

String Dualities and Ultraviolet Behaviour of Supergravity

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STRINGS 2010

Texas A+M, College Station March 19, 2010

Jorge Russo, Pierre Vanhove,

arXiv:1001.2535; 1002.3805

Stephen Miller, Jorge Russo, Pierre Vanhove, arXiv:1003.nnnn

Related recent and not-so-recent work:

Gutperle, Basu, Sethi, Kiritsis, Obers, Pioline, Waldron, Lambert, West, . . .

I) Exact dualities of low energy type II string theory.

Compactified on d-torus - rank-(d+1) dualities.

Very rich structure of Selberg/Langlands Eisenstein series
in super-graviton scattering.

II) The low energy limit and ultraviolet divergences of maximal supergravity in various dimensions.

UV divergences of supergravity determined by Duality;
Non-decoupling of supergravity

I

Dualities of type II theory

on $d=(10-D)$ -torus $\mathcal{T}^d$

Duality invariance implies relations between
perturbative and **nonperturbative** terms in the S-matrix.

Non-trivial dependence on the moduli (scalar fields),
- in contrast to classical supergravity

moduli spaces :

discrete identification-
breaks symmetry to
discrete subgroup

$$G(\mathbb{Z}) \backslash G(\mathbb{R}) / K$$

maximal compact subgroup

U-duality groups for type II in D dimensions

on $\mathcal{T}^d$ $D = 10 - d$

	Dimension	$G_d(\mathbb{Z}) = E_{d+1}$
 decompactification	10A	1
	10B	$SL(2, \mathbb{Z})$
	9	$SL(2, \mathbb{Z})$
	8	$SL(3, \mathbb{Z}) \times SL(2, \mathbb{Z})$
	7	$SL(5, \mathbb{Z})$
	6	$SO(5, 5, \mathbb{Z})$
	5	$E_{6(6)}(\mathbb{Z})$
	4	$E_{7(7)}(\mathbb{Z})$
	3	$E_{8(8)}(\mathbb{Z})$

...

Four (super)-Graviton Scattering in type II

$$A_D(s, t, u) = A_D^{analytic}(s, t, u) + A_D^{nonan}(s, t, u)$$

local term in eff. action

contains massless thresholds
- nonlocal terms in eff. action
(IR divergent for $D \leq 4$).

$$A_D^{analytic}(s, t, u) = \mathcal{R}^4 T_D(s, t, u)$$

$$s = -(k_1 + k_2)^2, \quad t = -(k_1 + k_4)^2, \quad u = -(k_1 + k_3)^2$$

$\mathcal{R}$ Linearised supercurvature – describes 256 physical states in supermultiplet.

$$T_D(s, t, u) = \sum_{p,q} \mathcal{E}_{(p,q)}^{(D)} \sigma_2^p \sigma_3^q$$

Power series in

$$\sigma_2 = s^2 + t^2 + u^2$$

$$\sigma_3 = s^3 + t^3 + u^3$$

Coefficients are automorphic functions of moduli

Duality - invariant effective IIB action

(Einstein frame)

Einstein-Hilbert

Higher-derivative interactions

$$S = \frac{1}{\ell_D^{D-2}} \int d^D x \sqrt{-G^{(D)}} R + S^{local}$$

$$S_D^{local} = \ell_D^{8-D} \int d^D x \sqrt{-G^{(D)}} \left(\mathcal{E}_{(0,0)}^{(D)} \mathcal{R}^4 + \ell_D^4 \mathcal{E}_{(1,0)}^{(D)} \partial^4 \mathcal{R}^4 + \ell_D^6 \mathcal{E}_{(0,1)}^{(D)} \partial^6 \mathcal{R}^4 + \dots \right)$$

$\sigma_2 \mathcal{R}^4$

$\sigma_3 \mathcal{R}^4$

1/2 BPS

1/4 BPS

1/8 BPS

Planck length

$$\ell_D^{D-2} = \ell_{D+1}^{D-1} \frac{1}{r_d}$$

Is this the complete list of "protected" terms??

$\mathcal{E}_{(p,q)}^{(D)}$ are E_{d+1} - invariant coefficients
functions of moduli

How may these be determined ??

Simple cases can be determined by supersymmetry:

e.g. **D=10 IIB**

duality group $SL(2, \mathbb{Z})$ modulus $\tau = \tau_1 + i\tau_2$ $\tau_2 \equiv e^{-\phi} = g_B^{-1}$
coupling

i) For $\mathcal{E}_{(0,0)}^{(10)} \mathcal{R}^4$, $\mathcal{E}_{(1,0)}^{(10)} \sigma_2 \mathcal{R}^4$

SUSY implies Laplace eigenvalue equation

$$\Delta^{(10)} = \tau_2^2 \partial_\tau \partial_{\bar{\tau}} \longrightarrow \boxed{\Delta \mathcal{E}_{(p,0)}^{(10)} = s(s-1) \mathcal{E}_{(p,0)}^{(10)}} \quad s = p + \frac{3}{2}$$

Invariant laplacian for $SL(2, \mathbb{R})/SO(2)$ $s = \frac{3}{2}$ for $\mathcal{R}^4$ $s = \frac{5}{2}$ for $\sigma_2 \mathcal{R}^4$

Unique solution is nonholomorphic Eisenstein series

$$\mathcal{E}_{(p,0)}^{(10)} = E_s = \sum_{(m,n) \neq (0,0)} \frac{\tau_2^s}{|m + n\tau|^{2s}}$$

$$E_s = \sum_{(m,n) \neq (0,0)} \frac{\tau_2^s}{|m + n\tau|^{2s}}$$

two power behaved terms

$$\sim 2\zeta(2s)\tau_2^s + (\dots)\zeta(2s-1)\tau_2^{1-s} + \sum_{k \neq 0} \mu(k, s) (e^{2\pi i k \tau} + c.c.) (1 + O(\tau_2^{-1}))$$

TREE-level term
($\tau_2 = g_s^{-1}$)

GENUS- $(s - \frac{1}{2})$ term

D-INSTANTON terms
Infinite no. of pert. Corrections-
non-zero Fourier modes.
Interesting measure

$\int d\tau_1$ projects onto "Constant Term" - zero Fourier mode - **power behaved terms,**
kills instantons

- NO HIGHER LOOP perturbative terms
- Non-renormalization at higher loops

examples:

$$E_{\frac{3}{2}} \mathcal{R}^4 ,$$

$$E_{\frac{5}{2}} s^2 \mathcal{R}^4$$

tree-level + one-loop

tree-level + two-loop

ii) Higher order: $\mathcal{E}_{(0,1)}^{(10)} \sigma_3 \mathcal{R}^4 \sim \mathcal{E}_{(0,1)}^{(10)} \partial^6 \mathcal{R}^4$

Inhomogeneous Laplace eigenvalue equation

$$(\Delta_\tau - 12) \mathcal{E}_{(0,1)}^{(10)} = -E_{\frac{3}{2}} E_{\frac{3}{2}} = - \left(\mathcal{E}_{(0,0)}^{(10)} \right)^2$$

$\mathcal{R}^4$ source

Not (yet) derived purely from supersymmetry but motivated by four-graviton scattering amplitude.

Constant term (zero Fourier mode) contains:

Genus 0, 1, 2, 3 + instanton anti-instanton pairs

Note:

Non-renormalisation. Interactions of the form $\partial^{2k} \mathcal{R}^4$ have contributions for genus- h with $h \leq k$
(for $h > 1$)

Does this continue to hold for $k > 3$??

[comments later]

Eisenstein series for higher-rank duality groups

General Eisenstein series depends on $r = \text{rank } G$
parameters $s_i \quad i = 1, \dots, r$

But Eisenstein series for a **maximal parabolic subgroup**, $P(\beta)$, defined with respect to a simple root β , depends on only **one** non-zero parameter

$$s = s_\beta \neq 0 \qquad s_i = 0, \quad (i \neq \beta)$$

$$E_\beta^G(g; \mu) = \sum_{\gamma \in P(\mathbb{Z}) \backslash G(\mathbb{Z})} e^{\langle \mu, H(\gamma g) \rangle}$$

$$P(\mathbb{Z}) = P(\mathbb{R}) \cap G(\mathbb{Z})$$

parabolic subgroup

$H(g)$ Cartan subalgebra

Automorphise – sum over orbits

$\mu = (\mu_1, \dots, \mu_r)$ Vector in the weight lattice, parameterising Cartan torus

e.g. $GL(d)$ Parabolic subgroup of matrices of form :

$$P(n_1, \dots, n_q) = \begin{pmatrix} U & * \\ 0 & a \end{pmatrix}$$

$d \times d$ matrix $U \in GL(d-1)$ β

is associated with Dynkin label $[0 \dots 01]$

NOTATION:

$\mathbf{E}_{[0 \dots 0 1 0 \dots 0];s}^G$ = Maximal Parabolic Eisenstein series for Parabolic subgroup $P(\beta)$ associated with Dynkin label $[0 \dots 0 1 0 \dots 0]$

index root β

[Standard $SL(2, \mathbb{Z})$ Eisenstein series: $E_s = \mathbf{E}_{[1];s}^{SL(2)}$]

These series satisfy Laplace equation - quadratic Casimir

$$(\Delta_{G/K} - \lambda) \mathbf{E}_{[0\dots 0\ 1\ 0\dots 0]}^G; s = 0$$

eigenvalue depends on G, β, s

$SL(d)$ and $SO(5,5)$ series can be analysed by explicit sums over integer lattice.

But such lattice sums are much more subtle for other duality groups E_6, E_7, E_8

Evaluate sums by “brute force” (Mathematica)

e.g, for $SL(d)$:

$$\mathbf{E}_{[10\dots 0];s}^{SL(d)} = \sum_{\mathbf{m} \in \mathbb{Z}^d} \frac{1}{(m^i G_{ij} m^j)^s}$$

Epstein Series

$$\mathbf{E}_{[01\dots 0];s}^{SL(d)} = \sum_{\mathbf{m}, \mathbf{n} \in \mathbb{Z}^d} \frac{1}{(m^{[i} n^{j]} G_{ik} G_{j\ell} m^{[k} n^{l]})^s}$$

antisymmetrised sums

.....

Lattice sums are much more subtle for other duality groups $SO(5,5), E_6, E_7, E_8$

Steve Miller

Evaluate sums by "brute force" (Mathematica)

Consistency conditions determine coefficients:

Laplace equations + boundary conditions

Laplace equations in various dimensions:

Decompactification of laplacian starting from D and using D=10 expressions as boundary conditions.

$$\Delta^{(D)} \equiv \Delta_{E_{11-D}/K_D}$$

$$R^4 \quad \left(\Delta^{(D)} - \frac{3(11-D)(D-8)}{D-2} \right) \mathcal{E}_{(0,0)}^{(D)} = 6\pi \delta_{D-8,0}$$

Singularity in D=8

"critical"
dimensions

$$\partial^4 R^4 \quad \left(\Delta^{(D)} - \frac{5(12-D)(D-7)}{D-2} \right) \mathcal{E}_{(1,0)}^{(D)} = 40\zeta(2) \delta_{D-7,0}$$

Singularity in D=7

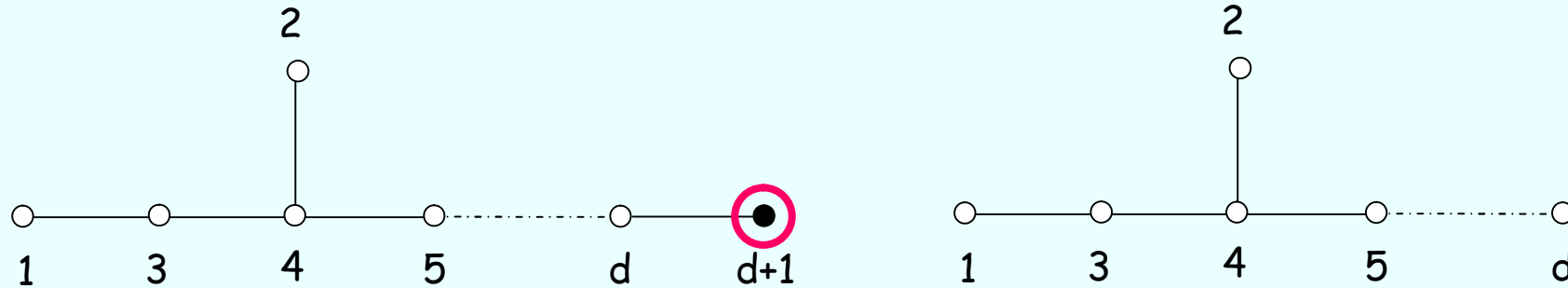
$$\partial^6 R^4 \quad \left(\Delta^{(D)} - \frac{6(14-D)(D-6)}{D-2} \right) \mathcal{E}_{(0,1)}^{(D)} = - \left(\mathcal{E}_{(0,0)}^{(D)} \right)^2 + 120\zeta(3) \delta_{D-6,0}$$

Singularity in D=6

Consistent degeneration limits of torus:

(i) Decompactify from D to $D+1$ dimensions

$$E_{d+1} \rightarrow E_d \quad (d = 10 - D) \quad r_d \rightarrow \infty$$



Power-behaved terms in $\int_{P(\alpha_{d+1})} \mathcal{E}_{(p,q)}^{(D)}$ (in r_d) zero Fourier mode - kills instantons

"Constant term in parabolic subgroup" $P(\alpha_{d+1})$

(i) Decompactification of circle radius r_d

Consistency conditions :

$D = 10 - d$

terms surviving in D+1

terms needed to reproduce thresholds (unitarity)

$$\begin{aligned}
 R^4 \quad \left(\frac{\ell_{D+1}}{l_D} \right)^{8-D} \int_{P(\alpha_d)} \mathcal{E}_{(0,0)}^{(D)} &= \frac{r_d}{\ell_{D+1}} \mathcal{E}_{(0,0)}^{(D+1)} + \left(\frac{r_d}{l_{D+1}} \right)^{8-D} \\
 \partial^4 R^4 \quad \left(\frac{\ell_{D+1}}{l_D} \right)^{12-D} \int_{P(\alpha_d)} \mathcal{E}_{(1,0)}^{(D)} &= \frac{r_d}{\ell_{D+1}} \mathcal{E}_{(1,0)}^{(D+1)} + \left(\frac{r_d}{l_{D+1}} \right)^{6-D} \mathcal{E}_{(0,0)}^{(D+1)} + \left(\frac{r_d}{l_{D+1}} \right)^{12-D} \\
 \partial^6 R^4 \quad \left(\frac{\ell_{D+1}}{l_D} \right)^{14-D} \int_{P(\alpha_d)} \mathcal{E}_{(1,0)}^{(D)} &= \frac{r_d}{\ell_{D+1}} \mathcal{E}_{(0,1)}^{(D+1)} + \left(\frac{r_d}{l_{D+1}} \right)^{4-D} \mathcal{E}_{(1,0)}^{(D+1)} \\
 &\quad + \left(\frac{r_d}{l_{D+1}} \right)^{8-D} \mathcal{E}_{(0,0)}^{(D+1)} + \left(\frac{r_d}{l_{D+1}} \right)^{12-D}
 \end{aligned}$$

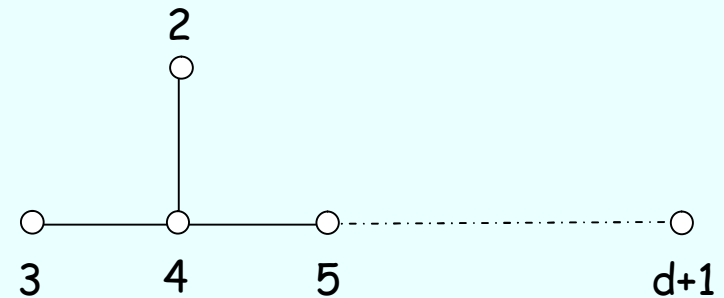
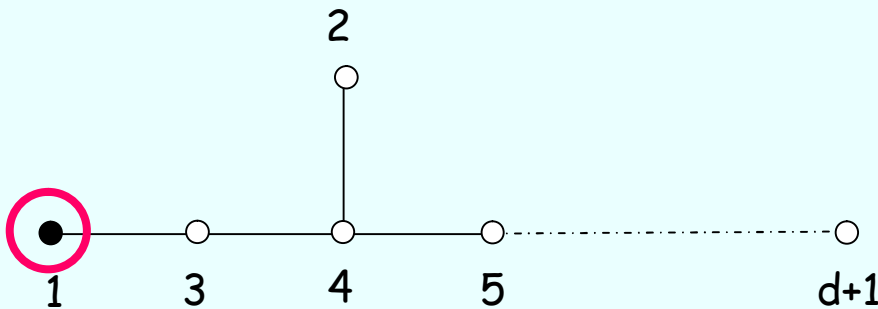
- Limited set of powers of r
- Note that all the **D+1** coefficients are contained in $\partial^6 R^4$

(ii) String perturbation expansion

weak
Coupling

$$\frac{1}{g_s^2} V_d = \frac{1}{y_D} \rightarrow \infty \quad \text{fixed } \frac{r_d}{l_s}$$

$$E_{d+1} \rightarrow SO(d, d) \quad \text{T-duality}$$



Perturbative terms in $\int_{P(\alpha_1)} \mathcal{E}_{(p,q)}^{(D)}$

Powers of y_D correspond to world-sheet genus

Constant term in parabolic subgroup $P(\alpha_1)$

(ii) String perturbation expansion

Powers of y_D correspond to world-sheet genus

genus 0 1 2 3

$$R^4 \int_{P(\alpha_1)} \mathcal{E}_{(0,0)}^{(D)} = \frac{l_s^{8-D}}{l_D^{8-D}} \left(\frac{2\zeta(3)}{y_D} + I_{(0,0)}^{(1)} \right)$$

$$\partial^4 R^4 \int_{P(\alpha_1)} \mathcal{E}_{(1,0)}^{(D)} = \frac{l_s^{12-D}}{l_D^{12-D}} \left(\frac{2\zeta(4)}{y_D} + I_{(1,0)}^{(1)} + y_D I_{(1,0)}^{(2)} \right)$$

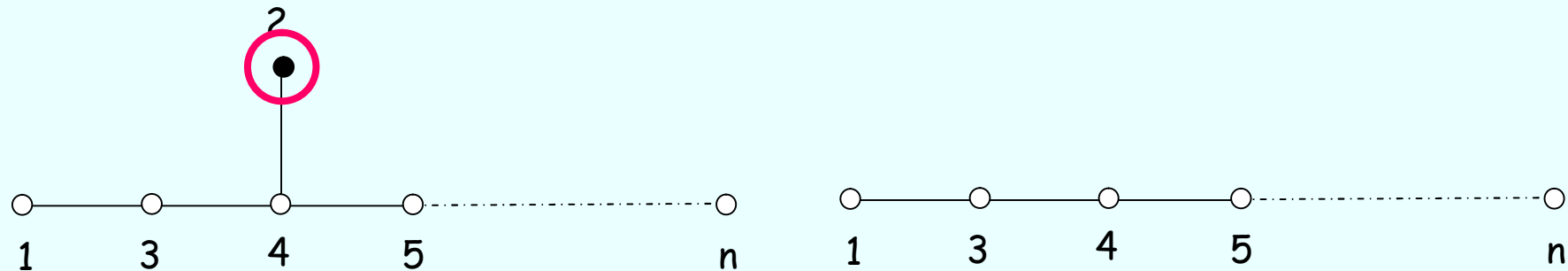
$$\partial^6 R^4 \int_{P(\alpha_1)} \mathcal{E}_{(0,1)}^{(D)} = \frac{l_s^{14-D}}{l_D^{14-D}} \left(\frac{2\zeta^2(3)}{3y_D} + I_{(0,1)}^{(1)} + y_D I_{(0,1)}^{(2)} + y_D^2 I_{(0,1)}^{(3)} + O(e^{-4/y_D}) \right)$$

(iii) 11-dimensional supergravity limit $\mathcal{V}_{d+1} \rightarrow \infty$
 vol. of M-theory torus

Large M-theory torus matches Feynman diagrams
 of eleven-dimensional supergravity

one, two loops

$E_{d+1} \rightarrow SL(d+1)$ geometric symmetry



Constant term in parabolic subgroup $P(\alpha_2)$

Perturbative terms in $\int_{P(\alpha_2)} \mathcal{E}_{(p,q)}^{(D)}$

SOLUTIONS to Laplace equations and b.c.'s:

e.g. $D = 3, (E_8)$

MBG, Miller, Russo, Vanhove 2010
(Pioline 2010)

$$R^4 \quad \mathcal{E}_{(0,0)}^{(3)} = \mathbf{E}_{[100000000]}^{E_8}; \frac{3}{2}$$

$$\partial^4 R^4 \quad \mathcal{E}_{(0,0)}^{(3)} = \mathbf{E}_{[100000000]}^{E_8}; \frac{5}{2}$$

More complicated for

$$\mathcal{E}_{(0,1)}^{(3)} \partial^6 \mathcal{R}^4$$

Perturbative expansions match in all three limits.

How can these expressions be interpreted in terms of sums over
1/2-BPS, 1/4-BPS, 1/8-BPS states ??

Solutions in all dimensions for $\mathcal{R}^4$ and $\partial^4 \mathcal{R}^4$

$G_d = E_{d+1(d+1)}$	$\mathcal{R}^4$	$\partial^4 \mathcal{R}^4$
$E_{8(8)}(\mathbb{Z})$	$\mathbf{E}_{[10000000]; \frac{3}{2}}^{E_8}$	$\frac{1}{2} \mathbf{E}_{[10000000]; \frac{5}{2}}^{E_8}$
$E_{7(7)}(\mathbb{Z})$	$\mathbf{E}_{[1000000]; \frac{3}{2}}^{E_7}$	$\frac{1}{2} \mathbf{E}_{[1000000]; \frac{5}{2}}^{E_7}$
$E_{6(6)}(\mathbb{Z})$	$\mathbf{E}_{[100000]; \frac{3}{2}}^{E_6}$	$\frac{1}{2} \mathbf{E}_{[100000]; \frac{5}{2}}^{E_6}$
$SO(5, 5, \mathbb{Z})$	$\mathbf{E}_{[10000]; \frac{3}{2}}^{SO(5,5)}$	$\frac{1}{2} \hat{\mathbf{E}}_{[10000]; \frac{5}{2}}^{SO(5,5)} + \frac{4}{45} \hat{\mathbf{E}}_{[00001]; 3}^{SO(5,5)}$
$SL(5, \mathbb{Z})$	$\mathbf{E}_{[1000]; \frac{3}{2}}^{SL(5)}$	$\frac{1}{2} \hat{\mathbf{E}}_{[1000]; \frac{5}{2}}^{SL(5)} + \frac{3}{\pi^3} \hat{\mathbf{E}}_{[0010]; \frac{5}{2}}^{SL(5)}$
$SL(3, \mathbb{Z}) \times SL(2, \mathbb{Z})$	$\hat{\mathbf{E}}_{[10]; \frac{3}{2}}^{SL(3)} + 2\hat{\mathbf{E}}_1(U)$	$\frac{1}{2} \mathbf{E}_{[10]; \frac{5}{2}}^{SL(3)} - 4 \mathbf{E}_{[10]; -\frac{1}{2}}^{SL(3)} \mathbf{E}_2(U)$
$SL(2, \mathbb{Z})$	$\mathbf{E}_{\frac{3}{2}}(\Omega) \nu_1^{-\frac{3}{7}} + 4\zeta(2) \nu_1^{\frac{4}{7}}$	$\frac{1}{2} \nu_1^{-\frac{5}{7}} \mathbf{E}_{\frac{5}{2}}(\Omega) + \frac{2\zeta(2)}{15} \nu_1^{\frac{9}{7}}$
$SL(2, \mathbb{Z})$	$\mathbf{E}_{\frac{3}{2}}(\Omega)$	$\frac{1}{2} \mathbf{E}_{\frac{5}{2}}(\Omega)$

$D = 8$

$D = 7$

$\nu_1 = \left(\frac{\ell_{10}}{r_B} \right)^2$

(i) Decompactification of circle radius r

from $D=3$ to $D=4$

$$R^4 \quad \mathbf{E}_{[10000000]; \frac{3}{2}}^{E_8} = r^6 \mathbf{E}_{[1000000]; \frac{3}{2}}^{E_7} + r^{10} \frac{3\zeta(5)}{\pi} \quad \text{+ instantons}$$

R^4 coefficient in $D=4$

contribute to thresholds

$\partial^4 R^4$

$$\mathbf{E}_{[10000000]; \frac{5}{2}}^{E_8} = r^{10} \frac{1}{2} \mathbf{E}_{[100000]; \frac{5}{2}}^{E_7} + r^{12} \frac{\zeta(3)}{\pi} \mathbf{E}_{[100000]; \frac{3}{2}}^{E_7} + r^{18} \frac{\pi^2 \zeta(9)}{15}$$

$\partial^4 R^4$ coefficient in $D=4$

Note: **Many** terms have to vanish for this to work!!

$$\mathbf{E}_{[10000000]; s}^{E_8} = 756r^{36} + 126r^{92-8s} + 576r^{60-4s} + 126r^{8s} + 576r^{14+4s}$$

(setting all coefficient series = 1 for simplicity)

(ii) D=3 String Perturbation Theory

e.g., D=3 (d=7) String coupling, $r^{-4} = y_3 = \frac{g_s^2}{V_7}$

$$R^4 \quad \mathbf{E}_{[10000000]; \frac{3}{2}}^{E_8} = r^{24} 2\zeta(3) + r^{20} \frac{3}{2\pi} \mathbf{E}_{[1000000]; \frac{5}{2}}^{SO(7,7)} + \text{instantons}$$

tree-level genus-one genus-two

$$\partial^4 R^4 \quad \mathbf{E}_{[1000000]; \frac{5}{2}}^{E_8} = r^{40} \zeta(5) + \frac{7}{24\pi} r^{36} \mathbf{E}_{[1000000]; \frac{9}{2}}^{SO(7,7)} + \frac{2}{3} r^{32} \mathbf{E}_{[0000010]; 2}^{SO(7,7)}$$

Verify by explicit genus-one and genus-two calculations

Note nonrenormalisation conditions

(iii) 11-dimensional supergravity limit $\mathcal{V}_{d+1} \rightarrow \infty$

e.g., D=3 (d=7)

$r^{1+d} = \mathcal{V}_{d+1}$ vol. of M-theory torus

$$R^4 \mathbf{E}_{[10000000]; \frac{3}{2}}^{E_8} = r^{32} 4\zeta(2) + r^{30} \mathbf{E}_{[1000000]; \frac{3}{2}}^{SL(8)} + \text{instantons}$$

$\partial^4 R^4$

$$\mathbf{E}_{[10000000]; \frac{5}{2}}^{E_8} = \frac{1}{2} r^{50} \mathbf{E}_{[1000000]; \frac{5}{2}}^{SL(8)} + \frac{2\zeta(3)}{\pi^2} r^{48} \mathbf{E}_{[0100000]; 2}^{SL(8)} + \frac{4\zeta(4)}{3} r^{54} \mathbf{E}_{[10000000]; -\frac{1}{2}}^{SL(8)}$$

Precisely reproduced by one-loop and two-loop supergravity in eleven dimensions on (d+1)-torus.

very

NOTE: Many terms have to vanish for this to work!!

For generic s

$$\mathbf{E}_{[10000000];s}^{E_8} \sim 64r^{6s+1} + 448r^{41-2s} + 280r^{4(s+2)} + 448r^{2(s+9)} \\ + r^{92-8s} + r^{8s} + 64r^{70-6s} + 280r^{54-4s} + 14r^{34} + 560r^{28}$$

(setting all coefficient series = 1 for simplicity)

All but 2 or 3 vanish for $s=3/2$ or $5/2$

II

Determining Supergravity UV divergences

MBG, Russo, Vanhove 2010

"Critical" Cases

Maximal SUGRA has logarithmic UV divergences at:

protected F-terms	{	One loop in D=8	R^4	}	$D = 4 + 6/L$
		Two loops in D=7	$\partial^4 R^4$		
		Three loops in D=6	$\partial^6 R^4$		
		Four loops in D=11/2	$\partial^8 R^4$		
		Five loops in D=???	$\partial^{??} R^4$		

These can be determined by duality.

(i) $D=8$ R^4

poles in ϵ cancel

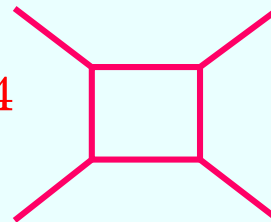
$$\mathcal{E}_{(0,0)}^{(8)} = \lim_{\epsilon \rightarrow 0} \left(\mathbf{E}_{[10]; \frac{3}{2} + \epsilon}^{SL(3)} + 2\mathbf{E}_{1-2\epsilon}^{SL(2)} \right) = \hat{\mathbf{E}}_{[10]; \frac{3}{2}}^{SL(3)} + 2\hat{\mathbf{E}}_1^{SL(2)}$$

- Poles in ϵ cancel (UV finiteness of string theory)
- Completely determined by finite expression in $D=7$
- String perturbation theory (in Einstein frame)

$$\int_{P(\alpha_1)} \mathcal{E}_{(0,0)}^{(8)} = \frac{2\zeta(3)}{y_8} + 2(\hat{\mathbf{E}}_1(T) + \hat{\mathbf{E}}_1(U)) + \frac{2\pi^2}{3} \log y_8$$

Recall maximal SUGRA

R^4



1-loop logarithm

$\log y_8$ must come from $\log(s l_s^2)$ in the string frame

$$\log(s l_s^2) = \log(s l_D^2) - \frac{1}{D-2} \log y_D$$

Weyl dilaton factor $l_s^{D-2} = l_D^{D-2} y_D^{-1}$

Coefficient is determined by unitarity and must be the same as the coefficient of $c \log(s/\Lambda^2)$ in supergravity.

Ultraviolet cutoff in supergravity

The coefficient of $\log y_8$ in $\mathcal{E}_{(0,0)}^{(8)}$ determines the UV logarithm in one-loop maximal supergravity in D=8 R^4

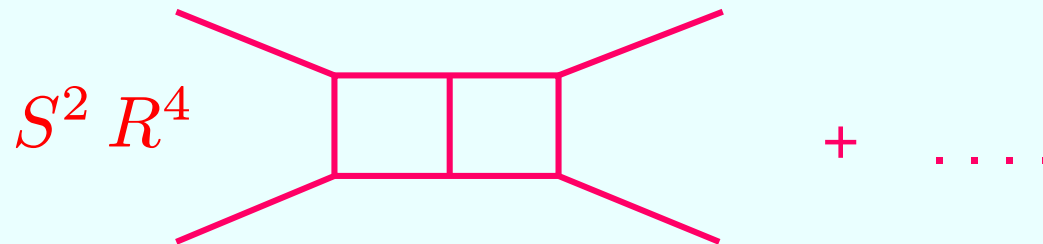
(ii) D=7 $\partial^4 R^4$

$$\mathcal{E}_{(1,0)}^{(7)} = \frac{1}{2} \hat{\mathbf{E}}_{[1000]; \frac{5}{2}}^{SL(5)} + \frac{3}{\pi^2} \hat{\mathbf{E}}_{[0010]1}^{SL(5)} \quad \text{poles cancel again}$$

- String perturbation theory

$$\int_{P(\alpha_1)} \mathcal{E}_{(1,0)}^{(7)} = \underbrace{\frac{\zeta(5)}{y_7^2}}_{\text{tree}} + \underbrace{\frac{1}{y_7}}_{\text{1-loop}} (\dots) + \underbrace{(\dots)}_{\text{2-loop}} + \underbrace{\frac{8\pi^2}{15} \log y_7}_{\text{2-loop logarithm}}$$

Agrees with **two-loop** supergravity UV divergence in D=7



Bern, Dixon, Dunbar, Perelstein, Rozowsky, 1998

(iii) D=6 $\partial^6 R^4$

$$\int_{P(\alpha_1)} \mathcal{E}_{(0,1)}^{(6)} = \underbrace{\frac{2\zeta(3)^2}{3 y_6^3}}_{\text{tree}} + \underbrace{\frac{1}{y_6^2} (\dots)}_{\text{1-loop}} + \underbrace{\frac{1}{y_6} (\dots)}_{\text{2-loop}} + \underbrace{(\dots)}_{\text{3-loop}} + \underbrace{15\zeta(3) \log y_6}_{\text{3-loop logarithm}} + n.p.$$

Agrees with **three-loop** supergravity UV divergence in D=8 *

$$S^3 R^4 \quad \text{[Diagram: a circle with three internal lines meeting at a central point and three external lines extending from the circumference]} \quad + \dots$$

Bern, Carrasco, Dixon, Johansson, Kosower, Roiban 2007

* Factor of 6 missing??

(iii) $D=8$ $\partial^6 R^4$ one-loop subdivergence
 R^4 coeff.

$$\mathcal{E}_{(0,1)}^{(8)} = \cdots + \frac{\pi}{9} \left(\frac{\pi}{6} + \mathbf{E}_{(0,0)}^{pert} \right) \log y_8 - \frac{\pi^2}{27} \log^2 y_8 + n.p.$$

two-loop

Agrees with $D=8$ two-loop supergravity calculations

Sum of double pole and single pole –
 includes effects of counterterm diagram

$$S^3 R^4 \left[\text{diagram 1} + \text{diagram 2} \right]$$

ε double pole

BUT

Supergravity limit of string theory

Ooguri, MBG, Schwarz 2006

MBG, Russo, Vanhove 2010

Whether or not there are ultraviolet divergences, supergravity does not decouple from string theory.

The supergravity limit:

Fixed $\ell_D^{D-2} = \ell_s^{D-2} y_D$ $\ell_s \rightarrow 0$ (decouples string excitations)

i.e. $y_D \rightarrow \infty$

Infinite towers of charged BPS black hole states/instantons become **massless** and/or instantons have **zero action**.

Wrapped p-branes; KK charges; KK monopoles.

Supergravity limit of string theory

Explicitly: e.g. D=8 R^4

String perturbation limit: $y_8 \rightarrow 0$

$$\mathcal{E}_{(0,0)}^{(8)} = \underbrace{\frac{2\zeta(3)}{y_8}}_{\text{tree}} + 2(\hat{\mathbf{E}}_1(T) + \hat{\mathbf{E}}_1(U)) \underbrace{+ \frac{2\pi}{3} \log y_8}_{\text{one loop}} + O(e^{-(y_8 T_2)^{-\frac{1}{2}}}, e^{-(y_8/T_2)^{-\frac{1}{2}}})$$

D-instantons
Wrapped Dp-string World-sheets

Supergravity limit $y_8 \rightarrow \infty$

action $\rightarrow 0$

Perturbative terms swamped by condensate of
zero action instantons - Poisson resum. gives

$$\mathcal{E}_{(0,0)}^{(8)} = y_8^{\frac{1}{2}} \mathbf{E}_{\frac{3}{2}}(T) - \frac{4\pi}{3} \log y_8 + O(e^{-\sqrt{y_8 T_2}}, e^{-\sqrt{y_8/T_2}}).$$

Comments on $\partial^8 \mathcal{R}^4$:

- Four-loop supergravity UV divergence in $D = \frac{11}{2}$

Bern, Carrasco, Dixon, Johansson, Roiban 2009

Is there a **FIVE-LOOP** contribution ??

- Indications of genus $h = 5$ contribution to $\partial^8 \mathcal{R}^4$

MBG, Russo, Vanhove 2010

Duality argument

MBG, Russo, Vanhove 2008

Pure spinor formalism

Berkovits, MBG, Russo, Vanhove 2009

[violates assertion that $\partial^{2k} \mathcal{R}^4$ only receives contributions from $k \leq 5$ when $h \leq k$ Berkovits 2006]

- Suggests $\partial^8 \mathcal{R}^4$ is a “D-term” contributions from all h

Would lead to **SEVEN-LOOP UV** divergence in N=8 SUGRA
(also suggested by certain superspace arguments)

$$D = 4$$

(NOTE: If $h \leq k$ for all k , UV divergences **absent** for $D < 4 + \frac{6}{h}$)