Recent Developments in Holographic Superconductors

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Outline

1) Review basic ideas behind holographic superconductors
2) New view of conductivity and the zero temperature limit (Roberts, G.H. 0908.3677)
3) Fermionic probes of holographic superconductors (Faulkner, McGreevy, Roberts, Vegh, G.H., 0911.3402. Related work by Chen, Kao, Wen; and Gubser, Rocha, Talavera.)

Superconductivity 101

In conventional superconductors (Al, Nb, Pb, …) pairs of elections with opposite spin can bind to form a charged boson called a Cooper pair.

Below a critical temperature T_c, there is a second order phase transition and these bosons condense.

The DC conductivity becomes infinite.

This is well described by BCS theory.

The new high T_c superconductors were discovered in 1986. These cuprates (e.g. YBaCuO) are layered and superconductivity is along CuO_2 planes.

Highest T_c today (HgBaCuO) is T_c = 134K.

Another class of superconductors discovered in 2008 based on iron and not copper FeAs(…) Highest T_c = 56K.

The pairing mechanism is not well understood. Unlike BCS theory, it involves strong coupling.

AdS/CFT is an ideal tool to study strongly coupled field theories.

Gravity Dual of a Superconductor

(Hartnoll, Herzog, and G.H., 2008)

Gravity	Superconductor
Black hole	Temperature
Charged scalar field	Condensate

Need to find a black hole that has scalar hair at low temperatures, but no hair at high temperatures.

Gubser (2008) argued that a charged scalar field around a charged black hole would have the desired property. Consider

$$\mathcal{L} = R + \frac{6}{L^2} - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} - |\partial\Psi - iqA\Psi|^2 - m^2|\Psi|^2$$

For an electrically charged black hole, the effective mass of Ψ is

$$m_{eff}^2 = m^2 + q^2 g^{tt} A_t^2$$

But the last term is negative. There is a chance for nontrivial hair.

General argument for instability

(Denef and Hartnoll, 2009)

Consider a scalar field with mass m and charge q in the near horizon geometry of an extremal Reissner-Nordstrom AdS black hole. Get a wave equation in AdS_2 with effective mass

$$m_{eff}^2 = \frac{m^2 - 2q^2}{6}$$

The extremal RN AdS black hole is unstable when this is below -1/4, the BF bound for AdS_2. The condition for instability is

$$m^2 - 2q^2 < -3/2$$

Hairy black holes

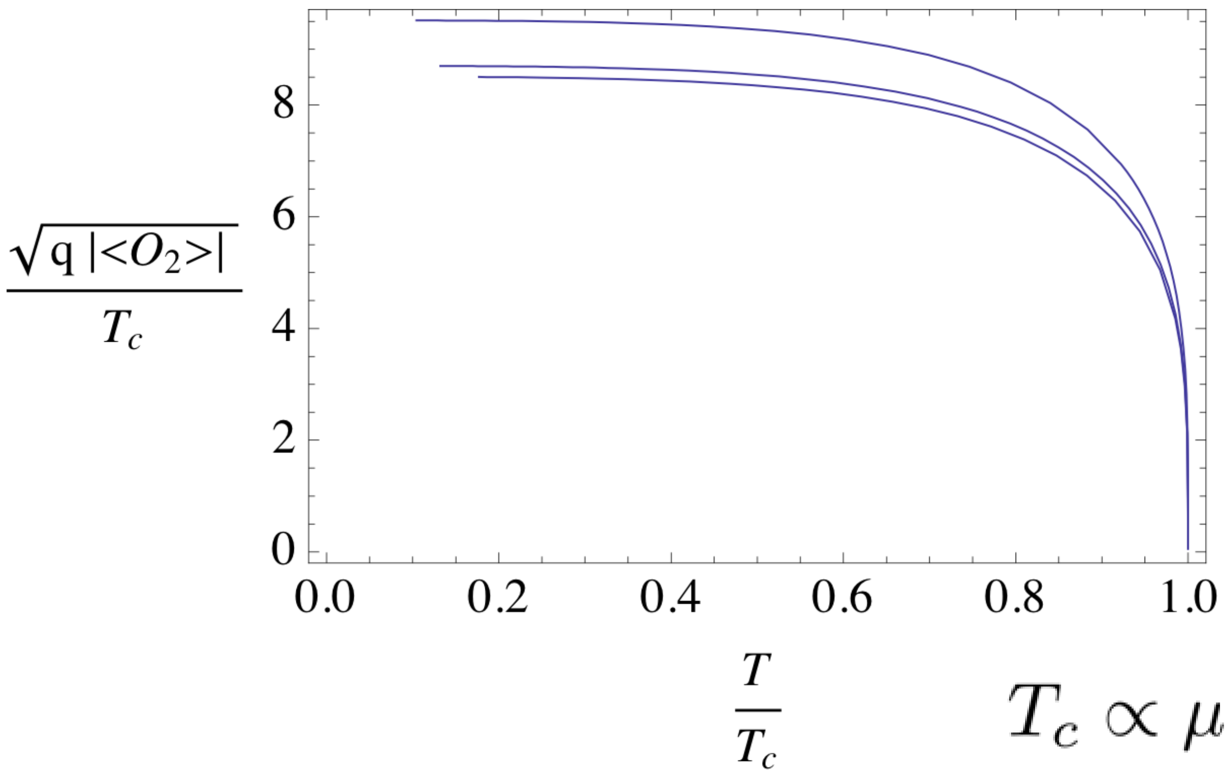
Look for static, homogeneous solutions:

$$ds^2 = -g(r)e^{-\chi(r)}dt^2 + \frac{dr^2}{g(r)} + r^2\left(dx^2 + dy^2\right)$$

$$A = \phi(r)dt, \quad \Psi = \psi(r)$$

Get four coupled nonlinear ODE's. At the horizon, $r = r_0$, g and Φ vanish, χ is constant. Asymptotically, metric approaches AdS_4 and

$$\phi = \mu - \frac{\rho}{r}, \qquad \psi = \frac{\psi_0}{r^\lambda}$$



$T_c \propto \mu$

Condensate as a function of temperature for $m^2 = -2$; q = 3,6,12.

(Hartnoll, Herzog, G.H., 2008)

Conductivity (as a function of ω)

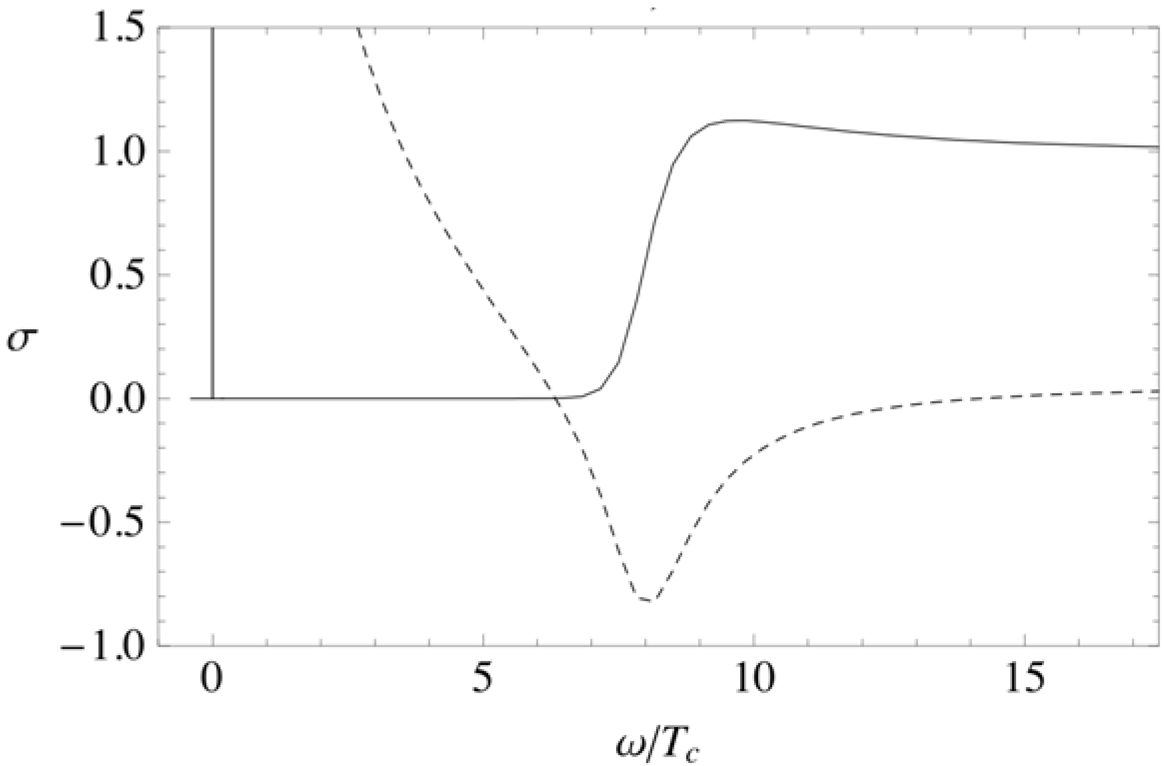
Solve for perturbation in A_x in the bulk with ingoing wave boundary conditions at the horizon. Maxwell's equation with zero spatial momentum and time dependence $e^{-i\omega t}$ gives

$$A_x'' + \left[\frac{g'}{g} - \frac{\chi'}{2}\right] A_x' + \left[\left(\frac{\omega^2}{g^2} - \frac{\phi'^2}{g}\right) e^{\chi} - \frac{2q^2\psi^2}{g}\right] A_x = 0$$

If $A_x = A_x^{(0)} + A_x^{(1)}/r$

The conductivity is

$$\sigma(\omega) = -\frac{i}{\omega}\frac{A_x^{(1)}}{A_x^{(0)}}$$



Low temperature limit of conductivity. Solid line is real part, dashed line is imaginary part. (Hartnoll, Herzog, G.H., 2008)

New View of Conductivity Calculation

(Roberts, G.H., 2009)

Introduce a new radial variable z so equation for the perturbed vector potential takes the form of a standard Schrodinger equation:

$$-A_{x,zz} + V(z)A_x = \omega^2 A_x$$

V vanishes at the horizon (z = - ∞) and at infinity (z = 0). Want ingoing wave boundary conditions at the horizon. Set V = 0 for z > 0, and send in a wave from the right:

$$A_x = e^{-i\omega z} + Re^{i\omega z} \text{ for } z > 0$$

Recall that if: $A_x = A_x^{(0)} + A_x^{(1)}/r$

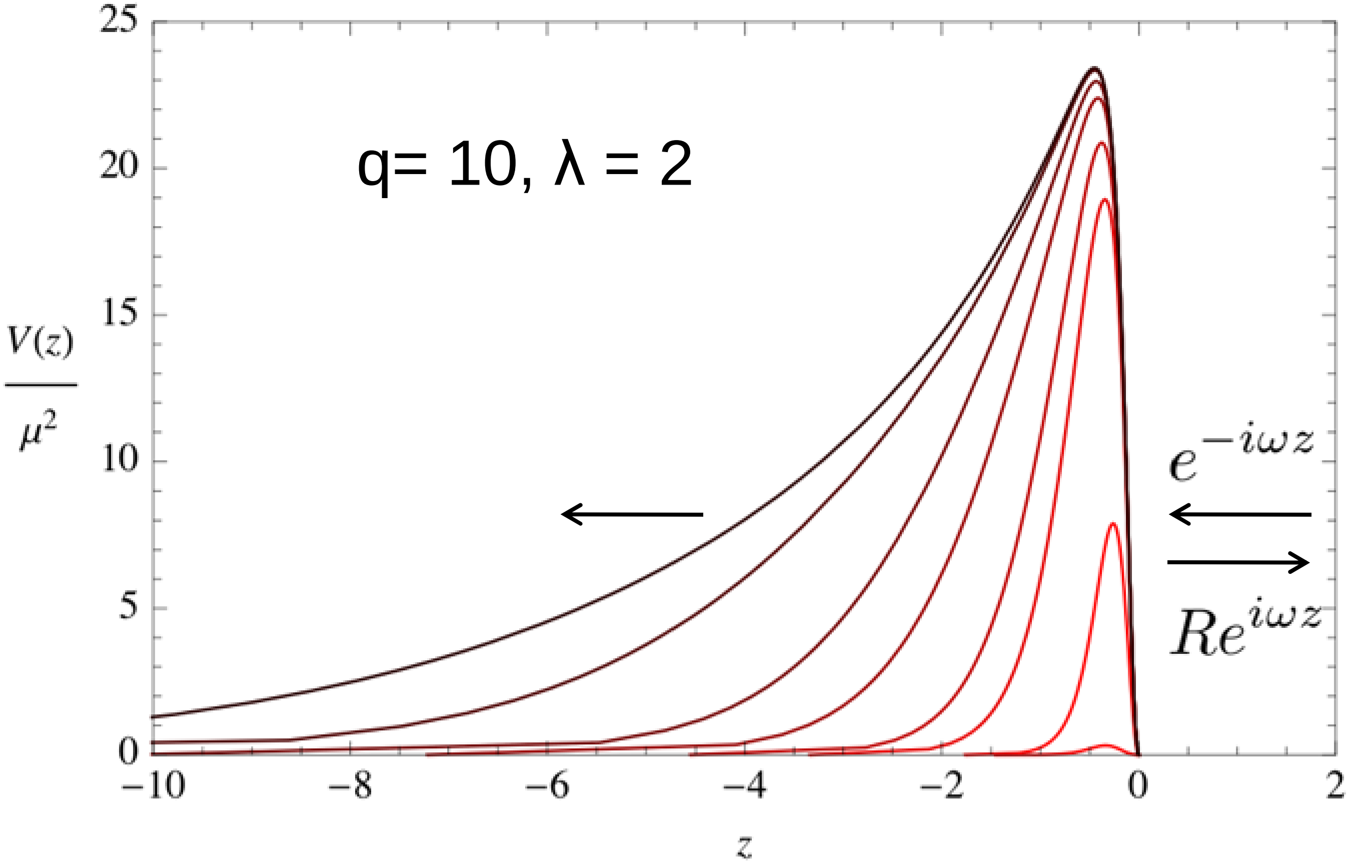
The conductivity is: $\sigma(\omega) = -\frac{i}{\omega}\frac{A_x^{(1)}}{A_x^{(0)}}$

In new formulation: $A_x^{(0)} = A_x(0) = 1 + R$

and $A_x^{(1)} = -A_{x,z}(0) = i\omega(1 - R)$

so

$$\sigma(\omega) = \frac{1 - R}{1 + R}$$



The potential grows as T/T_c gets smaller.

V remains finite at T = 0, so Re σ(ω) ≠ 0.

There is no hard gap in the conductivity.

(See also Basu, He, Mukherjee, Shieh, 2009)

Ground State of Holographic Superconductors (Roberts, G.H., 2009)

The Reissner Nordstrom AdS black hole has large entropy at T =0. If this was dual to a condensed matter system, it would mean the ground state was highly degenerate.

The extremal limit of the hairy black holes is not like Reissner Nordstrom. It has zero horizon area. It also has zero charge (except when q=0).

Case 1: $m = 0$ and $q^2 > 3/4$

These solutions approach AdS_4 near $r = 0$ (with the same value of the cosmological constant as infinity).

The holographic superconductor has emergent Poincare (and conformal) symmetry in the IR

Solution describes a charged scalar domain wall.

Case 2: $m^2 < 0$, $q^2 > |m^2|/6$

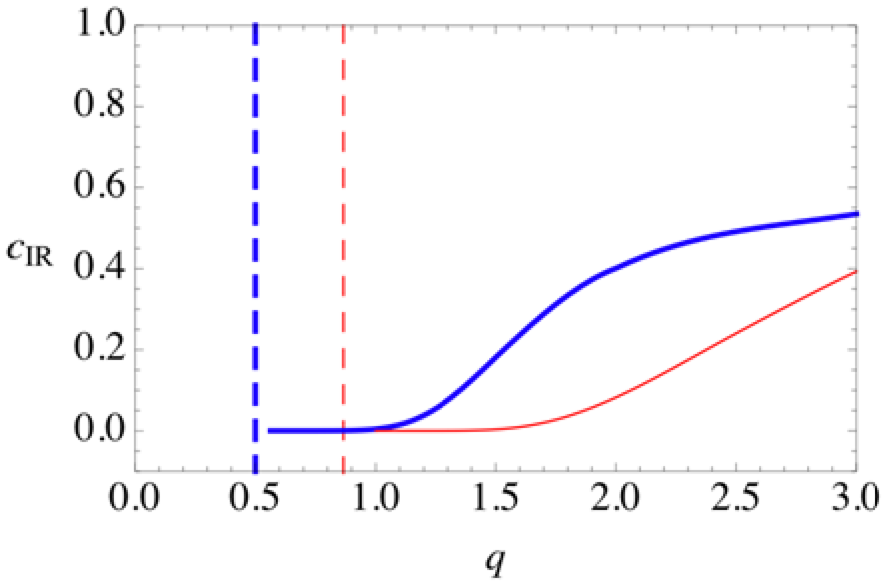
Near r = 0, the metric takes the form

$$ds^2 = r^2(-dt^2 + dx_i dx^i) + \frac{dr^2}{g_0 r^2(-\log r)}$$

There is a mild null singularity.

The holographic superconductor has emergent Poincare symmetry (but not conformal symmetry) in infrared.

The local speed of light in the IR is smaller than in the UV due to $g_{tt} = c^2 r^2$.



Blue: $m^2 = -1$

Red: $m^2 = 0$

It goes to zero in the limit that RN AdS becomes stable. Light cone in momentum space opens up.

Holographic Fermi surfaces

(Faulkner, Liu, McGreevy, and Vegh, 2009;
Cubrovic, Zaanen and Schalm, 2009)

Solved the Dirac eq. in extremal RN AdS black hole and computed the retarded Green's fn:

$G_R(\omega,k)$ = first subleading term / leading term

They found a Fermi surface, i.e., a static, normalizable mode at $k_i k^i = k_F^2$

=> pole at $\omega = 0$ in Re G_R
δ-fn in the spectral density Im G_R .

Fermionic probes of holographic superconductors

(Faulkner, McGreevy, Roberts, Vegh, G.H., 2009)

What is the effect of the condensate on the Fermi surface?

Solve the Dirac eq. in the background of a T = 0 hairy black hole. Choose $m_F = 0$ and $q_F = q/2$.

Key difference: We now have emergent Poincare symmetry in the IR.

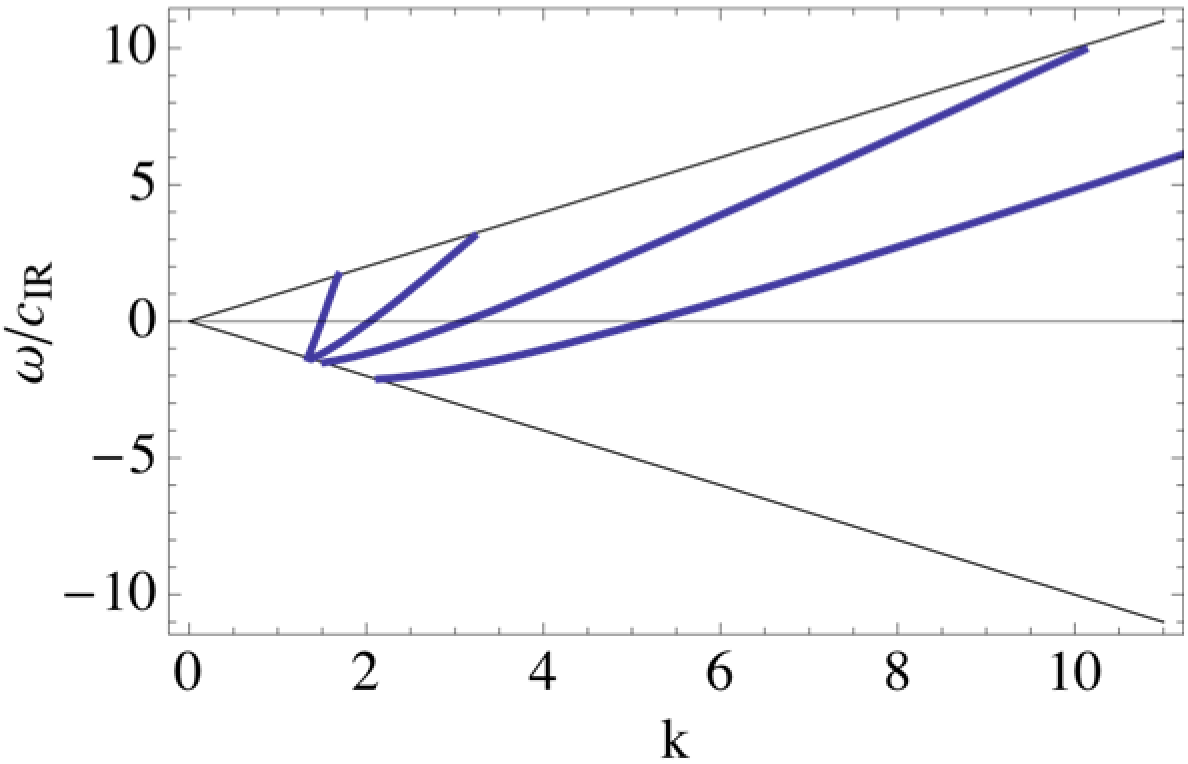
Consider a mode with e^{ikx} dependence near $r = 0$. If $k_\mu k^\mu < 0$, choose ingoing wave boundary conditions as before. This yields complex solutions, so

$$\text{Im } G_R \text{ is nonzero} \Rightarrow \text{continuous spectrum.}$$

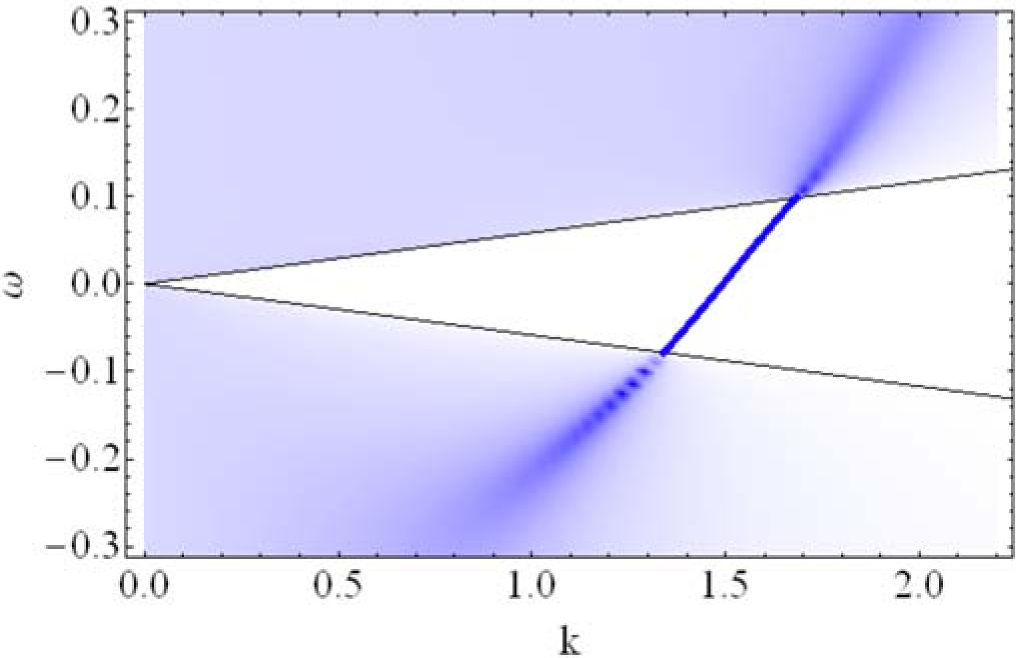
If $k_\mu k^\mu > 0$, modes are exponentially growing or damped. Normalizability is a real boundary condition. Solutions will be real and

$$\text{Im } G_R = 0$$

Not quite true: There are δ - fn contributions.

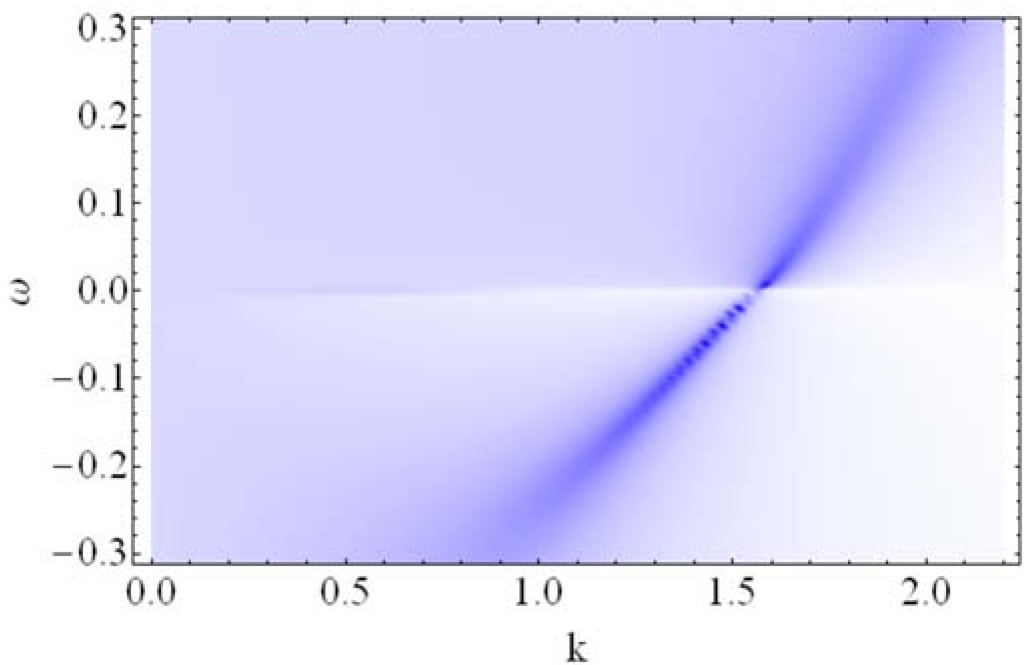


Boundstates outside the IR lightcone for
$m^2 = 0$, $q = 1.5, 2, 3, 5$. k_F is where $\omega = 0$.



Density plot of the fermion spectral density $A(k,\omega) = \mathrm{Im}\, G_R$ for $q = 3/2$, $m=0$.

Analogous plot for RN AdS



Now add coupling between scalar and fermion

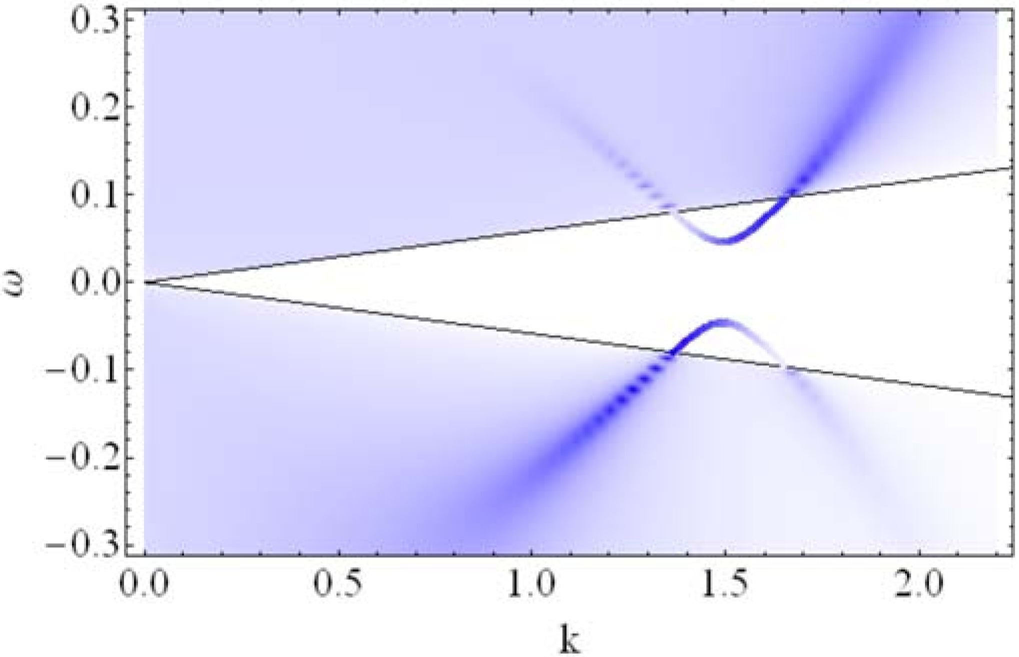
$$\eta \, \psi^* \, \zeta \, \zeta$$

coupling constant → η

scalar condensate → ψ^*

fermion → ζ

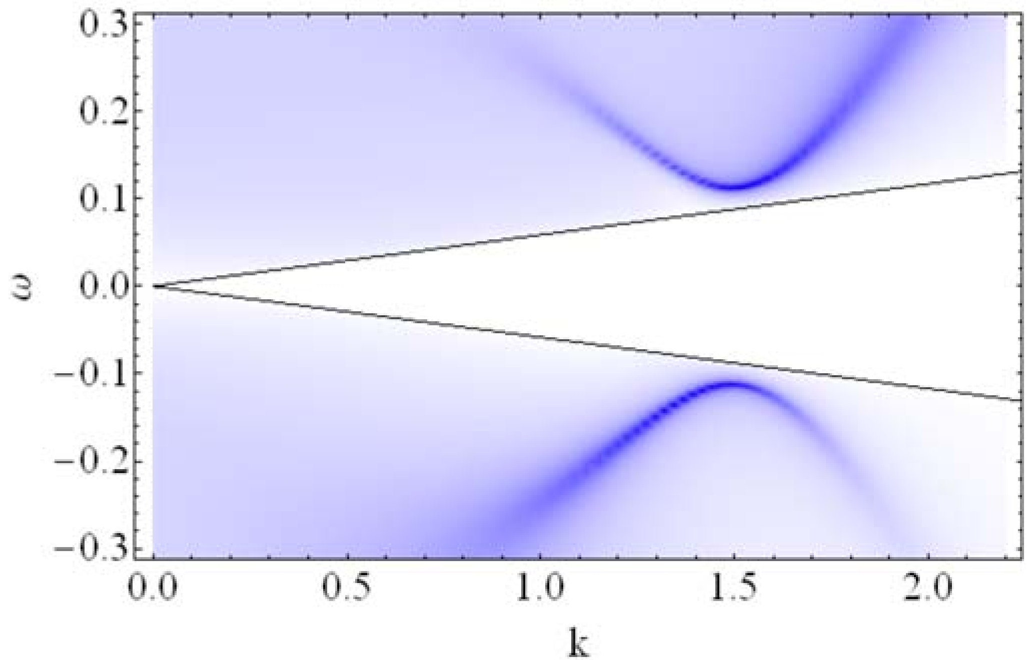
Like a Majorana mass term.
Analogous to the coupling between electrons and the condensate in BCS.



Any nonzero η introduces a gap!

η = 0.2

η = 1.5



Comments

This is very similar to what is seen in ARPES experiments on high T_c cuprates. There is a sharp quasiparticle peak in the superconducting phase, but not in the normal phase.

But full spectrum is not gapped. Still have continuum inside the light cone.

Do embeddings in string theory include this coupling between fermion and charged scalar?

Summary

- A simple gravitational theory can reproduce basic properties of a superconductor.
- The conductivity is directly related to a reflection coefficient in a 1D scattering problem.
- The ground state has been found, and there is no hard gap ($\mathrm{Re}\,\sigma(\omega) \neq 0$ at $T = 0$).
- The emergent IR Poincare invariance leads to stable quasiparticles near the Fermi surface.

Comments on AdS/CMT

- AdS/CFT is able to compute dynamical transport properties of strongly coupled systems at nonzero T. Condensed matter theorists have very few tools to do this.
- At present this can only be done for theories the condensed matter theorists don’t care about.
- Eventually, one might use gravity to predict new exotic states of matter and then try to look for them in the lab.

- Conversely, (if we are wildly optimistic) one might someday be able to test quantum gravity by doing condensed matter experiments!