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Interpolating geometries and gauge/gravity duality

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Based on:

[Maldacena,DM] JHEP 1001:104,2010,

[Gaillard,DM,Núñez,Papadimitriou] to appear

Texas A&M University, College Station – 19 March 2010

Plan of the talk

1. Supersymmetric geometries
2. “Interpolating” supersymmetric geometries
3. The unwarped resolved deformed conifold
4. The baryonic branch from fivebranes
5. Fivebranes from the baryonic branch
6. Adding flavours and a $\mathbf{G_2}$ story
7. Outlook

Motivations

- Systematic studies of supersymmetric geometries of String/M theory interesting mathematically and provide useful tools for addressing problems in string phenomenology (scanning the landscape) and gauge/gravity dualities

In the context of the gauge/gravity duality:

- New perspectives on familiar examples
- Methods to address more complicated models
- Can lead to the discovery of new gauge/gravity duals

General supersymmetric geometries

- Supersymmetric geometries of $\mathbf{d = 11, 10}$ and $\mathbf{d < 10}$ supergravities may be analysed systematically in the framework of **G**-structures/generalised geometry

Input: general metric ansatz plus Killing spinors

Output: set of equations for RR+NS fluxes and (multi-) forms

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In Type IIB supergravity:

$$\mathbf{ds^2 = e^{2\Delta}\left[dx^2_{1,3} + ds^2_6\right]}$$

$$\mathbf{F_5 = e^{4\Delta+\Phi}(1 + *_{10})\mathrm{vol}_4 \wedge f}$$

$$\mathbf{F_1 = 0 \quad (\text{for simplicity})}$$

Type IIB supersymmetric geometries

[see also talk of D. Lüst]

$$e^{-2\Delta+\Phi/2}(d - H_3\wedge)e^{2\Delta-\Phi/2}\Psi_1 = d\left(\Delta + \tfrac{\Phi}{4}\right)\wedge\bar{\Psi}_1 + \frac{ie^{\Delta+5\Phi/4}}{8}[f - *_6F_3]$$

$$(d - H_3\wedge)e^{2\Delta-\Phi/2}\Psi_2 = 0$$

[Graña, Minasian, Petrini, Tomasiello]

- Ψ_1, Ψ_2 are "pure spinors" in the sense of generalised geometry. Alternatively: multi-forms

- We restrict to the case when these take the form

$$\Psi_1 = -e^{i\zeta}e^{\Delta+\Phi/4}\left(1 - ie^{2\Delta+\Phi/2}J - \frac{1}{2}e^{4\Delta+\Phi}J\wedge J\right)$$

$$\Psi_2 = -e^{4\Delta+\Phi}\Omega$$

- J, Ω define a more familiar **SU(3)** structure. Non-constant phase ζ allows interpolation between different classes

Interpolating SU(3) structures

"Geometry"

$$\mathrm{d}\left(e^{6\Delta+\Phi/2}\Omega\right) = 0$$

$$\mathrm{d}\left(e^{8\Delta} J \wedge J\right) = 0$$

$$\mathrm{d}\left(e^{2\Delta-\Phi/2}\cos\zeta\right) = 0$$

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"Fluxes"

$$*_6\mathbf{F}_3 = -\mathbf{e}^{-2\Delta-3\Phi/2}\sec\zeta\,\mathbf{d}\left(\mathbf{e}^{4\Delta+\Phi}\mathbf{J}\right)$$

$$\mathbf{H}_3 = -\sin\zeta\,\mathbf{e}^{\Phi} *_6 \mathbf{F}_3$$

$$\mathbf{f} = -\mathbf{e}^{-4\Delta-\Phi}\mathbf{d}\left(\mathbf{e}^{4\Delta}\sin\zeta\right)$$

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"Fluxes"

$$*_6F_3=-e^{-2\Delta-3\Phi/2}\sec\zeta\, d\left(e^{4\Delta+\Phi}J\right)$$
$$H_3=-\sin\zeta e^{\Phi}*_6F_3$$
$$f=-e^{-4\Delta-\Phi}d\left(e^{4\Delta}\sin\zeta\right)$$

- $\sin\zeta\longrightarrow 1$: warped Calabi-Yau with ISD 3-form $dJ=d\Omega=0$, $G_3=F_3+iH_3=i*G_3$, $e^{\Phi}=g_s$ [Giddings,Kachru,Polchinski]
  E.g.: Klebanov-Strassler

- $\cos\zeta\longrightarrow 1$: "superstrings with torsion" [Strominger]
  E.g.: Maldacena-Núñez

Non-Kähler geometries (Type I)

"Geometry" ($\zeta = 0$)

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$$\Delta = \Phi/4$$

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"Fluxes" ($\zeta = 0$)

$$*_6 F_3 = -e^{-2\Phi} d\left(e^{2\Phi} J\right)$$
$$H_3 = 0$$
$$F_5 = 0$$

[Gauntlett,DM,Waldram]

- S-dual version involves only: metric, dilaton, NS 3-form $\mathbf{H_3}$ $\longrightarrow$ Type I/Heterotic solutions
- $\mathbf{M_6}$ is complex but non-Kähler. Killing spinors preserved by connection $\hat{\nabla} = \nabla_{\mathrm{spin}} + H_3$ with torsion

Generating new solutions

[Minasian,Petrini,Zaffaroni], [Gaillard,DM,Núñez,Papadimitriou]

Simple solution generating method

In: solution to "non-Kähler" equations $\longrightarrow$ **out**: general solution

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$$\Phi^{\rm new} = \Phi^{\rm old} \qquad \sin\zeta = \kappa_2 e^{\Phi^{\rm old}}$$

$$e^{2\Delta} = \frac{\kappa_1}{\cos\zeta} e^{\Phi^{\rm old}/2} \qquad F_3^{\rm new} = \frac{1}{\kappa_1^2} F_3^{\rm old}$$

$$\text{non trivial } F_5,\ H_3 \text{ generated}$$

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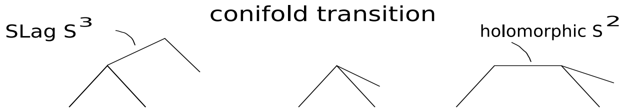
- Key point: Bianchi identity of simpler system $\Rightarrow$ Bianchi of more general system $\Rightarrow$ equations of motion (integrability results)
- Applies also with supersymmetric sources [Koerber,Tsimpis]
- Application to gauge/gravity duality: connection between wrapped fivebranes and Klebanov-Strassler theory

The conifold and the conifold transition

- The (Calabi-Yau) conifold singularity: $z_1^2 + z_2^2 + z_3^2 + z_4^2 = 0$
- Two desingularisations of the tip preserving the CY condition
  1. "Deformation": $\sum_i z_i^2 = \epsilon^2$ $\qquad T^*S^3 \simeq S^3 \times \mathbb{R}^3$
  2. "Resolution": $\mathcal{O}(-1) \oplus \mathcal{O}(-1) \to \mathbb{C}P^1 \quad \simeq S^2 \times \mathbb{R}^4$



- [Vafa]: for large N, N D5 branes wrapped on S^2 in the resolved conifold $\leftrightarrow$ deformed conifold with N units of RR F_3 through S^3
- Is it possible to see the geometric transition purely in the context of (Type IIB) supergravity?

The unwarped resolved deformed conifold

- **M** fivebranes wrapped on the $\mathbf{S^2}$ of the resolved conifold. Back-reaction of branes (**M** large) modifies the geometry $\longrightarrow$ work with "non-Kähler" equations

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- [Papadopoulos,Tseytlin] ansatz $\longrightarrow$ back-reacted solution

$$ds^2_{str} = dx^2_{3+1} + \frac{M}{4} ds^2_6 \qquad H_3 = \frac{M}{4} w_3$$

$$e^{2\phi(t)} = e^{2\phi_0} \frac{\sqrt{f(t)}c(t)'}{\sinh^2 t}$$

- $\mathbf{ds^2_6}$ depends (simply) on a function $\mathbf{c(t)}$. $\mathbf{w_3}$ is a 3-form

- Solution explicit, up to 1st order ODEs:

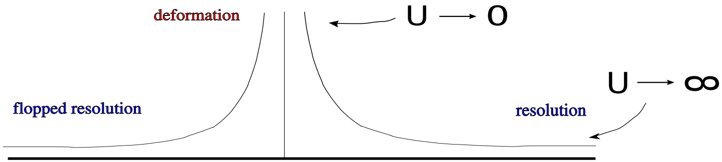
$$f' = 4 \sinh^2 t\, c \qquad c' = \frac{1}{f}[c^2 \sinh^2 t - (t \cosh t - \sinh t)^2]$$

- Parameters: $\mathbf{M}, \phi_0$, $\mathbf{0 < U < \infty}$. $\mathbf{U}$ is defined at large $\mathbf{t}$ and matched (numerically) to a parameter $\gamma^2 \geq 1$ near $\mathbf{t} \sim \mathbf{0}$

Realising the geometric transition

- $\mathbf{t} \to \mathbf{0}$: $\mathbf{r}^2_{\mathbf{S}^3} \sim \mathbf{M}\gamma^2 \ \to$ radius of $\mathbf{S}^3$
- $\mathbf{t} \to \infty$: $\mathbf{r}^2_{\mathbf{S}^2} \sim \mathbf{M} \log \mathbf{U} \ \to$ radius of $\mathbf{S}^2$

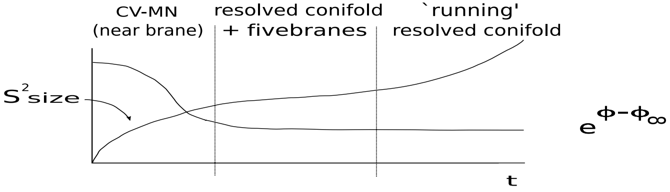
- Parameter **U** *interpolates* between deformation and resolution



- $\mathbf{U} \to \mathbf{0}$: $\approx$ deformed conifold with large $\mathbf{S}^3 + \int \mathbf{H}_3 = \mathbf{M}$ flux

- $\mathbf{U} \to \infty$:$\approx$ resolved conifold + **M** NS5 branes (far from branes)

Field theory (decoupling) limit: large U



- Generalised GVW superpotential $\longrightarrow$ define a "gauge coupling":

$$\mathbf{W} = \int_{M_6} e^{-2\phi} \Omega \wedge (H_3 + i dJ) \qquad \Rightarrow \qquad \beta_{\frac{8\pi^2}{g^2_{YM}}} = 3M$$

- Decoupling limit (near brane): $\mathbf{U} \longrightarrow \infty \quad \Rightarrow$ field theory

$$\lambda_{'t\ \mathrm{Hooft}} = g^2_{YM} M \sim \frac{1}{\log U} \ll 1$$

$\Rightarrow$ [Maldacena,Núñez] (CV-MN) solution: $\mathbf{SU(M)}\ \mathcal{N} = 1$ SYM

The baryonic branch from fivebranes

- Using the generating technique, we can add **D3** branes and **B**-field to the "unwarped resolved deformed conifold"

- Warp factor generated: $\mathbf{h = 1 + \cosh^2 \beta (e^{2(\phi - \phi_\infty)} - 1)}$

- Transformed solution has all fluxes (except $\mathbf{F_1}$) and depends on one new parameter (β): $\mathbf{M}, \phi_0, \mathbf{U}, \beta$

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- Solution incorporates various limits. In a "decoupling limit" $\beta \to \infty$ we recover the baryonic branch of the KS theory [Butti,Graña,Minasian,Petrini,Zaffaroni]

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New perspective on the baryonic branch of Klebanov-Strassler

Fivebranes on $\mathbf{S^2}$ of resolved conifold $+$ **B**-field: for large **U** we expect fivebranes on a fuzzy $\mathbf{S^2}$

Fuzzy two-sphere from the baryonic branch of KS

- Klebanov-Strassler theory: $\mathbf{SU(Mk)} \times \mathbf{SU(M(k+1))}$ quiver

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- Classical baryonic branch vacuum [Dymarsky,Klebanov,Seiberg]

$$\mathbf{A_i = C\,\Phi_i \otimes 1_{M\times M}}\ , \qquad \mathbf{B_i = 0}$$

$$\Phi_1 = \begin{pmatrix} \sqrt{k} & 0 & 0 & \cdot & 0 & 0 \\ 0 & \sqrt{k-1} & 0 & \cdot & 0 & 0 \\ 0 & 0 & \sqrt{k-2} & \cdot & 0 & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \\ 0 & 0 & 0 & \cdot & 1 & 0 \end{pmatrix} \qquad \Phi_2 = \begin{pmatrix} 0 & 1 & 0 & \cdot & 0 & 0 \\ 0 & 0 & \sqrt{2} & \cdot & 0 & 0 \\ 0 & 0 & 0 & \sqrt{3} & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \\ 0 & 0 & 0 & 0 & \cdot & \sqrt{k} \end{pmatrix}$$

$$\mathbf{D - terms :} \left\{ \begin{array}{rcl} A_1A_1^\dagger + A_2A_2^\dagger - B_1^\dagger B_1 - B_2^\dagger B_2 & = & (k+1)|C|^2 1_k \\ A_1^\dagger A_1 + A_2^\dagger A_2 - B_1 B_1^\dagger - B_2 B_2^\dagger & = & k|C|^2 1_{k+1} \end{array} \right.$$

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- From the $\mathbf{k \times (k+1)}$ matrices $\mathbf{\Phi_i}$ we construct matrices spanning two irreducible representations of $\mathbf{SU(2)}$

Fuzzy two-sphere from the baryonic branch of KS

$$\mathbf{L}_1 = \frac{1}{2}(\Phi_1\Phi_2^\dagger + \Phi_2\Phi_1^\dagger) \qquad \mathbf{L}_2 = \frac{i}{2}(\Phi_1\Phi_2^\dagger - \Phi_2\Phi_1^\dagger)$$

$$\mathbf{L}_3 = \frac{1}{2}(\Phi_1\Phi_1^\dagger - \Phi_2\Phi_2^\dagger)$$

Define a spin $\mathbf{j} = \frac{\mathbf{k}-1}{2}$ irreducible representation of $\mathbf{SU(2)}$

$$\mathbf{R}_1 = \frac{1}{2}(\Phi_1^\dagger\Phi_2 + \Phi_2^\dagger\Phi_1) \qquad \mathbf{R}_2 = \frac{i}{2}(\Phi_2^\dagger\Phi_1 - \Phi_1^\dagger\Phi_2)$$

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Define a spin $\mathbf{j} = \frac{\mathbf{k}}{2}$ irreducible representation of $\mathbf{SU(2)}$

- Looks like these define $\mathbf{SU(2)} \times \mathbf{SU(2)} \simeq \mathbf{SO(4)}$, but in fact they define a fuzzy super two-sphere

The fuzzy sphere spectrum (weak coupling)

- Fluctuations: $\mathbf{A_i} = \Phi_\mathbf{i} + \delta\mathbf{A_i}$, $\mathbf{B_i} = \delta\mathbf{B_i}$, $\mathbf{a}_\mu^{\mathrm{L,R}} = \delta\mathbf{a}_\mu^{\mathrm{L,R}}$

fields	on $\mathbf{S^2}$	$\mathbf{SU(2)}$ spin	$\mathcal{N} = 1$ multiplet	eigenvalues
$\mathbf{a}_\mu^\mathbf{L}, \mathbf{a}_\mu^\mathbf{R}$	scalar	$\mathbf{j = l}$	1 vector	$\lambda_{\mathbf{l},-}, \lambda_{\mathbf{l},+}$
$\delta\mathbf{A_i}$	vector	$\mathbf{j = l}$	1 vector	$\lambda_{\mathbf{l},-}, \lambda_{\mathbf{l},+}$
$\mathbf{B_i}$	spinor	$\mathbf{j = l} + \frac{1}{2}$	2 chiral	$\lambda_{\mathbf{l,B}}$

- Eigenvalues for $\mathbf{l} \ll \mathbf{k}$

$$\lambda_{\mathbf{l},-} \sim \frac{\mathbf{g}_+^2 |\mathbf{C}|^2}{\mathbf{2k+1}} \mathbf{l(l+1)} , \qquad \lambda_{\mathbf{l,B}} = |\mathbf{C}|^4 \mathbf{h}^2 (\mathbf{l}+1)^2$$

- Agrees with spectrum of Maldacena-Núñez compactification of D5 branes wrapped on $\mathbf{S^2}$ [Andrews,Dorey]

Parameters: $\theta_{\mathbf{Fuzzy}} \propto \frac{1}{\mathbf{k}}$ $\qquad |\mathbf{C}|^2 \mathbf{R}^2_{\mathbf{Fuzzy}} \propto \frac{\mathbf{k}}{\mathbf{g}_+^2}$

Comparison with gravity (strong coupling)

- Compare with the parameters computed in the gravity solution (in the intermediate "fivebrane" region)

$$\int_{S^2} B \propto g_s M \log U \equiv k \qquad \# \text{ cascade steps}$$

$$|C|^2 \propto M U \Lambda_0^2 \qquad \text{[Dymarsky,Klebanov,Seiberg]}$$

- Large **B**-field $\Rightarrow$ use open string metric: $r_{\text{open}} \sim \dfrac{B}{r_{\text{closed}}}$ [Seiberg,Witten]

Parameters: $\theta_{\text{NC}} \sim \dfrac{1}{B} \qquad \dfrac{m_{KK}^2}{|C|^2} \sim \dfrac{g_+^2}{k}$

The flavoured, deformed, resolved, conifold

[Gaillard,DM,Núñez,Papadimitriou]

- Branes wrapped on an infinitely extended surface ⟶ effective 4d coupling constant vanishes ⟶ "flavours" [Karch,Katz]

- Back-reacted solution with $\mathbf{N_f} \sim \mathbf{N_c}$ smeared D5 "flavour branes" constructed in [Casero,Nuñez,Paredes]

- **SU(3)** structure transformation ⟶ "flavoured warped resolved deformed conifold", includes: $\mathbf{N_c}$ "colour D5", $\mathbf{N_f}$ "flavour D5", plus bulk *and* mobile D3 branes

- Different from previous "flavoured" solutions, obtained with D7 branes. Possible because D5 probes (with D3 charge) are supersymmetric on the "resolved deformed conifold"

A G_2 story

[in progress]

- Flash out a similar construction in Type IIA supergravity

$$ds^2 = h^{-1/2} dx_{1,2}^2 + h^{1/2} ds_7^2$$

$$F_2 = 0 \,, \qquad F_4, H_3 \;\; \textbf{unconstrained}$$

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$$\cos^2 \zeta = h$$

"Fluxes"

$$F_4 = \mathrm{vol}_3 \wedge dh^{-1} + c_2 d(e^{-2\Phi}\phi)$$
$$*_7 H_3 = c_1 e^{2\Phi} d(e^{-2\Phi}\phi)$$

- $\zeta \rightarrow 0$: **7d** counterpart of "non-Kähler" geometries [GMPW]
- $\zeta \rightarrow \pi/2$: warped $\mathbf{G_2}$ holonomy manifold [Cvetic,Lu,Pope]

A G_2 story

- Solution generating method: start with $\mathbf{M}$ fivebranes wrapped on the $\mathbf{S^3}$ inside the $\mathbf{G_2}$-manifold $\mathbf{X = S^3} \times \mathbb{R}^4$. After backreaction the geometry is "$\mathbf{G_2}$ with torsion": there is an interpolating parameter $\mathbf{U}$

- $\mathbf{U} \to \mathbf{0}$: $\mathbf{G_2}$-manifold $\mathbf{X}$ with large $\mathbf{S^3}$ + $\mathbf{H_3}$ flux

- $\mathbf{U} \to \infty$: $\mathbf{G_2}$-manifold $\tilde{\mathbf{X}}$ + NS5 branes on $\tilde{\mathbf{S}}^3$

- Realises $\mathbf{G_2}$ geometric transition in Type IIA supergravity

- Decoupling limit (near brane): $\mathbf{U} \to \infty \quad \Rightarrow \quad$ field theory

$\Rightarrow$ T-dual [Maldacena,Nastase]: $\mathbf{SU(M)_{\frac{M}{2}}}$ $\mathcal{N} = \mathbf{1}$ Chern-Simons

- $\mathcal{N} = \mathbf{1}$, $\mathbf{3d}$ field theory dual to warped $\mathbf{G_2}$ manifold not known

$\longrightarrow$ presumably it is a Chern-Simons theory

Outlook

- Interpolating geometries I: solution generating method for classes of supersymmetric geometries of Type IIA/IIB. Applications to gauge/gravity
- Interpolating geometries II: solutions realising geometric transitions in Type IIB (torsional **SU(3)**) and Type IIA (torsional $\mathbf{G_2}$). Applications to gauge/gravity
- Perhaps there exist other classes with similar features, besides **SU(3)** and $\mathbf{G_2}$. Eleven dimensions?
- Relation between baryonic branch of KS and fuzzy two-sphere may be explored for more general quiver theories
- The $\mathbf{G_2}$ story: the geometry works as in the **SU(3)** case. It would be nice to have a field theory picture

Thank you!

15 March 2010

Alice Martelli says hello to the world!