The uses of Ω backgrounds

Nikita Nekrasov

Strings'2010

A&M College Station

Texas

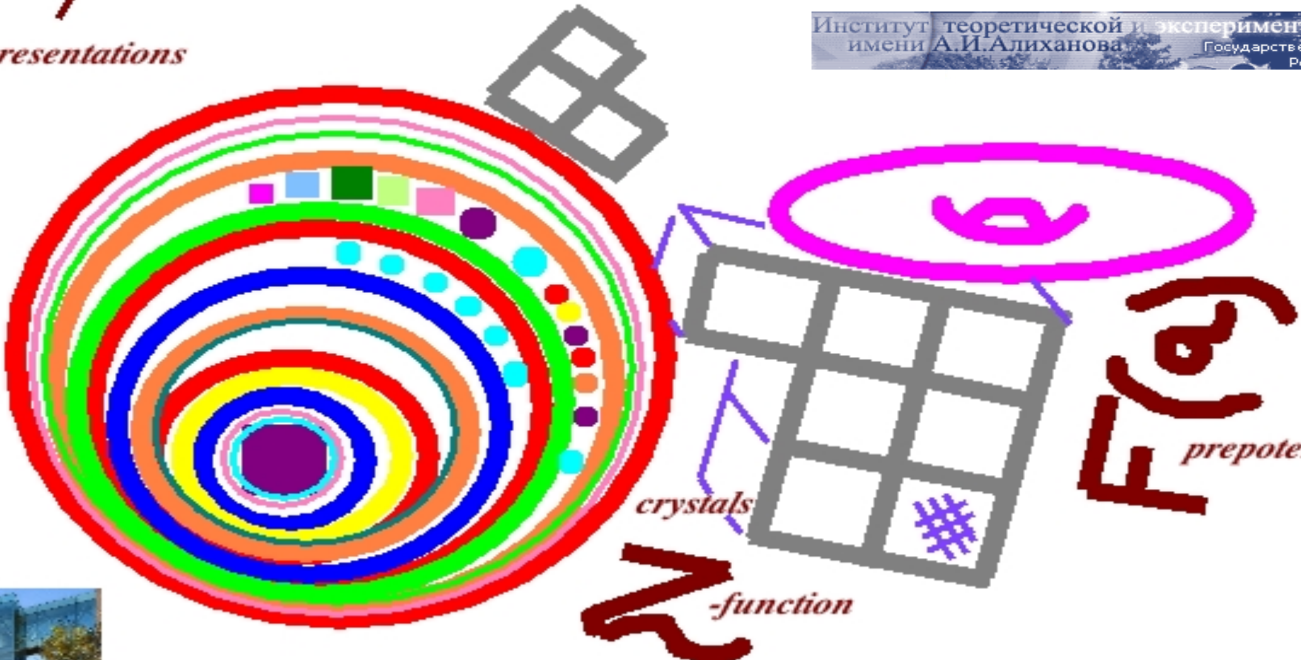
measures

gauge theory

correlators

representations

Институт теоретической и экспериментальной физики
имени А.И.Алиханова
Государственный научный центр
Российской Федерации



4-manifolds

Simons Center for Geometry and Physics

topological strings

quantum geometry

E_1 E_2

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$$\mathcal{F}_g(a;\Lambda) = \sum_{\beta\in H_2(X;\mathbf{Z})} e^{\beta.a}\Lambda^{\beta_0}\int_{\overline{\mathcal{M}_g(X;\beta)}} 1$$

$$Z = \sum_{n=0}^{\infty} q^n \int_{\mathcal{M}_n} \mathcal{X}_n$$

Based

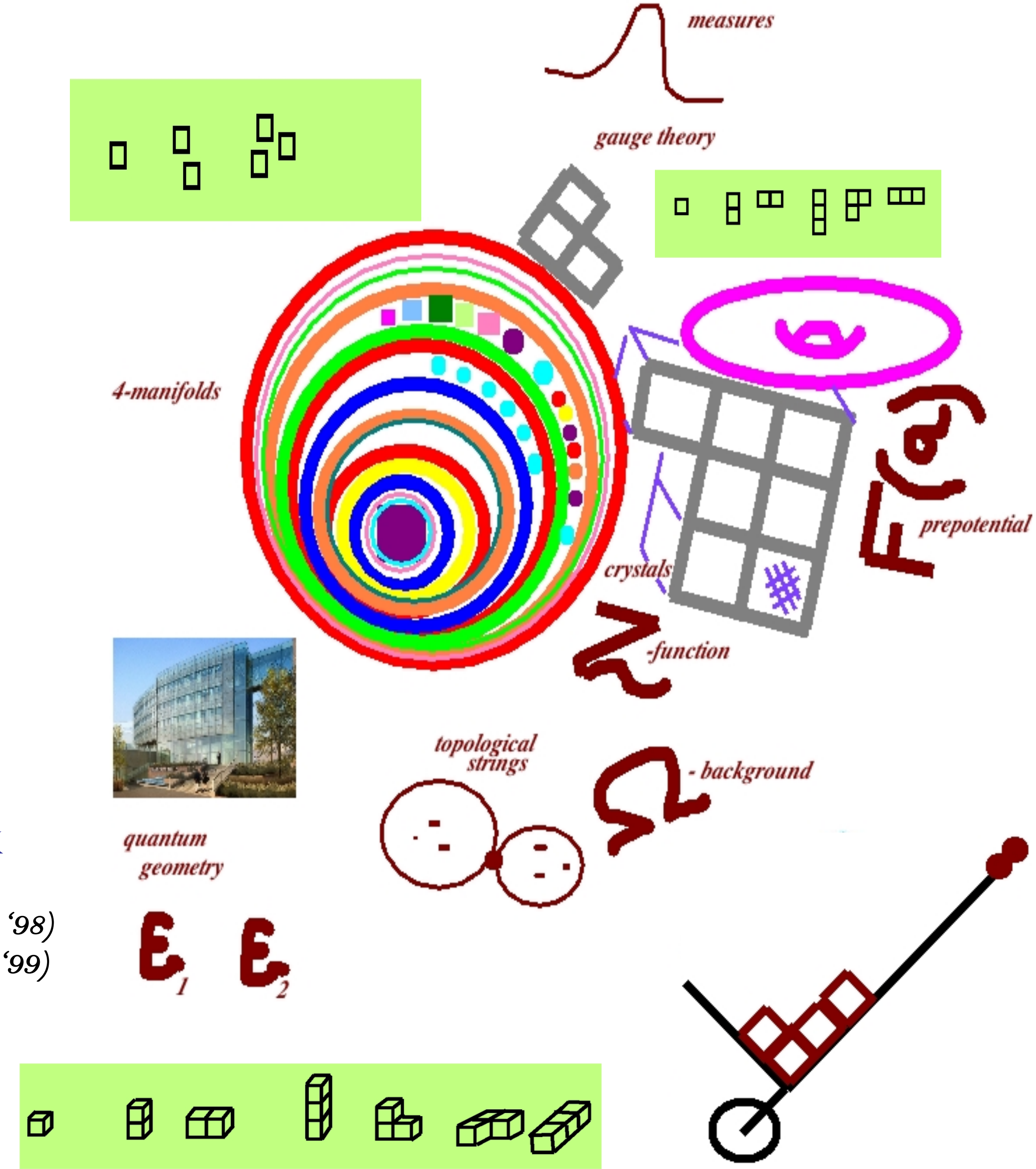
on

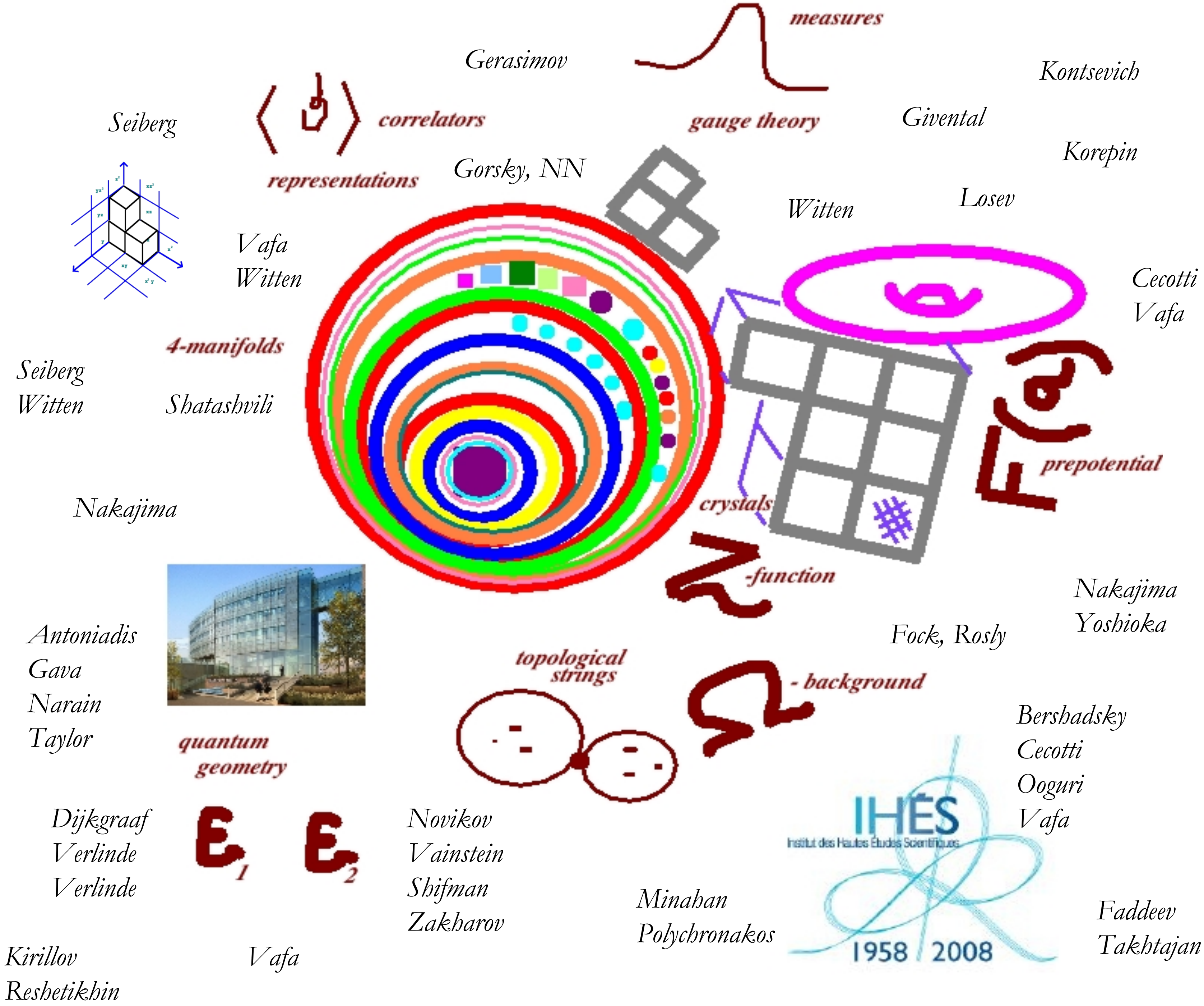
RECENT WORK

NN, S.Shatashvili
arXiv:0901.4744,
arXiv:0901.4748,
arXiv:0908.4052,
NN, E.Witten
arXiv:1002.0888

EARLIER WORK

G.Moore, NN, S.Shatashvili ('97, '98)
A.Losev, NN, S.Shatashvili ('97, '99)
NN ('02, '05,'08)
NN, A.Okounkov ('03)




measures
Gerasimov
Kontsevich
Seiberg
correlators
gauge theory
Givental
Korepin
representations
Gorsky, NN
Losev
Witten
Vafa
Witten
Cecotti
Vafa
4-manifolds
Seiberg
Witten
Shatashvili
prepotential
Nakajima
crystals
-function
Nakajima
Yoshioka
Antoniadis
Gava
Narain
Taylor
Fock, Rosly
topological
strings
- background
Bershadsky
Cecotti
Ooguri
Vafa
quantum
geometry
Dijkgraaf
Verlinde
Verlinde
Novikov
Vainstein
Shifman
Zakharov
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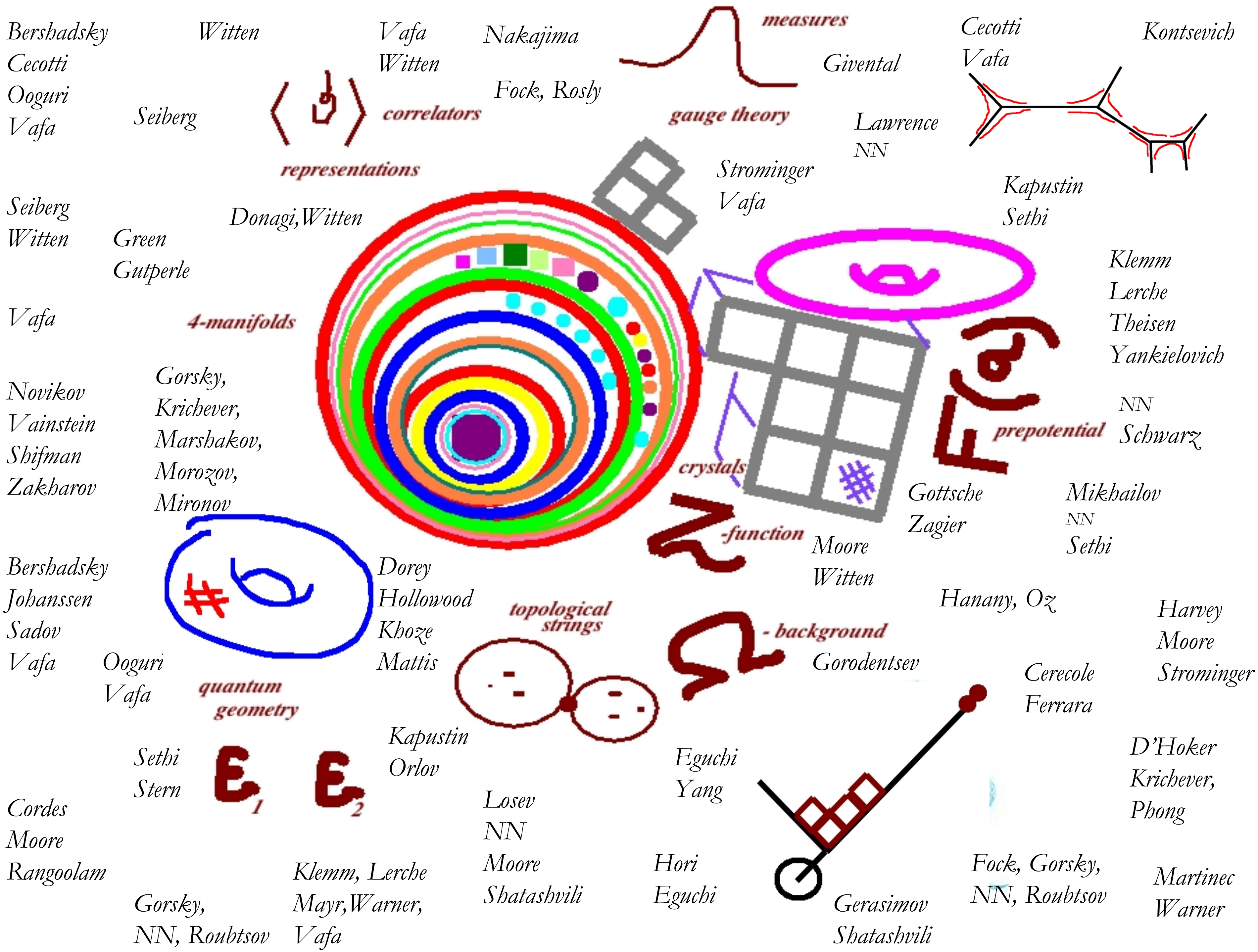
1958 2008
Minahan
Polychronakos
Faddeev
Takhtajan
Kirillov
Reshetikhin
Vafa



Bershadsky
Cecotti
Ooguri
Vafa
Witten
Seiberg
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correlators
representations
Nakajima
Fock, Rosly
measures
gauge theory
Givental
Lawrence
NN
Cecotti
Vafa
Kontsevich
Strominger
Vafa
Kapustin
Sethi
Seiberg
Witten
Green
Gutperle
Donagi, Witten
Vafa
4-manifolds
Klemm
Lerche
Theisen
Yankielovich
Novikov
Vainstein
Shifman
Zakharov
Gorsky,
Krichever,
Marshakov,
Morozov,
Mironov
F(a)
prepotential
NN
Schwarz
crystals
Gottsche
Zagier
Mikhailov
NN
Sethi
-function
Moore
Witten
Bershadsky
Johanssen
Sadov
Vafa
Dorey
Hollowood
Khoze
Mattis
Hanany, Oz
Harvey
Moore
Strominger
topological strings
Ω - background
Gorodentsev
Ooguri
Vafa
quantum geometry
Cerecole
Ferrara
Kapustin
Orlov
Sethi
Stern
E1
E2
Eguchi
Yang
D'Hoker
Krichever,
Phong
Cordes
Moore
Rangoolam
Losev
NN
Moore
Shatashvili
Hori
Eguchi
Klemm, Lerche
Mayr, Warner,
Vafa
Gorsky,
NN, Roubtsov
Fock, Gorsky,
NN, Roubtsov
Martinec
Warner
Gerasimov
Shatashvili

Bershadsky
Cecotti
Ooguri
Vafa

Witten

Vafa
Witten

Nakajima

Fock, Rosly

measures

Givental

Cecotti
Vafa

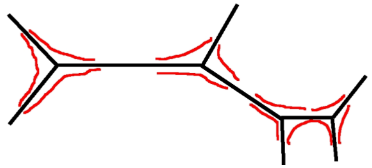
Kontsevich
Soibelman

Seiberg

correlators

gauge theory

Nakajima
Yoshioka

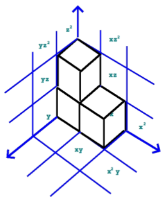


representations

Kac, Smilga

Strominger
Vafa

Seiberg
Witten



Donagi, Witten

Gerasimov
Shatashvili

NN
Okounkov

Vafa

4-manifolds

NN, Okounkov
Carlsson

Vainstein
Shifman
Yung

Moore
NN
Shatashvili

prepotential

crystals

Denef

Gukov
Witten

Kapustin
Witten

Gaiotto
Neitzke
Moore

Gottsche
Nakajima
Yoshioka

-function

Losev
NN
Marshakov

Pestun

Iqbal, NN,
Okounkov,
Vafa

NN
Shadchin

topological strings

Maulik, NN
Okounkov
Pandharipande

Harvey
Moore
Strominger

- background

NN
Ooguri
Vafa

quantum geometry

Gorodentsev
Leenson

Kapustin
Orlov

Gaiotto

Antoniadis
Hohenegger
Narain
Taylor

Alday
Gaiotto
Tachikawa

Baulieu
Losev
NN

E_1 E_2

Dijkgraaf
Vafa

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1958 2008

Ellingsrud
Stromme

Beilinson
Drinfeld

Aganagic, Klemm,
Marino, Vafa

Okounkov
Reshetikhin
Vafa

Gorsky,
NN, Roubtsov

Gerasimov
Shatashvili

Braverman

Fock
Goncharov

*Bershadsky
Cecotti
Ooguri
Vafa*

Witten

*Vafa
Witten*

*Nakajima
Gerasimov*

Fock, Rosly

*Gukov
Gorsky
Mironov*

Givental

*Cecotti
Vafa*

*Kontsevich
Soibelman*

Seiberg

*Cherkis
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*Becker
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Nakajima

*Alekseev
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*Gottsche
Zagier*

*NN
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*Gorsky,
Krichever,
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Morozov,
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Fock, Rosly

*Connes
Douglas
Schwarz*

Nakajima

*Aganagic, Klemm,
Marino, Vafa*

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Tachikawa*

Faddeev

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Hanany, Oz

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Witten*

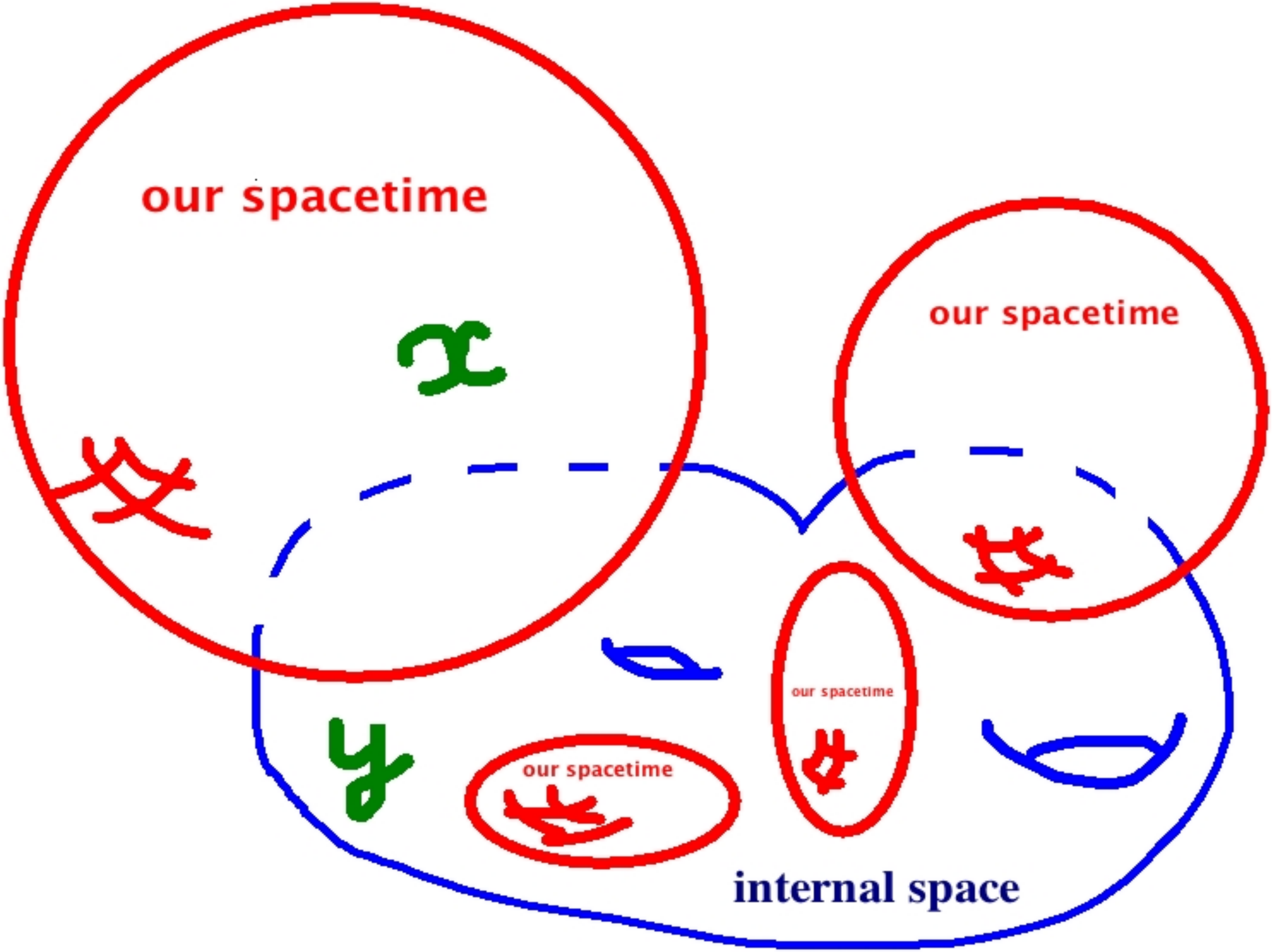
*Gorsky,
NN, Roubtsov*

Douglas, Moore

*Gerasimov
Shatashvili*

The **Omega**-backgrounds are the particular (super)gravity backgrounds

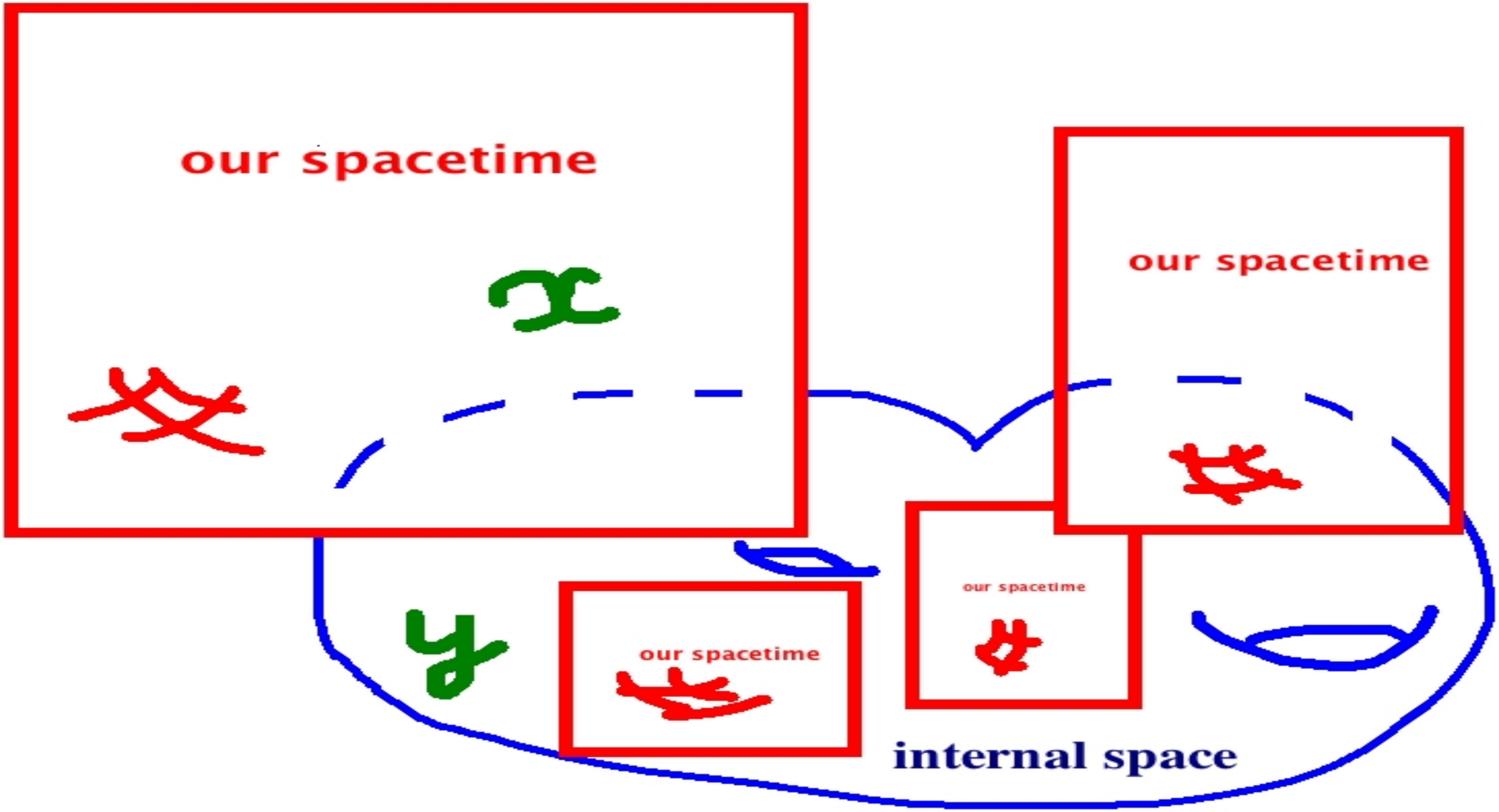
Generalization of a warped compactification


our spacetime
x
our spacetime
our spacetime
y
our spacetime
internal space

Omega-background

can be introduced when

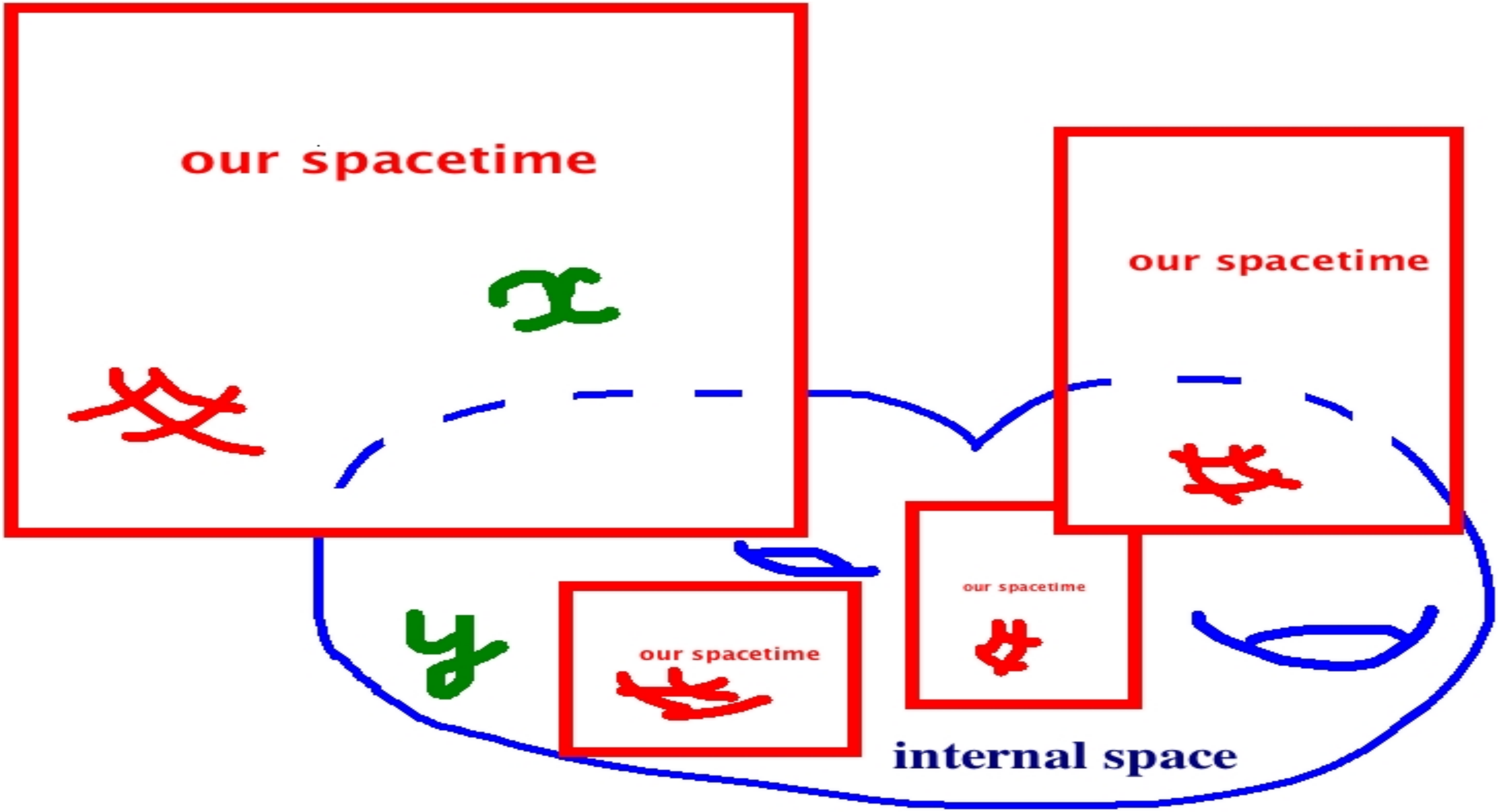
the spacetime has isometries



Omega-background

can be introduced when

the QFT has global symmetries



Supersymmetric Theory

With non-anomalous R-symmetry
can be subject to
the Omega-background
while preserving
some fraction of
supersymmetry.

Supersymmetric Theory

The explicit construction

Start with ***N=2*** supersymmetric theory
(in two dimensions or in four dimensions)

Supersymmetric Theory

The explicit construction

Lift it to ***N=1*** supersymmetric theory

in two dimensions up

i.e. four or six dimensions

Supersymmetric Theory

The explicit construction

For example, promote the gauge group **G**

to the group

L_2G = Maps (T^2, G) X T^2

Supersymmetric Theory

The explicit construction

Now **compactify** on

$\mathbf{T^2 = S^1 (r_1) \times S^1 (r_2)}$

with the twisted boundary conditions on T^2

rotating the space-time (e.g. $\mathbf{R^{2d}}$) by the angles

$$\mathrm{Re}(\varepsilon^i r_1),\ \mathrm{Im}(\varepsilon^i r_2)$$

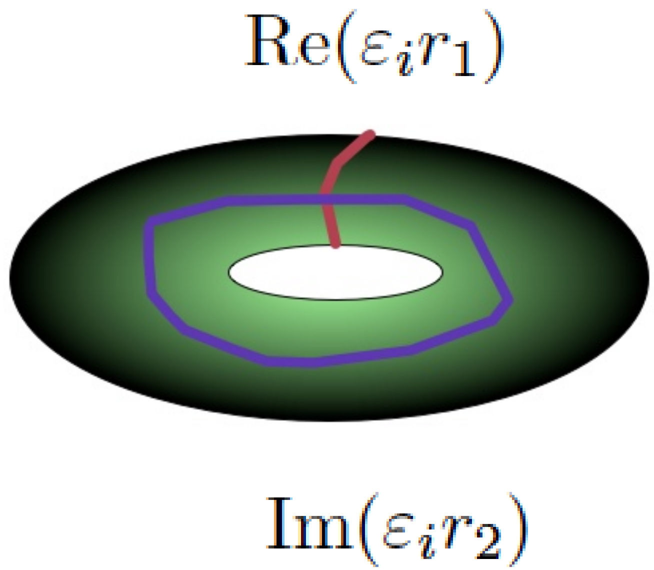
$$i = 1, \ldots, d$$

The twisted boundary conditions on T^2

Rotate the space-time $\mathbf{R}^{2d}$

as you go around

the **A** and **B** cycle on T^2



Omega-background

Now eliminate T^2

keeping the twisted boundary conditions:

send $r_1, r_2 \to 0$

and keep

$$\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_d$$

finite

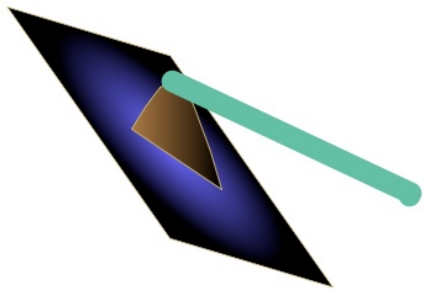
The result:
a **MASSIVE DEFORMATION**
of the original theory

The complex parameters:

$$\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_d$$

have dimension of mass

The supersymmetry



is broken

Indeed, the translational invariance in $\mathbf{R}^{2d}$ is broken so the original Super-Poincare algebra must be broken

The supersymmetry

***need not be completely broken*!**

Indeed, the original ***N=2*** theory
has an
R-symmetry,
so the susy generating spinors
transform both under the spacetime rotations
and the **R**-symmetry

$$Q^i_\alpha \, , \qquad \overline{Q}^i_{\dot\alpha}$$

The supersymmetry

is not completely broken.

once we supplement the geometric twist with an **R**-symmetry Wilson loop, proportional to

$$\sum_{i=1}^{d} \varepsilon^i$$

The supersymmetry of the Omega-background

Depends on the epsilon-parameters.

If all of them are non-zero then the resulting susy algebra is generated by two supercharges, closes on the spacetime rotation (similar to the AdS superalgebra)

$\{ Q, Q^{*} \} = \varepsilon^{i}$ times (x_{2i}, x_{2i+1}) - **rotation**

The supersymmetry of the Omega-background

If some of ε^i are zero

then the resulting susy algebra is larger:

N=2 *super-Poincare* in

lower spacetime dimensions, where

$$\varepsilon = 0$$

with the ***central extension*** given by

the rotation in the directions where $\varepsilon \neq 0$

The ground states in the Omega-background

Start with $N=2$ gauge theory
in four dimensions,
and turn on the Omega-deformation
in two dimensions

The ground states in the Omega-background

Start with N=2 gauge theory
in four dimensions,
and turn on the Omega-deformation
in two dimensions

The unbroken supersymmetry is that of a

two dimensional N=2 theory.

The ground states in the Omega-background

N=2 gauge theory in four dimensions,
the Omega-deformation in two dimensions.
The unbroken supersymmetry is that of a

two dimensional N=2 theory.

On the general grounds the susy vacua of such theory
are given by the eigenstates of a

quantum integrable system.

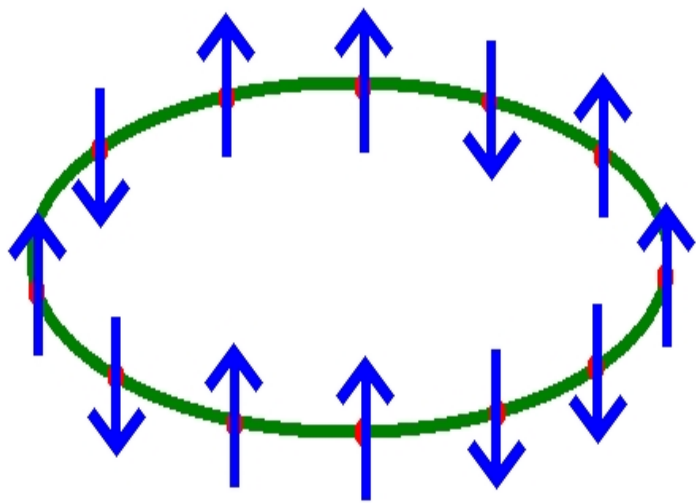
The quantum Hamiltonians are the generators of the (twisted) chiral ring

The ground states in the Omega-background

The susy vacua of such theory are given by the eigenstates of a

quantum integrable system.

The quantum Hamiltonians are the generators of the (twisted) chiral ring



Quantization of Seiberg-Witten integrable system

It is remarkable that the susy vacua of our theory are
the eigenstates of
the quantum integrable system
which is
The quantum version of the integrable system
governing the special geometry of
the moduli space of vacua of the four dimensional theory.
The role of the Planck constant is played by

$$\varepsilon$$

Pure N=2 SYM in 4d:

periodic Toda chain

$$H_2 = \frac{1}{2}\sum_i p_i^2 + U(x_1, \ldots, x_N)$$

$$U(x_1, \ldots, x_N) = \Lambda^2 \left(\sum_{i=1}^{N-1} e^{x_i - x_{i+1}} + e^{x_N - x_1} \right)$$

$$p_i = \varepsilon \partial_{x_i}$$

Pure N=2 SYM in 4d: periodic Toda chain: Type A and Type B spectral problems

$$U(x_1,\ldots,x_N)=\Lambda^2\left(\sum_{i=1}^{N-1}e^{x_i-x_{i+1}}+e^{x_N-x_1}\right)$$

Type A: L^2 - normalizable function with x_i noncompact

Type B: periodic wavefunctions with the period $2\pi i$

The $\mathcal{N}=2^*$ SYM in 4d:

elliptic Calogero-Moser system

Type A and Type B

spectral problems

$$\hat{H}_2 = \frac{\varepsilon^2}{2} \sum_{i=1}^{N} \frac{\partial^2}{\partial x_i^2} - m(m+\varepsilon) \sum_{i<j} \wp(x_i - x_j)$$

The A or B cycle periodicity of the wavefunctions

The choice between the Type A or Type B spectral problems

$$\hat{H}_2 = \frac{\varepsilon^2}{2} \sum_{i=1}^{N} \frac{\partial^2}{\partial x_i^2} - m(m+\varepsilon) \sum_{i<j} \wp(x_i - x_j)$$

The supersymmetric boundary conditions at infinity in gauge theory

The general asymptotically conformal N=2 theory

The ADE (0,2) six dimensional theory compactified on a Riemann surface $\mathcal{C}$.

We now have a field theory construction for A_1 and A_2

The dual quantum integrable system:

The quantum Hitchin system on $\mathcal{C}$

Quantum Hitchin system

is not unique

There are many Type A or Type B models,
distinguished by a choice of a real slice

Supersymmetric ground states of the (0,2) theory compactified on a Riemann surface C and subject to the Omega-background in R^2

Depend on the choice of boundary conditions.
There exist roughly
the electric-type and the magnetic-type
boundary conditions for each gauge group factors,
with some consistency requirements.

The Lagrangian

The effect of the Omega-deformation on the ***N*=2** theory **Lagrangian** is simple:

Shift the **complex adjoint Higgs field**

$$\sigma \mapsto \sigma - \sum_{i=1}^{d} \varepsilon^i V_i^\mu D_\mu$$

The Lagrangian

Thus the complex adjoint Higgs field becomes a differential operator

$$\sigma \mapsto \sigma - \sum_{i=1}^{d} \varepsilon^i V_i^\mu D_\mu$$

The « new Higgs scalar »

$$\sigma - \sum_{i=1}^{d} \varepsilon^i V_i^\mu D_\mu$$

Where, e.g. for rotational symmetry:

$$V_i^\mu \frac{\partial}{\partial x^\mu} = x^{2i} \frac{\partial}{\partial x^{2i+1}} - x^{2i+1} \frac{\partial}{\partial x^{2i}}$$

The transformation

$$\sigma \mapsto \sigma - \sum_{i=1}^{d} \varepsilon^i V_i^\mu D_\mu$$

is not a field redefinition,

so the theory

IS

deformed

The Omega-deformation can be undone

by a field redefinition, so that the theory

IS NOT

deformed

The Omega-deformation can be undone

by a field redefinition, so that the theory

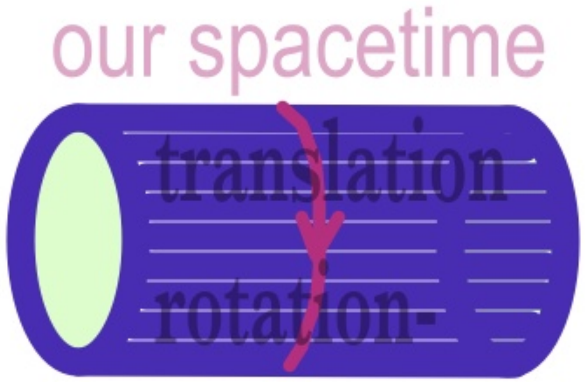
IS NOT

deformed, when the

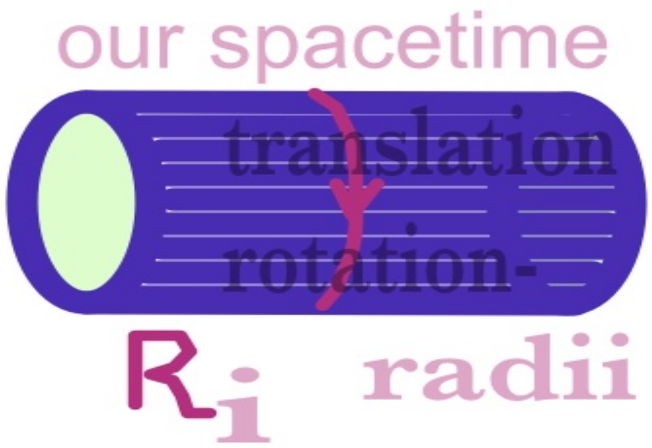
isometries involved in the Omega-deformation act freely, without fixed points

The Omega-deformation can be (sometimes) undone

The isometries without fixed points

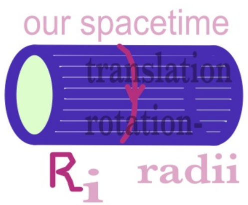


The torus compactification



$$\mathbf{g}_{ij} = R_i^2 \delta_{ij}$$

Omega deformation parameters ε^i

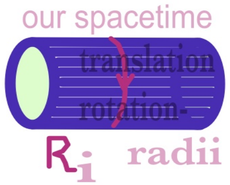


The field redefinition

$$\Sigma = \sigma - \sum_{i=1}^{d} \varepsilon^i (\partial_i + A_i)$$

i=*1, … , d*

$$\mathbf{X}^i = \frac{1}{R_i} (\partial_i + A_i)$$



The Lagrangian in the SO(d+2)-covariant form

$$\frac{1}{2}\sum_{i<j}^{d} \mathrm{tr}\,[\mathbf{X}^i, \mathbf{X}^j]^2 +$$

$$\frac{1}{2}\sum_{i=1}^{d} \mathrm{tr}\,[\Sigma, \mathbf{X}^i][\overline{\Sigma}, \mathbf{X}^i] +$$

$$\frac{1}{2}\,\mathrm{tr}\,[\Sigma, \overline{\Sigma}]^2$$

The SO(d+2) rotation

$$\begin{pmatrix} \mathbf{X}^1 \\ \mathbf{X}^2 \\ \cdots \\ \mathbf{X}^d \\ \mathrm{Re}\Sigma \\ \mathrm{Im}\Sigma \end{pmatrix} \mapsto \begin{pmatrix} \mathbf{Y}^1 \\ \mathbf{Y}^2 \\ \cdots \\ \mathbf{Y}^d \\ \mathrm{Re}\sigma \\ \mathrm{Im}\sigma \end{pmatrix}$$

The SO(d+2) rotation

$$\begin{pmatrix} \mathbf{X}^1 \\ \mathbf{X}^2 \\ \dots \\ \mathbf{X}^d \\ \mathrm{Re}\Sigma \\ \mathrm{Im}\Sigma \end{pmatrix} \mapsto \begin{pmatrix} \mathbf{Y}^1 \\ \mathbf{Y}^2 \\ \dots \\ \mathbf{Y}^d \\ \mathrm{Re}\sigma \\ \mathrm{Im}\sigma \end{pmatrix}$$

$$\mathbf{Y}^i = \frac{1}{R_i'}(\partial_i + \widetilde{A}_i)$$

The effective T^d geometry

$$\begin{pmatrix} \mathbf{X}^1 \\ \mathbf{X}^2 \\ \ldots \\ \mathbf{X}^d \\ \mathrm{Re}\Sigma \\ \mathrm{Im}\Sigma \end{pmatrix} \mapsto \begin{pmatrix} \mathbf{Y}^1 \\ \mathbf{Y}^2 \\ \ldots \\ \mathbf{Y}^d \\ \mathrm{Re}\sigma \\ \mathrm{Im}\sigma \end{pmatrix}$$

$$\mathbf{Y}^i = \frac{1}{R'_i}(\partial_i + \widetilde{A}_i)$$

$$R'_i = R'_i(R, \varepsilon)$$

$$G^{ij} = \mathbf{g}^{ij} + \mathrm{Re}(\varepsilon^i \overline{\varepsilon}^j)$$

The new Lagrangian

$$\frac{1}{2}\sum_{i<j}^{d} \mathrm{tr}\,[\mathbf{Y}^i, \mathbf{Y}^j]^2 +$$

$$\frac{1}{2}\sum_{i=1}^{d} \mathrm{tr}\,[\sigma, \mathbf{Y}^i][\overline{\sigma}, \mathbf{Y}^i] +$$

$$\frac{1}{2}\,\mathrm{tr}\,[\sigma, \overline{\sigma}]^2$$

The new Lagrangian equals

$$G^{ij} G^{i'j'} \,\mathrm{tr}\, \widetilde{F}_{ii'} \widetilde{F}_{jj'} +$$

$$G^{ij} \,\mathrm{tr}\, \widetilde{D}_i \sigma \widetilde{D}_j \overline{\sigma} +$$

$$\mathrm{tr}\, [\sigma, \overline{\sigma}]^2$$

The new Lagrangian equals

$$G^{ij} G^{i'j'} \operatorname{tr} \widetilde{F}_{ii'} \widetilde{F}_{jj'} +$$

$$G^{ij} \operatorname{tr} \widetilde{D}_i \sigma \widetilde{D}_j \overline{\sigma} +$$

$$\operatorname{tr} [\sigma, \overline{\sigma}]^2$$

$$\widetilde{D}_i = \partial_i + \widetilde{A}_i$$

$$G^{ij} = \mathbf{g}^{ij} + \mathrm{Re}(\varepsilon^i \overline{\varepsilon}^j)$$

Using this rotation one can map the gauge theory in the **Omega-background**

*To the configuration of **D-branes** of the **(A,B,A)**- or **(B,A,A)**-type*

To the arrangement of D-branes

of the (A,B,A)- or (B,A,A)-type

in

the two dimensional sigma model with

the <u>hyperkahler</u> target space

$$M_H$$

the Higgs branch of

the three dimensional gauge theory

aka the Hitchin moduli space

Other uses of Omega-backgrounds

The space of supersymmetric ground states
of the (asymptotically) conformal four dimensional N=2 theory
subject to the generic Omega-background on S^3
is identified with
the space of conformal blocks of Liouville
(and ADE Toda) with

$$b^2 = \varepsilon_1 / \varepsilon_2$$

Other uses of Omega-backgrounds

When

$$\varepsilon_2 = -\varepsilon_1 = \hbar$$

The instanton partition function

is identified with the partition function of

a topological string on a local Calabi-Yau

With $\hbar$ *being the topological string coupling constant*

Other uses of Omega-backgrounds

In the context of

the quantum integrable systems

the localization with respect to

the supercharge preserved by the

Omega-deformation

leads to the explicit formulae for the

Yang function of the quantum system

(determines the spectrum)

Other uses of Omega-backgrounds

More generally, the correlation functions of
observables invariant under the supersymmetry of the Omega-deformed theory
(local observables, susy Wilson loops)
can be effectively computed, using localization
and related to the
correlation functions in some two dimensional (conformal) field theories.

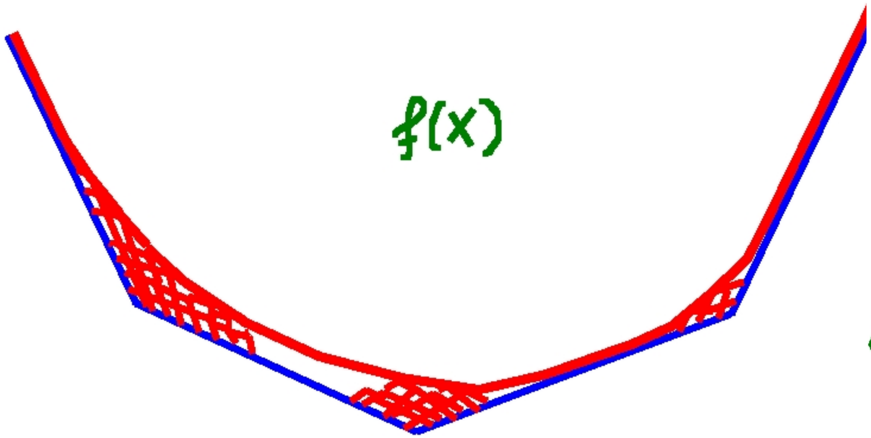
Other uses of Omega-backgrounds

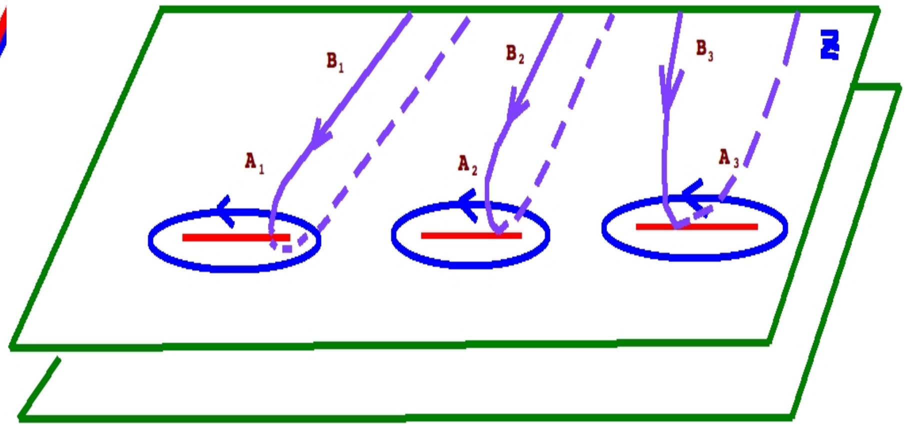
*In particular, the very **Seiberg-Witten geometry** can be derived in a mathematically satisfactory fashion using this analysis.*

Other uses of Omega-backgrounds

Seiberg-Witten geometry

can be derived





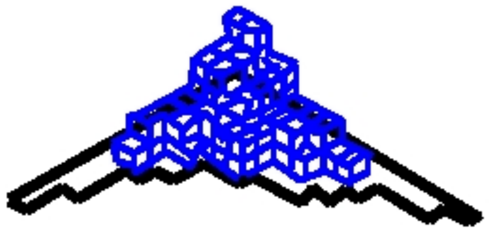
Other uses of Omega-backgrounds

Computations of the
« bulk contribution to the index »
of the quantum mechanics of D0-branes

Other uses of Omega-backgrounds

The ***six-dimensional Omega-backgrounds***
lead to the notion of
K-theoretic equivariant vertex
(generalized melting crystal model)
and could be used to test the M-theory predictions

The
K-theoretic equivariant vertex



$$Q = \frac{q}{(q_1 q_2 q_3)^{\frac{1}{2}}}$$

$$Z(q_1, q_2, q_3, q) = \sum_{\pi} q^{|\pi|} \mu_\pi(q_1, q_2, q_3) =$$

$$\exp\left(-\sum_{n=1}^{\infty} \frac{Q^n}{n(1-Q^n)(1-Q^n q_1^n q_2^n q_3^n)} \frac{(1-q_1^n q_2^n)(1-q_1^n q_3^n)(1-q_2^n q_3^n)}{(1-q_1^n)(1-q_2^n)(1-q_3^n)}\right)$$

The M-theory Omega-deformation

An SU(5) twist

The M-theory Omega-deformation

A route to E_{10} symmetry?

Superstrings in
Ramond-Ramond
Backgrounds
March 22-26 2010
Simons Center
Stony Brook
SIMONS CENTER FOR GEOMETRY AND PHYSICS
Participants: Y. Aisaka, G. Arutyunov, N. Beisert, A. Dymarsky, D. Gaiotto, P. Grassi, V. Kazakov, W. Linch, A. Mikhailov,
G. Moore, H. Ooguri, V. Pestun, A. Polyakov, R. Roiban, V. Schomerus, S. Shatashvili, K. Skenderis, D. Sorokin,
M. Staudacher, A. Tseytlin, B. Vallilo, P. Vieira, K. Zarembo.
Organizers: N. Berkovits, L. Mazzucato, N. Nekrasov.

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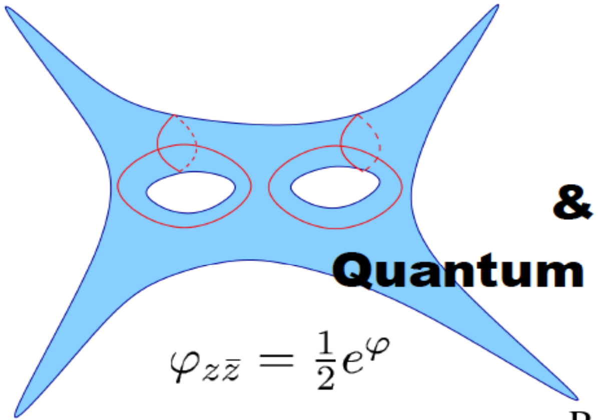

Perspectives, Open Problems & Applications of Quantum Liouville Theory

$$\varphi_{z\bar{z}} = \frac{1}{2} e^{\varphi}$$

$$T(z) = \varphi_{zz} - \frac{1}{2}\varphi_z^2$$

$$\langle e^{\alpha_1 \varphi(P_1)} \cdots e^{\alpha_n \varphi(P_n)} \rangle = \int_{\mathscr{C}\mathscr{M}_\alpha} e^{-\frac{1}{2\pi\hbar} S_\alpha(\varphi)}\, \mathscr{D}\varphi$$

Simons Center Workshop
March 29 – April 3, 2010
Stony Brook University

STONY BROOK
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Organizers: N.Nekrasov, L.Takhtajan