Multiloop systematics in pure spinor field theory

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With **Jonas Bjornsson**

Over the past 15 years there have been impressive advances in evaluating multiloop field theory amplitudes – especially in maximally supersymmetric Yang—Mills and supergravity (so far up to four loops).

$L = 4$

But going to higher orders, specially for supergravity, and understanding the general structure seems a daunting task.

The motivation for this work is to develop an understanding of how the relative simplicity of string perturbation diagrams may be mimicked in the field theory limit – what is the field theory limit of string perturbation theory?

Recall:

Maximally supersymmetric Yang-Mills is UV finite in $D = 4$ dimensions. Interesting systematics of UV divergence as function of D and loop number, L (and colour).

e.g. $\partial^2 \mathrm{Tr} F^4$ is renormalized beyond L=2 but $\partial^2 (\mathrm{Tr} F^2)^2$ is not for any D.

Heroic calculations in maximal supergravity determine that, for $L < 5$, the leading low energy behaviour of the L-loop four-graviton amplitude is $\partial^{2L} R^4$.

esp. Bern, Carrasco, Dixon, Johansson, Kosower, Roiban

Various arguments suggest that for $L > 4$ the leading behaviour is $\partial^8 R^4$. This would imply the presence of a seven-loop logarithmic divergence when $D = 4$.

Supersymmetry counterterm arguments — Bossard, Howe, Stelle, Vanhove; Elvang, Kiermaeir

Arguments based on string theory — Berkovits, MBG, Russo, Vanhove

Some disagreement, however. — Kallosh

In string theory perturbative loop amplitudes can be formulated in an efficient manner. RNS or Pure Spinor

Pure spinor formalism – superstring perturbation theory in a manifestly supersymmetric manner. Berkovits

No sum over spin structures.

Powerful calculational tool.

viz. explicit string loop calculations — Berkovits, Gomez, Mafra

Multiparticle string tree amplitudes — Mafra, Schlotterer, Stieberger

The talk will sketch an efficient way of describing some systematics of multiloop field theory computations using a pure spinor formalism that makes supersymmetry manifest.

- UV behaviour determined by counting fermion zero modes
- Suggestive insights into $L \geq 5$

Modeled closely on Berkovits' string formalism

Field Theory Loop Calculations Using Pure Spinors.

- World-line action

"Non-minimal" pure spinor

$$S = \int d\tau \left(\dot{X} P - \frac{1}{2} P^2 + \dot{\theta} p \underbrace{+ \dot{\lambda} w + \bar{w} \dot{\bar{\lambda}} - s \dot{r}}_{\text{extra fermionic and bosonic coordinates}} \right)$$

Fermionic zero modes (in pink) need to be saturated

- First quantized (world-line) version of the pure spinor string theory formalism of Berkovits.

- Manifestly space-time supersymmetric.

- The geometric origins are still mysterious (but see recent suggestions for how it may arise from gauge fixing classical pure spinor theory. Berkovits; Oda)

World-line coordinates in the non-minimal version

		classical coordinates		non-minimal coordinates	
Bosons	World-line scalars	X^m,	λ^α,	$\bar\lambda_\alpha$	
	no. of components	10	11	11	
	World-line vectors	P_m,	w_α,	$\bar w^\alpha$	
		10	11	11	$\times L$ zero modes in L loops
Fermions	World-line scalars	θ^α,		r_α	
		16		11	
	World-line vectors	d_α		s^α	
		16		11	$\times L$

$d_\alpha = p_\alpha + P_m(\gamma^m\theta)_\alpha/2$

Pure spinors

$$\lambda\gamma^m\lambda = 0$$
$$\bar\lambda\gamma^m r = 0$$

etc.

For supergravity these are doubled $\hat\theta\,,\ \hat d\,,\ \hat\lambda\,,\ \hat{\bar\lambda}\,,\ \hat w\,,\ \hat{\bar w}\,,\ \hat r\,,\ \hat s$

BRST operator SYM $Q = \lambda d + \bar{w} r$ $\hat{Q} = \hat{d}\hat{\lambda} + \hat{\bar{w}}\hat{r}$

SUGRA $Q_{tot} = Q + \hat{Q}$

so that

$$[Q_{tot}, (b + \hat{b})] = H\,, \qquad [Q_{tot}, (b - \hat{b})] = 0$$

with composite b ghost

$$b = -\frac{1}{4}\left(\frac{P^m \bar{\lambda}\gamma_m d}{(\lambda\bar{\lambda})} + \frac{\bar{\lambda}\gamma_{mnp} r (d\gamma^{mnp} d)}{384(\lambda\bar{\lambda})^2} + \ldots\right)$$

$[Q, b] = H$

$b^2 = 0$

one d one P two d's no P 's

note potential small $\lambda, \bar{\lambda}$ singularities

Similarly for $\hat{b}$

Trees and loops

Amplitudes constructed by tying together vertices with propagators, mimicking string theory.

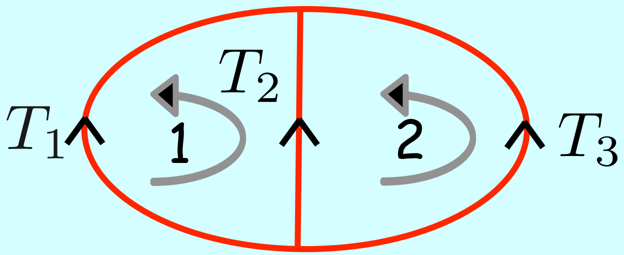
Structure of loop amplitudes.

Consider ϕ^3 L-loop $(L>1)$ skeleton diagram b_I - cycles

$(3L-3)$ moduli propagator lengths T_i $(i=1,\dots,3L-3)$

Basis of one forms $\omega_I = a_I^i\, d\tau_i$ $(a_I^i = \pm 1)$ $I=1,\dots,L$

Period matrix $\Omega_{IJ} = \oint_{b_I} w_J$

e.g. 

$$\Omega_{IJ} = \begin{pmatrix} T_1+T_2 & -T_2 \\ -T_2 & T_2+T_3 \end{pmatrix}$$

L-loop SUGRA amplitude $(L > 1)$:

$$A = \sum_{F_L} \int_0^\infty dT_1 \ldots dT_{3L-3} \int_{F_L} \prod_{r=1}^{N} d\tau_r \, K(\{T_i\}, \{\tau_r\}, \{k_r\})$$

sum over skeletons

$$K = \int \underbrace{\mathcal{D}\Phi\, \mathcal{D}\hat{\Phi}}_{\text{measure}} \left(\underbrace{\mathcal{N}\hat{\mathcal{N}}}_{\text{regulator}} \prod_{i=1}^{3L-3} \underbrace{\left(\int_0^{T_i} \frac{d\tau_i}{T_i} b \int_0^{T_i} \frac{d\tau_i}{T_i} \hat{b} \right)}_{\text{b-ghost insertions}} \int \prod_{r=1}^{N} d\tau_r \, \underbrace{V(k_1, \tau_1) \ldots V(k_N, \tau_N)}_{\text{plane-wave vertices}} \, e^{-S} \right)$$

Saturation of fermionic zero modes requires consideration of modes coming from :

(i) integrated vertex operators, V

(ii) the regulator $\mathcal{N}\hat{\mathcal{N}}$ of large $\lambda, \bar{\lambda}, \hat{\lambda}, \bar{\hat{\lambda}}$ divergences,

(iii) composite b-ghost insertions

Constrains pattern of diagrams that contribute to the amplitude.

(i) Vertex operators

Unintegrated

$$U = \lambda^\alpha \,\hat\lambda_\beta \, A_\alpha{}^\beta(X,\theta,\hat\theta)$$

$$[Q,U] = 0 = [\hat Q, U]$$

superfield $\qquad A \sim \cdots + \theta^3\hat\theta^3\, R + \ldots$

Integrated

internal momenta $\qquad$ d modes

$$V \sim P^m P^n\, G_{mn}(X,\theta,\hat\theta) + d^\alpha \hat d_\beta\, W_\alpha{}^\beta(X,\theta,\hat\theta) + \ldots$$

$G \sim D\bar D A \sim \cdots + \theta^2\hat\theta^2\, R + \ldots \qquad W \sim D\bar D G \sim \cdots + \theta\hat\theta\, R + \ldots$

metric $\qquad$ bispinor

Superfields satisfy linearized eqs. of motion

(ii) Regulator for large $\lambda, \bar{\lambda}$ divergences (as well as source of s)

$$\mathcal{N} = e^{-\int [Q,\chi] d\tau} \sim 1 - [Q,\chi] \quad \text{BRST exact}$$

$$= e^{-\int (\lambda\bar{\lambda} + \theta r - \lambda d s + \dots) d\tau}$$

powers of θ correlated with powers of r

The only source of s

Integration over s zero modes pulls out a factor $(\lambda\, d\, s)^{11L}$

NOTE:

Each d has $16L$ zero modes.

$11L$ of these are can be soaked up by the d's in the factor $(\lambda\, d\, s)^{11L}$

i.e. $5L$ d zero modes must be soaked up by the b insertions and the vertex operators (5 for each loop).

(iii) The term in b zero mode containing two d zero modes is

$$\frac{1}{T_i}\int_0^{T_i} d\tau_i b_{zero} = b^{IJ}\,\frac{\partial\Omega_{IJ}}{\partial T_i} \sim d^I_\gamma d^J_\delta\,\frac{\partial\Omega_{IJ}}{\partial T_i}$$

The term with one less d zero mode has a ℓ^I_m factor

loop momentum (zero mode of P^m)

NOTE condition:

If no. of independent components of $\frac{\partial\Omega_{IJ}}{\partial T_i}$ is $< 3L-3$

not all the d zero modes can be supported

In that case other terms in b^{IJ} must contribute.
(and more d^I's must come from elsewhere).

BRST invariance and contact terms.

J. Bjornsson

Check that a BRST – exact vertex decouples.

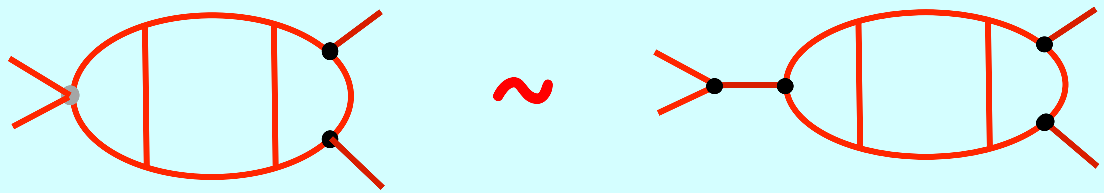
$[Q,\rho]$ for SYM $\qquad [Q_{tot},\chi]$ for SUGRA

Certain "contact terms" need to be cancelled with new interactions.

- No internal contact vertices needed.
- Contact vertices are needed for external states for $L > 2$.

e,g.

Three loops



These vertices encode nonlinear corrections to the SYM and SUGRA superfield equations.

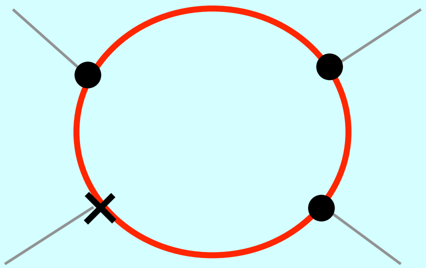
They modify the UV behaviour in SYM but not in SUGRA

Do not contribute to non-planar SYM 4-point amplitude.

Consider four-graviton scattering : Maximal supersymmetry

Amplitude $\sim R^4 \, I(s,t,u)$

ONE LOOP



R^4

$\times$ = unintegrated vertex $A\lambda\hat{\lambda}$

$\bullet$ = integrated vertex $W d\hat{d}$

Scalar field theory box diagram

$$R^4 \int d^D k \left(\frac{1}{k^2}\right)^4$$

cutoff

$R^4 \, \Lambda^{D-8}$

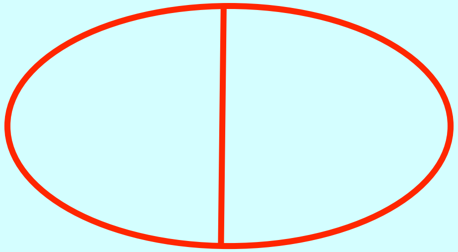
$(D\hat{D})^8 \, G$

Log divergence in $D=8$

$\frac{1}{2}$ - BPS

[Yang-Mills case: $F^4 \, \Lambda^{D-8}$]

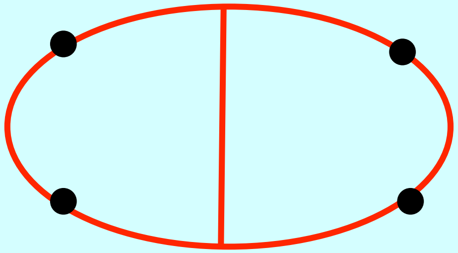
TWO LOOPS

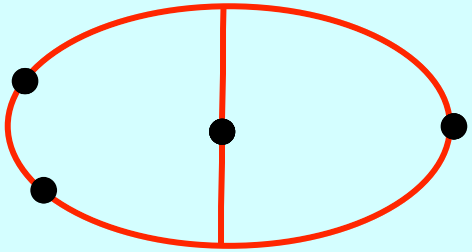


Two-loop skeleton

$c_2^D \, \partial^4 R^4 \Lambda^{2(D-7)}$

$(D\hat{D})^{12} \, G$



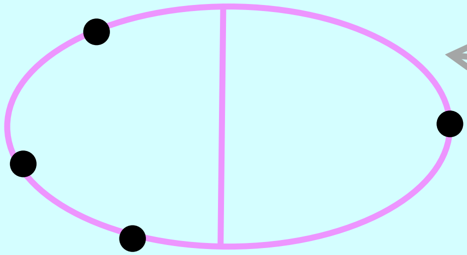


$\frac{1}{4}$ - BPS

The two types of allowed contributions

Log divergence in $D = 7$

Forbidden diagram = 0



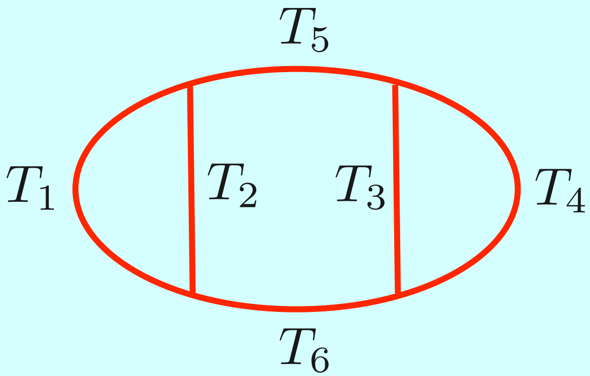
note triangle

[Yang-Mills case: $F^4 \Lambda^{2(D-7)}$]

$c_2^{(7+\epsilon)} \sim$ Vacuum amplitude in $(3+\epsilon)$ dimensions

THREE LOOPS - period matrices

Degenerate case

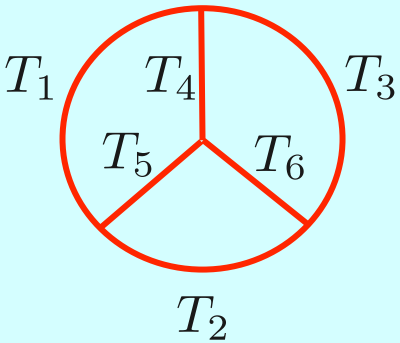


$$\Omega_{IJ} = \begin{pmatrix} T_1 + T_2 & -T_2 & 0 \\ -T_2 & T_2 + T_3 + T_5 + T_6 & -T_3 \\ 0 & -T_3 & T_3 + T_4 \end{pmatrix}$$

Ω_{IJ} only depends on five combinations of T_i

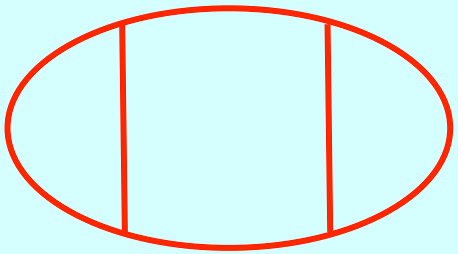
(on $T_5 + T_6$ not $T_5 - T_6$)

The two distinct three-loop skeletons

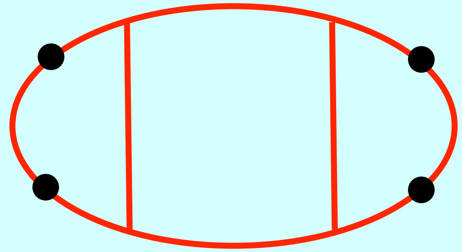


$$\Omega_{IJ} = \begin{pmatrix} T_1 + T_4 + T_5 & -T_5 & -T_4 \\ -T_5 & T_2 + T_5 + T_6 & -T_6 \\ -T_4 & -T_6 & T_3 + T_4 + T_6 \end{pmatrix}$$

THREE LOOPS



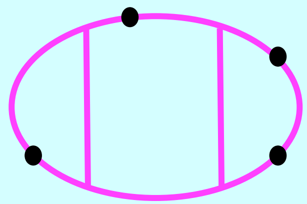
$\partial^8 R^4 \Lambda^{3D-20}$



Ladder amplitude

The two distinct
three-loop skeletons

not allowed



$c_3^D \, \partial^6 R^4 \Lambda^{3(D-6)}$

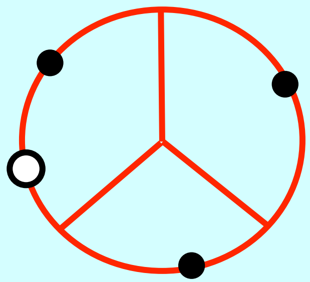
= integrated vertex $G \, P P$

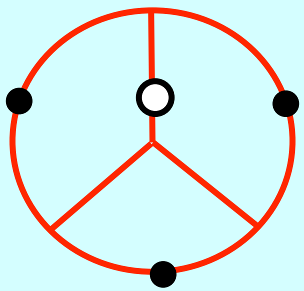
$(D\hat{D})^{14} \, G$

$\frac{1}{8}$- BPS

Log divergence in
$D = 6$

P^2





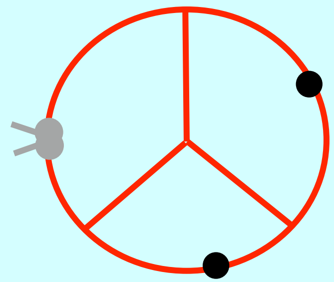
Examples of contributions

$\log\Lambda$

Coefficient $c_3^{(6+\epsilon)}$ $=$ vacuum amplitude for ϕ^3 scalar field theory in $(4+\epsilon)$ dimensions

Comment on contact interaction :

 = contact vertex



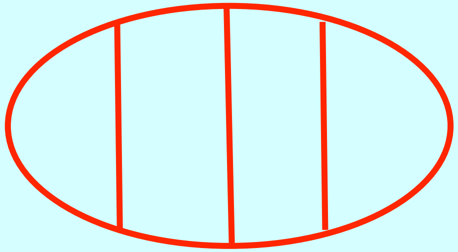
Arises in YM planar term (and SUGRA) only

- not in nonplanar term, giving milder divergence.

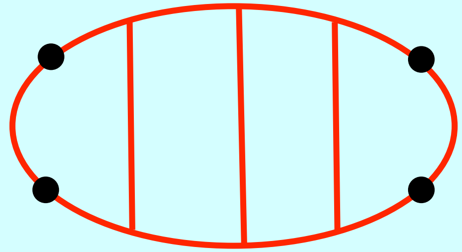
$\partial^2\,\mathrm{Tr}\,F^4\,\Lambda^{3D-18}$

$\partial^4\,(\mathrm{Tr}\,F^2)^2\,\Lambda^{3D-20}$

FOUR LOOPS

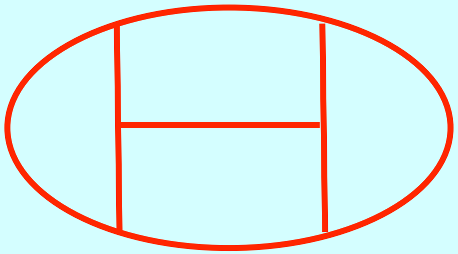


$\partial^{12} R^4 \Lambda^{4D-26}$



Ladder amplitude

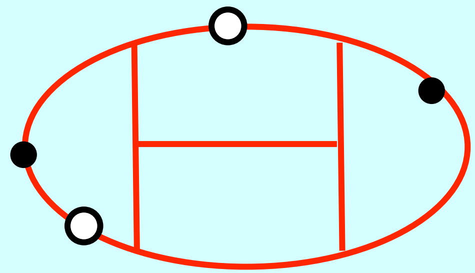
Two of the five
four-loop skeletons



$c_4^D \, \partial^8 R^4 \Lambda^{4D-22}$

0 - BPS

Log divergence in
$D = 11/2$



Example of a leading
amplitude

- Results up to four loops agree with the suggestion that ultraviolet divergence first occurs as log divergence at L loops in dimension

$$D = 4 + \frac{6}{L}$$ Planar SYM and SUGRA]

c.f. Bern, Carrasco, Dixon, Johansson, Kosower, Roiban

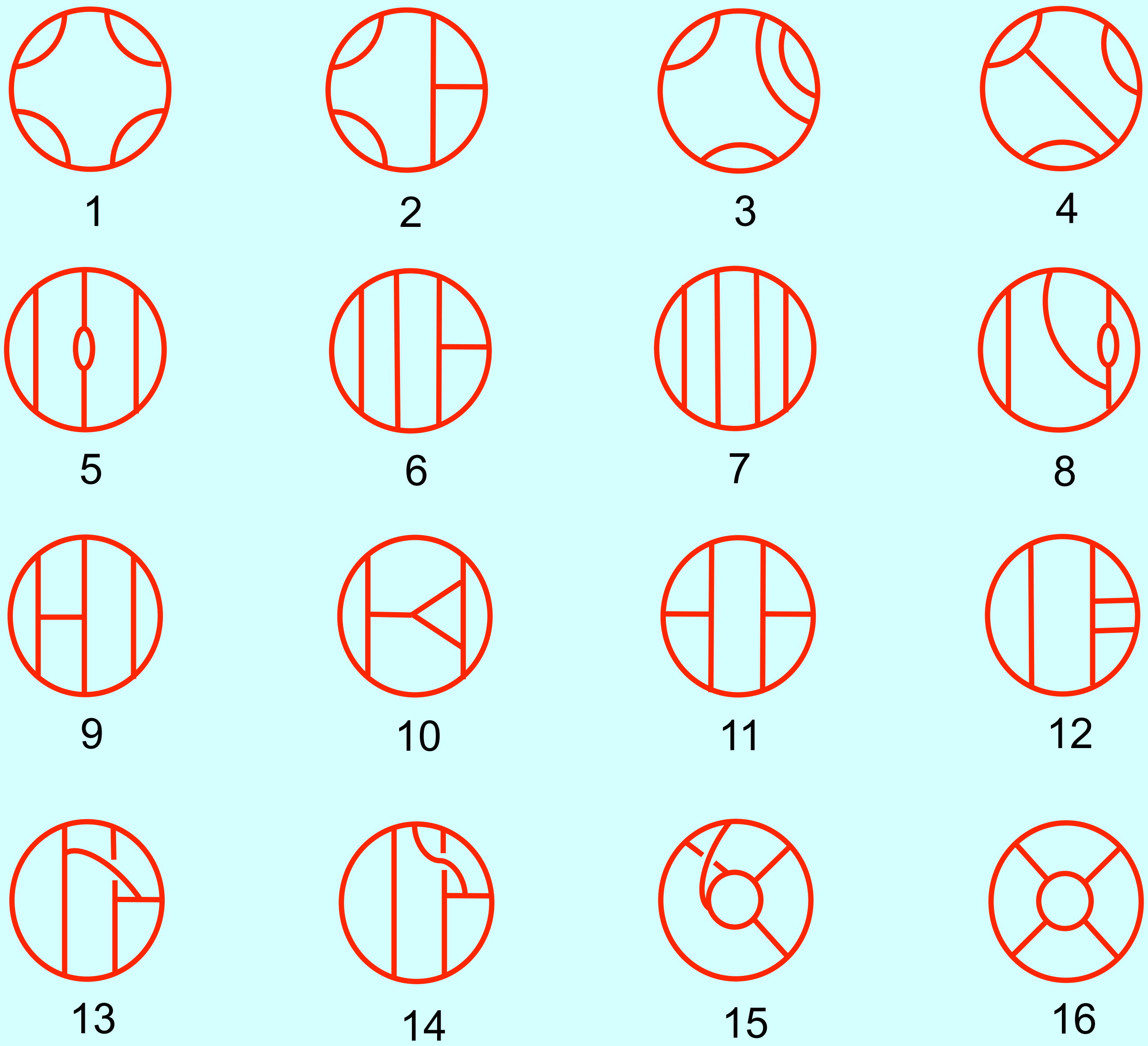
Would lead to finiteness in $D = 4$ if true for all loops

- This behaviour followed by dimensional analysis once the fermionic zero modes have been extracted.

What happens at five loops?

FIVE LOOPS

The sixteen five-loop skeletons
to which vertices must be attached



New feature arises at five loops. In this case require $3L-3=12$ b insertions. Can now get a new singularity

$$\left(\frac{r}{\lambda\bar{\lambda}}\right)^{12} d^{24}$$

Small-$\lambda\,,\,\bar{\lambda}$ singularity and more than $11 r$'s gives $0/0$.
Needs new regulator for tip of the cone. Berkovits and Nekrasov.

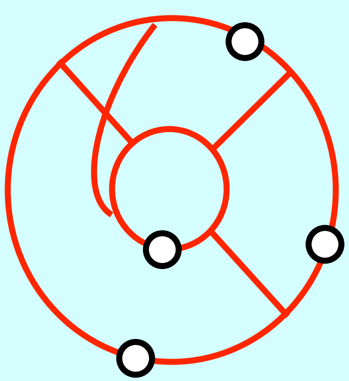
Effectively exchanges a power of $r/\lambda\bar{\lambda}$ for a d giving a total of $24+1=25$ d's, as required. But subtle details need to be resolved !

This only affects the leading divergence, which arises entirely from the last two skeleton diagrams. No triangles.

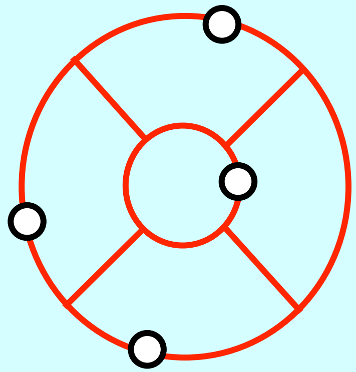
All four vertices can now be $G_{mn}\,P^m P^n$.

Only the last two diagrams give leading contribution:

(Skeletons with no sub-triangles)



$$c_5^{(D)}\,\partial^8\,R^4\,\Lambda^{5D-24}$$



Nonplanar

Planar

The vertices contain momentum factors that cancel four propagators, leaving the vacuum (skeleton) diagrams

$$\partial^8 R^4 \int d^{5D}k\,\left(\frac{1}{k^2}\right)^{12}$$

SUGGESTS that at FIVE LOOPS leading divergence is

$$c_5^{(D)}\,\partial^8 R^4\,\Lambda^{5D-24}$$

- Would give a 5-loop log divergence in $D = 24/5$ dimensions. (but $c_5^{(24/5+\epsilon)}$ is unknown)

- Earlier formula suggests $\partial^{10} R^4\,\Lambda^{5D-26}$- logarithm in dimensions.

$$D = 4 + 6/L = 26/5$$

- In D=4 dimensions ($N = 8$ supergravity) this gives the L-loop behaviour

$$\partial^8 R^4\,\Lambda^{L(D-2)-14}$$

which would lead to a log divergence in four dimensions at seven loops $D = 4, L = 7$.

- It would be a surprise if its coefficient were to vanish since there is no (obvious) supersymmetry protection.

A more complete 5-loop calculation is of interest.

Questions

- Does perturbative supergravity make sense in isolation from string theory? At what order do UV divergences arise?

- Can the perturbation expansion of supergravity be obtained from string theory?

  Can the onset of UV divergences of multi-loop supergravity be determined as a limit of multi-loop superstring theory?

- How is the structure of perturbative supergravity embedded in the non-perturbative duality symmetries of string theory?

  Discrete versus continuous duality symmetry groups
  Role of non-perturbative states.