Perturbative features of the wavefunction of the universe for pure gravity

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Outline

- The wavefunction of the universe in EAdS and dS
- 4d de Sitter or EAdS and conformal gravity.
- The wavefunction for 5d de Sitter

De Sitter space

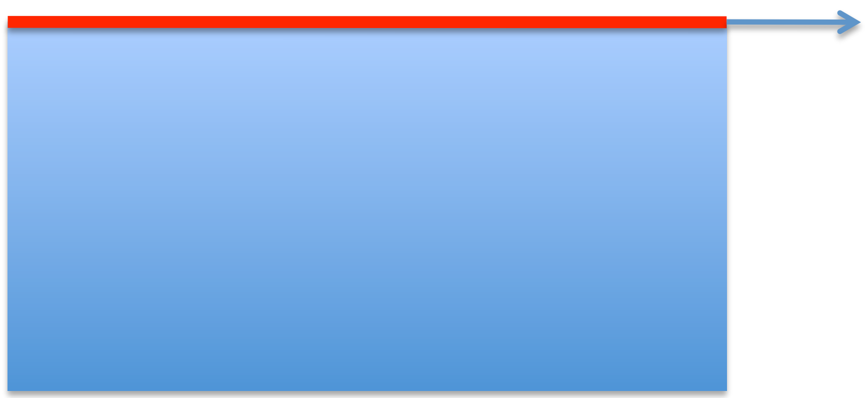
- Expanding universe (Poincare patch)

$$ds^2 = -dt^2 + e^{2t} dx^2$$

Proper time

$$ds^2 = \frac{-d\eta^2 + dx^2}{\eta^2}$$

Conformal time



Asymptotic future η=0

past

Comoving vs Physical distances.

- x is ``comoving position''. Physical distance is exponentially growing. (x = constant, geodesic of a particle ''at rest''.)

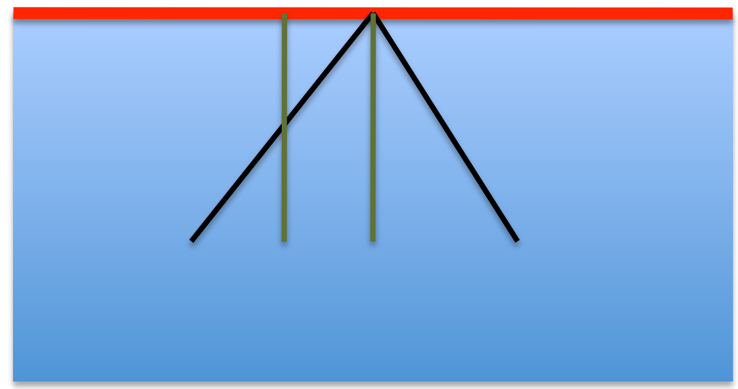
- Fixed comoving distance Δx , gives an exponentially growing physical distance.

- Translation symmetry → momentum is conserved. e^{ikx}

- Fixed coming momentum k, gives a physical momentum that increases to the past and decreases to the future.

$$ds^2 = \frac{-d\eta^2 + dx^2}{\eta^2}$$

Horizon



Crossed at $\eta = x$, or $k\eta = 1$

Follow a fixed k mode
Early times, large physical momentum, like plane waves in flat space → Bunch Davies vacuum.

Looking at a fixed k mode at late times → looking at superhorizon distances.

Pure gravity → Look at metric fluctuations.

$$ds^2 = \frac{-d\eta^2 + dx^2 + h_{ij}dx^i dx^j}{\eta^2}$$

Gravity wave fluctuations become constant at late times →
Wavefunction becomes ``scale independent'' for large scale factors:

$$\Psi(\frac{h}{\eta^2}) \to \Psi(h)$$

For length scales >> η

For each physical mode, the leading approximation to the wavefunction is a gaussian

$$\Psi \sim e^{-\frac{M^2}{H^2}k^3h^2}$$

For superhorizon distances

Consider computing corrections to the gaussian approximation.

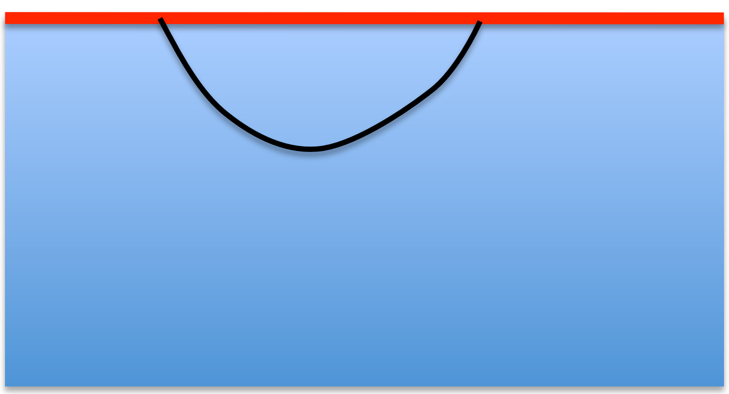
Simplest → Three point function.

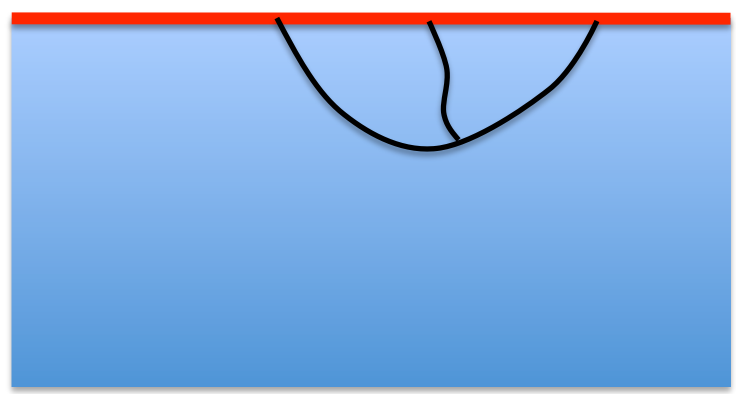
Can be computed directly by expanding the Einstein action to cubic order.

Conformal symmetry restricts its form → Only 3 possible shapes

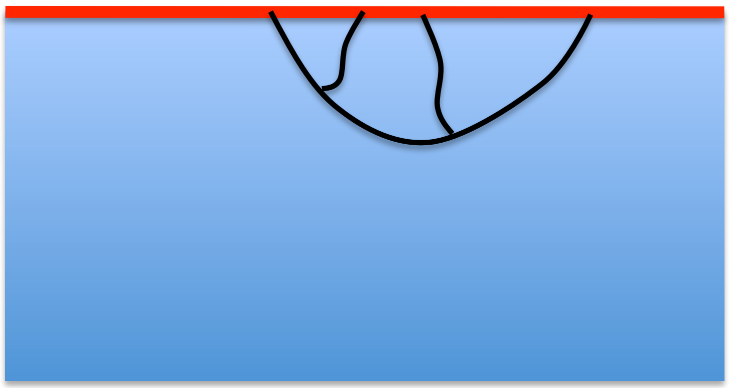
Einstein gravity produces only one of these shapes.

JM & Pimentel





Imagine computing all tree diagrams → Leading contribution to higher point functions.



Contained in a classical solution of Einstein's equation with fixed future (and past BD) boundary conditions.

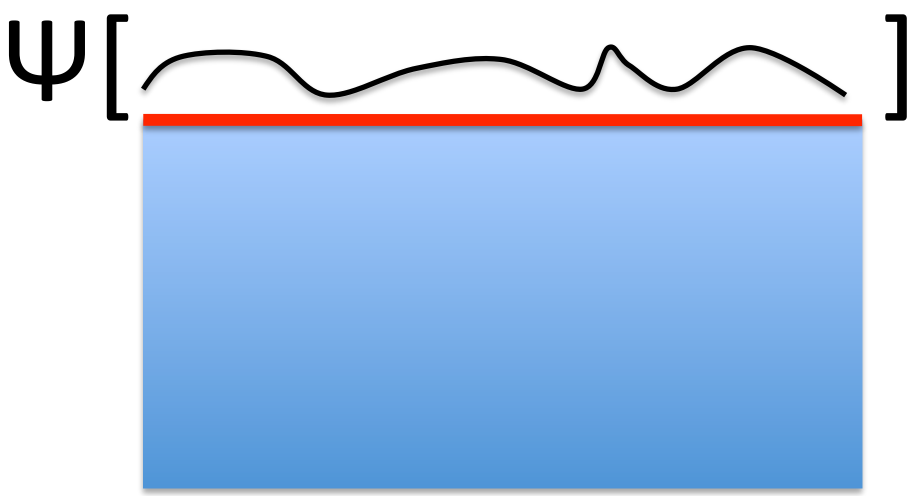
Fix the boundary conditions for the metric in the future to an arbitrary shape.

Impose (interacting) Bunch Davies boundary conditions in the past.
→ Solution decays when $\eta \to -(1 + i\varepsilon)\infty$. Feynman boundary conditions in flat space. This prescription works to any order in perturbation theory.

$$\Psi \approx e^{iS} = e^{i\frac{M^2}{H^2}\int\sqrt{g}(R+12)}$$

Focus on one of the oscillating Factors in the Hartle Hawking picture (as we usually do when looking at the Klein Gordon equation).

Evaluate the classical action on a classical solution



Late time behavior

$$\Psi \approx e^{iS} = e^{i\frac{M^2}{H^2}\int\sqrt{g}(R+12)} = e^{i\frac{const}{\eta^3}\int\sqrt{1+h}+\ldots}\Psi_R(h)$$

Interesting part

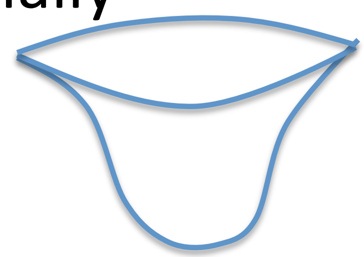
Divergent "counterterms" → Pure phases, drop out from $|\Psi|^2$

Klein Gordon norm → gives $|\Psi|^2$

EAdS vs. dS

- The computation of the dS wavefunction is very similar to the computation of the EAdS wavefunction.

- In EAdS: Also evaluate the ``wavefunction'', as in Hartle-Hawking. We focus on the exponentially increasing wavefunction in this case.



(Generating function of correlation functions)

- In perturbation theory, they are related in a very simple way.

EAdS → dS analytic continuation

$$z \to -i\eta \;, R_{AdS} = -iR_{dS} \;,$$

$$ds^2 = R_{AdS}^2 \frac{dz^2 + dx^2}{z^2} \;, \qquad \to \qquad ds^2 = R_{dS}^2 \frac{-d\eta^2 + dx^2}{\eta^2}$$

The boundary conditions also transform properly:

$$g \sim e^{-wz} \;, \quad z \to \infty \qquad \to \qquad g \sim e^{iw\eta} \;, \quad \eta \to -\infty$$

Decaying → oscillating with one frequency

In flat space → continuation from Euclidean space

In de Sitter → continuation from EAdS.

- This works also at loop level.

- Expectation values vs. Wavefunctions:

$$\langle BD|\phi\phi|BD\rangle \longrightarrow \text{Analytic continuation from Sphere}$$

$$\Psi[g_b] = \langle g_b|BD\rangle \longrightarrow \text{Analytic continuation from EAdS.}$$

JM
Harlow, Stanford

Turning EAdS computations into dS ones

- We could consider the action for an S^3 boundary in EAdS. (The CFT partition function on S^3) → Gives usual Hartle-Hawking factor for S^3

$$|\Psi|^2 \sim e^{S_{dS}}$$ ← De Sittter entropy

- Black hole free energies in EAdS → Give Hartle-Hawking factors for $S^2 x S^1_\beta$. Metrics are complex !.

JM

Similar to Hartle, Hawking, Hertog

dS/CFT

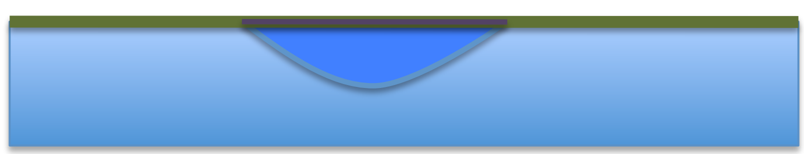
Strominger
Witten
(JM)

- The wavefunction $\Psi[g_b] = Z[g_b]_{CFT}$
- At one loop we start getting exponential suppressions

$$\Psi \sim e^{i \frac{M^2}{H^2} \frac{1}{\eta_c^3} \int \sqrt{g_b} - \frac{1}{\eta_c^3} \int \sqrt{g} + \cdots}$$

- Suppression of fluctuations at short distances.

- Like an exclusive amplitude in a massless gauge theory.
- Objections to dS/CFT go away. (Bubble decays → field theories with boundaries, etc..)



$EAdS_4$ or dS_4 gravity wavefunction

- Can be evaluated at tree level using the classical solution.
- We will show that the whole computation could also be viewed as a problem in conformal gravity.

Conformal Gravity

- Gravity that involves only the "conformal class" of the metric.
- Overall rescalings of the metric (or Weyl transformations of the metric) do not matter.

$$g_{\mu\nu} \rightarrow \Omega^2 g_{\mu\nu}$$

- Action depends only on the Weyl tensor

$$S = \int W^2$$

- Equations of motion → 4^{th} order in derivatives
- → Leads to ghosts.
- Around flat space, the solutions go like

e^{iEt}, te^{iEt} and complex conjugates.

Flat space hamiltonian → non diagonalizable

Two properties:

- Solutions of pure gravity Einstein's equations with a cosmological constant are also solutions of the equations of motion of conformal gravity.

- Renormalized action on dS → Same as action of conformal gravity on a solution of Einstein's equations.

Anderson
Miskovic Olea
Aros Contreras Olea
Troncoso, Zanelli

Useful identity:

$$\int Euler = \int W^2 + 2\int Ricci^2 - \frac{1}{3}R^2$$

Equations of motion of Weyl gravity → Involves Ricci tensor.
For Einstein spaces: $R_{\mu\nu} \propto g_{\mu\nu}$

$$\frac{\delta S_{\text{conformal}}}{\delta g^{\mu\nu}} \propto g_{\mu\nu} \to 0$$

Conformal gravity lagrangian ~ (Einstein equations)^2

Evaluating the Einstein action on an Einstein space → Same as evaluating the 4 volume.

$$S_E \propto \int \sqrt{g} \propto \int (W^2 - E)$$

$$S_{\text{E,Renormalized}} = \int d^4x\sqrt{g} - \text{Boundary} = \int d^4x\sqrt{g}W^2 - (\text{Euler Number})$$

- If we can select the Einstein solutions from the more numerous solutions of conformal gravity→ we can forget about the Einstein action and compute everything in terms of the conformal gravity action.
- We get an explicitly IR finite computation.

- A simple boundary condition on the fields of conformal gravity selects the Einstein gravity solutions.
- Conformal gravity equations: 4th order. 2 boundary conditions in the past from Bunch Davies (or EAdS conditions). Two in the future:

$$g_{ij}(\eta = 0) = g^b_{ij} \ , \qquad \partial_\eta g_{ij}(\eta = 0) = 0$$

$$ds^2 = \frac{-d\eta^2 + (g^0 + \eta^2 g^2 + \eta^3 g^3 + \cdots)dxdx}{\eta^2}$$

Einstein solutions.

Starobinski
Fefferman Graham

No time derivative

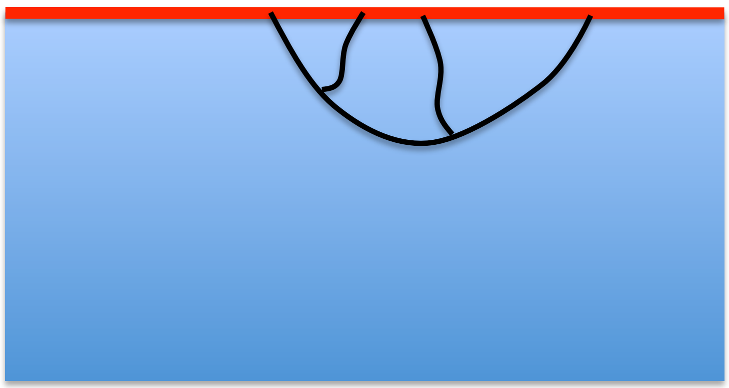
$$\Psi_{Conformal}[h, h' = 0] = \Psi_{Einstein,\mathrm{Re}\,normalized}[h]$$

$$e^{c\int W^2 - E} = e^{S_{E,\mathrm{Renormalized}}} \qquad c = \frac{M^2}{H^2}$$

-We get the ``right'' sign for the conformal gravity action for dS and the ``wrong'' one for EAdS

-The overall constant is simply the ``central'' charge, or the de Sitter entropy, which is given by M^2/H^2

-This is also the only dimensionless coupling constant for pure gravity in dS (or AdS) …(at tree level).



Ordinary de-Sitter wavefunctions:

$$h = (1 - ik\eta)e^{ik\eta}$$

Can be viewed as the combination of conformal gravity wavefunctions obeying the Neumann boundary condition.

We can use the propagators of conformal gravity with a Neumann condition + the vertices of conformal gravity

Or

The usual propagators of Einstein gravity

Ghosts?

- With a boundary condition, conformal gravity gave the same results as ordinary gravity. Thus we got rid of the ghosts.
- All we did, was to evaluate the ghost wavefunctions at zero values for the ghost fields.
- A quartic action + conformal couplings to background curvature → to an action in dS or AdS, which is the sum of two quadratic fields, one with positive norm one with negative norm. We are simply putting zero boundary conditions for the negative norm one.

Quartic Scalar field

$$S = \frac{1}{2}\int d^4x\sqrt{g}\left[(\nabla^2 C)^2 - 2(R_{\mu\nu} - \frac{1}{3}g_{\mu\nu}R)\partial_\mu C\partial_\nu C\right]$$

$$S = -\frac{1}{2}\int_{AdS_4}\sqrt{g}\,[(\nabla^2 C)^2 - 2(\nabla C)^2]$$

$$S = \int_{AdS_4}\sqrt{g}\,\{[\nabla(C+\varphi)]^2 - [(\nabla\varphi)^2 - 2\varphi^2]\}$$

Massive (tachyonic in AdS) field

Massless field

(setting this to zero at the boundary)

Quantum Questions

- Some versions of N=4 conformal sugra appear to be finite.
  Fradkin Tseytlin
- (one of these appears from the twistor string theory)
  Berkovits Witten
- Can this truncation be extended to the N=4 theory? Do we get an ordinary O(4) gauged sugra?
  (suggested by Berkovits)

$$\int e^{C}(W^{2}+C\nabla^{4}C)$$

In N=4 conformal supergravity, the coupling
Constant is the vev of a field → sets the ratio
of the Planck scale to the cosmological constant scale
We can get large hierarchies from a not so large C.

Quantum questions…

- Can the quantum theory with a Neumann boundary condition be interpreted as the result of a Unitary bulk theory ?

- Note that we would only get the wavefunction at one time. Only superhorizon wavefunction.

- We expect problems with unitarity → how do these appear.

- Gravity + Pauli-Villars ghost field → Making mass comparable to AdS (or dS) scale → gives conformal gravity.

Conclusions

- Conformal gravity with Neumann boundary conditions is equivalent (at tree level) to ordinary gravity on superhorizon distances.
- In AdS: The partition function of conformal gravity with Neumann boundary conditions is the same as that of ordinary gravity
- Gives a different way to compute AdS gravity correlation functions. Connections with Twistor string?
- This is non-linear, but classical (or semiclassical) relation
- It would be interesting to see what happens in the quantum case. One probably needs to do it for N=4 conformal sugra, which is finite.

Side remark

Einstein gravity in flat space

In progress

JM, Pimentel, Raju,…

- Limit from dS (or EAdS) gravity
- Compute correlation functions of stress tensors

Polchinski
Giddings
Penedones….

- We do not have ``energy'' conservation
- Singularity of the AdS (or dS) tree amplitude is the flat space tree amplitude.

(Both delta Function stripped)

$$\langle T(1)\cdots T(n)\rangle \sim \frac{\prod_{i=1}^{n}|\vec{k}_i|}{(|k_1|+\cdots+|k_n|)^{n-1}}\mathcal{A}_{n,\mathrm{Flat}}$$

Another application of conformal gravity

Solution of the tree level 5d measure for pure 5d gravity.

Finding the probability for different shapes for the spatial sections.

5d pure gravity in de Sitter

- Gravity with positive cosmological constant
- Consider the BD vacuum in the weakly coupled regime, $\frac{R^3}{G_N} \gg 1$

$$ds^2 = \frac{-d\eta^2 + g_{ij}dx^i dx^j}{\eta^2} \qquad g_{ij} = \delta_{ij} + h_{ij}$$

$\Psi(g_{ij})$ Wavefunction of the universe

- Similar to 4d case.
- Use the EAdS → dS analytic continuation.
- One crucial difference:

Starobinski,
Fefferman-Graham
Heningson-Skenderis

In Euclidean space, we have a real answer:

$$\Psi(\frac{1}{\epsilon^2} g_{ij}) = e^{c_{AdS}\left[\frac{1}{\epsilon^4}\int\sqrt{g}+\frac{1}{\epsilon^2}\int\sqrt{g}R+\log\epsilon\int W^2-E+\text{Finite}(g)\right]}$$

$$c_{AdS} = \frac{R^3_{AdS}}{G_N} \to i\frac{R^3_{dS}}{G_N}$$

All terms become purely imaginary, including the finite term. The only real part arises via

$$\log\epsilon \to \log|\eta_0| + i\frac{\pi}{2}$$

$$|\Psi|^2 = e^{-c_{dS}\pi\int d^4x\sqrt{g}(W^2-E)}$$

(Depends on the metric of the four dimensional slice)

Action of conformal gravity

Gives a topological term, the Euler number.

It is completely local

It was non-local in even bulk dimensions.

In three bulk dimensions, or dS_3 gravity, we get only the Euler number $\rightarrow$ only the topology of the space matters.

Conclusions, 5d

- In five dimensional de-Sitter there is a huge simplification if we compute the wavefunction.
- We simply get the action of conformal gravity in 4d. This is the 4d spatial slice of the 5d geometry at superhorizon distances.

Strings and Rigid strings

(Pointed out by Polyakov)

$$S_{usual} = \int \sqrt{g}$$

Polyakov

$$S_{rigid} = \int \sqrt{g}\hat{K}^{i}_{ab}\hat{K}^{i,ab}$$

Induced metric

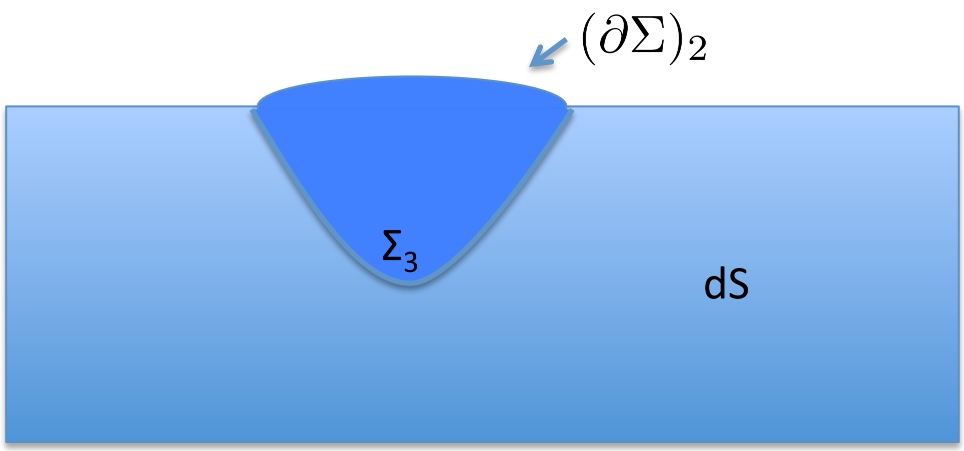
Weyl invariant in target space.

The problem of computing a Wilson loop in AdS is equivalent to computing a Wilson loop in flat space with the rigid string action, with an extra Neuman boundary condition on the fields. $X^{\mu}(\sigma=0) = f^{\mu}(\tau)$, $\partial_{\sigma} X^{\mu}(\sigma=0) = 0$

Alexakis

Value of the Wilson loop - counterterm = Value of the rigid string action.

Membranes in dS and rigid strings



Membrane (domain wall in 4d) is created in the probe approximation. (Or connecting same energy vacua). Its dS boundary is a two dimensional surface. The tree level probability that this surface has a given shape → Given by the rigid string action.

$$|\Psi(X)|^2 = e^{-R^3 T\pi(S_{rigid} - Euler)}$$

Same argument using the conformal anomaly for the membrane action

Berenstein, Corrado, JM
Fischler
Graham, Witten

The End