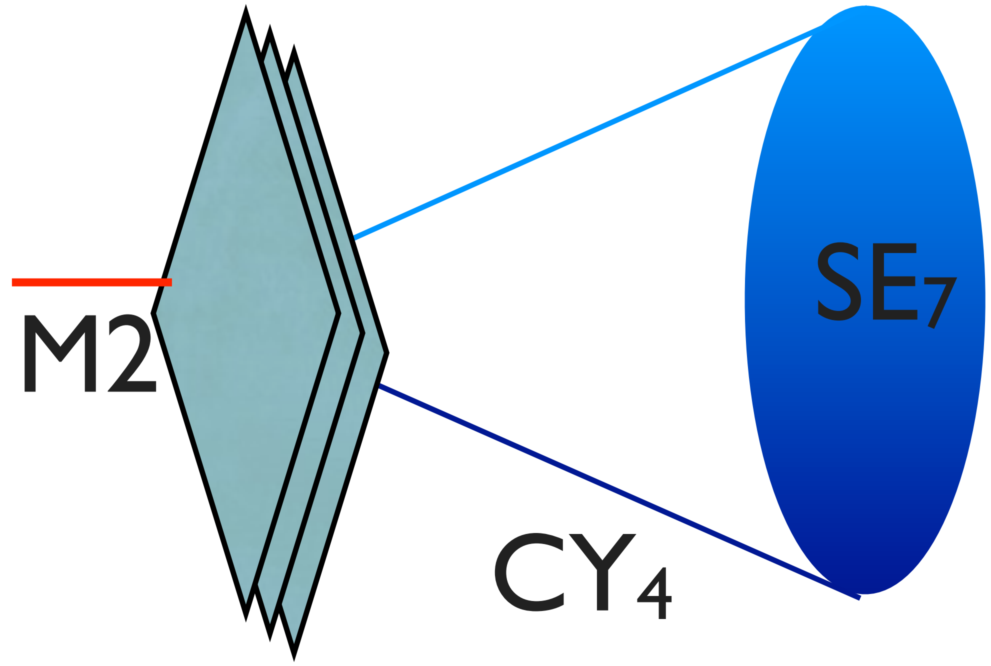
e.g. S^7 is U(1) fibration over CP^3



Near horizon limit we obtain $AdS_4 \times SE_7$

$$
\begin{aligned}
ds^2 &= \tfrac{1}{4} ds^2(AdS_4) + ds^2(SE_7) \\
G_4 &= +\tfrac{3}{8} Vol(AdS_4)
\end{aligned}
$$

Dual to an N=2 SCFTs in d=3.
Special case $SE_7 = S^7$ then have N=8 SCFT in d=3.



Near horizon limit we obtain "skew whiffed" $AdS_4 \times SE_7$

$$
\begin{aligned}
ds^2 &= \tfrac{1}{4} ds^2(AdS_4) + ds^2(SE_7) \\
G_4 &= -\tfrac{3}{8} Vol(AdS_4)
\end{aligned}
$$

Generically dual to an N=0 SCFTs in d=3.

Solutions are perturbatively stable.

Special case $SE_7 = S^7$ then have N=8 SCFT in d=3.

Consistent KK Truncations

- Consider KK reduction of some high D dimensional theory on some internal manifold M. Obtain a low d dimensional theory with an infinite number of fields.
- A consistent KK truncation is one where we can keep a finite set of fields such that any solution of the low d theory involving these fields uplifts to an exact solution of the high D theory.
- E.g. KK reduction on S^1 - keep $U(1)$ invariant modes
- However, it cannot usually be done.

Some general results known in context of AdS/CFT

- There is a consistent KK reduction of D=11 supergravity on any SE_7 space to minimal D=4 N=2 gauged supergravity (g , A)

Keeps the fields dual to the superconformal current multiplet [JPG,Varela][Buchel, Liu]

- There is a more general consistent KK reduction of D=11 supergravity on any SE_7 space to D=4 N=2 gauged supergravity coupled to a vector multiplet and a hypermultiplet.

Also keeps the fields dual to the breathing mode supermultiplet. [JPG,Varela,Kim,Waldram]

- There is an additional sub-truncation that is adapted to the skew-whiffed vacua...

- Metric, g , abelian gauge field A , charged scalar χ and an extra neutral scalar h with D=4 action:

$$S=\frac{1}{16\pi G}\int d^4x\sqrt{-g}\Big[R-\frac{(1-h^2)^{3/2}}{1+3h^2}F_{\mu\nu}F^{\mu\nu}-\frac{3}{2(1-\frac{3}{4}|\chi|^2)^2}|D\chi|^2-\frac{3}{2(1-h^2)^2}(\nabla h)^2 -\frac{24(-1+h^2+|\chi|^2)}{(1-\frac{3}{4}|\chi|^2)^2(1-h^2)^{3/2}}\Big]+\frac{1}{16\pi G}\int\frac{2h(3+h^2)}{(1+3h^2)}F\wedge F$$

Any solution to equations of motion gives an infinite number of exact solutions of D=11 supergravity, one for each choice of SE_7

- $h=0\ (F\wedge F=0)$ is approximately [HHH] model if $\chi<<1$
- Skew-whiffed AdS_4 vacuum: $h=\chi=0$ uplifts to the skew-whiffed solution $AdS_4\times SE_7$
- Pope-Warner AdS_4 vacuum: $h=0,\ |\chi|^2=\frac{2}{3}$ uplifts to Pope-Warner solution

Skew-Whiffed Solution in D=11

$$
\begin{aligned}
ds^2 &= \tfrac{1}{4} ds^2(AdS_4) + ds^2(KE_6) + \eta \otimes \eta \\
G_4 &= -\tfrac{3}{8} Vol(AdS_4)
\end{aligned}
$$

Skew-whiffed solution has N=0 susy and is perturbatively stable

Pope Warner Solution in D=11

$$
\begin{aligned}
ds^2 &= \tfrac{3}{16} ds^2(AdS_4) + \tfrac{1}{2} ds^2(KE_6) + \eta \otimes \eta \\
G_4 &= -\tfrac{9}{64} Vol(AdS_4) + \tfrac{1}{\sqrt{2}}(\eta \wedge \Omega + c.c)
\end{aligned}
$$

Pope-Warner solution has N=0 susy.   Stability??

There is a domain wall solution describing RG flow between CFTs

Holographic Superconductors

- Ansatz for D=4 fields to get electrically charged black holes (branes) at finite T

$$ds_4^2 = -e^{-\beta(r)}g(r)dt^2 + \frac{dr^2}{g(r)} + r^2(dx^2 + dy^2)$$

$$A = \phi(r)dt \qquad \chi = \chi(r) \in \mathbb{R} \qquad h = h(r)$$

- Boundary conditions at infinity

$$g = 4r^2 + \dots$$

$$\beta = 0 + \dots$$

$$\phi = \mu - \frac{q}{r} + \dots$$

Skew-whiffed CFT at finite T and μ

χ dual to operator $\mathcal{O}_\chi$ with $\Delta = 2$ - order parameter of superconductvity

$$\chi = \frac{\chi_1}{r} + \frac{\chi_2}{r^2} + \dots$$

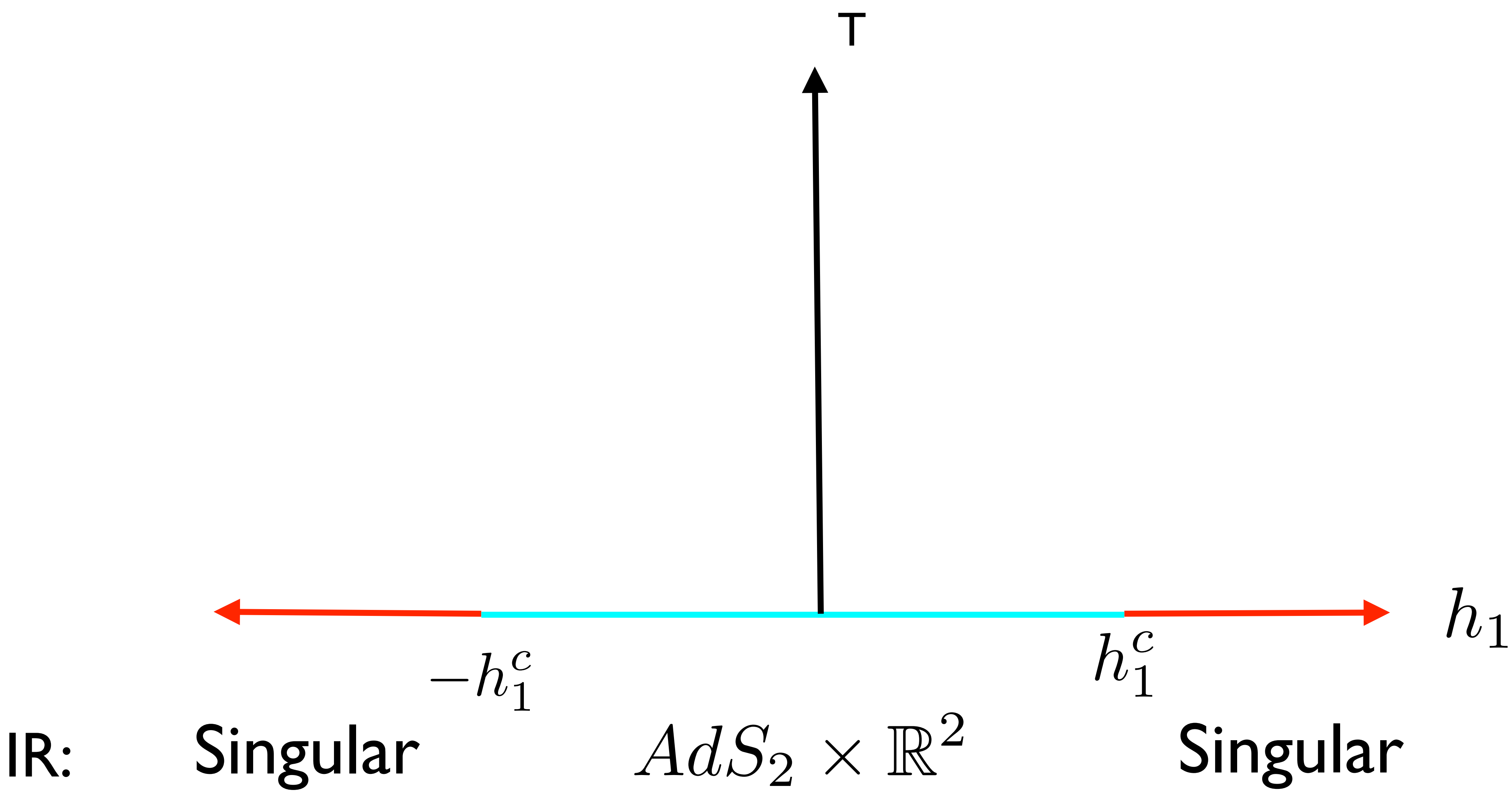
- $\chi_1 \neq 0$ corresponds to deforming Skew-Whiffed CFT by $\mathcal{O}_\chi$
- $\chi_2 \neq 0$ corresponds to $< \mathcal{O}_\chi > \neq 0$
- Want solutions with spontaneously broken $U(1)$ so we set $\chi_1 = 0$

h dual to relevant operator $\mathcal{O}_h$ with $\Delta = 2$

$$h = \frac{h_1}{r} + \frac{h_2}{r^2} + \dots$$

- $h_1 \neq 0$ corresponds to deforming Skew-Whiffed CFT by $\mathcal{O}_h$
- $h_2 \neq 0$ corresponds to $< \mathcal{O}_h > \neq 0$

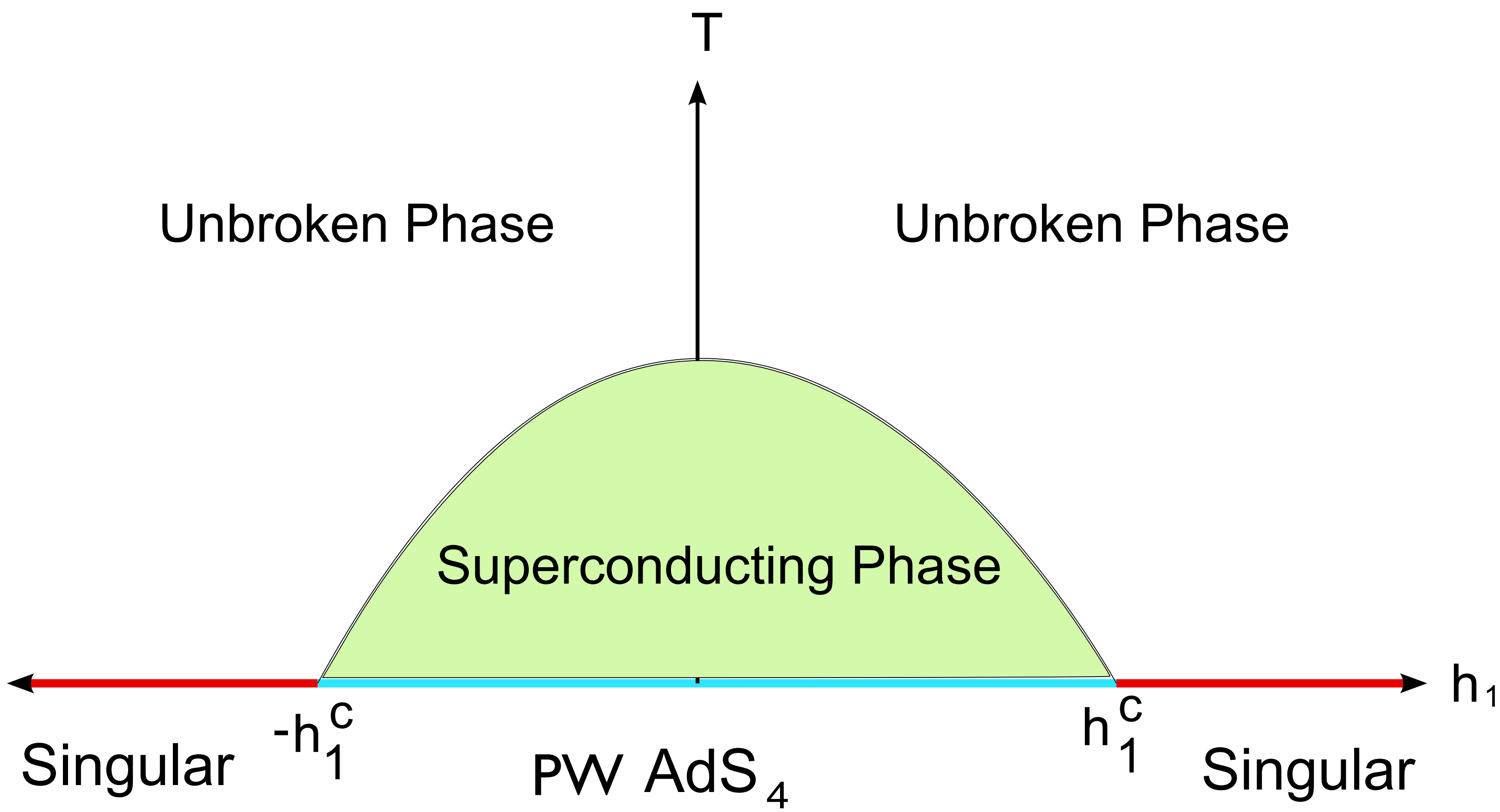
- Have found high temp Unbroken Phase Black holes with $\chi = 0$ that generalise AdS-RN for all values of T and h_1 ($\mu = 1$)



- For $h_1 = 0$ we have the exact AdS-Reissner-Nordstrom solution
- For $|h_1| < h_1^c$ we have finite entropy density at T=0 (unphysical?)

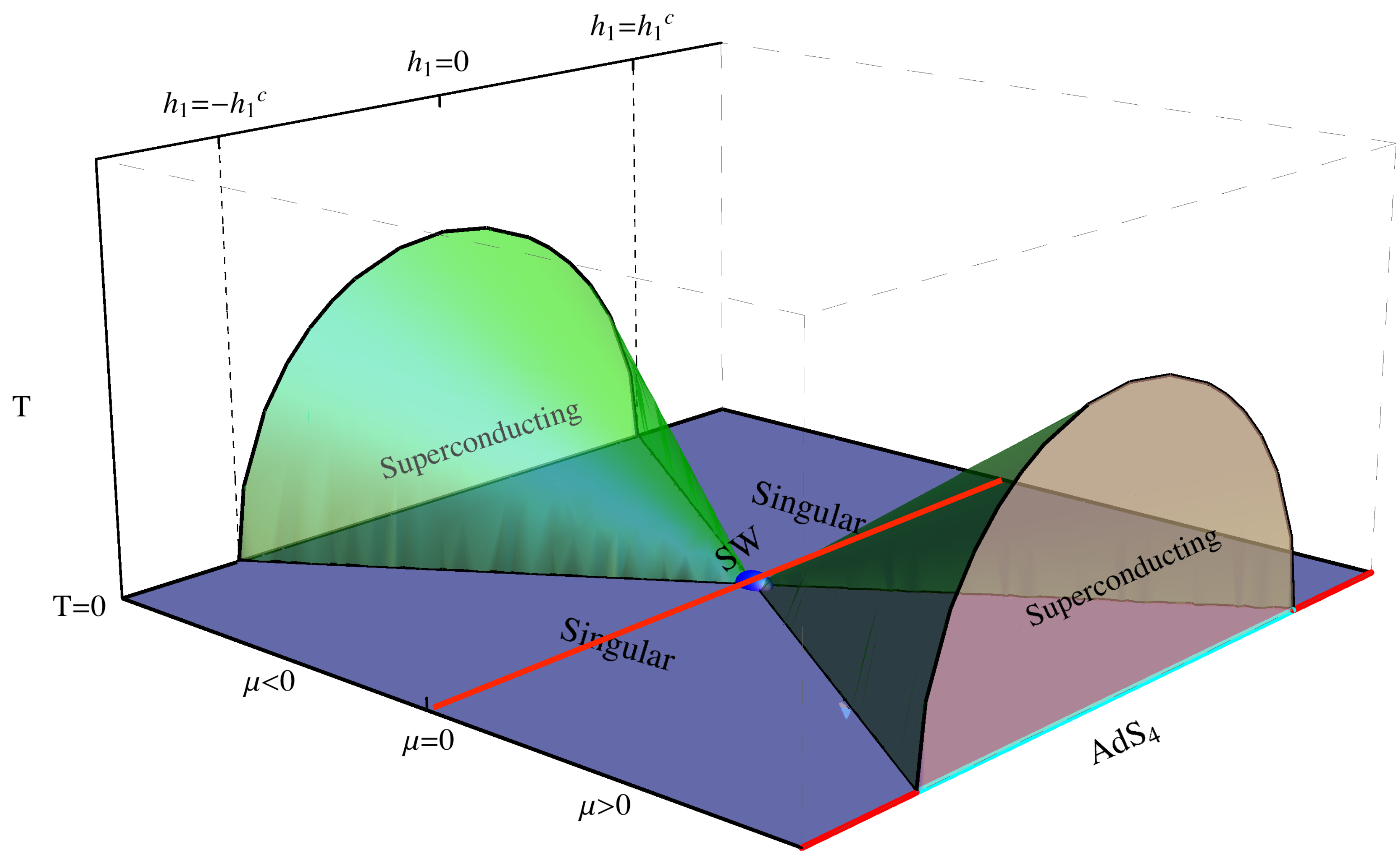
- Have also found superconducting black holes with $\chi \neq 0$ when $|h_1| < h_1^c$ that dominate the free energy.

Phase diagram for SW CFT deformed by $\mathcal{O}_h$ with $\mu \neq 0$:

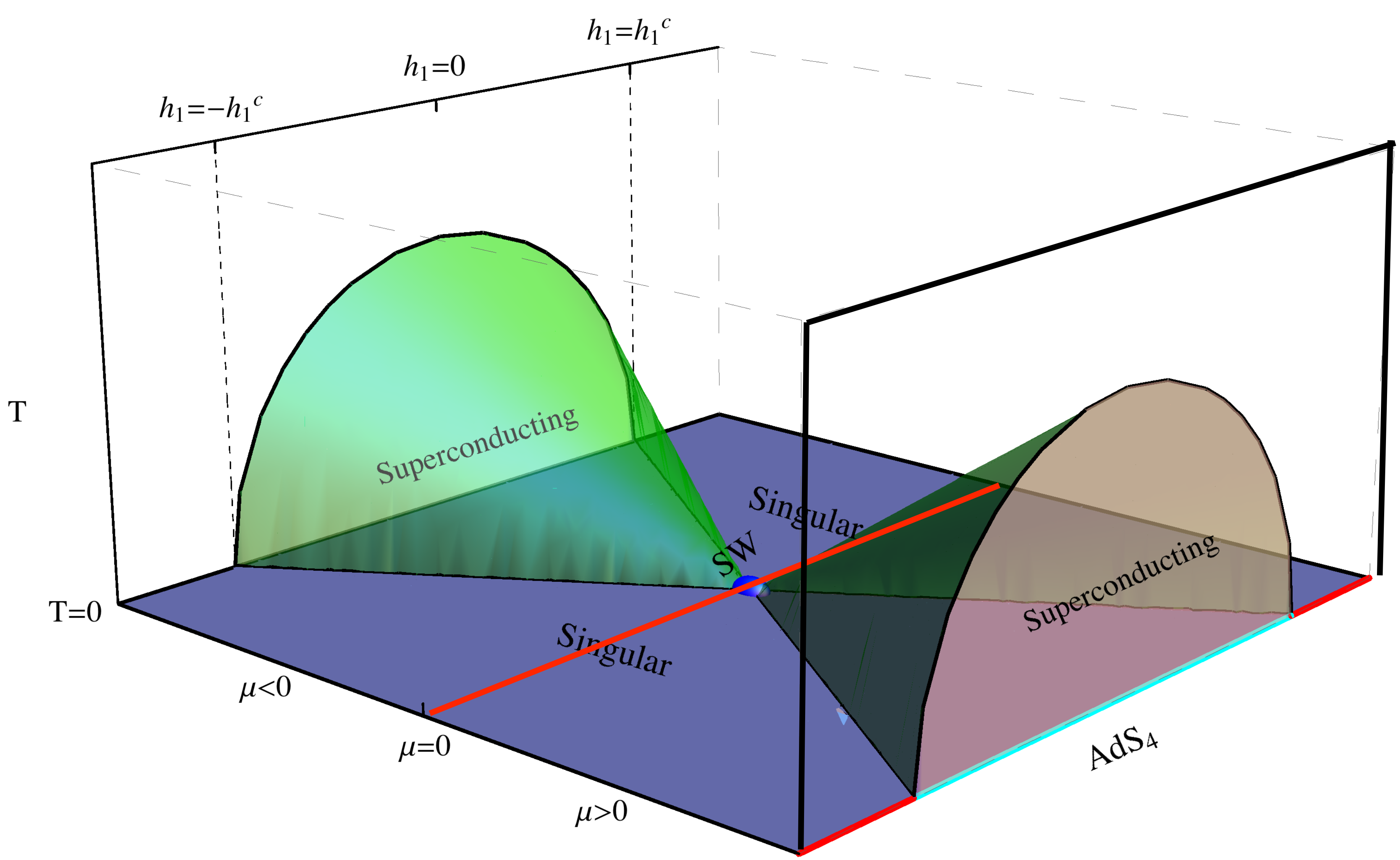


- Superconducting phase cloaks $AdS_2 \times \mathbb{R}^2$ at T=0
- At T=0 superconducting black holes become charged domain walls and approach Pope-Warner AdS_4 behaviour in far IR: universal invariant critical behaviour. (Note [HHH] model is singular in IR).
- At T=0 $c_{UV}/c_{IR} \to 0$ monotonically as $|h_1| \to h_1^c$

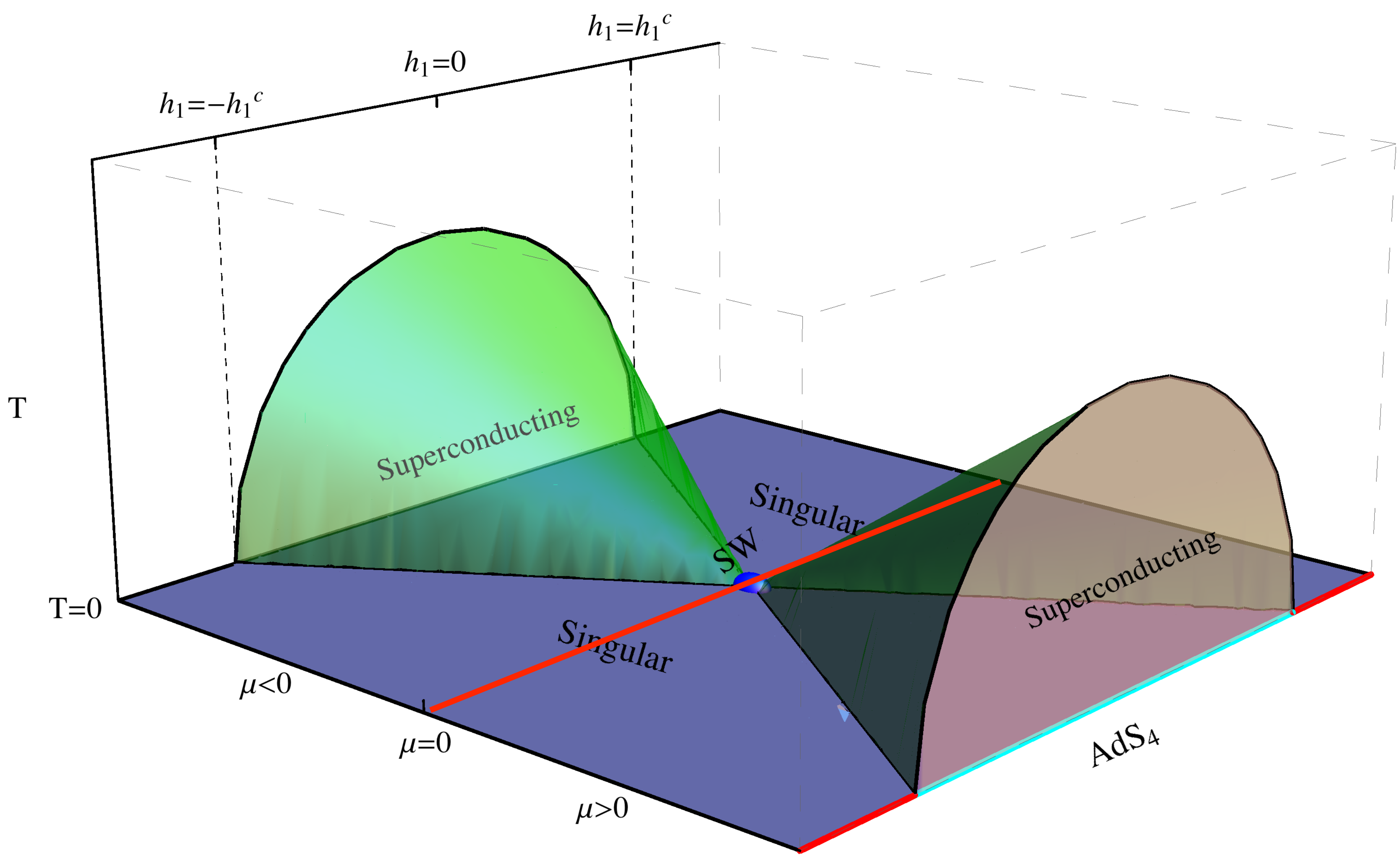
Phase Diagram including $\mu = 0$



Phase Diagram including $\mu = 0$

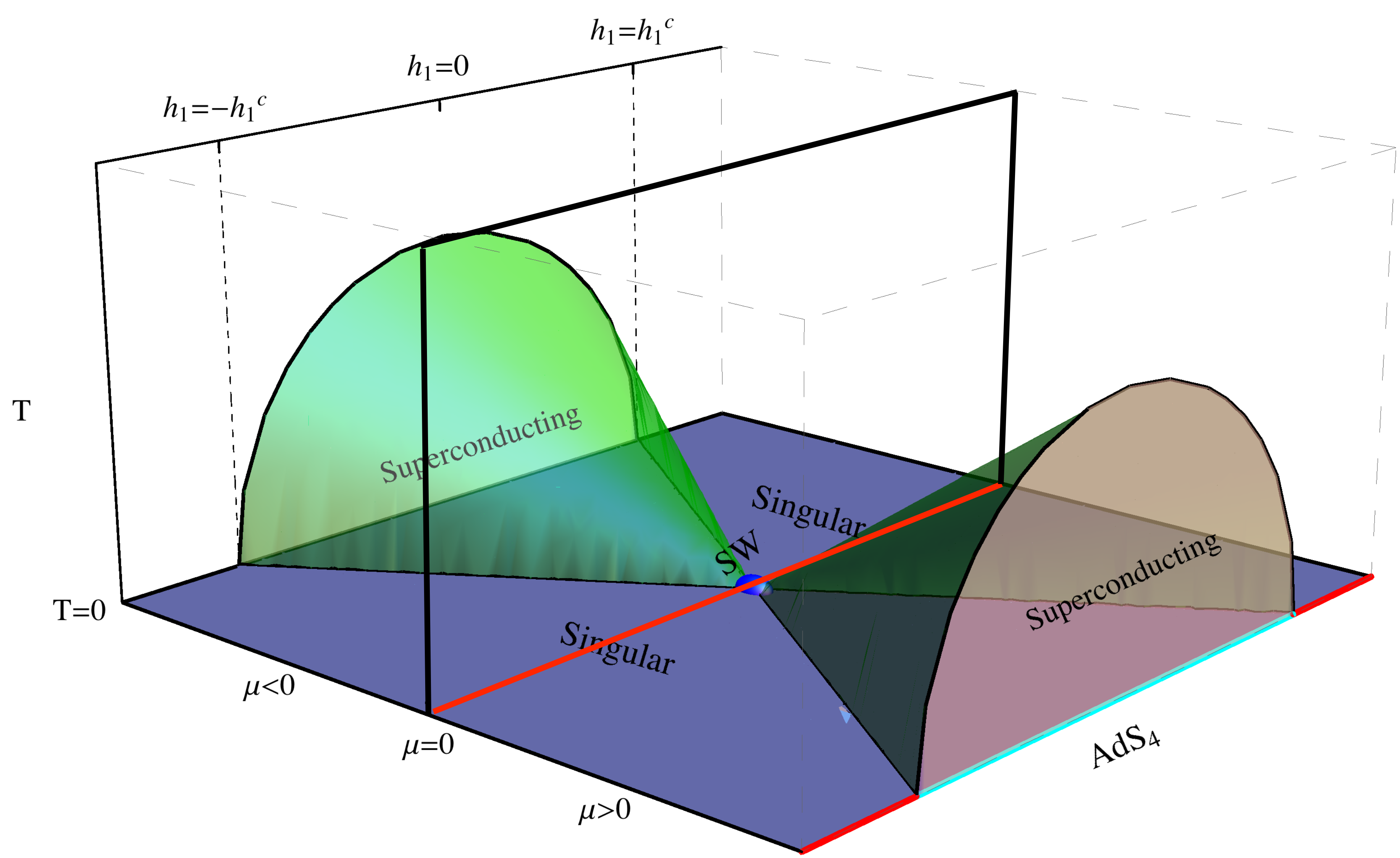


Phase Diagram including $\mu = 0$



Phase Diagram including $\mu = 0$

Further black hole solutions with $A = \chi = 0,\ h \neq 0$



Final Comments

- New consistent Kaluza-Klein truncation of D=11 SUGRA on arbitrary D=7 Sasaki-Einstein spaces. Parent truncation preserves N=2 susy and for skew-whiffed case only there is an interesting simpler truncation involving g, A, χ, h

- Why do consistent truncations work?

- Have used the truncation to construct first examples in M-theory of holographic superconductors in d=3.
  Relevant operator gives rise to a superconducting dome.
  The superconductors have an interesting universal behaviour at zero temperature described by the Pope-Warner CFT.

- Many extensions possible: magnetic fields, fermions.

- Can we make contact with real materials?