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Higher Spin Gauge Theory and Holography

Xi Yin
IAS & Harvard U.

Based on work with Simone Giombi, arXiv:0912.3462
and previous work by Fradkin, Vasiliev, Mikhailov, Sezgin, Sundell, Klebanov, Polyakov, Petkou.....

Outline

- Sezgin-Sundell-Klebanov-Polyakov conjecture
- What is Vasiliev theory?
- The 3-point functions
- Comments

Motivational remarks

$\mathcal{N}=4$ U(N) SYM $\longleftrightarrow$ IIB $AdS_5 \times S^5$

λ, N — R, ℓ_s, ℓ_p

$$(R/\ell_s)^4 \sim \lambda, \qquad (R/\ell_p)^4 \sim N$$

- Keep N>>1, $R \gg \ell_p$: weakly coupled gravity
- $\lambda \gg 1 \leftrightarrow R \gg \ell_s$ - supergravity limit
- $\lambda \ll 1 \leftrightarrow \ell_s \gg R (\gg \ell_p)$ - weakly coupled gauge theory dual; the bulk is highly

  stringy, but still weakly coupled as a string field theory. We want to understand this limit.
- $\ell_s \to \infty$, bulk $\Rightarrow$ higher spin gauge theory (+matter) in AdS

Free Theory: can construct higher spin conserved currents

$J_{\mu_1 \ldots \mu_s} = \mathrm{Tr}\, \Phi D_{(\mu_1} \ldots D_{\mu_s)} \Phi + \ldots$

of each integer spin s, due to HS gauge fields in the bulk.

In a large N theory with adjoint fields, there are also operators like $\mathrm{Tr}(\Phi^n)$, which are dual to massive bulk fields.

So, the bulk $AdS_5 x S^5$ in the tensionless string limit, dual to free N=4 U(N) SYM, is described by a HS gauge theory coupled to massive matter fields.

Mikhailov, Sezgin, Sundell, Klebanov, Polyakov, Beisert, Bianchi, Morales, Samtleben, ...

What is the simplest HS gauge theory?

- Vasiliev's minimal bosonic HS gauge theory

Fradkin+Vasiliev, Vasiliev '88-'99

- An infinite tower of gauge fields in AdS_4, one of each even integer spin s = 0, 2, 4, ...
- Essentially unique construction, up to 1 overall coupling constant
- Highly non-local: arbitrarily high # derivative interactions at any given order in coupling
- Possibly UV complete due to the constraints on the interactions by HS gauge symmetry; contains gravity. What is its CFT_3 dual?

Sezgin-Sundell, Klebanov-Polyakov '02:

Vasiliev's HS gauge theory is dual to free (UV) or critical (IR) O(N) vector model in 2+1 dimensions.

- Free O(N) model: N massless scalars ϕ_i (i=1,...,N) in the O(N) singlet sector. This free CFT has conserved currents of the form $J_{\mu_1\ldots\mu_1} = \phi_i\partial_{(\mu_1}\ldots\partial_{\mu_n)}\phi_i + \ldots$ for each <u>even</u> integer n.

- Critical O(N) model: the IR fixed point of a relevant deformation of the free theory. It can be described using the Lagrangian $L=(\partial_\mu\phi_i)^2+\alpha(\phi_i\phi_i-1/g)$, where α is a Lagrangian multiplier, and g is a coupling constant. The critical point is achieved by tuning $g\to\infty$ and $\langle\alpha\rangle\to 0$.

comments

- Why vectors? - the absence of long "single trace" operators (as opposed to e.g. adjoint matter) - dual to a pure HS gauge theory. Operators such as $\phi_i\partial_\mu\ldots\phi_i\phi_j\partial^\mu\ldots\phi_j$ decouple from the "single trace" currents in the infinite N limit, and are dual to bound states in the bulk.

- For an operator like $\mathcal{O}=N^{-1/2}\phi_i\phi_i$ (w/ normalized two-point function), the three point function $\langle\mathcal{O}\mathcal{O}\mathcal{O}\rangle \sim N^{-1/2} \sim g_{bulk}$. $N\to\infty$ limit is dual to the free HS theory in the bulk. For spin s, $\Delta(J_s)=s+1$, agrees with AdS/CFT dictionary (in fact, follows from representation of conformal algebra).

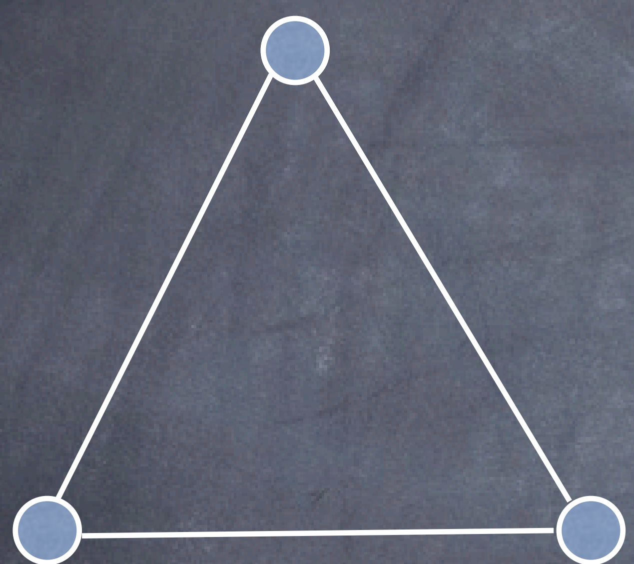
- Bulk scalar φ has $m^2=-2/R^2$, dual to a scalar operator of dimension $\Delta=2$ or 1, depending on the choice of boundary condition $\varphi \sim z^\delta$ $(z\rightarrow 0)$.
- $\Delta=1$ boundary condition: dual to free O(N) theory. $\varphi \leftrightarrow \mathcal{O}=\phi_i\phi_i$.
- $\Delta=2$ boundary condition: dual to critical O(N) model. $\varphi \leftrightarrow \alpha$. Classically $(N=\infty)$, $\Delta(\alpha)=2$. At finite N, $\Delta(\alpha)=2+\mathcal{O}(1/N)$.
- HS symmetry broken by 1/N corrections in the critical O(N) model. Will return to this point later.

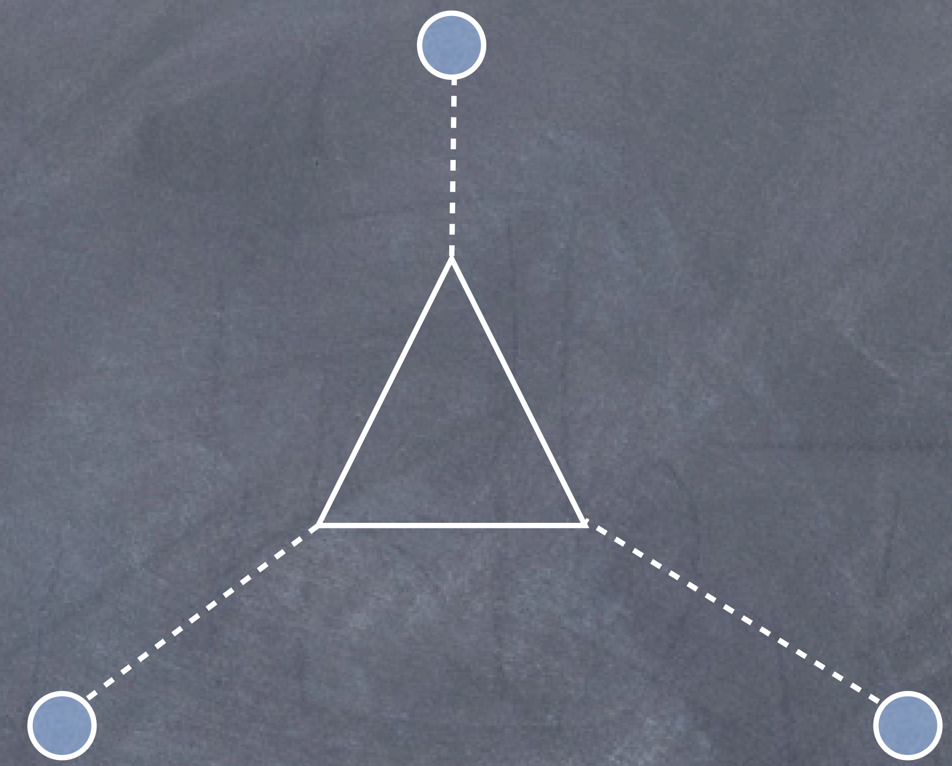
Evidence beyond N=∞?

- Petkou, Sezgin+Sundell: consider $\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\mathcal{O}(x_3)\rangle$. Spatial dependence fixed by conformal symmetry. We are interested in the coefficient. It turns out that Vasiliev's theory does not have scalar^3 coupling with any number of derivatives.

- Δ=1 case: the integration over boundary-to-bulk propagators would diverge, e.g. for $\lambda\varphi^3$ bulk coupling. So λ better be zero, to be consistent with a finite $\langle \mathcal{O}\mathcal{O}\mathcal{O}\rangle$.

- Δ=2 case: no divergence from integration over boundary-to-bulk propagators. So the duality implies $\langle \alpha\alpha\alpha\rangle=0$ at leading order in 1/N. This is indeed the case for the d=3 critical O(N) model.





So, the scalar 3-point function gives a consistency check of the conjecture: "0=0". We want more.

Want to study $\langle J_{s_1}(x_1)J_{s_2}(x_2)J_{s_3}(x_3)\rangle$. The spatial and polarization dependence constrained by conformal symmetry. We will compute the coefficients of these 3-point functions from the bulk theory directly, and compare to free/critical O(N) model at leading order in 1/N expansion.

Mikhailov: if the 3-point functions of the HS currents are of the same form as in free d=3 scalar field theory, then all higher n-point functions are constrained by Ward identities up to finitely many constant coefficients at each given n.

What is Vasiliev’s theory

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What is Vasiliev's theory?

- x^μ - AdS_4 coordinates. $(y_A, \bar{y}_{\dot{A}}, z_A, \bar{z}_{\dot{A}})$ - bosonic auxiliary variables, $A,\dot{A}=1,2$. Introduce $*$ product on the auxiliary variables, e.g. $y_A * y_B = y_A y_B + \epsilon_{AB}$.
- Master fields: $W_\mu(x|y,\bar{y},z,\bar{z})$, $S_A(x|\ldots)$, $S_{\dot{A}}(x|\ldots)$, $B(x|\ldots)$. $W=W_\mu dx^\mu$ is a 1-form in x^μ, $S=S_A dz^A+S_{\dot{A}}d\bar{z}^{\dot{A}}$ a 1-form in $(z,\bar{z})$, and B a scalar.
- Expand the master fields in powers of $(y,\bar{y},z,\bar{z})$. The components will be related to various derivatives of the physical HS fields.

Vasiliev's equations

$$d_x W + W * W = 0,$$
$$d_z W + d_x S + W * S + S * W = 0,$$
$$d_z S + S * S = B * (K dz^2 + \bar{K} d\bar{z}^2),$$
$$d_x B + W * B - B * \pi(W) = 0,$$
$$d_z B + S * B - B * \pi(S) = 0.$$

- $K = e^{z^A y_A}$ (similarly $\bar{K}$) - Kleinian. $K * K = 1$. $K * f(y,\bar{y},z,\bar{z}) * K = f(-y,\bar{y},-z,\bar{z})$. π acts like $K * (\ldots) * K$ together with flipping the sign of dz (not $d\bar{z}$).
- Gauge symmetry: $\delta W = d\varepsilon + [W,\varepsilon]_*$ etc., $\varepsilon = \varepsilon(x|y,\bar{y},z,\bar{z})$. When the master fields are expressed in terms of the physical d.o.f., the gauge symmetry will reduce to HS symmetry.

Why is it a field theory in AdS_4?

- AdS_4 vacuum: $W=W_0(x|y,\bar{y})$, $S=0$, $B=0$. The equation $dW_0=W_0*W_0$ is solved by $W_0 = (\omega_0)_{AB}y^Ay^B + (\omega_0)_{\dot{A}\dot{B}}\bar{y}^{\dot{A}}\bar{y}^{\dot{B}} + (e_0)_{A\dot{B}}y^A\bar{y}^{\dot{B}}$, where ω_0 and e_0 are the spin connection and vierbein of AdS_4. The bilinears of y's form generators of so(3,2) under $*$ product.

Why is it a HS gauge theory?

- Linearized equations

  $D_0\hat{W} = 0,$

  $d_z\hat{W} + D_0 S = 0,$

  $d_z S = B * (K dz^2 + \bar{K} d\bar{z}^2),$

  $\tilde{D}_0 B = 0,$

  $d_z B = 0,$

  where $W = W_0 + \hat{W}$, $D_0 = d_x + [W_0, \cdot]_*$

- $d_z B = 0 \Rightarrow B = B(x|y,\bar{y}) \Rightarrow$ solve S in terms of $B \Rightarrow$

  solve the $(z,\bar{z})$-dependence of $\hat{W}$ in terms of B. $D_0\hat{W} = 0$ restricted to $z_A = \bar{z}_{\dot{A}} = 0$ then gives the free HS equations of motion.

Where are the physical degrees of freedom?

- $\hat{W}|_{z=\bar{z}=0} = \sum \Omega_{A_1 \dots A_n \dot{B}_1 \dots \dot{B}_m}(x) y^{A_1} \dots y^{A_n} \bar{y}^{\dot{B}_1} \dots \bar{y}^{\dot{B}_m}$,
  $B = \sum B_{A_1 \dots A_n \dot{B}_1 \dots \dot{B}_m}(x) y^{A_1} \dots y^{A_n} \bar{y}^{\dot{B}_1} \dots \bar{y}^{\dot{B}_m}$.
  Spin-s gauge field $\subset \Omega_{(s-1+k,s-1-k)}$ and $B_{(2s+n,n)}$, $B_{(n,2s+n)}$
  Here k=1-s, …, s-1; n=0,1,2,….

- $\Omega_{(s-1,s-1)} = \Omega_{A\dot{B}|A_1 \dots A_{s-1} \dot{B}_1 \dots \dot{B}_{s-1}}(x) dx^{A\dot{B}} y^{A_1} \dots y^{A_{s-1}} \bar{y}^{\dot{B}_1} \dots \bar{y}^{\dot{B}_{s-1}}$
  contains the symmetric traceless s-tensor.
  $B_{(2s,0)}$, $B_{(0,2s)}$ are the HS generalizations of the Weyl tensor.

Computing the 3-point function

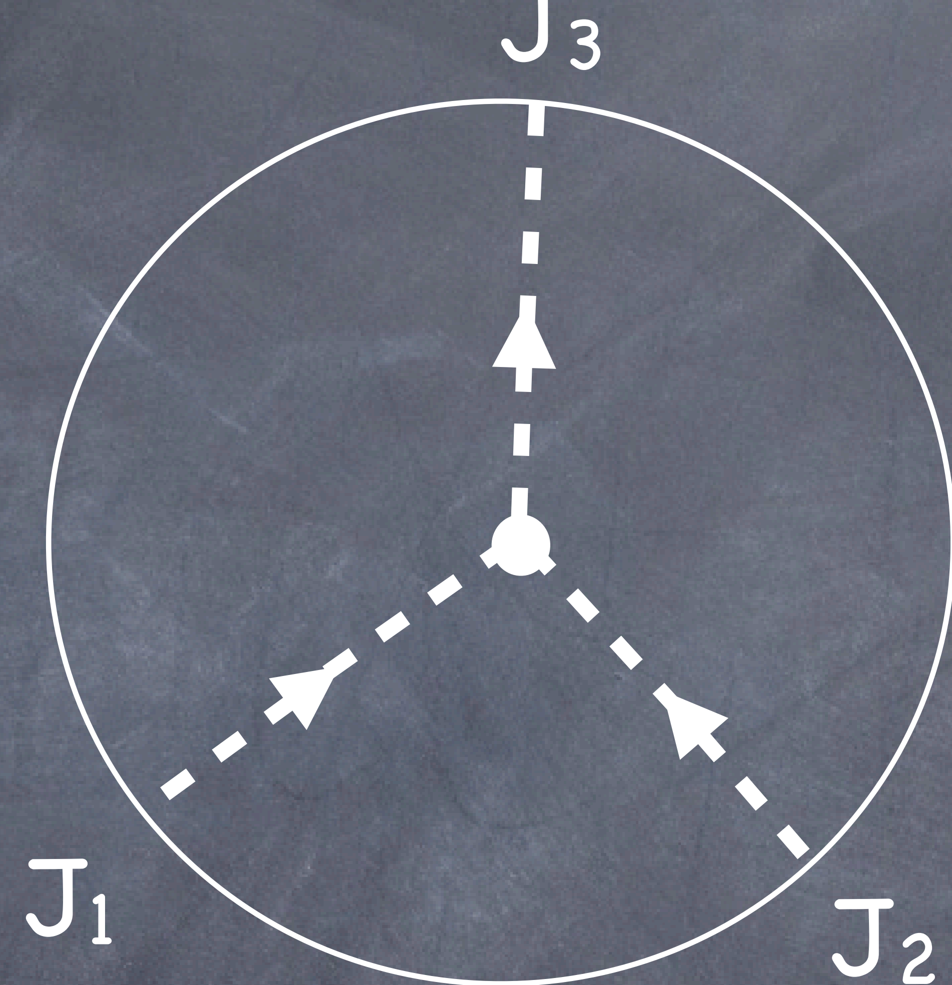
S. Giombi + XY

- Write $J_s(\vec{x},\vec{\varepsilon})=J_{\mu_1\ldots\mu_s}(\vec{x})\varepsilon^{\mu_1}\ldots\varepsilon^{\mu_s}$, for a null polarization vector $\vec{\varepsilon}$. We want to compute
  $\langle J_{s_1}(\vec{x}_1,\vec{\varepsilon}_1)J_{s_2}(\vec{x}_2,\vec{\varepsilon}_2)J_{s_3}(\vec{x}_3,\vec{\varepsilon}_3)\rangle$

- We do not know a useful form of the Lagrangian. Will compute $<J_1J_2J_3>$ directly from the equations of motion.

- Take J_1 and J_2 to be sources ➥ bulk fields φ_1 and φ_2 (concretely, we solve the boundary-to-bulk propagators of the corresponding master fields.)

- Solve the equation of the form (schematically) $D\varphi = \varphi_1 * \varphi_2$.


J3
J1
J2

- The spin-s component of the solution φ has boundary behavior $\varphi(\vec{x},z) \to z^{\delta}\, \varphi_3(\vec{x})$.

  (z is radial coordinate in AdS_4 here, not to be confused with the auxiliary variable z_A)

- With a standard normalization of the boundary-to-bulk propagator, the boundary expectation value of the outcoming field is related to <JJJ> by a factor

  $\varphi_3(\vec{x}_3) \sim \langle J_3 J_3\rangle^{-1} \langle J_3(\vec{x}_3) J_1(\vec{x}_1) J_2(\vec{x}_2)\rangle$.

- By comparing the channels <1,2➠3> and <1,3➠2>, we can determine $\langle J_2J_2\rangle/\langle J_3J_3\rangle$, and hence determine the normalization of <JJJ> up to one overall constant g.

- g is the coupling constant of Vasiliev theory (absent from the equations of motion).

Some technical remarks

- In practice, starting with a spin-s source at $\vec{x}=0$, polarization $\vec{\varepsilon}$, we solve for the linearized master fields $B^{(1)}(\vec{x},z|y,\bar{y})$, $\Omega^{(1)}(\vec{x},z|y,\bar{y})$, etc.

- Next, at quadratic order, we can solve the $(z,\bar{z})$-dependence of $B^{(2)}$ via the equation $d_z B^{(2)} = -S^{(1)} * B^{(1)} + B^{(1)} * \pi(S^{(1)})$.

- Then, we can solve the x^μ-dependence of $B^{(2)}$ via $\tilde{D}_0 B^{(2)} = -\hat{W}^{(1)} * B^{(1)} + B^{(1)} * \pi(\hat{W}^{(1)})$. Here $\hat{W}^{(1)} = \Omega^{(1)} + W'^{(1)}$, $W'^{(1)}$ determined from $B^{(1)}$ by linearized eqn.

- Solve $B^{(2)}$ by inverting $\tilde{D}_0^\dagger \tilde{D}_0 B = \ldots$, look at $B^{(2)}(\vec{x},z\to 0|y, \bar{y}=z=\bar{z}=0) \sim z^\delta B^{(2)}{}_{A_1\ldots A_{2s}}(\vec{x})\, y^{A_1}\ldots y^{A_{2s}}$. The 3-point function is extracted from $B^{(2)}{}_{A_1\ldots A_{2s}}(\vec{x})$.

Results

- We computed explicitly the case with one scalar operator and two general HS currents J_{s_1}, J_{s_2}. To fix the relative normalization of two-point functions, we compare two channels $\langle 0, s_1 \Rightarrow s_2 \rangle$ and $\langle s_1, s_2 \Rightarrow 0 \rangle$.

- The corresponding 3-point function coefficients $C(0, s_1 \Rightarrow s_2)$ and $C(s_1, s_2 \Rightarrow 0)$ are computed in the case $s_1 > s_2$. The case $C(s_1 = s_2 \Rightarrow 0)$ is naively zero, but we believe it is in fact singular, and should be regularized; one way to do this is to take the answer for $s_1 \neq s_2$ and analytically continue to $s_1 = s_2$.

- We find $C(0, s_1 \Rightarrow s_2) \sim \Gamma(s_1+1/2)\Gamma(s_2+1/2)$,

  $C(s_1, s_2 \Rightarrow 0) \sim \Gamma(s_1+s_2+1/2)$.

- These are in complete agreement with the normalized 3-point functions of free O(N) theory, provided the identification $g=\#N^{-1/2}$. We can fix #.

- With the alternative $\Delta=2$ boundary condition, the bulk scalar propagator is now different. Analogous computation of $C(0,s_1 \Rightarrow s_2)$ and $C(s_1,s_2 \Rightarrow 0)$ again agrees with the result of the (leading) $\mathcal{O}(N^{-1/2})$ result of the corresponding 3-point functions in the critical O(N) model. (Lang & Rühl, 91,92)

- In the computation, we have assumed the spatial & polarization dependence of the 3-point functions of the CFT_3, and only extracted the coefficients in the limit where two sources collide (i.e. OPE coefficient).

$\langle J_{s_1} J_{s_2} J_{s_3} \rangle$ for general nonzero s_1, s_2, s_3?

- Spatial & polarization dependence nontrivial. For example, there are two possible structures for $\langle TTT \rangle$ allowed by conformal symmetry in 3d.
- Even the tree-level computation in Vasiliev theory in this case is more involved - work in progress.

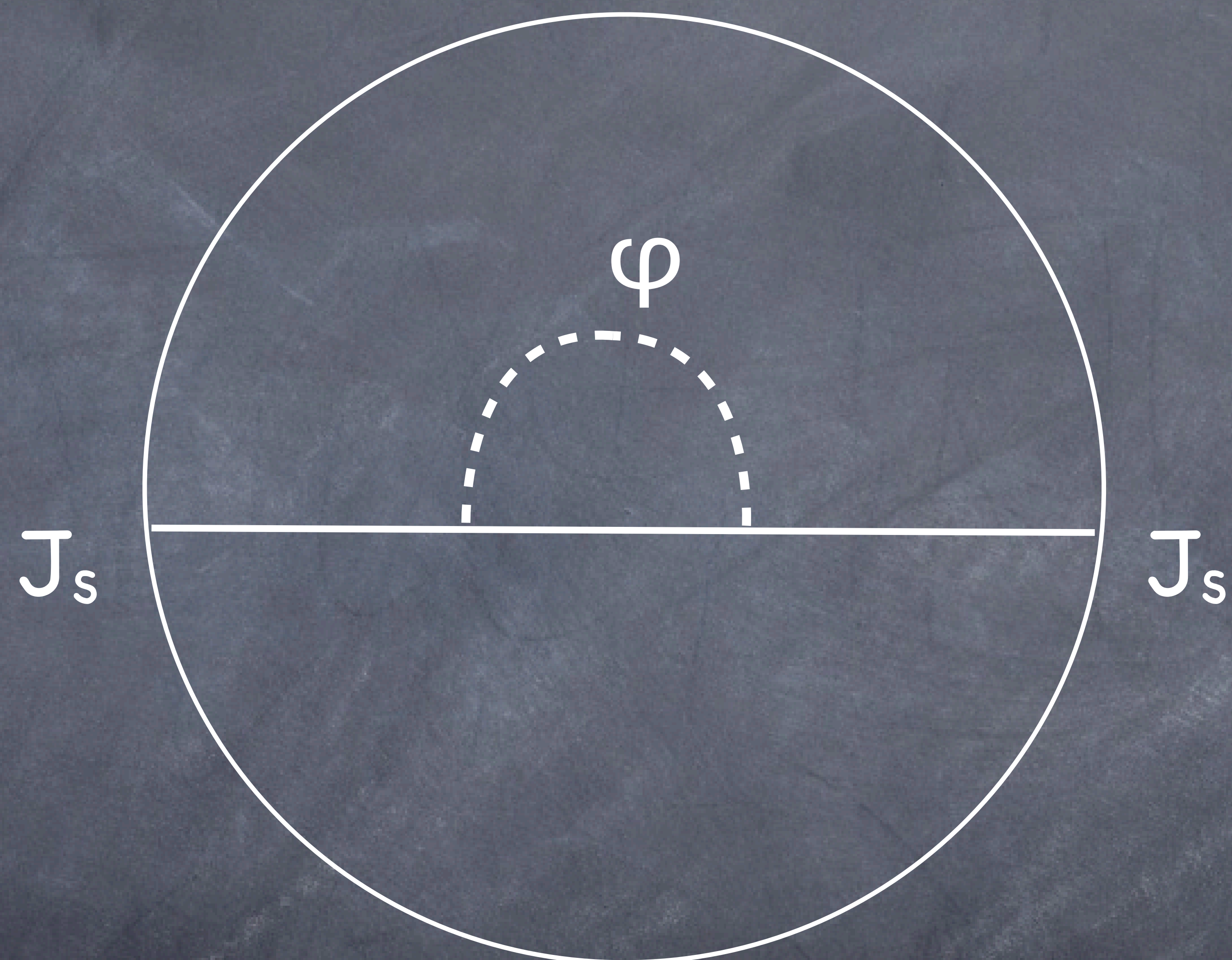
Breaking of HS symmetry?

- While the HS symmetry is an exact symmetry of the free theory (and an exact gauge symmetry of the bulk Vasiliev theory), it is not an exact symmetry of the critical O(N) model - broken by 1/N effects. How is the HS symmetry broken in the bulk?

- With Δ=1 boundary condition, bulk scalar on equal footing as HS fields. With Δ=2 boundary condition, this is no longer the case. Bulk propagator for the scalar modified by this boundary condition: $G_\Delta(x,x') = \xi^\Delta/(1-\xi^2)$, $\xi=1/\cosh(d(x,x'))$.

- Loops with scalars break HS symmetry! (details in progress)

Similar mechanism previously suggested by Girardello, Porrati, Zaffaroni '02



φ
J_s
J_s

Comments on some generalizations

- Vasiliev's non-minimal bosonic HS theory: both even and odd integer spins included. Two possible boundary conditions for the bulk scalar. Also, can impose different boundary conditions on the bulk U(1) gauge field, e.g. $\Delta=2$ boundary condition on the electric-magnetic dual $\check{A}_\mu$ rather than A_μ. These are naturally conjectured to be dual to critical QED coupled to N charged scalars, and the critical CP^{N-1} model.

see work of Witten '03, Leigh, Petkou '03

- The minimal bosonic theory we have discussed so far is called "type A" model by Sezgin-Sundell. There is also "type B" model, in which the bulk scalar is parity odd. This HS theory has been conjectured to be dual to free O(N) fermions/critical Gross-Neveu model.

- Matrix generalization: replace Vasiliev's $*$ algebra by its tensor product with the algebra of k$\times$k matrices ➡ nonabelian HS gauge theory in AdS_4. In the non-minimal case, the natural conjectural dual is the free theory of kN complex scalars in the SU(N) singlet sector, and the critical U(k) QCD with N scalar flavors, depending on the choice of boundary condition for the bulk U(k) gauge field. (finite k, large N)

- Potentially exact holographic duals to CMT systems. However, these holographic descriptions are much more complicated than the CFT themselves - no better (in fact, much harder) than doing 1/N expansion directly in the CFT. More likely, the insight will go the other way around: from the CFT to the bulk.

Conclusion

- Vasiliev theory makes sense
- Sezgin-Sundell-Klebanov-Polyakov conjecture partially confirmed at (bulk) tree level
- Much more to be understood
- Connection to HS/tensionless limits of string theories?