Recent results for

Holographic Three-Point Functions

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27. June, Strings 2011, Uppsala

Correlation Functions in CFT

[Osborn, Petkou Ann.Phys., 9307010]

The space-time dependence of two- and three-point functions of Scalar Conformal Primary Operators is fixed by conformal symmetry:

Here sits the interesting data!

$$\langle \mathcal{O}_I(x_1)\mathcal{O}_J(x_2)\rangle = \frac{\delta_{IJ}}{|x_{12}|^{\Delta_I+\Delta_J}}$$

$$\langle \mathcal{O}_I(x_1)\mathcal{O}_J(x_2)\mathcal{O}_K(x_3)\rangle = \frac{C_{IJK}}{|x_{12}|^{\Delta_I+\Delta_J-\Delta_K}\,|x_{23}|^{\Delta_J+\Delta_K-\Delta_I}\,|x_{31}|^{\Delta_K+\Delta_I-\Delta_J}}$$

$$\langle \mathcal{O}_I(x_1)\mathcal{O}_J(x_2)\mathcal{O}_K(x_3)\mathcal{O}_L(x_4)\rangle = C_{IJKL}\, f\left(\frac{x_{12}x_{34}}{x_{13}x_{24}}, \frac{x_{12}x_{34}}{x_{14}x_{23}}\right) \prod_{i<j} \frac{1}{|x_{ij}|^{\Delta_i+\Delta_j-\frac{1}{3}\Delta}}$$

follows from $\{\Delta_J\}$ and $\{C_{IJK}\}$ and OPE

$\{\Delta_J\}$ from Integrability (@ N=∞)

[Minahan, Zarembo JHEP, 0212208] [Bena, Polchinski, Roiban Phys.Rev., 0305116]

[Beisert, Staudacher Nucl.Phys., 0504190] [Beisert, Eden, Staudacher J.Stat.Mech., 0610251] [many maaaany more XYZ, yymmnnn]

[Arutyunov, Frolov JHEP, 0901.1417] [Gromov, Kazakov, Vieira Phys.Rev.Lett., 0901.3753] [Bombardelli, Fioravanti, Tateo J.Phys., 0902.3930] [Arutyunov, Frolov JHEP, 0903.0141]

[Beisert et. al. 1012.3982]

Solve# $Y_{1,0}(u_{4,j}) = -1$ and plug in# ...

$$\Delta_J = J + \sum_j \epsilon(u_{4,j}) + \sum_{a=1}^{\infty} \int_{-\infty}^{\infty} \frac{du}{2\pi i} \frac{\partial \epsilon_a^*}{\partial u} \ln\left(1 + Y_{a,0}^*(u)\right)$$

Well, it's actually not that easy, and you probably don't want to see the full form of the Y's and their relations.

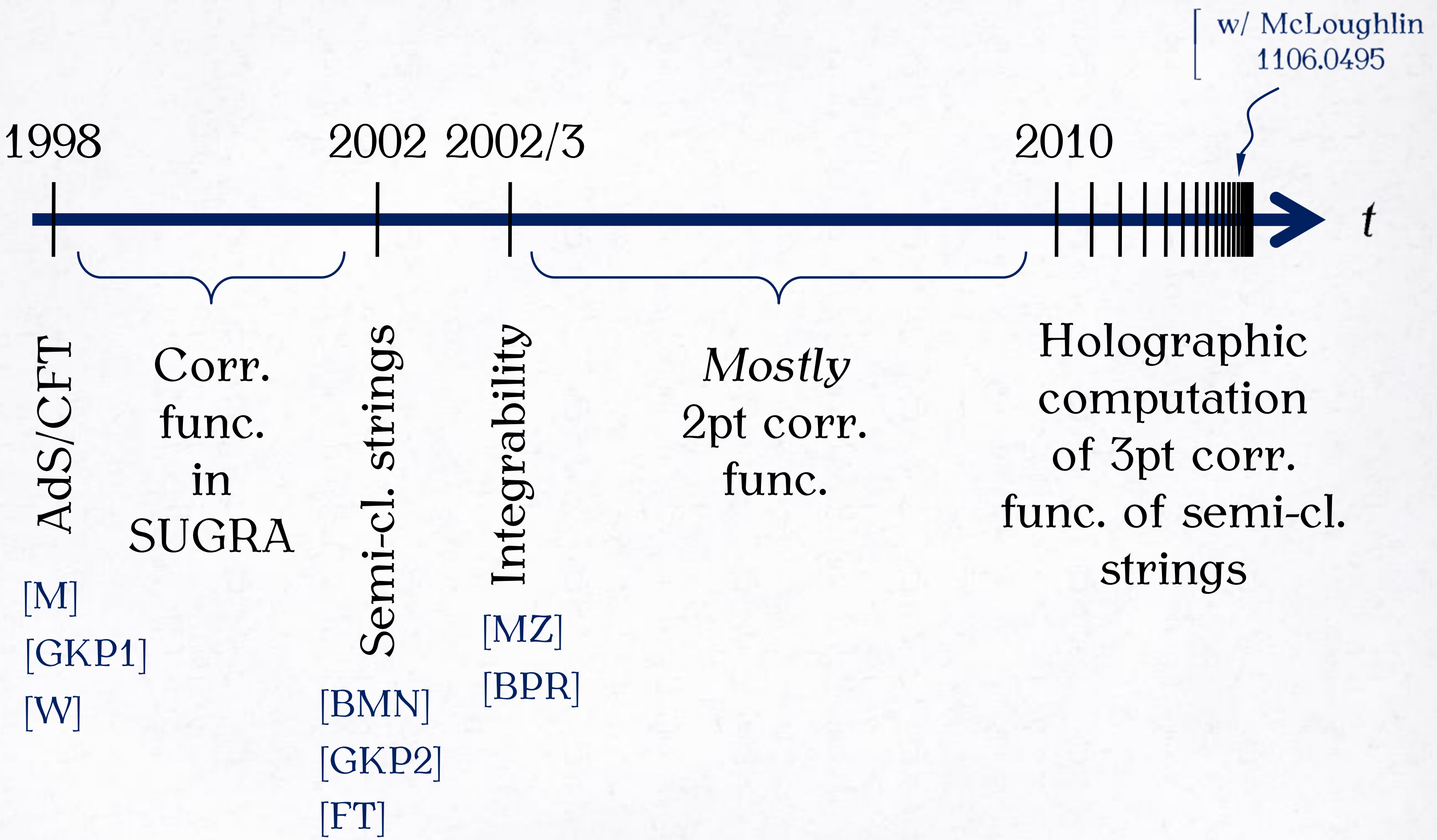
$\{ C_{IJK} \}$ from Integrability

The goal—or rather the *dream*—would be to find:

Solve $\boxed{Z_{2,1,0}(v_{4,k,\ell}) = -2}$ and plug in ...

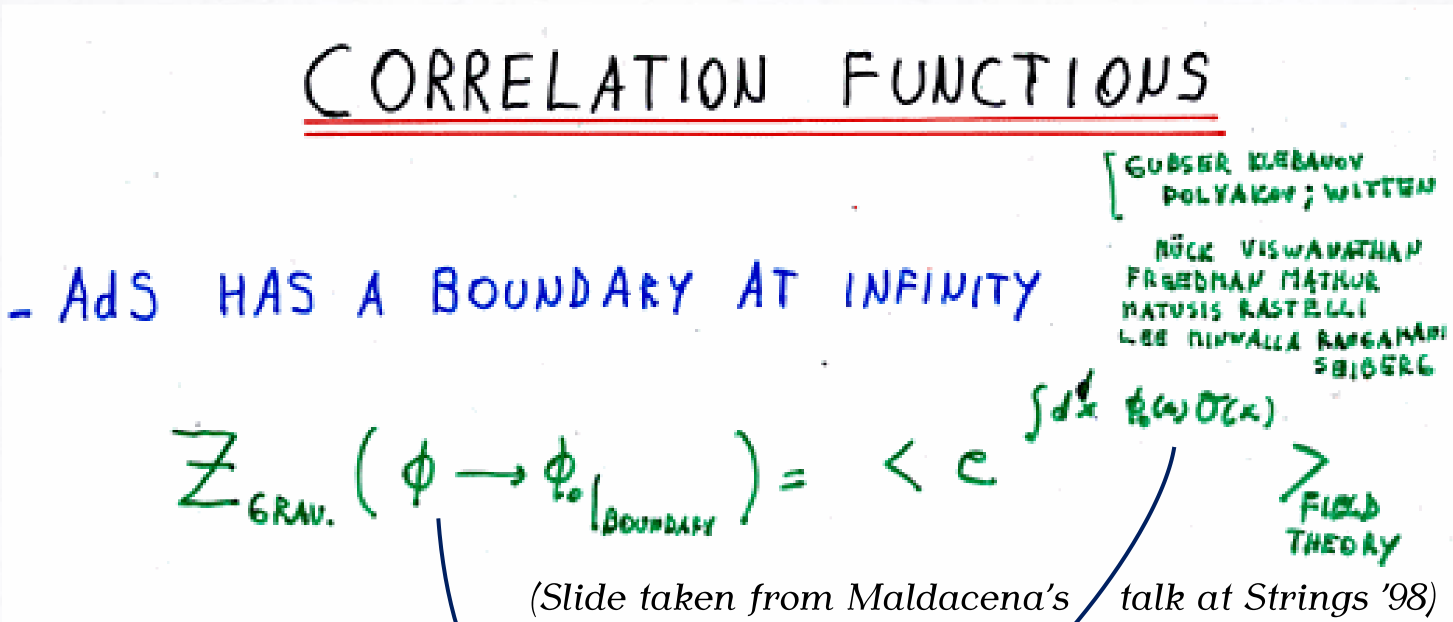
$$C_{IJK} = \frac{\sqrt{\Delta_I \Delta_J \Delta_K}}{N} + \sum_{k,\ell} \delta(v_{4,k,\ell}) + \sum_{a,b=0}^{\infty} \int_{-\infty}^{\infty} \frac{dv}{2\pi i} \frac{\partial \delta^*_{a,b}}{\partial v} \exp(1 - Z^*_{a,b,0}(v))$$

Overview and Plan of the Talk

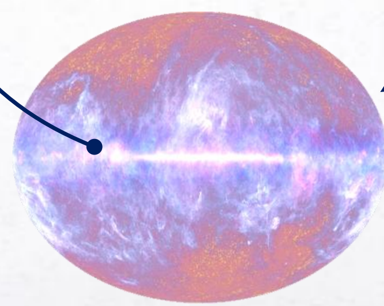


AdS/CFT and Holography

[Maldacena Adv.Theor.Math.Phys., 9711200] [Gubser, Klebanov, Polyakov Phys.Lett., 9802109] [Witten Adv.Theor.Math.Phys., 9802150]



(Slide taken from Maldacena's talk at Strings '98)



Holographic Computation

[Maldacena GKP Witten] [Freedman, Mathur, Matusis, Rastelli Nucl.Phys., 9804058]

Poincare coordinates $ds^2 = \frac{dz^2 + d\vec{x}^{\,2}}{z^2}$

Bulk scalar field $\phi(z, \vec{x})$ with mass $m^2 = \Delta(\Delta - 4)$

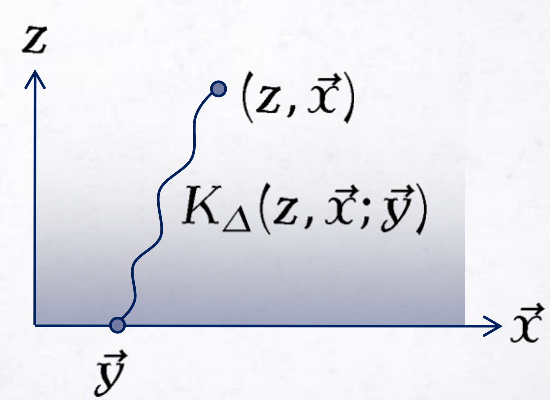
Boundary condition $\lim_{z \to 0} \phi(z, \vec{x}) = z^{4-\Delta} \phi_0(\vec{x})$

Bulk-to-boundary propagator

$$K_\Delta(z, \vec{x}; \vec{y}) = \mathcal{N}_\Delta \left(\frac{z}{z^2 + (\vec{x} - \vec{y})^2} \right)^\Delta$$

$$\phi(z, \vec{x}) = \int d^4y\, K_\Delta(z, \vec{x}; \vec{y}) \phi_0(\vec{y})$$

$$Z_{\text{gravity}}[\phi_0] = e^{-S_{\text{gravity}}[\phi_0]} \stackrel{!}{=} Z_{\text{gauge}}[\phi_0]$$



SUGRA / CPO Approximation

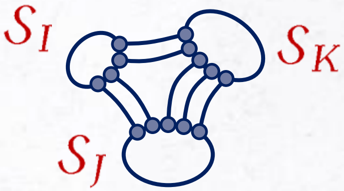
Lee, Minwalla
Rangamani, Seiberg
Adv.Theor.Math.Phys., 9806074

Supergravity ("Massless string modes") ↔ Chiral Primary Operators

SYM: Chiral primary operator

$$\mathcal{S}_I = C_I^{i_1,\dots,i_k}\, \mathrm{tr}\, \phi_{i_1} \cdots \phi_{i_k}$$

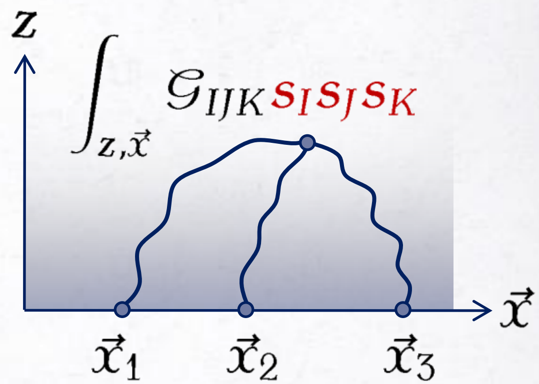
SO(6) irrep



SUGRA: Dual scalar field

$$h_{\alpha\beta} \sim g_{\alpha\beta}\, Y_I\, s_I + \dots \qquad \text{(Graviton)}$$

$$a_{\alpha\beta\gamma\delta} \sim \epsilon_{\alpha\beta\gamma\delta\varepsilon}\, \nabla^{\varepsilon} Y_I\, s_I + \dots \qquad \text{(5-form)}$$



SYM @ λ=0 and SUGRA @ λ=∞ :

$$\langle \mathcal{S}_I(x_1)\mathcal{S}_J(x_2)\mathcal{S}_K(x_3)\rangle = \frac{1}{N}\frac{\sqrt{\Delta_I\Delta_J\Delta_K}\langle C_I C_J C_K\rangle}{|x_{12}|^{\Delta_I+\Delta_J-\Delta_K}\,|x_{23}|^{\Delta_J+\Delta_K-\Delta_I}\,|x_{31}|^{\Delta_K+\Delta_I-\Delta_J}}$$

⟿ Non-renormalisation theorems (result is true for any λ)

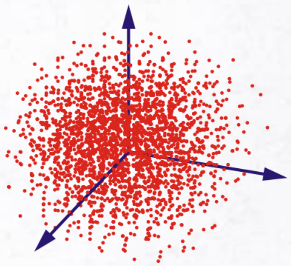
Stringy States / Non-CPO Operators

Massive string modes ↔ Unprotected operators

- Konishi operator ↔ First excited string level

$\mathrm{tr}\,\phi_i\phi_i$

$$\Delta = \begin{cases} 2 + \frac{3}{4\pi^2}\lambda + \frac{3}{16\pi^4}\lambda^2 + \ldots\ldots\ldots \\ 2\sqrt[4]{\lambda} - 2 + \frac{2}{\sqrt[4]{\lambda}} + \ldots \end{cases}$$

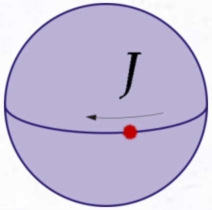


Semi-classical strings ↔ Operators with large charges/spin

- BMN operator ↔ nearly point-like string orbiting S^5

[Berenstein, Maldacena, Nastase JHEP, 0202021]

$$\sum_{p=0}^{J} e^{2\pi i p n/J}\,\mathrm{tr}\,\phi_1 Z^p \phi_2 Z^{J-p}$$

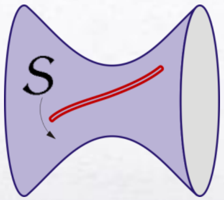


$J \sim \sqrt{\lambda} \gg 1$

- GKP operator ↔ Folded string spinning on AdS_5

[Gubser, Klebanov, Polyakov Nucl.Phys., 0204051]

$\mathrm{tr}\,ZD^SZ$ + permutations



$S \sim \sqrt{\lambda} \gg 1$

Two-point Functions

The "trick" for computing $\{\Delta_J\}$ was to compute NOT DIRECTLY:

$$\langle \bar{\mathcal{O}}_J(\vec{x}_1)\mathcal{O}_J(\vec{x}_2)\rangle = \frac{1}{|\vec{x}_1 - \vec{x}_2|^{2\Delta_J}}$$

but to compute:

- Eigenvalues of the dilatation operator

[Beisert Phys.Rept., 0407277]

$$\mathcal{D}\mathcal{O}_J = \Delta_J \mathcal{O}_J$$

- String energies in semi-classical quantization

[Tseytlin Int.J.Mod.Phys., 0209116]

$$\Delta_J = J + E_{\text{light-cone}}$$

because these computations can be re-phrased in the language of integrable spin chains.

[Minahan, Zarembo JHEP, 0212208]

[Arutyunov, Frolov, Staudacher JHEP, 0406256]

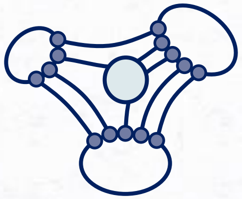
Three-point Functions in SYM

- BMN operators. Straightforward Feynman diagram computatioms.

[Kristjansen, Plefka, Semenoff, Staudacher Nucl.Phys., 0205033] [Constable, Freedman, Headrick, Minwalla JHEP, 0205089] [Chu, Khoze, Travaglini JHEP, 0206005] [Beisert, Kristjansen, Plefka, Semenoff, Staudacher Nucl.Phys., 0208178]

[Constable, Freedman, Headrick, Minwalla JHEP, 0209002]

- Spin-chain approach. Integrability.



[Roiban, Volovich JHEP, 0407140] [Alday, David, Gava, Narain JHEP, 0502186] [Alday, David, Gava, Narain JHEP, 0502186]

- Recent data collection.

[Georgiou, Gili, Russo JHEP, 0907.1567] [Grossardt, Plefka 1007.2356]

$$\langle \mathcal{O}_{\text{CPO}}\, \mathcal{O}_{\text{CPO}}\, \mathcal{O}_{\text{non-CPO}} \rangle \leftrightarrow \Delta_{\text{non-CPO}}$$

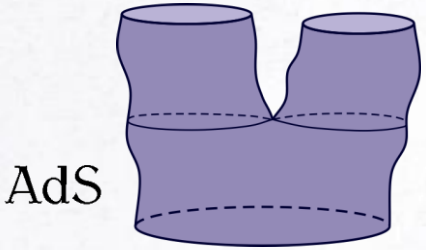
- Algebraic Bethe ansatz approach.

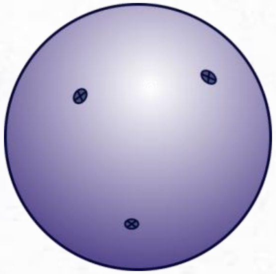
[Escobedo, Gromov, Sever, Vieira 1012.2475, 1104.5501]

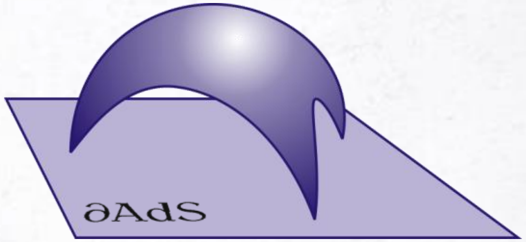
Label Operators by Bethe roots. Free theory. Combinatorics. Including Long-Long-Short.

Correlation Functions from Strings

String amplitude $\neq$ Correlation function







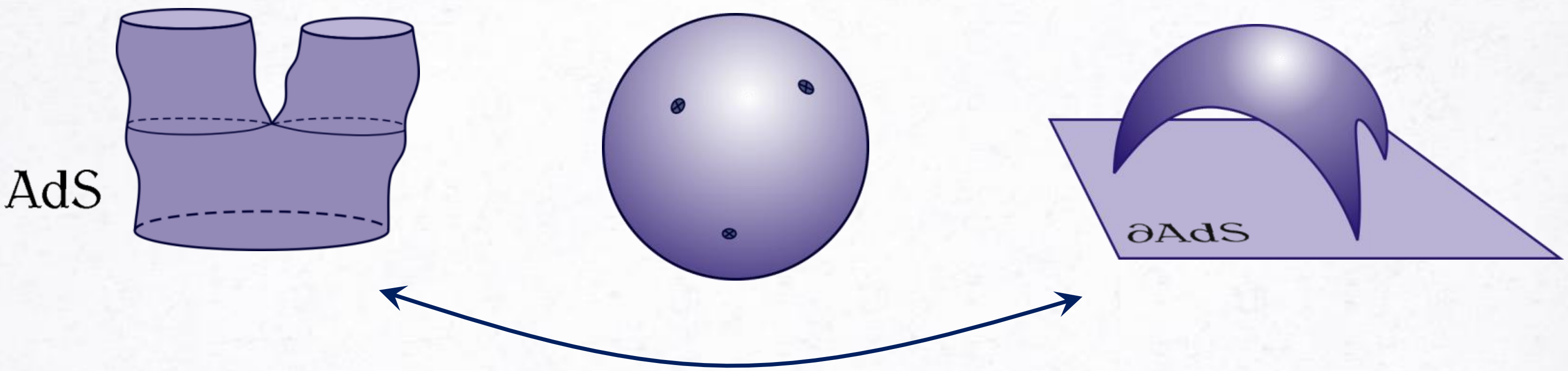
[Peeters, Plefka, Zamaklar JHEP, 0410275] Splitting of spinning strings in $\mathbb{R} \times S^5$

[Murchikova 1104.4804] Splitting of spinning strings in AdS_3

[Vicedo 1105.3868] Splitting of any string in $\mathbb{R} \times S^3$

Correlation Functions from Strings

String amplitude $\neq$ Correlation function



"Tunneling" or "Wick rotation"

[Dobashi, Shimada, Yoneya Nucl.Phys., 0209251] Holography in the BMN limit $J \sim \sqrt{\lambda} \gg 1$

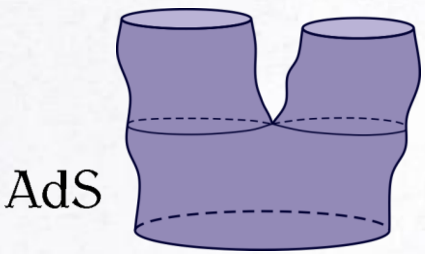
WKB: $\phi(z) \sim e^{iS(z)}$ $\quad S(z) = \pm \int^{z} \sqrt{\lambda\omega^2 - J^2/z'^2}\, dz' \xrightarrow{z\to 0} \pm iJ \ln z$

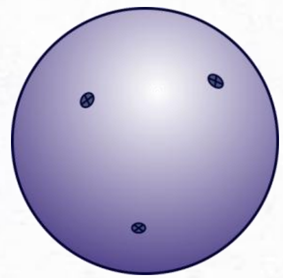
Imaginary momentum $\Rightarrow$ Wick rotate, or allow complex solutions

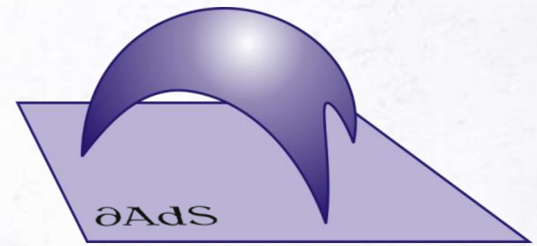
[Tsuji Prog.Theor.Phys., 0606030] Two point correlator as tunneling amplitude

Correlation Functions from Strings

String amplitude $\neq$ Correlation function







[Janik, Surowka, Wereszczynski JHEP, 1002.4613]	Convolution with wave-functions, $e^{i\int p\,dx}$, changes $\mathcal{L}$ to $\mathcal{H}$
[Bak, Chen, Wu 1103.2024]	Legendre transformation to Routhian
[Buchbinder, Tseytlin JHEP, 1005.4516]	Vertex operator insertions on world-sheet in conf. gauge
[Buchbinder JHEP, 1002.1716]	Virasoro constraints $\leftrightarrow$ Marginality condition
[TK, McLoughlin 1106.0495]	Vertex operators as boundary actions in LC gauge. Virasoro constraints build in.

Vertex Operators

Def.: Marginal perturbation of string sigma model.

General structure: $\mathrm{e}^{i\,\text{Charge}\cdot\text{Coordinate}}\,\text{Polynomial(Coordinates)}$

Flat-space example: $e^{ik\cdot X}\,(\partial X\cdot\bar\partial X)^{S/2}$

AdS: Global $\leftrightarrow$ Poincaré $\leftrightarrow$ Embedding coordinates

$$\mathrm{e}^{iEt}\longleftrightarrow\left(\frac{z}{z^2+\vec{x}^{\,2}}\right)^{\Delta}\longleftrightarrow(Y_+)^{-\Delta}$$

Semiclassical vertex operators in AdS: [Polyakov Int.J.Mod.Phys., 0110196] [Tseytlin Nucl.Phys., 0304139]

$$V_J^{\text{dilaton}}=(Y_+)^{-\Delta}(X_x)^J\left(\partial Y_M\bar\partial Y^M+\partial X_k\bar\partial X_k\right)+\ldots\quad\leftrightarrow\quad\mathrm{tr}(F_{\mu\nu}^2Z^J+\ldots)$$

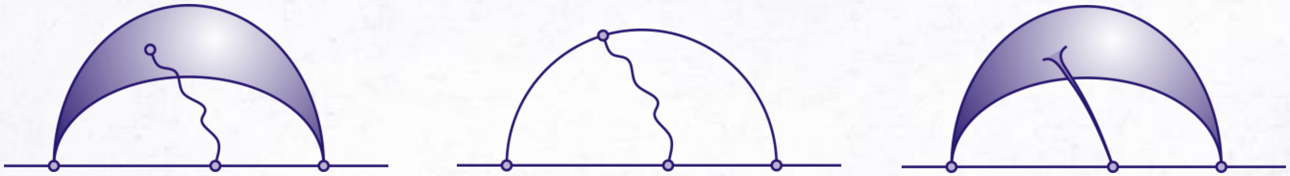
$$V_J^{\text{primary}}=(Y_+)^{-\Delta}(X_x)^J\left(\frac{\partial\vec{x}\cdot\bar\partial\vec{x}-\partial z\bar\partial z}{z^2}-\partial X_k\bar\partial X_k\right)+\ldots\quad\leftrightarrow\quad\mathrm{tr}(Z^J)$$

$$V_S^{\text{Regge}}=(Y_+)^{-\Delta}(\partial Y_x\bar\partial Y_x)^{S/2}+\ldots$$

$$V_J^{\text{Regge}}=(Y_+)^{-\Delta}(\partial X_x\bar\partial X_x)^{J/2}+\ldots$$

Heavy-Heavy-Light

Two semiclassical strings + one massless/light string



Hybrid formulation of String theory and Supergravity [Tseytlin Phys.Lett. 1990]

$$\frac{1}{\mathcal{N}}\int \mathcal{D}\mathbb{X}\,\mathcal{D}\Phi\,\ldots\,e^{-S_{\rm str}[\mathbb{X},\Phi]-S_{\rm sugra}[\Phi]} \qquad \partial\mathbb{X}^{\mathbb{M}}\partial\mathbb{X}^{\mathbb{N}}\,g_{\mathbb{M}\mathbb{N}}(\mathbb{X},s)\,e^{\varphi/2}$$

CPO dual — Dilaton

[Zarembo JHEP, 1008.1059] Folded spinning string + CPO dual. Also $1 \ll J \ll \sqrt{\lambda}$

[Costa, Monteiro, Santos, Zoakos, JHEP, 1008.1070] Various strings + Dilaton. Also gauge theory. $C_{HH\mathcal{L}} \sim \lambda\frac{\partial}{\partial\lambda}\delta\Delta_H(\lambda,J)$

[Roiban, Tseytlin Phys.Rev., 1008.4921] Various combinations $\langle V_{H_1}V_{H_2}V_{L_1}V_{L_2}V_{L_3}V_{L_4}\ldots\rangle$

Heavy-heavy-light activity: Spinning strings, Giant magnon, Giant graviton, Wilson loops, Winding, Finite-size, ...

[Hernandez J.Phys., 1011.0408] [Ryang JHEP, 1011.3573] [Arnaudov, Rashkov Phys.Rev., 1011.4669] [Georgiou JHEP, 1011.5181] [Park, Lee 1012.3293] [Buchbinder, Tseytlin JHEP, 1012.3740] [Bissi, Kristjansen, Young, Zoubos 1103.4079]

[Bak, Chen, Wu 1103.2024] [Arnaudov, Rashkov, Vetsov 1103.6145] [Hernandez 1104.1160] [Bai, Lee, Park 1104.1896] [Alday, Tseytlin 1105.1537] [Ahn, Bozhilov 1105.3084] [Lee, Park 1105.3279]

Light-Cone Approach

[TK, McLoughlin 1106.0495]

- Integrate out world-sheet metric components, which are Lagrange multipliers for Virasoro constraints.

[Callan, McLoughlin, Swanson Nucl.Phys., 0404007]

- Fix AdS-light-cone gauge $x^{\pm} = \frac{1}{\sqrt{2}}(x^3 \pm i x^0)$

[Metsaev, Thorn, Tseytlin Nucl.Phys., 0009171]

$$x^+ = \tau \qquad p_- = s$$

- Effective path integral for physical degrees of freedom

$$\langle \ldots \rangle = \int \mathcal{D}\mathbb{X}^A \, \mathcal{D}\mathbb{p}^A \, ds \, (\ldots) \, e^{\underbrace{-\frac{\sqrt{\lambda}}{2\pi}\int_0^{2\pi} d\sigma \int_{\tau_1}^{\tau_2} d\tau \left[\mathbb{p}_A \dot{\mathbb{X}}^A + s \dot{\mathbb{X}}^- - \mathcal{H}_{lc}(\mathbb{p}_A, \mathbb{X}^A, s) \right]}}$$

"World-sheet bulk action" $\mathcal{S}$

- Semiclassical vertex operators supply world-sheet boundary action

$$V \sim (..\mathbb{X}..)^Q \quad \text{so that} \quad V e^{-\mathcal{S}} = e^{-(\mathcal{S}+\mathcal{B})} \quad \text{with} \quad \mathcal{B} = -\ln V$$

$Q \sim \sqrt{\lambda}$

Toy Model with Boundary Action

Consider the phase space action $\mathcal{S} = \int_{\tau_1}^{\tau_2} d\tau \left[p\dot{x} - \mathcal{H}(p,x)\right]$

and add a "vertex operator" $V_Q(\tau) = e^{-Qx(\tau)}$ to either end of $[\tau_1, \tau_2]$

This yields the boundary action:

$$\mathcal{B} = -\sum_{i=1,2} \ln V_{Q_i}(\tau_i) = \int_{\tau_1}^{\tau_2} d\tau \sum_{i=1,2} Q_i x(\tau)\delta(\tau - \tau_i)$$

Take the variation of the total action: $\delta(\mathcal{S} + \mathcal{B}) = 0$

$$\frac{\delta}{\delta p}: \quad \dot{x} - \frac{\delta\mathcal{H}}{\delta p} = 0$$

$$\frac{\delta}{\delta x}: \quad -\dot{p} - \frac{\delta\mathcal{H}}{\delta x} + (-p + Q_1)\delta(\tau - \tau_1) + (p + Q_2)\delta(\tau - \tau_2) = 0$$

$\Rightarrow$ Bulk eq. of motion & Bdry conditions: $\quad p(\tau_1) = Q_1 \quad p(\tau_2) = -Q_2$

Classical Two-point Function

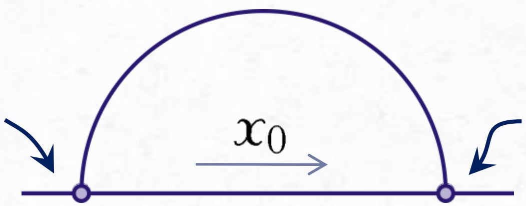
Vertex operator: $V^{\rm BMN}_{\Delta,J}(\vec{a},\underline{n}) = \left(\frac{|\underline{z}|}{\underline{z}^2+(\vec{x}-\vec{a})^2}\right)^{\Delta}\left(\frac{\underline{n}\cdot\underline{z}}{|\underline{z}|}\right)^{J}$

Insertion point in AdS

vector in $\mathbb{R}^6 = z \otimes S^5$

specifies plane of rotation e.g. $\underline{n} = (1, i, 0, 0, 0, 0)$

$V_{\Delta_1,J_1}(\vec{a}_1 = a, \underline{n}_1)$



$V_{\Delta_2,J_2}(\vec{a}_2 = b, \underline{n}_2)$

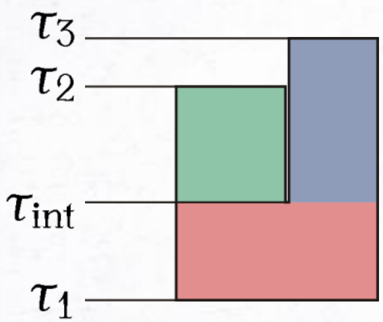
Saddle point solution: $\underline{z}_{\rm cl} = \frac{1}{2}(x_0-a)e^{-\phi}\underline{n}^* + \frac{1}{2}(b-x_0)e^{\phi}\underline{n}$

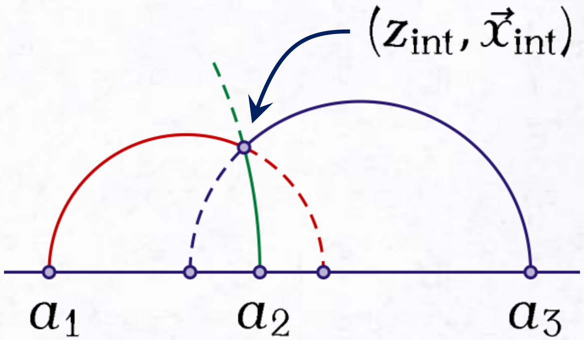
if and only if $J_1 = \Delta_1 = \Delta_2 = -J_2$ and $\underline{n}_1 = \underline{n}_2 = \underline{n}$

$$\langle V_1 V_2\rangle_{\rm cl} = \underbrace{1}_{e^{-S}} \times \underbrace{\frac{e^{-\phi J_1}}{(b-a)^{\Delta_1}}\lim_{x_0\to a}\left(\frac{x_0-a}{b-x_0}\right)^{\frac{\Delta_1-J_1}{2}}}_{e^{-\mathcal{B}_1}} \times \underbrace{\frac{e^{-\phi J_2}}{(b-a)^{\Delta_2}}\lim_{x_0\to b}\left(\frac{b-x_0}{x_0-a}\right)^{\frac{\Delta_2+J_2}{2}}}_{e^{-\mathcal{B}_2}} = \frac{1}{|b-a|^{2\Delta}}$$

Classical Three-point Function

Saddle point solution of bulk and boundary action is given by the combination of three geodesics





The bulk action evaluates again to zero on the classical solution.

The boundary action evaluates to

extremal
$\underline{n}_1 = \underline{n}_2 = \underline{n}_3$

$$\mathcal{B} = \sum_{i=1}^{3} \ln \left(\frac{z_{\rm int}}{z_{\rm int}^2 + (\vec{x}_{\rm int} - \vec{a}_i)^2} \right)^{\Delta_i} + (\Delta_1 - \Delta_2 - \Delta_3)\varphi_{\rm int}$$

non-extremal
$\underline{n}_1 \nparallel \underline{n}_2 \nparallel \underline{n}_3$

$$\mathcal{B} = \sum_{i=1}^{3} \ln \left(\frac{z_{\rm int}}{z_{\rm int}^2 + (\vec{x}_{\rm int} - \vec{a}_i)^2} \right)^{\Delta_i}$$

intersection point on the sphere

Classical Three-point Function

Need to solve

$$\delta\mathcal{B} = 0 \qquad \text{where} \qquad \mathcal{B} = \sum_{i=1}^{3} \ln\left(\frac{z_{\text{int}}}{z_{\text{int}}^2 + (\vec{x}_{\text{int}} - \vec{a}_i)^2}\right)^{\Delta_i}$$

[Dobashi, Yoneya
Nucl.Phys., 0406225]

"It is easy to convince oneself that, for generic configurations of three boundary points, there is no solution to these equations."

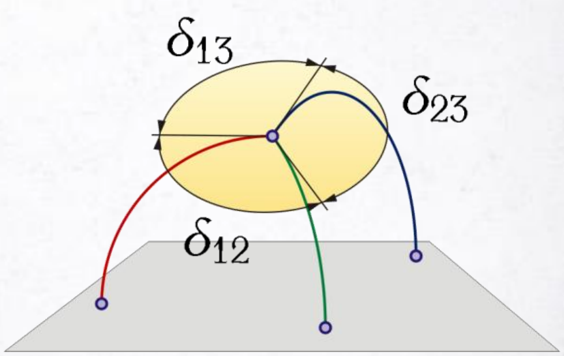
[Janik, Surowka, Wereszczynski
JHEP, 1002.4613] "Now we have to find the saddle point w.r.t. $\vec{x}$ and z. This is very difficult, if not impossible, to do explicitly."

[Minahan
to appear] "I found it anyway!"

$$\vec{x}_{\text{int}} = \frac{\alpha_2\alpha_3\,\vec{a}_{23}^{\,2}\,\vec{a}_1 + \alpha_1\alpha_3\,\vec{a}_{13}^{\,2}\,\vec{a}_2 + \alpha_1\alpha_2\,\vec{a}_{12}^{\,2}\,\vec{a}_3}{\alpha_2\alpha_3\,\vec{a}_{23}^{\,2} + \alpha_1\alpha_3\,\vec{a}_{13}^{\,2} + \alpha_1\alpha_2\,\vec{a}_{12}^{\,2}}$$

$$z_{\text{int}} = \frac{\sqrt{\alpha_1\alpha_2\alpha_3(\alpha_1+\alpha_2+\alpha_3)}\,|\vec{a}_{23}||\vec{a}_{13}||\vec{a}_{12}|}{\alpha_2\alpha_3\,\vec{a}_{23}^{\,2} + \alpha_1\alpha_3\,\vec{a}_{13}^{\,2} + \alpha_1\alpha_2\,\vec{a}_{12}^{\,2}}$$

with $\alpha_1 = \Delta_2 + \Delta_3 - \Delta_1$ etc.



$$\cos\delta_{12} = \frac{\Delta_3^2 - \Delta_1^2 - \Delta_2^2}{2\Delta_1\Delta_2}$$

Classical Three-point Function

The minimum of the boundary action is thus

$$\langle V_1 V_2 V_3 \rangle_{\rm cl} = e^{-\mathcal{B}} = \frac{\sqrt{\dfrac{\alpha_1^{\alpha_1}\alpha_2^{\alpha_2}\alpha_3^{\alpha_3}(\alpha_1+\alpha_2+\alpha_3)^{\alpha_1+\alpha_2+\alpha_3}}{(\alpha_1+\alpha_2)^{\alpha_1+\alpha_2}(\alpha_2+\alpha_3)^{\alpha_2+\alpha_3}(\alpha_3+\alpha_1)^{\alpha_3+\alpha_1}}}}{|\vec{a}_1-\vec{a}_2|^{\alpha_3}|\vec{a}_1-\vec{a}_3|^{\alpha_2}|\vec{a}_2-\vec{a}_3|^{\alpha_1}}$$

where $\alpha_1 = \Delta_2 + \Delta_3 - \Delta_1 = J_2 + J_3 - J_1$ etc.

This agrees with [Lee, Minwalla Rangamani, Seiberg Adv.Theor.Math.Phys., 9806074] for $J \sim \sqrt{\lambda} \gg 1$

What else?

[TK, McLoughlin 1106.0495]

- *Quantum fluctuations*
  - AdS light-cone gauge particularly nice for semi-cl. quantization
  - Fluctuation expansion $\mathbb{X}^{\mathbb{M}} = \mathbb{X}^{\mathbb{M}}_{\text{cl.}} + \frac{1}{\sqrt[4]{\lambda}}\tilde{\mathbb{X}}^{\mathbb{M}}$
  - τ-dependent redefinition of the fluctuations, then redefinition of τ

$$\mathcal{S}_{\text{fl}} = \frac{1}{4\pi}\int_0^{2\pi} d\sigma \int_{-\infty}^{\infty} d\tilde{\tau} \left(\dot{\tilde{\mathbb{X}}}^2 + \acute{\tilde{\mathbb{X}}}^2 + \mu^2\tilde{\mathbb{X}}^2\right) \qquad \mu = \frac{\Delta}{\sqrt{\lambda}}$$

  - Plane-wave action for fluctuations on each segment—incl. fermions

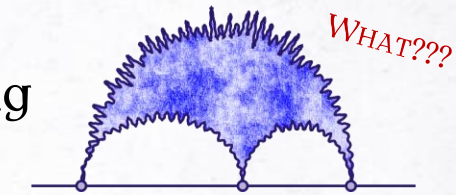
- *Circular winding string*
  - Vertex operators from [Buchbinder JHEP, 1002.1716] and [Ryang JHEP, 1011.3573], although quite different, give same classical answer.
  - Now, contribution from world-sheet bulk *and* boundary action

Summary

Want to find the ***structure constants*** $\{C_{IJK}\}$,
or rather ***a neat method for computing them***.

Supergravity

Quantum String



Semi-classical Strings