3d theories from 3d manifolds

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to appear
Introduction

Previous talk by prof. Gukov:

The T[M] theories

• 3d N=2 SCFT associated to 3-manifold
• IR limit of $A_1$ 6d (2,0) SCFT on $M$
Expected properties of $T[M]$

Space of $R^2 \times S^1$ vacua of $T[M]$

$\iff$

$SL(2,C)$ flat connections on $M$

- Follows from 6d on $S^1 \iff 5d$ SYM
Expected properties of $T[M]$ 

Ellipsoid partition function of $T[M]$ 

$\Longleftrightarrow$ 

SL(2) CS partition function on $M$ 

• Motivated by AGT, Nekrasov-Witten
Motivation

The 6d theory is mysterious

Can we define $T[M]$ directly in 3d?
Simple cutting

Cut $M$ along Riemann surfaces?

Too many unknown building blocks
Main result

Explicit three-dimensional construction of a class of 3d N=2 SCFTs labeled by the same data as SL(2) Chern-Simons wavefunctions
Labels

- 3d manifold M with boundary + knots
- triangulation of boundary
- "polarization" of the boundary
Polarization

\[ [X,Y] = h \]
\[ [Z,Y] = h \]
\[ [X,Z] = 0 \]

\[ Y \Psi(X,Z,\ldots) = -h(\partial_X + \partial_Z) \Psi(X,Z,\ldots) \]
Main result

$\text{SL}(2) \text{ CS wavefunction on } M$

equals

Ellipsoid partition function of 3d SCFT
Main conjecture

If $M$ has no boundary, only knots

3d SCFT labeled by $M$

coincides with $T[M]$
$\partial M \quad M$

4d theory \hspace{1cm} 3d b.c.
Main conjecture

If M has boundary $\partial M$

$3d$ SCFT $[M] + 4d$ SW theory $[\partial M]$

coincides with the

IR limit of $A_1 6d$ $(2,0)$ SCFT on $M$
Main tool

Decompose M into tetrahedra

- SL(2) CS wavefunction from glued tetrahedra
- 3d theory from “glued” chiral multiplets.
  - Abelian gauge fields and superpotentials
Consistency conditions

Different decompositions

$\iff$

mirror 3d theories
Consistency conditions

$N_f = 1$ SQED $\iff$ 3 chirals; $W=XYZ$

Aharony, Hanany, Intriligator, Seiberg, Strassler
Consistency conditions

\[ U(1)_{1/2} + 1 \text{ chiral} \equiv 1 \text{ chiral} \equiv U(1)_{-1/2} + 1 \text{ chiral} \]
Generalizations

Line defects in 3d SCFT

- Labeled by CS Wilson loops in $M$

Higher rank?
Ellipsoid partition function

- 3d N=2 SUSY gauge theory on ellipsoid
  \[ b^2 |z|^2 + b^{-2} |w|^2 = 1 \]
- Computable in UV by localization
- Will denote as \( \Psi_b \)

Kapustin, Willet, Yaakov
Hama, Hosomichi, Lee
Ellipsoid partition function

• U(1) Flavor symmetry $\Rightarrow$ parameter $x$

  • $x = m + i (b + b^{-1}) R$

  • $m$: twisted mass

  • $R$: R-symmetry assignment

• $\Psi_b(x)$ holomorphic in $x$
\[ \Psi_b \text{ in Abelian 3d theories} \]

Chiral multiplet partition function

\[ \Psi_b(x_a)_{\text{chiral}} = s_b(i \frac{Q}{2} - x_a) \]

\[ Q = b + b^{-1} \]

\[ s_b(x) \equiv \prod_{m,n \in \mathbb{Z}_{\geq 0}} \frac{mb + nb^{-1} + \frac{Q}{2} - ix}{mb + nb^{-1} + \frac{Q}{2} + ix}, \]
Ellipsoid partition function

- Almost unaffected by superpotential $W$
- $W$ can break flavor symmetries
- $W$ must have R-charge 2
Ellipsoid partition function

- Example: \( W = XYZ \)
  - \( x, y, z \) parameters for \( U(1)_x \) \( U(1)_y \) \( U(1)_z \)
  - \( x + y + z = 0 + i (b + b^{-1}) = i Q \)
  - Adding \( W \) constrains \( z = iQ - x - y \)
  - \( \Psi_b = \Psi_b(x)^{\text{chiral}} \Psi_b(y)^{\text{chiral}} \Psi_b(iQ - x - y)^{\text{chiral}} \)
\( \Psi_b \) in Abelian 3d theories

- Gauge multiplets \( \implies \) \( y_i \) scalar fields
- \( q^i_a \) charge of chiral multiplet "a"
- \( x_a = q^i_a y_i + q^f_a z_f \)
- Topological currents \( * F_i \) give extra flavor
- FI parameters are twisted masses \( z'_i \)
\( \Psi_b \) in Abelian 3d theories

- \( \Psi_b(z,z') = \int \Psi_b(x_a)_{\text{chiral}} e^{-i\pi(y,y)} - 2i\pi z'.y \, dy \)
- \( (y,y) \) Chern-Simons pairing
  - \( (y,y) = k \, y^2 \) for gauge field level \( k \)
$N_f=1$ SQED

$$\int \Psi_b(z+y)^{\text{chiral}} \Psi_b(z-y)^{\text{chiral}} e^{-i\pi(y,y)} - 2i\pi z'.y \ dy$$
Things to remember

• Gauging a flavor symmetry
• Fourier transform with gaussian kernel
• $\text{Sp}(2N,\mathbb{Z})$ on $(2\pi x, -i\partial_x)$
• Adding superpotential
• linear constraint.
Comparison with wavefunctions

- Dimofte rules
  - tetrahedron $\Rightarrow$ quantum dilogarithm
  - gluing $\Rightarrow$ linear constraints on arguments
  - changes of polarization $\Rightarrow$ Fourier transform
$\Psi_b$ in Abelian 3d theories

tetrahedron $\Rightarrow$ quantum dilogarithm $e_b(i \frac{Q}{2} - x_a)$

- $e_b(x) = s_b(x) \exp i \pi x^2/2$
  - Chiral multiplet
  - background CS coupling level $-1/2$
  - physically required: cancel anomaly
Comparison with wavefunctions

- Pick a 3-manifold decomposed into tetrahedra
- Build 3d theory with $\Psi_b(z) = \text{wavefunction}$
  - tetrahedron $\Rightarrow$ chiral multiplet (+CS term)
  - changes of polarization $\Rightarrow$ gauging
  - internal edges $\Rightarrow$ superpotential terms
Conclusions

• We conjecture a 3d definition of $T[M]$
  • Many mirror descriptions
  • All are Abelian CSM theories. Why?
• 3d Field theories as 3-manifold invariants!
• We have no direct 6d to 3d derivation
Refined statement

Boundary conditions for $N=2$

4d Abelian gauge theories

• 3d manifold $M$ with boundary + knots
• triangulation of boundary
• polarization
Refined statement

Boundary conditions for N=2
4d SW theories (with BPS particles)

- 3d manifold $M$ with boundary + knots
- triangulation of the boundary