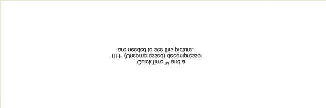
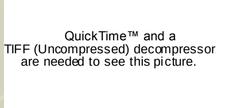
Non-Fermi Liquids and Holography

Joseph Polchinski





Strings2010

MARCH 15-19, 2010 COLLEGE STATION, TEXAS

Naturalness in condensed matter physics

Typical scale of electronic excitations:

$1\ \text{eV} \sim 10^4\ \text{K}$ —— ~ Planck scale

~ Standard Model

$10^2\ \text{K}$ ——

$1\ \text{K}$ ——

Why should there be any excitations lighter than this?

Naturalness in condensed matter physics

Typical scale of electronic excitations:

$1\ \text{eV} \sim 10^4\ \text{K}$ —— ~ Planck scale

~ Standard Model

$10^2\ \text{K}$ ——

$1\ \text{K}$ ——

Why should there be any excitations lighter than this?

In insulators there are not, aside from phonons, which are light because they are Goldstone bosons.

But conductors are common, at room temp. and down to much lower temperatures. There must be a `natural' effective theory of low energy charge excitations.

But conductors are common, at room temp. and down to much lower temperatures. There must be a `natural' effective theory of low energy charge excitations. There is: Landau-Fermi liquid theory.

$$H = \int d^d\mathbf{k}\ \psi_{s,\mathbf{k}}^{\dagger}(\varepsilon_{\mathbf{k}} - \mu)\,\psi_{s,\mathbf{k}}$$

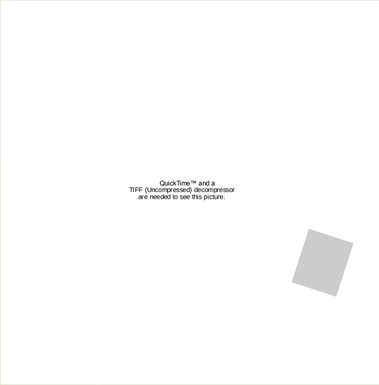
$\varepsilon_{\mathbf{k}} < \mu$: filled

$\varepsilon_{\mathbf{k}} > \mu$: empty

Low lying states: moving an electron from just below the Fermi surface to just above.

But what about interactions?

$$H = \int d^d\boldsymbol{k}\ \psi_{s,\boldsymbol{k}}{}^{\dagger}(\varepsilon_{\boldsymbol{k}} - \mu)\psi_{s,\boldsymbol{k}}$$



Interactions are irrelevant in the RG sense for kine-
matic reasons (Landau), except for special points ---

Zero momentum charge 2 (BCS)
Zero momentum charge 0 (zero sound,
Pomeranchuk instability)

Allows precise calculations, e.g. BCS gap/T_c.

If this LFL `Standard Model' were the whole story, AdS/CM would have little use here. But there is evidence for beyond-the-LFL physics.

Heavy fermion materials, high-T_c cuprates near critical doping have low-energy properties not described by this effective theory.

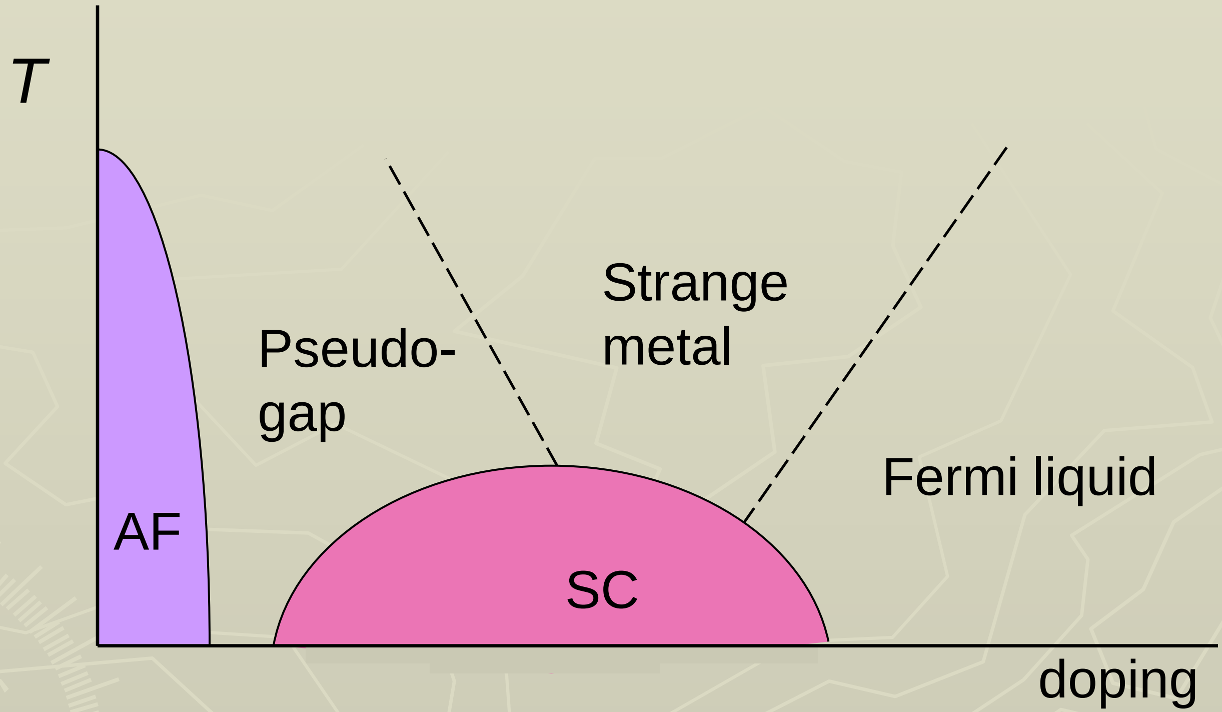
E.g. resistivity $\propto T^2$ in LFL, $\propto T$ in cuprates at critical doping: low energy interactions are *stronger*.

Hall conductivity $(\sigma^{xy}/\sigma^{xx})_{\text{Hall}}/(\sigma^{xx})_{B=0} \propto T^0$ in LFL, $\propto T^{-1}$ in cuprates.

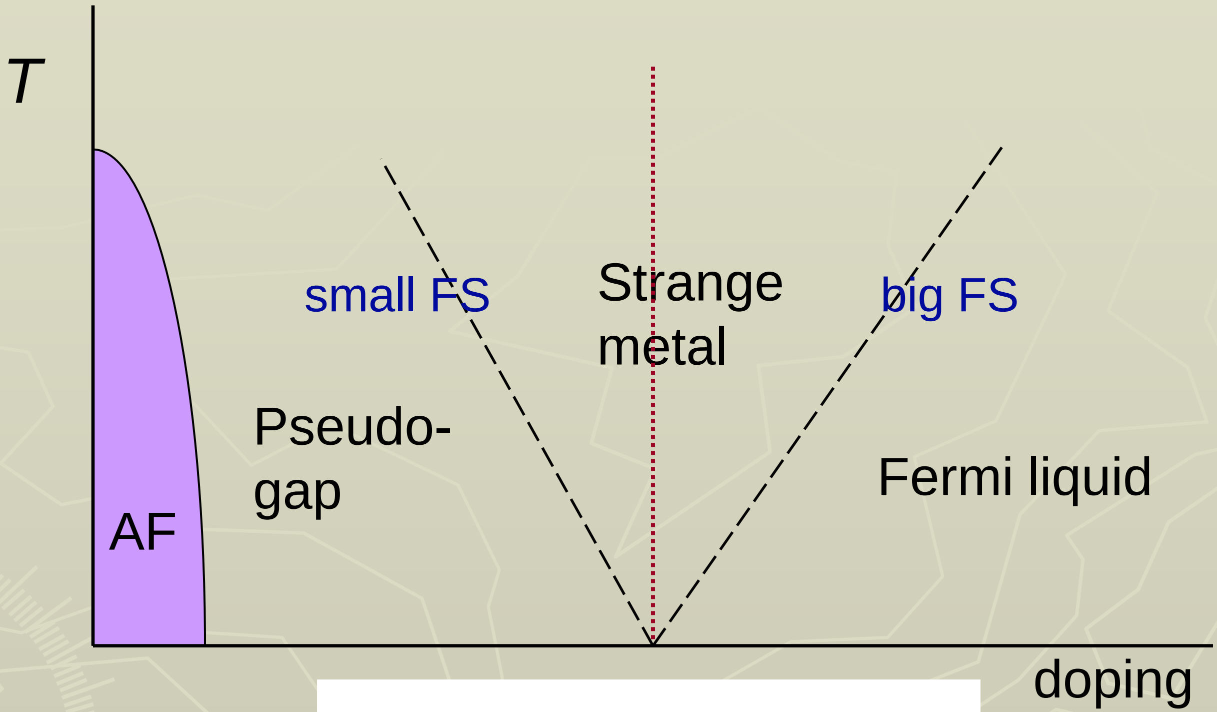
Also, there is evidence for a Fermi surface, which disappears continuously at the transition.

"Non-Fermi liquid" or "strange metal."

canonical cuprate phase diagram



with superconductivity suppressed



QuickTime™ and a
TIFF (Uncompressed) decompressor
are needed to see this picture.

Comment: it is not clear why the strange normal state should be connected with superconductivity.

A logical possibility would have been a low energy LFL theory with a strong attractive BCS interaction generated by complicated UV effects, but this does not seem to be realized in nature.

Searching for a framework: some proposals

- Scaling theory (Senthil, 0803.4009) ~ critical phenomena before the renormalization group.

- Marginal Fermi liquid theory (Varma, et al., 1989): a phenomenology with field theoretic underpinnings.

- Fractionalization of electron into spinon+holon coupling to emergent gauge field (long history, see Lee, 0905.4532): need to solve strong-coupling problem.

General feature: fermions in interaction with some other low energy degree of freedom.

An opportunity for AdS/CM?

Outline of the remainder:

- Holographic models
- Semiholographic models
- Nonholographic models

General feature: fermions in interaction with some other low energy degree of freedom.

An opportunity for AdS/CM?

Outline of the remainder:

2. Holographic models

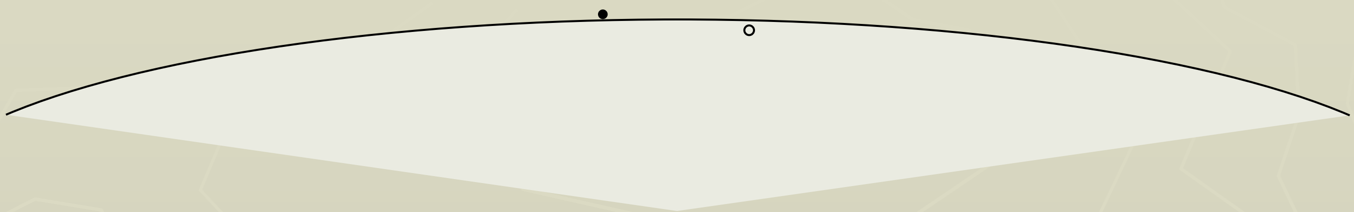
3. Semiholographic models

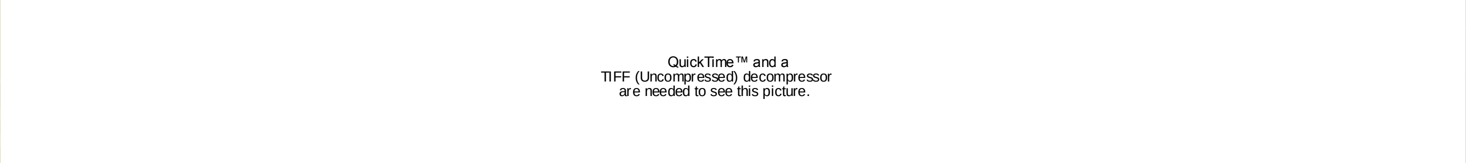
1. Nonholographic models

General feature: fermions in interaction with some other low energy degree of freedom.

Nonholographic (i.e. field theory) models

Look near a point on the Fermi surface in 2+1:





(Fairly generic.) Look at regime $t \to \varepsilon^{-2} t,\ x^1 \to \varepsilon^{-1} x^1,$
$x^2 \to \varepsilon^{-2} x^2,\ \psi \to \varepsilon^{3/2} \psi,\ a \to \varepsilon^{3/2} a$:
interaction ~ $\varepsilon^{-5+9/2} = \varepsilon^{-1/2}$: relevant

Look at many-flavor (or spin) limit for fermions. Fermion loop ~ $i|\omega|/q$ (Landau damping): more relevant than previous kinetic term for a.

QuickTime™ and a
TIFF (Uncompressed) decompressor
are needed to see this picture.

Interaction is now marginal, under $t \to \varepsilon^{-2} t,\ x^1 \to \varepsilon^{-1} x^1,\ x^2 \to \varepsilon^{-2} x^2,\ \psi \to \varepsilon^{3/2} \psi,\ a \to \varepsilon^2 a$, effective coupling of order $1/N$.

Look at many-flavor (or spin) limit for fermions (JP '93). Fermion loop ~ ω/q (Landau damping): more relevant than previous kinetic term for a.

QuickTime™ and a
TIFF (Uncompressed) decompressor
are needed to see this picture.

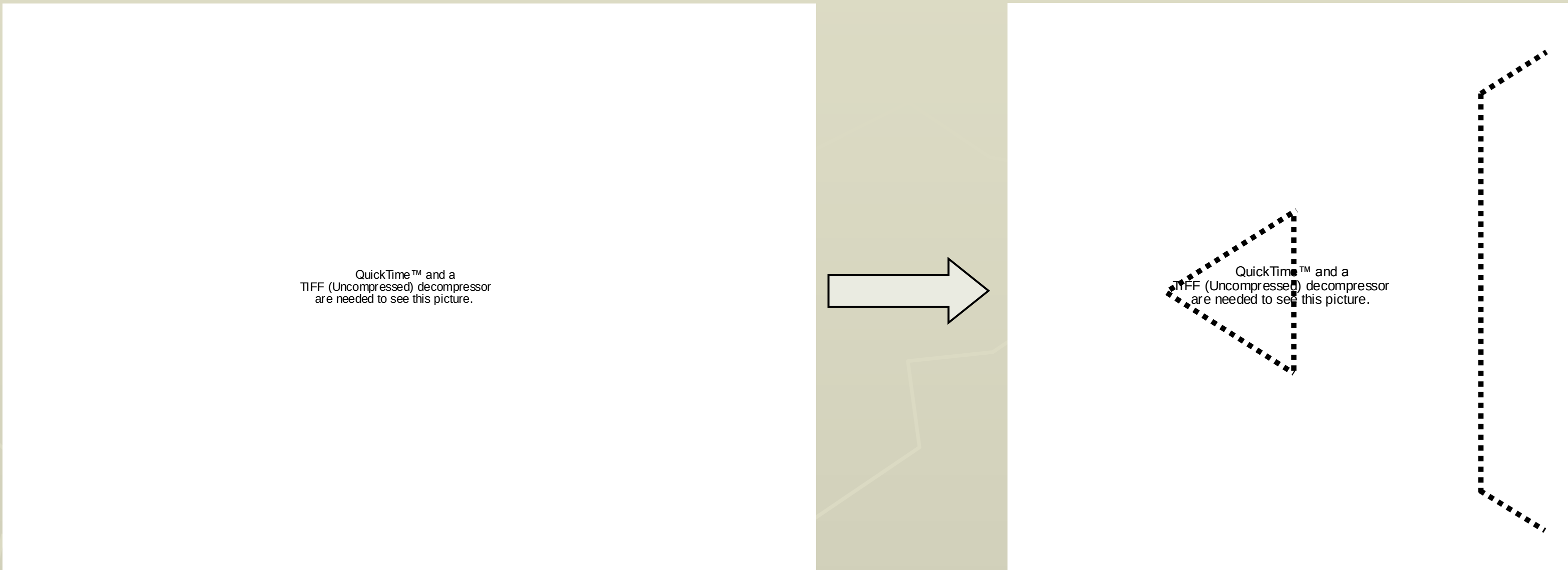
Interaction is now marginal, under $t \to \varepsilon^{-2}t,\ x^1 \to \varepsilon^{-2}x^1,$ $x^2 \to \varepsilon^{-1}x^2,\ \psi \to \varepsilon^{3/2}\psi,\ a \to \varepsilon^2 a,$ effective coupling of order $1/N$.

Not so simple, though: marginal under a whole range of scalings, $t \to \varepsilon^{-z}t,\ x^1 \to \varepsilon^{-2}x^1,\ x^2 \to \varepsilon^{-1}x^2,\ \psi \to \varepsilon^{(z+1)/2}\psi,$ $a \to \varepsilon^2 a,$ for $2 \le z \le 3$.

Enhancement of many graphs (S.-S. Lee), e.g.

Naively N^{-2} due to internal bosonic propagators, but actually N^{-0}.

Enhancement of many graphs (S.-S. Lee), e.g.



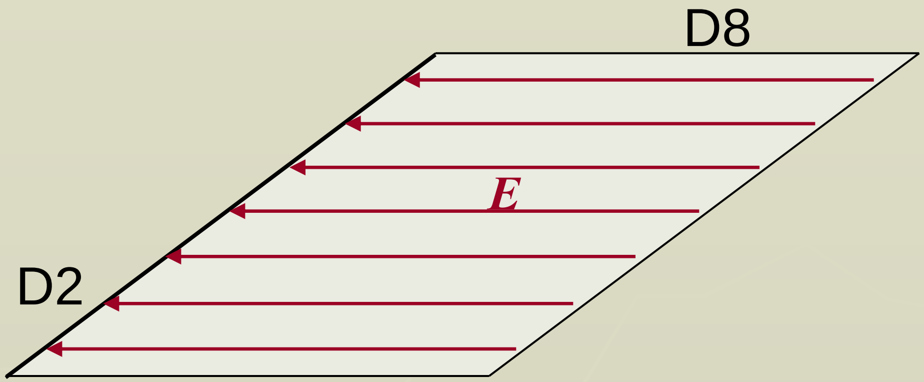
Naively N^{-2} due to internal bosonic propagators, but actually N^{-0}. Proposal: introduce double line notation (extra line is location around Fermi surface) and all planar graphs survive. (Recent work: Melitski & Sachdev; Mross, McGreevy, Liu, Senthil).

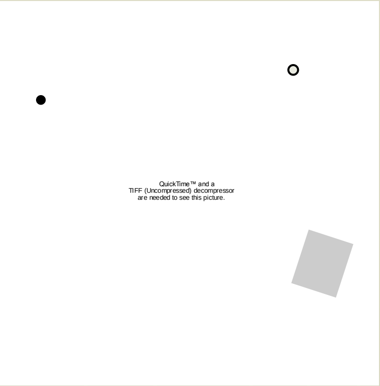
Holographic models

Find a brane configuration whose weakly coupled physics is fermi sea + massless modes and follow it to strong coupling, e.g.

	0	1	2	3	4	5	6	7	8	9
D2	x	x	x							
D8	x	x	x	x	x	x	x	x	x	

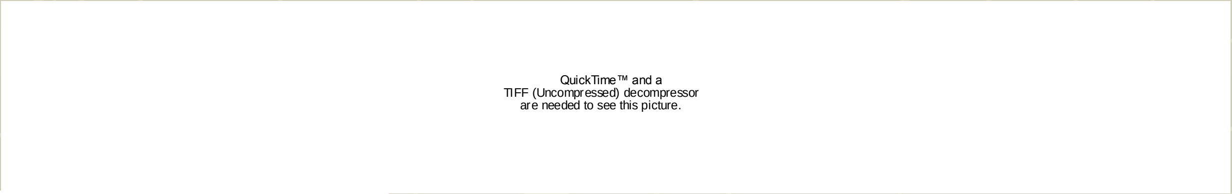
(2+1) 2-8 massless strings are fermionic, coupled to 2-2 gauge fields. Turn on chemical potential for 2-8 strings. Vacuum dynamics is conformal, at least for $N_8 >> N_2$.

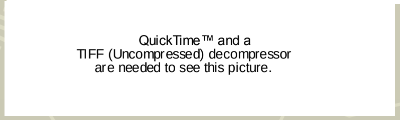




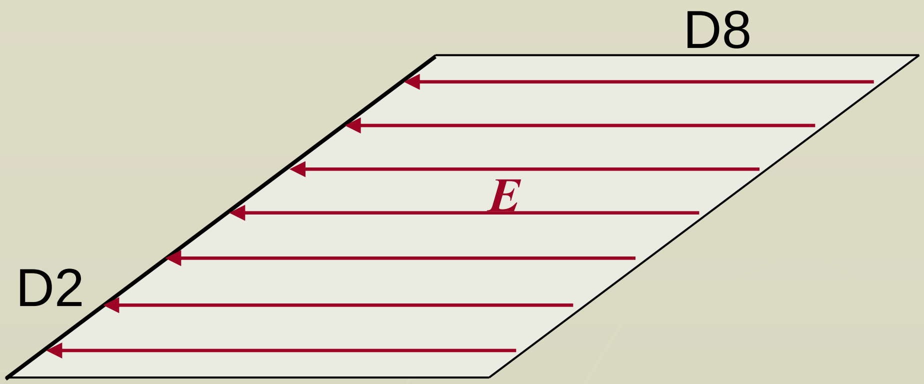
One nice result (Kulaxizi & Parnachev): Fermi-surface like behavior in current-current correlator (imaginary part down to $\omega = 0$ for finite momentum q).

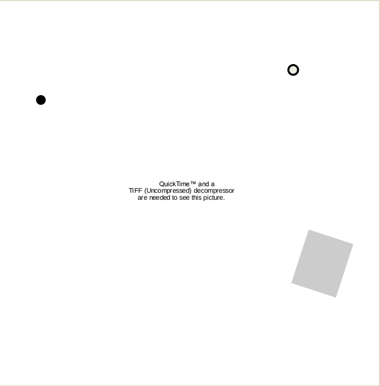
From expanding DBI action:





q-dependent terms suppressed at horizon $y \to \infty$.





Puzzles:

- No $2k_{\mathrm{F}}$ singularities (strong coupling?)
- Seems to work even for brane configurations with charged bosons.
- Breakdown of picture as $\omega \to 0$:
    - Backreaction on bulk fields blows up at the horizon, difficult to cure the singularity. General problem with top-down constructions.
    - Even before backreaction on bulk becomes large, DBI field theory becomes stringy because $2\pi\alpha' E \to 1$.

Forge ahead optimistically to transport properties (Hartnoll, JP, Silverstein & Tong, generalizing Karch, Kulaxizi & Parnachev). Ignore backreaction, e.g. by taking $N_{\rm color}$ large (valid over a range of scales). Take probe branes in general conformal background,

$$ds^2 = r^2 dt^2 + r^{2/z} dx \cdot dx + dr^2/r^2$$

$z =$ dynamical exponent (Kachru, Liu & Mulligan)

Dimensional analysis gives resistivity $\rho \propto T^{z/2}/J^t$ when the charge density J^t is sufficiently small. Model confirms this, and range of validity larger than expected. Plausible value $z = 2$ gives linear resistivity. Hall conductivity does not work in probe (Drude) limit, needs more.

A semiholographic model

(Lee, 0809.3402; Liu, Mcgreevy, Vegh, 0903.2477; Cubrovic, Zaanen, Schalm, 0904.1993; Faulkner, Liu, Mcgreevy, Vegh, 0907.2694; Faulkner & JP 1001.5049)

Bulk theory: metric with negative c.c. + $U(1)$ gauge field with Einstein-Maxwell action + massive charged fermion ψ.

Dual theory has operators $T_{\mu\nu}, j_\mu, O_s$, specific Lagrangian not known.

At finite charge density, ground state is a charged AdS black brane.

FLMV show that the O_s correlator from AdS/CFT is

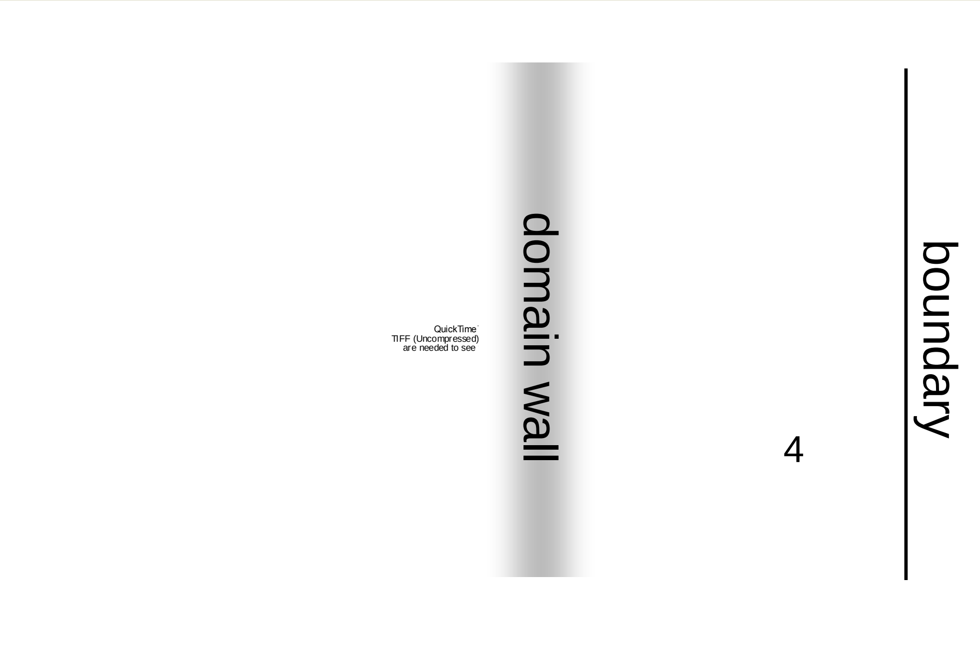
$$G_O = \frac{1}{\omega - v_F k_\parallel - C\omega^{2\nu}}$$

$k_\parallel =$ distance from Fermi surface. v_F, C, ν are constants due to rotational symmetry, but would generically vary along the FS.

The exponent ν would be 1 for an LFL, $1/2$ for a marginal FL, and is continuously variable from 0 to ∞ here, depending on the charge-to-mass ratio of the bulk fermion ψ.

So this system has a Fermi surface with non-Fermi liquid behavior. How does it work?

Geometry at finite charge density:

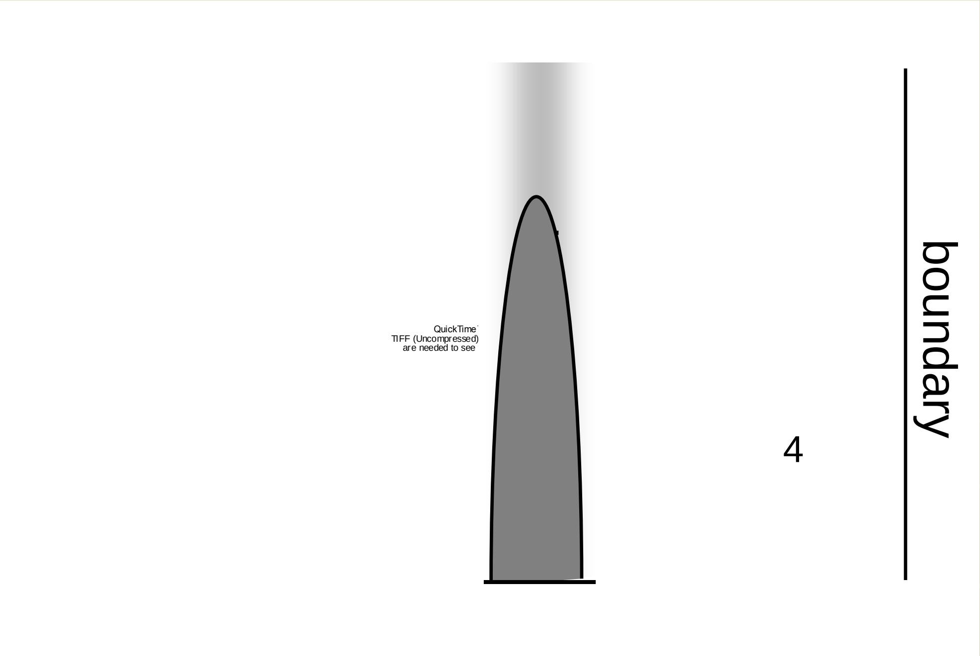


AdS_4: relativistic scaling (space ~ time)

$AdS_2 \times R^2$: local criticality: only time scales

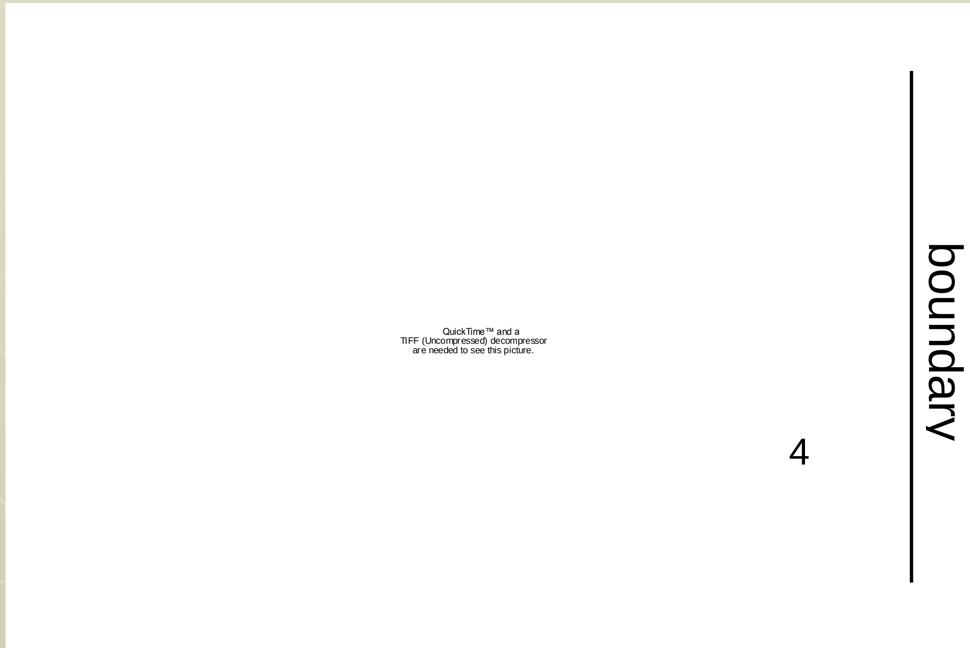
Lifshitz: space ~ time^z

Fermion density in bulk:



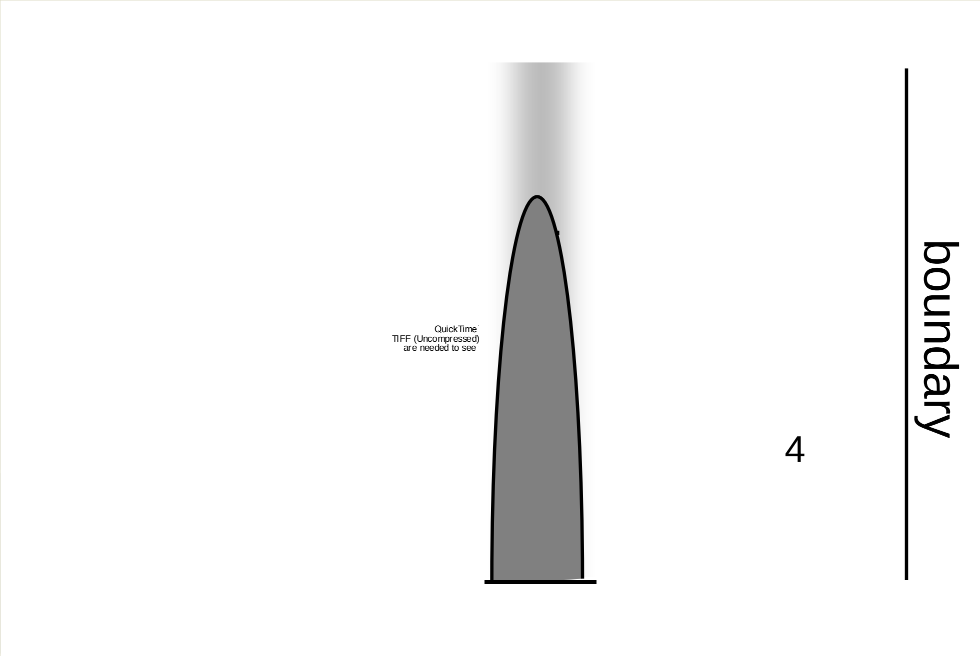
Shown is the local WKB $k_F \sim (-V)^{1/2}$. There is a Fermi liquid in the bulk, localized on the domain wall.

An aside:



Almost everywhere in the FLMV parameter space there is also a Fermi sea in the AdS_2 bulk. The total charge is divergent $\sim \int dr/r$ (AdS_2 spatial volume). Backreaction converts geometry to Lifshitz (Hartnoll, JP, Silverstein, Tong, 0912.1061).

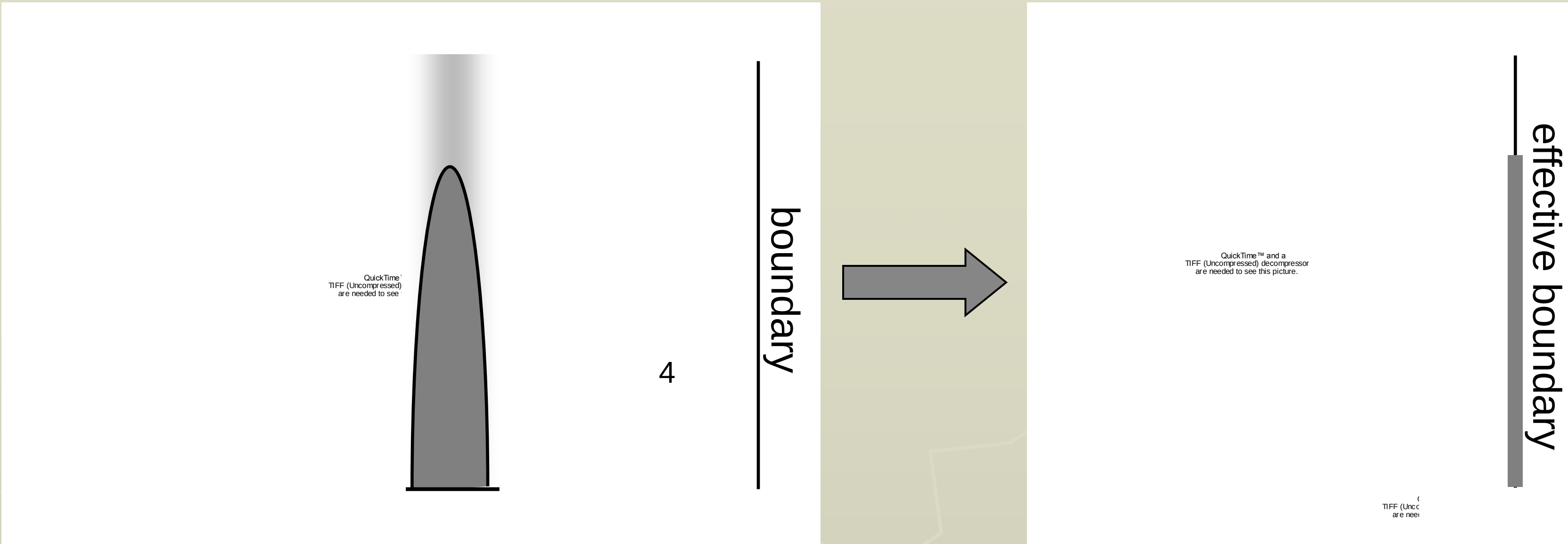
Low energy effective theory



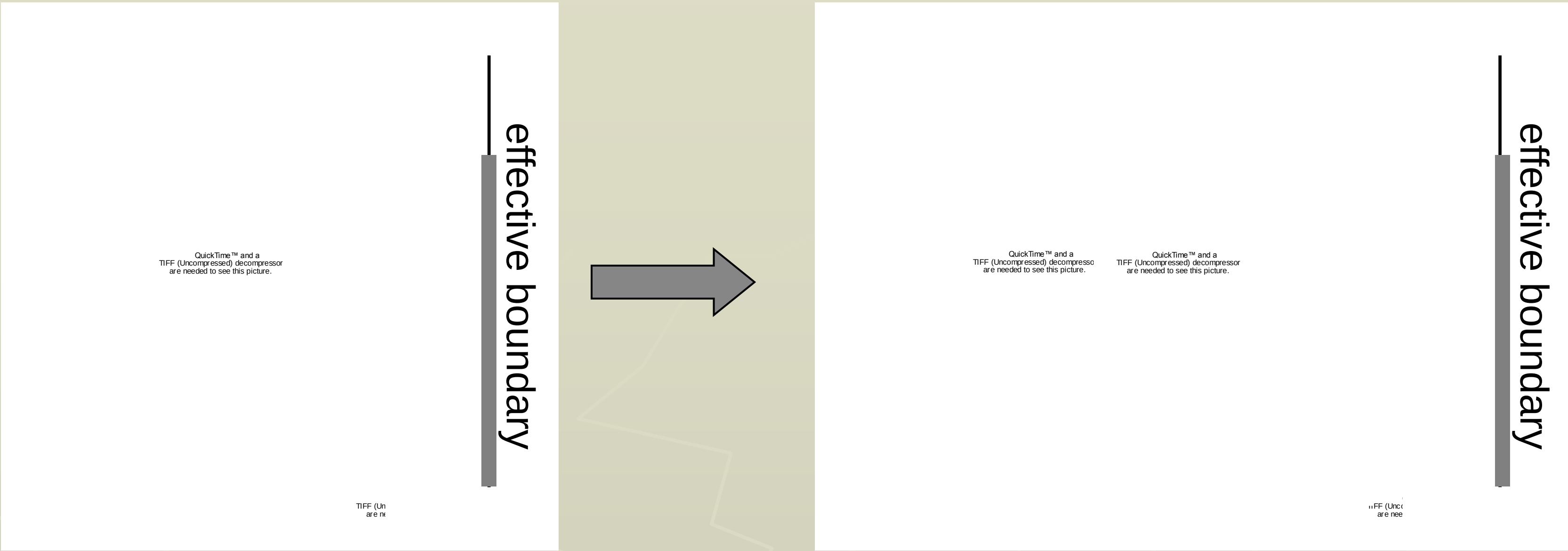
Two kinds of low energy excitation:

- Deformations of the Fermi surface
- States near the horizon

Let us write an effective theory with *only* these…



- Drop nonuniversal UV region and introduce boundary in place of domain wall
- Couple explicit boundary FL theory to the strongly coupled theory CFT described by the bulk dual.
- Move boundary to $r = \infty$ (matching)



• Drop nonuniversal UV region and introduce boundary in place of domain wall
• Couple explicit boundary FL theory to the strongly coupled theory CFT described by the bulk dual.
• Move boundary to $r = \infty$ (matching)

Same universality class

Effective theory:

QuickTime™ and a
TIFF (Uncompressed) decompressor
are needed to see this picture.

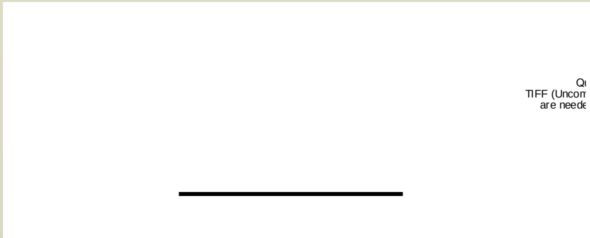
QuickTime™ and a
TIFF (Uncompressed) decompressor
are needed to see this picture.

Singlet fermion $\chi_{k,s}$, at finite density, coupled through fermion bilinear to strongly coupled CFT with charged fermionic operator $\Psi_{k,s}$ (dimension $\Delta_k \equiv \nu_k + 1/2$) and $AdS_2 \times R^2$ dual.

"Semi-holographic"

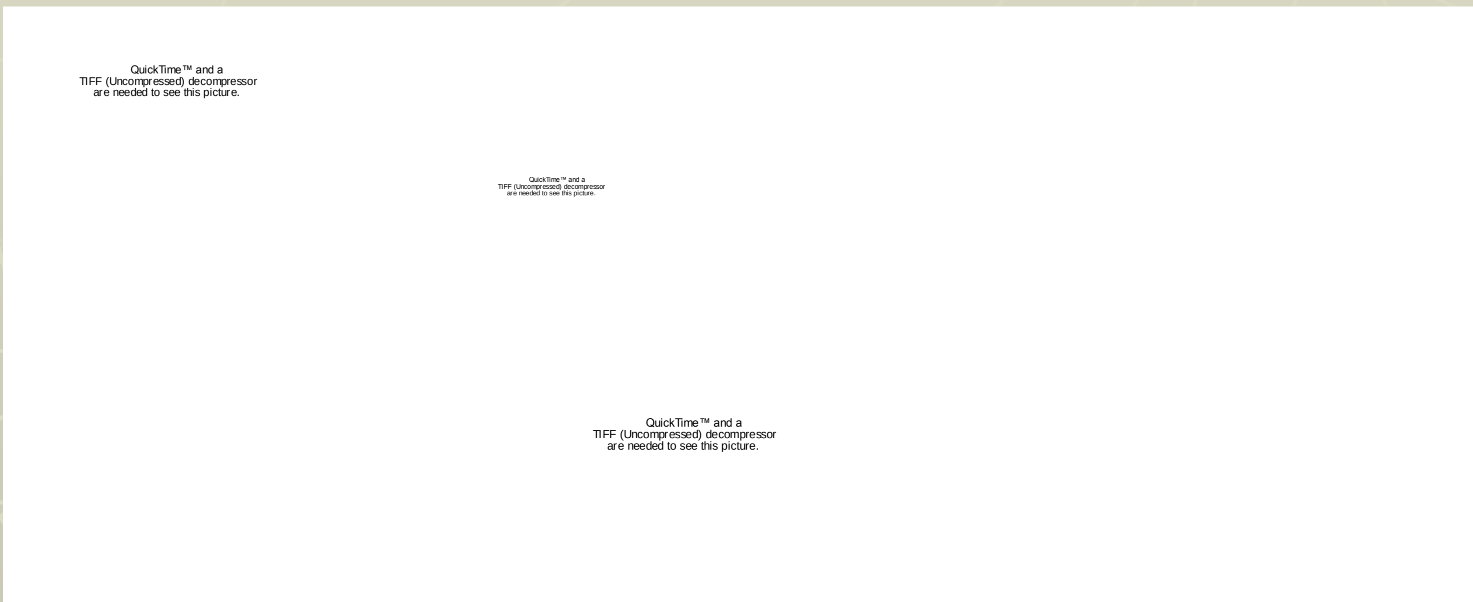
Propagators in decoupled theories:

χ :



Ψ :

Full χ propagator:



Reproduces non-Fermi liquid behavior of FLMV.

Generalization: more general critical sector

$$ds^2 = r^2 dt^2 + r^{2/z} dx \cdot dx + dr^2/r^2$$

$z = \infty$: AdS$_2$ x R^d

$z = 1$: AdS$_{d+2}$

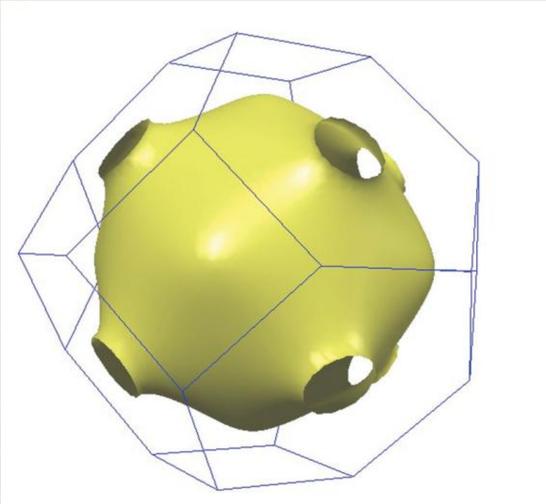
$1 < z < \infty$: Lifshitz

Kachru, Liu, Mulligan, 0808.1725

NFL behavior requires G_0 to be critical for $\omega \to 0$ at *finite* k, this holds for $z = \infty$ only, AdS$_2$ x R^d. For $z < \infty$ critical behavior is at $\omega, k \to 0$.

Generalization: spacetime lattice (important, e.g., for proper treatment of conductivity).

$$\mathcal{L}_\chi =$$



Translation invariance broken to discrete lattice symmetries $\rightarrow \vec{k}$ is only defined mod reciprocal lattice vectors $\vec{K}$. E.g. in a 1-dimensional lattice of spacing a, $k \sim k + 2\pi/a$.

Also in coupling:

QuickTime™ and a
TIFF (Uncompressed) decompressor
are needed to see this picture.

Generalization: Impurities

Add momentum-violating terms

$$\chi_{\vec{k}}^{\dagger}\chi_{\vec{k}+\vec{q}}\,, \quad \Psi_{\vec{k}}^{\dagger}\chi_{\vec{k}+\vec{q}}\,, \quad \chi_{\vec{k}}^{\dagger}\Psi_{\vec{k}+\vec{q}}$$

Other approaches to lattices, impurities:
Hellerman, hep-th/0207226
Kachru, Karch, Yaida, 0909.2639
Hartnoll, Herzog, 0801.1693

Summary:

- Fermi liquid coupled to strongly coupled CFT through fermion mixing term, rather than bosonic coupling - a new framework?

- Connects with electron fragmentation ideas: Ψ creates a composite color singlet state. What are the fragments? At strong coupling, there are no `partons.' But coupling likely ~1 in the real systems, as at RHIC.

- Fragments = unquasiparticles. Quasiparticle: particle-like excitation in CM effective field theories. Unparticle: non-particle excitations in CFTs.

Summary, continued:

Puzzles:

• Requires $AdS_2 \times R^2$ scaling - can this occur in realistic systems, e.g. at finite N?

• Requires $\Delta_{\vec{k}} = 1$; why is this value preferred? Generically $\Delta_{\vec{k}}$ will vary over the Fermi surface, and get $1/N$ corrections.

Note: Marginal Fermi liquid also needs $AdS_2 \times R^2$ scaling (singularities at $\omega = 0$ for nonzero k), but coupled through bosonic currents.

Conclusion

There appears to be a fascinating new low energy fixed point, which can appear in many interesting materials. Holographic methods may provide new insights.