Gravity as an Effective Field Theory

$$I_\Lambda[g] = -\int d^4x \, \sqrt{-\mathrm{Det} g} \Big[\Lambda^4 g_0(\Lambda) + \Lambda^2 g_1(\Lambda) R$$
$$+ g_{2a}(\Lambda) R^2 + g_{2b}(\Lambda) R^{\mu\nu} R_{\mu\nu}$$
$$+ \Lambda^{-2} g_{3a}(\Lambda) R^3 + \Lambda^{-2} g_{3b}(\Lambda) R R^{\mu\nu} R_{\mu\nu} + \ldots \Big] \, .$$

Here either

- Λ is an ultraviolet cutoff, $g_n(\Lambda)$ are dimensionless unrenormalized couplings,

  or

- Λ is a sliding renormalization scale, $g_n(\Lambda)$ are dimensionless renormalized couplings (and counterterms are needed).

Either way, the Λ-dependence of the dimensionless couplings $g_n(\Lambda)$ is such that physics is independent of Λ.

$$\Lambda \frac{d}{d\Lambda} g_n(\Lambda) = \beta_n\Big(g(\Lambda)\Big)$$

There is no obstacle to letting Λ go to infinity if there is a fixed point with $\beta(g_*) = 0$, and if $g(\Lambda)$ is on a trajectory attracted to the fixed point. This is "asymptotic safety." (SW, 1976)

Trajectories with $g \to g_*$ for $\Lambda \to \infty$ form the *ultraviolet critical surface*. The physical requirement that actual couplings lie on UV critical surface may play the same role for theories including gravitation as does renormalizability in QCD.

Near a fixed point

$$\Lambda \frac{d}{d\Lambda} g_n(\Lambda) = \sum_m B_{nm} \Big(g_n(\Lambda) - g_{n*} \Big) ,$$

where

$$B_{nm} \equiv \left. \frac{\partial \beta_n(g)}{\partial g_m} \right|_{g=g_*}$$

so for $\Lambda \to \infty$,

$$g_n(\Lambda) \to g_{n*} + \sum_r u_n^{(r)} \Lambda^{\lambda_r}$$

where

$$\sum_m B_{nm} \, u_m^{(r)} = \lambda_r \, u_n^{(r)} .$$

The dimensionality of the ultraviolet critical surface equals the number of eigenvalues with $\mathrm{Re}\lambda_r < 0$.

Even with an infinite number of couplings $g_n(\Lambda)$, it's not surprising to find a finite-dimensional critical surface.

Indications of Asymptotically Safe Gravitation

- Dimensional Continuation ($d = 2+\epsilon$)
  - SW 1979
  - Kawai, Kitazawa, & Ninomiya, 1993, 1996
  - Aida & Kitazawa, 1997 (2 loops)
  - Niedermaier 2003
- $1/N$ Expansion
  - Smolin 1982 ($R + C^2$)
  - Percacci, 2006
- Lattice Quantization
  - Ambjørn, Jurkewicz, & Loll, 2004, 2005, 2006, 2008

- Truncated 'Exact" Renormalization Group
  - Wegener & Houghton, 1973
  - Polchinski, 1984
  - Wetterich, 1993

  (Exact renormalization group equations link all $g_n(\Lambda)$. Truncate equations by setting all but a finite number of $g_n(\Lambda)$ equal zero, ignore the non-zero value of the other $\beta_n(g)$.)

  - Reuter, 1998
  - Dou & Percacci, 1998
    (gravity + free matter)
  - Souma, 1999 ($R^0 + R$)
  - Lauscher & Reuter, 2001 ($R^0 + R$)
  - Reuter & Saueressig, 2002 ($R^0 + R$)

– Lauscher & Reuter, 2002
$(R^0 + R + R^2)$

– Reuter & Saueressig, 2002

– Percacci & Perini, 2002, 2003
(constraints on free matter)

– Perini, 2004

– Litim, 2004

– Codello & Percacci, 2006

– Reuter & Saueressig, 2007

– Machado & Saueressig, 2007

– Litim, 2008

With only 2 non-zero couplings, UV critical surface is 2 dimensional.
With only 3 non-zero couplings, UV critical surface is 3 dimensional.
This was not encouraging.

Good News! Calculations with $N > 3$ non-zero couplings:

– Codello, Percacci, & Rahmede, 2008
$2 \leq N \leq 9\,(R^0,\ R^1,\ \ldots R^{N-1}$; also with matter)

– Benedetti, Machado, & Saueressig, 2009
$N = 4\ (R^0,\ R,\ R^2,\ C^2$; also with matter)

Both groups find a 3-dimensional UV critical surface in all cases.

PROBLEMS

1. Must the couplings must lie on the ultraviolet critical surface? Yes, if otherwise observables blow up at finite Λ.
(Landau pole, φ^4 "triviality")

2. Does the truncation converge?

$$R^0, R, R^2, \ldots, R^{N-1}$$

UV attractive eigenvalues:

$N = 3$	$-1.38 \pm 2.32i$	-26.8
$N = 4$	$-2.71 \pm 2.27i$	-2.07
$N = 5$	$-2.86 \pm 2.45i$	-1.55
$N = 6$	$-2.53 \pm 2.69i$	-1.78
$N = 7$	$-2.41 \pm 2.42i$	-1.50
$N = 8$	$-2.51 \pm 2.44i$	-1.24
$N = 9$	$-2.41 \pm 2.55i$	-1.40

3. How do we use Γ_Λ?

For processes at energy (or rate) E:

- If $\Lambda \gg E$, higher terms in truncation may be negligible, but radiative corrections are important. (E.g. for $\Lambda \to \infty$, tree approximation for H gives $H \propto \Lambda$, but physical quantities should be Λ-independent.)
- If $\Lambda \ll E$, radiative corrections negligible, but truncation doesn't work.

So take $\Lambda \approx E$, & hope for the best. Since $H_{\rm true} = H_{\rm tree}(\Lambda) + H_{\rm rad}(\Lambda)$, to minimize $H_{\rm rad}(\Lambda)$ we take Λ so that

$$dH_{\rm tree}(\Lambda)/d\Lambda = 0 \ .$$

4. What about ghosts?

Problem arises with truncation of action; otherwise propagator denominator is $k^2 \times$ power series in k^2, power series may not have zeroes. But with truncation, power series is polynomial.

Example: Only 2nd and 4th derivatives of metric: (Stelle, 1977, 1978).

The denominator of the spin two propagator is, schematically,

$$\frac{1}{ak^4 + bk^2} = \left(\frac{1}{b}\right) \left(\frac{1}{k^2} - \frac{1}{k^2 + b/a}\right) ,$$

(with b from R, and a from R^2 and $R_{\mu\nu}R^{\mu\nu}$), so there are two poles in k^2, at 0 and $-b/a$. Whatever the sign of b, one of the poles seems to have a residue of the wrong sign.

Solution?: Couplings like a, b run with $\Lambda \approx \sqrt{k^2}$. It may be that either:

- There is no root of $k^2 = -b(k^2)/a(k^2)$ (Benedetti, Machado, & Saueressig, 2009)
- There is a root of $k^2 = -b(k^2)/a(k^2)$, but at this root $b(k^2)$ has a different sign from $b(0)$. (Niedermaier 2009)

Both papers consider

$$I_\Lambda[g] = -\int d^4x \sqrt{-\mathrm{Det}\, g} \Big[\Lambda^4 g_0(\Lambda) + \Lambda^2 g_1(\Lambda) R + g_{2a}(\Lambda) R^2 + g_{2b}(\Lambda) R^{\mu\nu} R_{\mu\nu} \Big] .$$

Benedetti et al.: Truncated exact renormalization group

Niedermaier: Perturbation theory

A Cosmological Application (SW 2010)

The perturbations to a de Sitter solution have $\dot{a}/a \propto e^{\xi Ht}$. For a Lagrangian density that depends only on $g_{\mu\nu}$ and $R_{\mu\nu\rho\sigma}$ but not on $R_{\mu\nu\rho\sigma;\rho}$, etc.

$$\xi^2 + B\xi - A = 0$$

For the truncated action

$$I_\Lambda[g] = -\int d^4x \sqrt{-\mathrm{Det}g}\Big[\Lambda^4 g_0(\Lambda) + \Lambda^2 g_1(\Lambda) R + g_{2a}(\Lambda) R^2 + g_{2b}(\Lambda) R^{\mu\nu} R_{\mu\nu}\Big] \, .$$

we have

$$B = 3 \, , \quad A = \frac{3g_1^2}{g_0(3g_{2a} + g_{2b})} \, .$$

If $0 < A \ll 1$, the roots are $\xi \simeq -3$ and $\simeq A/3$, so $\approx 3/A$ e-foldings.

Benedetti et al.: At fixed point

$$g_0 = -0.00442, \quad g_1 = -0.0101$$

$$g_{2a} = -0.0109, \quad g_{2b} = 0.01$$

so $A = 3.05$, & inflation ends immediately.

Niedermaier:

$$A \to 36.4 / \ln(\Lambda/M)$$

We don't know what value to give Λ/M.