THE KERR-FERMI SEA

STRINGS 2010
MITCHELL INSTITUTE
TEXAS A&M

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MARCH 2010

Another chapter in the fascinating saga of Kerr

"...But Kerr's solution has also surpassing theoretical interest; it has many properties that have the aura of the miraculous about them..."

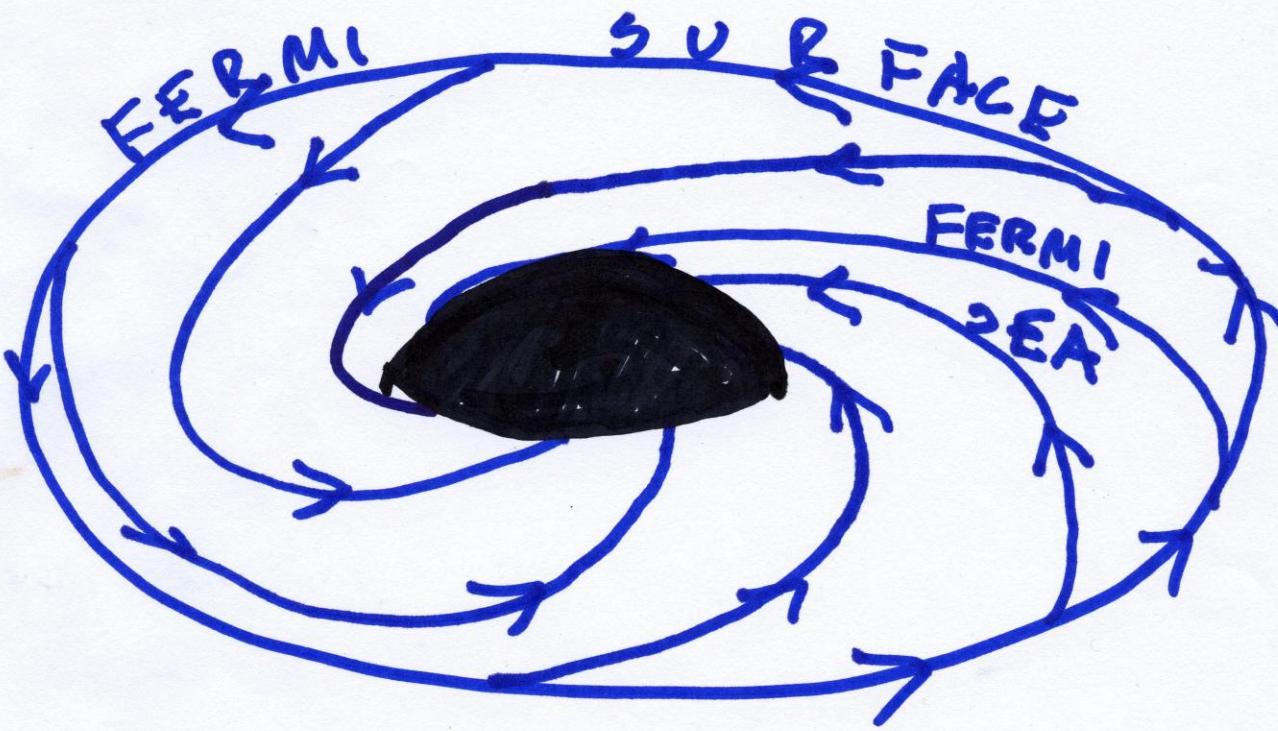
S. Chandrasekhar, 1979

Existence of exact solution, separability of wave equation, consistency of energy extraction, Killing-Yano symmetry, ~~no~~ SUSY, near-horizon conformal symmetry ....

What is the source of these miracles?

MAIN RESULT

In the presence of FREE massive fermions, in its $\hbar \to 0$ semiclassical ground state a rotating black hole



soaks in a whirlpool bath filled with Fermi liquid!

(not empty space)

MOTIVATION/INTEREST

(i) String theory → CFT dual of Kerr = Fermi liquid
(still not understood)

(ii) Interesting parallels/ connections to recent work on AdS/CMT:
Our techniques perhaps applicable. Duals of rotating atomic condensates,
c.f. Schwaikhard et. al.

(iii) Astrophysical black holes potentially bathing in neutron liquid. observable consequences?

Surprising & basic feature of Kerr ground state which might have been noticed 30 years ago.

Black Hole Bomb

Press, Teukolsky '72 Eardley Zouros '7

A scalar wave with

$\boxed{\omega < m\,\Omega_H}$ scatters

energy ang mom. horizon velocity

superradiantly, extracting

energy from the black hole.
Consistency constraints
$\Rightarrow$ Second Law of Black Hole Mechanics

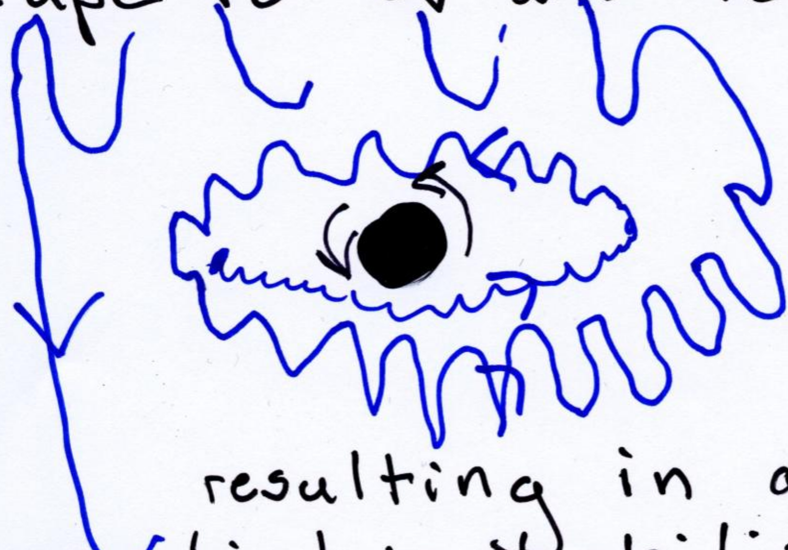
If the wave has

$\boxed{\omega < m\Omega_H}$ (superradiant) and

$\boxed{\omega < \mu}$ (bound) it can't

mass

escape to ∞ and rescatters



resulting in an exponential instability or

BLACK HOLE BOMB

(i) Kerr + massive scalar = unstable

(ii) endpoint unknown

(iii) ~~graviton, photon bomb~~

(iv) small Kerr-AdS = unstable

(v) $\simeq$ holographic superconductor

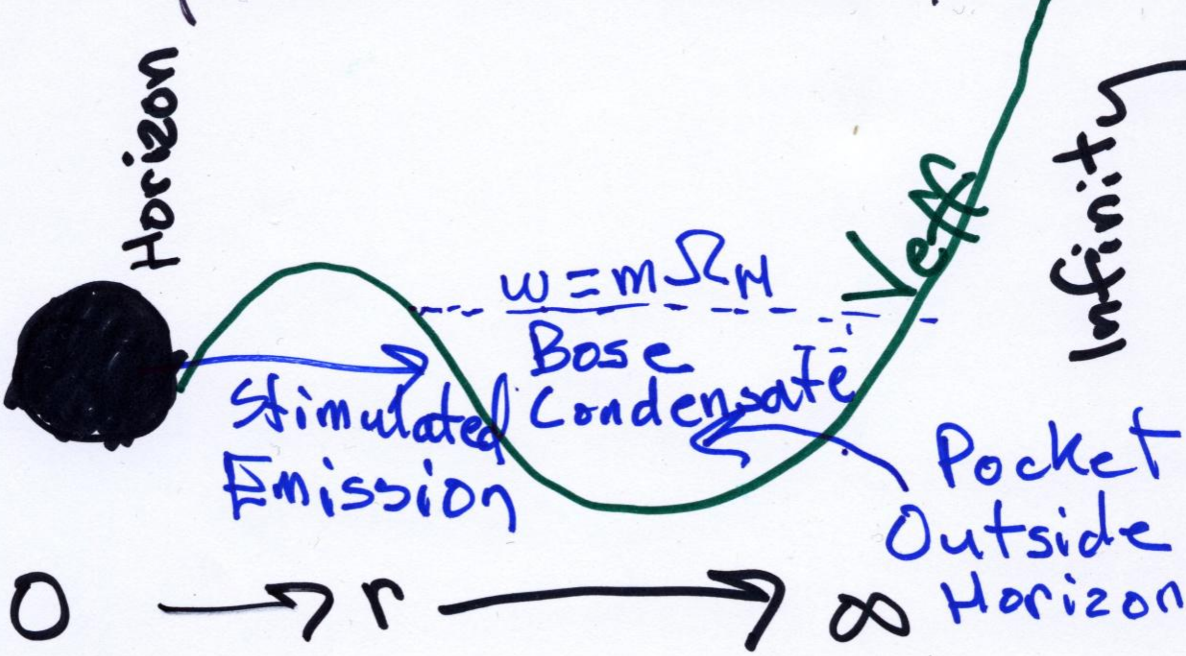
(vi) Constraints on light axions

Arvanitaki Dimopolous Dubovsky Kaloper March-Russell

Particle Picture

$\hbar \to 0$ particle limit μ fixed $\neq$ $\hbar \to 0$ field limit $\frac{\mu}{\hbar}$ fixed

Superradiance $\to$ Penrose process



Black hole = ~~th~~ thermal reservoir with Boltzman factor $e^{-\frac{\omega - m\Omega_H}{T_H}}$.

Bound superradiant particles cannot get across horizon or to ∞. They form condensate in pocket which stimulates emission, which feeds condensate

Runaway $\Rightarrow$ BLACK HOLE BOMB

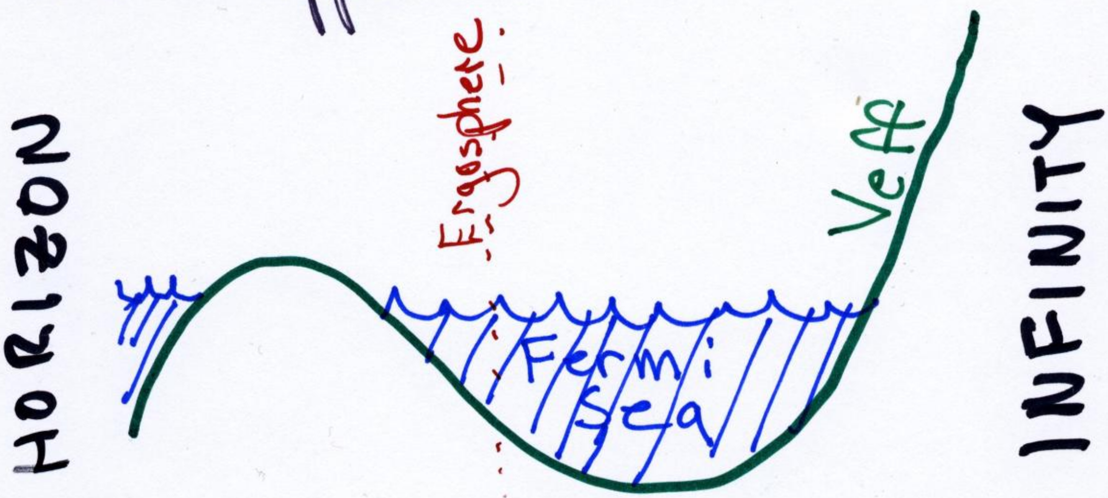
OUR QUESTION:

What is the fermionic version of the black hole bomb?

Real world has light $\mu \neq 0$

FERMIONS

What happens?



KERR BLACK HOLE GROUND STATE

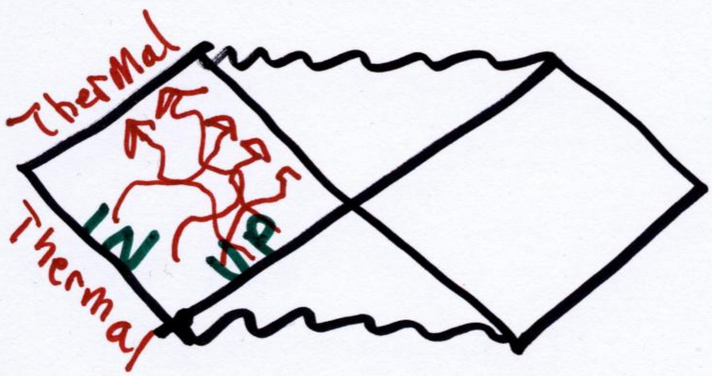
has a filled fermi sea extending well outside ergosphere, and light waves persist in the classical limit.

Due to the "miraculous" properties of Kerr, the properties of this Fermi sea are analytically calculable in the WKB limit.

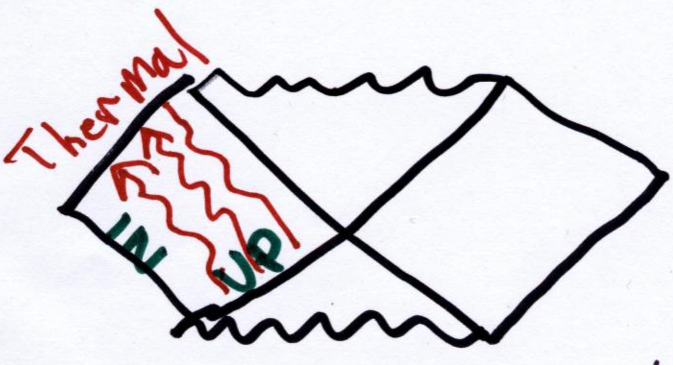
Note $\dfrac{M_p^2}{M_{sun} M_{neutron}} \sim 10^{-19}$.

Unruh Vacuum

is the asymptotic state approached by matter fields around a black hole formed by gravitational collapse.

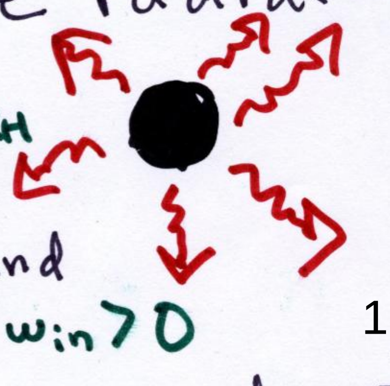


Hartle -Hawking Vacuum



Unruh Vacuum

It describes a black hole radiating into empty space, with Boltzmann factor $e^{-\frac{\omega - m\Omega_H}{T_H}}$



For extreme Kerr $T_H \to 0$ and

(i) IN modes are empty for $\omega_{in} > 0$

(ii) UP modes are full if superradiant $\omega_{up} - m_{up}\Omega_H < 0$

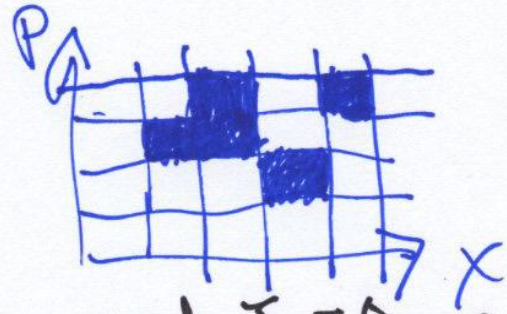
WKB Particle Limit

$\hbar \to 0$ mass $\equiv m$ fixed

Instead of using quantum field operators, states ~~are~~ described in terms of the single-particle phase space =

$$(r, \theta, \phi, p_r, p_\theta, m)$$

with cells of volume $\hbar^3$. p_ϕ

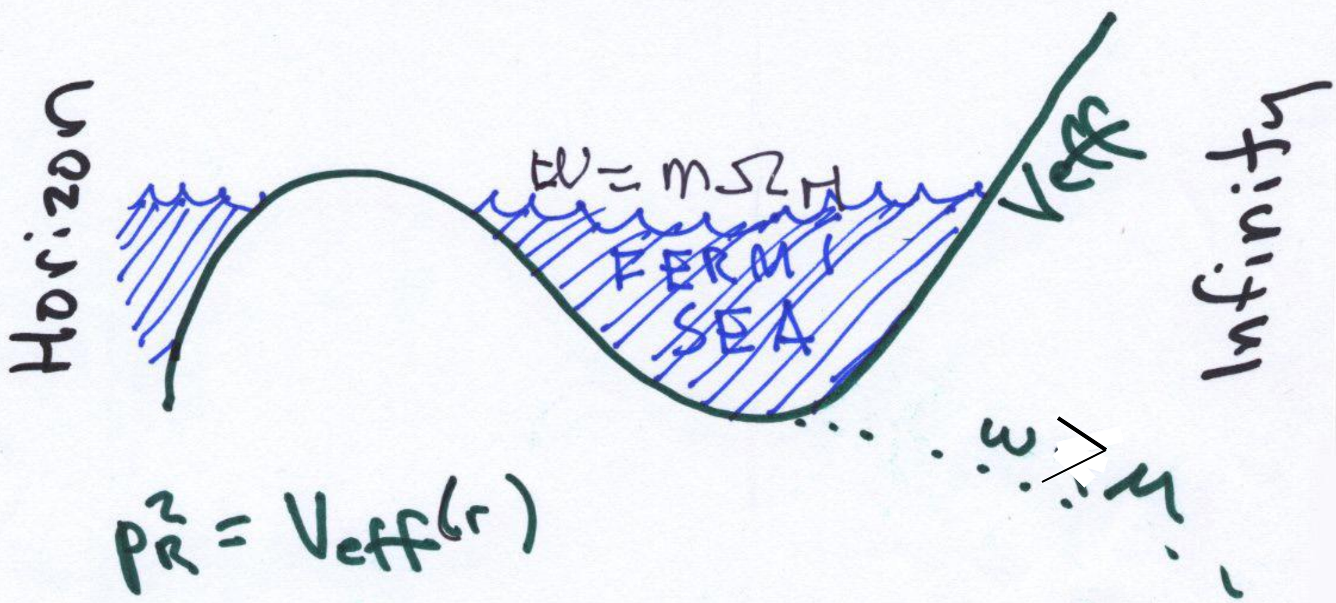


For fermions and $T_H = 0$, a multi-particle state is specified by saying which cells are filled & which empty. (4 copies for Dirac fermions $e\uparrow, e\downarrow, p\uparrow, p\downarrow$)

An orbit in phase space is <u>filled</u> in the WKB limit of the Unruh vacuum iff it ~~is the~~ corresponds to the limit of a <u>filled</u> WKB wavefunction.

Orbits turn out to be filled if they are bound & superradiant (no surprise)

$$\boxed{\omega < m\Omega_H \text{ and } \omega < \mu}$$



$$p_R^2 = V_{eff}(r)$$

$$-\partial_r^2 \psi_{WKB} = V_{eff}\, \psi_{WKB}$$

Detailed analysis is a bit involved, but answer here is simple.

To be more precise, choose Boyer-Lindquist coordinates

$$ds^2 = -\frac{\Delta}{\rho^2}(dt - M\sin^2\theta\, d\phi)^2 + \frac{\rho^2}{\Delta}dr^2 + \frac{\sin^2\theta}{\rho^2}((r^2+M^2)d\phi - M\,dt)^2 + \rho^2 d\theta^2$$

$\rho^2 \equiv r^2 + a^2\cos^2\theta$ $\qquad$ $\Omega_H = \frac{1}{2M}$

$\Delta = (r-M)^2$ $\qquad$ $T_H = 0$ (extreme)

Then a particle orbit in Kerr has THREE constants of motion

$m = p_\phi$ angular momentum

$-p_t = \omega =$ solution of quadratic equation $p^\mu p_\mu = -\mu^2$ energy

$$Q = p_\theta^2 + \frac{(m - M\omega\sin^2\theta)^2}{\sin^2\theta} + M^2\mu^2\cos^2\theta - (m-\omega M)^2$$

Carter constant – surprise

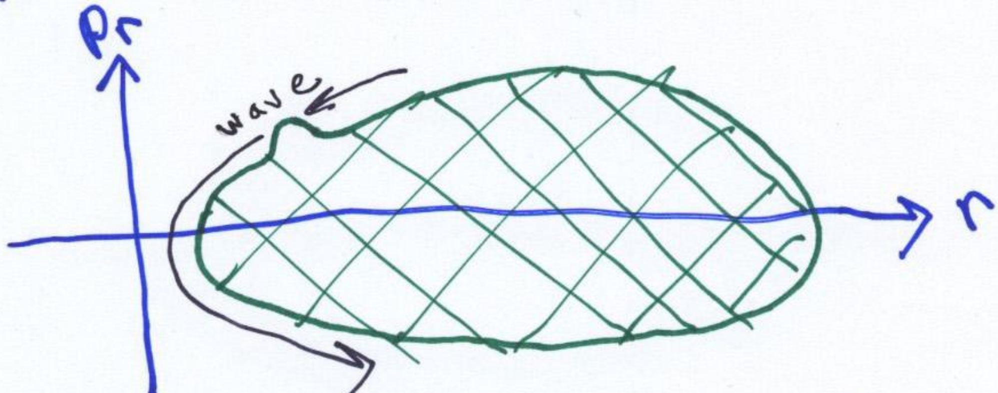
The FERMI SURFACE then has two pieces described by the 5-dimensional hypersurfaces in phase space $(r, \theta, \phi; p_r, p_\theta, p_\phi)$

$\omega = \mu$ $\qquad$ $\hookrightarrow (r, \theta, \phi; \omega, Q, m)$

$\omega = m\Omega_H$

After some algebra, the $\omega = m\Omega_H$ superradiant hypersurface can be described by

$$p_r = \pm \frac{\sqrt{\frac{r^2+2Mr}{4M^2} m^2 - \mu^2 r^2 - Q}}{r-M} \equiv \pm \sigma_0$$



(i) for each Q, m separate Fermi surface in ordinary space

(ii) waves on top Fermi surface $\tilde{\sigma} = p_r - \sigma_0$ obey fluid continuity equation

$$\partial_t \tilde{\sigma} + \partial_i (v^i \tilde{\sigma}) = 0,$$

$$v^i = v^i(r, \theta, m, Q, M) = \frac{\partial \omega}{\partial p_i}$$

They can be excited by a particle incident on the sea.

Properties

Number density

$$\text{measure} = \frac{1}{(2\pi)^3}\, dr \wedge d\theta \wedge d\phi \wedge dp_r \wedge dp_\theta \wedge dp_\phi$$

$\hbar = 1 = G = c$
$= M_p$

density at (r, θ, ϕ) is

$$N(r,\theta,\phi) = \frac{1}{(2\pi)^3 \sqrt{h}} \int_{sea} dp_r \wedge dp_\theta \wedge dp_\phi$$

$$= \frac{\mu^3}{(2\pi)^2 (r-M)\sqrt{h}} \left(\frac{4M^3\chi}{3} - M(r^2 + a^2\cos^2\theta) + \frac{1}{3\sqrt{\chi}}\left(r^2 + a^2\cos^2\theta\right)^{3/2} \right)$$

$$\sqrt{h} \equiv \rho \sin\theta \sqrt{\frac{(r^2+M^2)^2}{\Delta} - M^2\sin^2\theta}$$

$$\chi \equiv \frac{5}{4} + \frac{r}{2M} + \frac{r^2}{4M^2} - \csc^2\theta - \frac{1}{4}\sin^2\theta$$

For $r \sim M$

$N \sim \mu^3$ one fermion/Compton cube

Also

$$E(r < M) \sim \mu^4 M^3$$

REAL WORLD

Do observed black holes like to bathe in neutron (or other) liquids?

If a Kerr black hole is formed by the collapse of ~~eg~~ photons

the pocket is created devoid of fermions. Populating it via Unruh-Starobinski-Hawking radiation takes a time of order $e^{+\frac{1}{m}} \to \infty$.

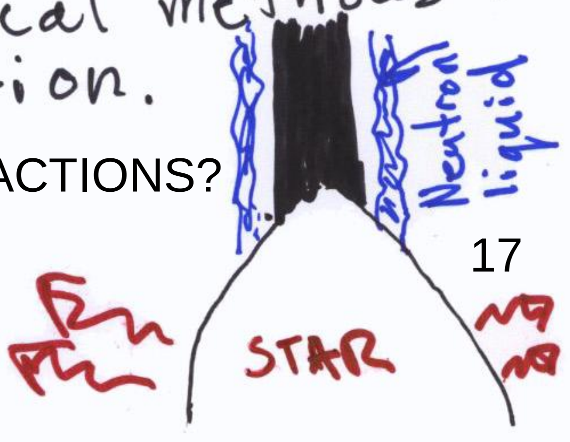
On the other hand consider an aging star with

M > Oppenheimer-Volkov, Chandrasekhar limits

Assuming not too much matter is blown off as it burns out and begins to collapse, it cannot evolve into a neutron star or white dwarf. It is usually assumed to become an ordinary Kerr black hole. However it may also become a bathing black hole if baryons are trapped in the pocket. This can be addressed possibly by analytic/numerical methods - or even observation.

ARE THEY STABLE WITH INTERACTIONS?

Effects of accretion disc???



Note that the ratio of neutron energy inside $r \lesssim 10M$ to the mass M of the black hole itself goes as

$$\text{Ratio} \simeq \frac{m^4_{\text{neutron}} M^2_{\text{star}}}{M^6_{\text{Planck}}}$$

$$\frac{m_{\text{neutron}}}{M_{\text{planck}}} \sim 10^{-19}$$

$$\frac{M_{\text{sun}}}{M_{\text{planck}}} \sim 10^{38}$$

So the backreaction is large for $M_{\text{star}} \sim M_{\text{sun}}$ (This is the same counting which puts the Oppenheimer-Volkoff limit for a neutron star of order M_{sun}.) and might have observable consequences.

Can we find the back-reacted bathing black hole solution?

Can we see black holes bathing in the sky?

CONCLUSION

The mathematical theory of Kerr black holes has a rich structure with diverse connections to CM holography, astrophysics, string theory, supersymmetry, black hole puzzles and gravity/field theory dualities. Much has been learned, but much remains mysterious & waiting to be understood.