EXACT RESULTS AND STRINGY EFFECTS IN ABJM THEORY

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mostly [Drukker-M.M.-Putrov, 1007.3837 & 1103.4844]

review [M.M., 1104.0783]

A new interpolating function in AdS/CFT

In this talk I will focus on the *free energy on the three-sphere* of ABJM theory and related 3d CFTs:

$$F(\lambda, N) = \sum_{g \geq 0} F_g(\lambda) N^{2-2g}$$

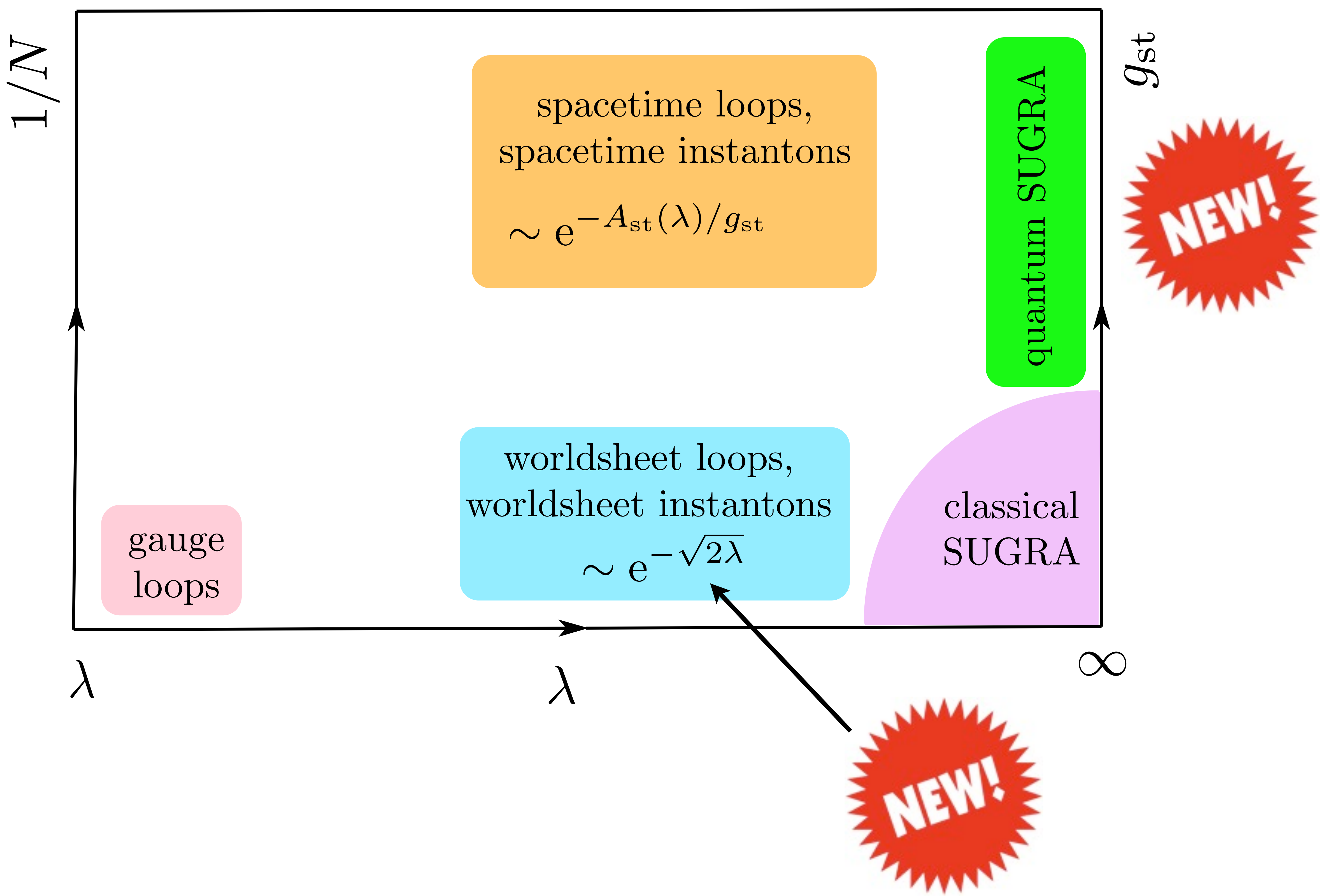
λ 't Hooft parameter

This quantity has many interesting properties:

• It is a *good measure of the number of degrees of freedom* of the 3d CFT. In fact, it is a candidate for a 3d c-function [Jafferis, Klebanov, ...]

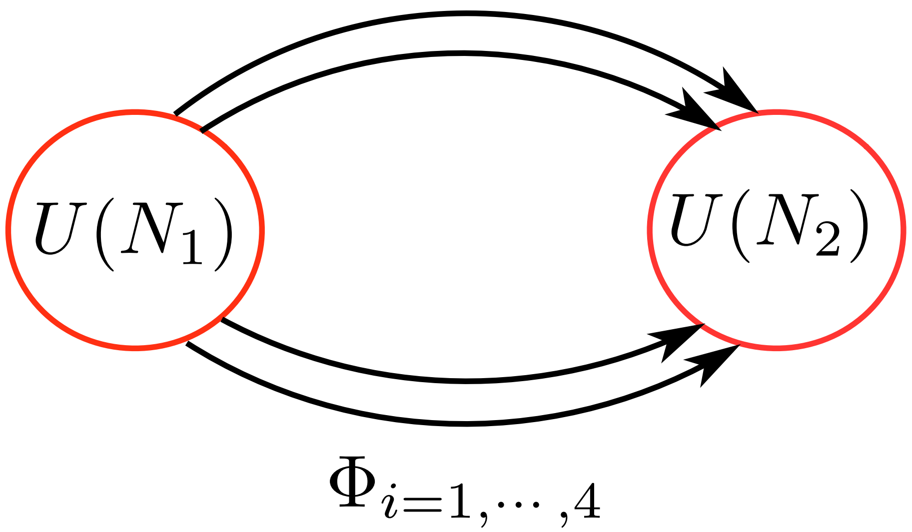
• The $F_g(\lambda)$ can be *effectively* calculated order by order in the $1/N$ expansion, and *exactly* as a function of λ

The parameter space of string/gauge dualities



A B J M theory

2 CS theories + 4 hypers in the bifundamental



$$U(N_1)_k \times U(N_2)_{-k}$$

two 't Hooft couplings $\lambda_i = \frac{N_i}{k}$

In this talk we restrict to the "ABJM slice" $\lambda_1 = \lambda_2 = \lambda = \frac{N}{k}$

This is a 3d SCFT which (conjecturally) describes *N* M2 branes probing a $\mathbb{C}^4/\mathbb{Z}_k$ singularity

Gravity dual

M-theory on $\mathrm{AdS}_4 \times \mathbb{S}^7/\mathbb{Z}_k$ $\xrightarrow{\text{Hopf reduction}}$ type IIA theory/$\mathrm{AdS}_4 \times \mathbb{P}^3$

$$\mathrm{d}s^2 = \frac{L^2}{4\ell_s^2}\left(\mathrm{d}s^2_{\mathrm{AdS}_4} + 4\mathrm{d}s^2_{\mathbb{CP}^3}\right)$$

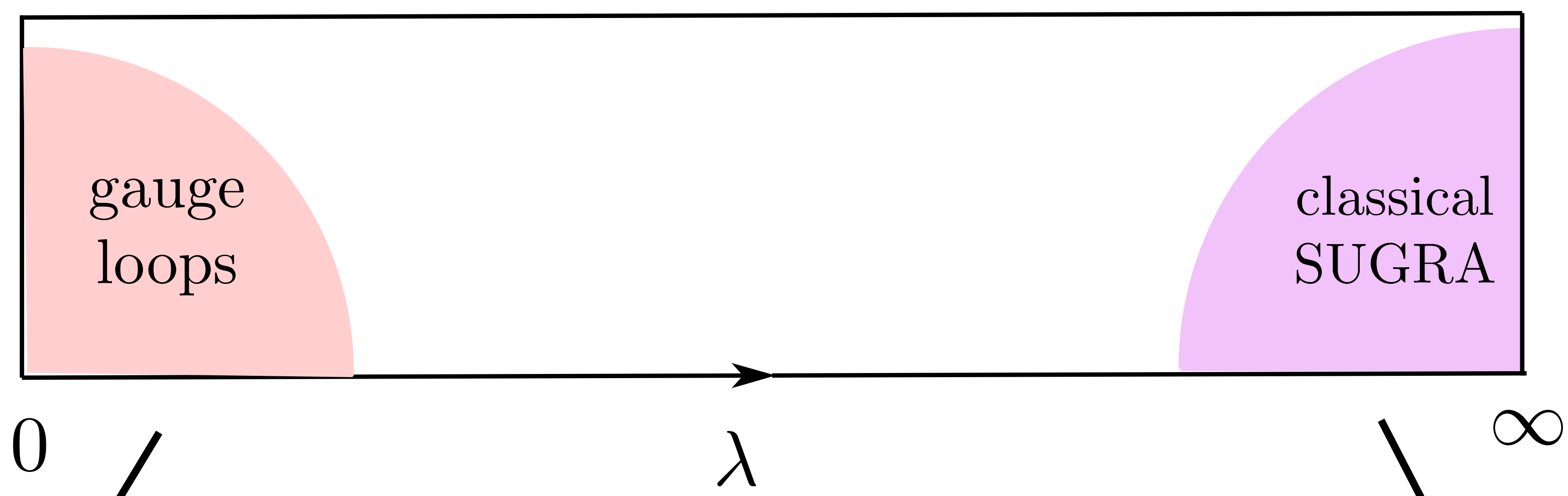
Gauge/gravity dictionary:

$$k^2 = g_{\mathrm{st}}^{-2}\left(\frac{L}{\ell_s}\right)^2$$

$$\hat{\lambda} = \lambda - \frac{1}{24} = \frac{1}{32\pi^2}\left(\frac{L}{\ell_s}\right)^4\left(1 - \frac{4\pi^2 g_{\mathrm{st}}^2}{3}\left(\frac{\ell_s}{L}\right)^6\right)$$

corrections to the planar/strong coupling dictionary [Bergman-Hirano, Aharony et al.]

(Planar) free energy at weak and strong coupling



$$\log\left(\det \mathcal{D}_{\mathbb{S}^3}\right)$$
$$= N^2 \left\{ \log(2\pi\lambda) - \frac{3}{2} - 2\log 2 \right\}$$

one-loop calculation

$$-I_{\mathrm{AdS}_4} = -N^2 \frac{\pi\sqrt{2}}{3\sqrt{\lambda}}$$
$$\sim N^{3/2} k^{1/2}$$

gravitational action

There should be a non-trivial function of the 't Hooft coupling interpolating between these two results!

Localization and matrix models

In supersymmetric theories one can often reduce the path integral to an integral over supersymmetric configurations/ vacua. On spherical spacetimes all the modes are massive and the vacua simplify dramatically: the path integral reduces to a *matrix model!*

Example: in N=4 SYM the path integral calculating the vev of the 1/2 BPS Wilson loop reduces to a *Gaussian, Hermitian matrix model* [Ericksson-Semenoff-Zarembo, Drukker-Gross, Pestun]

Reduction to a matrix model in ABJM

Localization techniques were applied to the ABJM theory in a beautiful paper by [Kapustin-Willett-Yaakov]. The partition function on $\mathbb{S}^3$ is given by the following matrix integral:

contribution CS gauge fields

$$Z_{\mathrm{ABJM}}(N_1, N_2, g_{\mathrm{top}})$$
$$= \frac{1}{N_1! N_2!} \int \prod_{i=1}^{N_1} \frac{\mathrm{d}\mu_i}{2\pi} \prod_{j=1}^{N_2} \frac{\mathrm{d}\nu_j}{2\pi} \frac{\prod_{i<j} \left(2 \sinh\left(\frac{\mu_i - \mu_j}{2}\right)\right)^2 \left(2 \sinh\left(\frac{\nu_i - \nu_j}{2}\right)\right)^2}{\prod_{i,j} \left(2 \cosh\left(\frac{\mu_i - \nu_j}{2}\right)\right)^2} \mathrm{e}^{-\frac{1}{2 g_{\mathrm{top}}}\left(\sum_i \mu_i^2 - \sum_j \nu_j^2\right)}$$

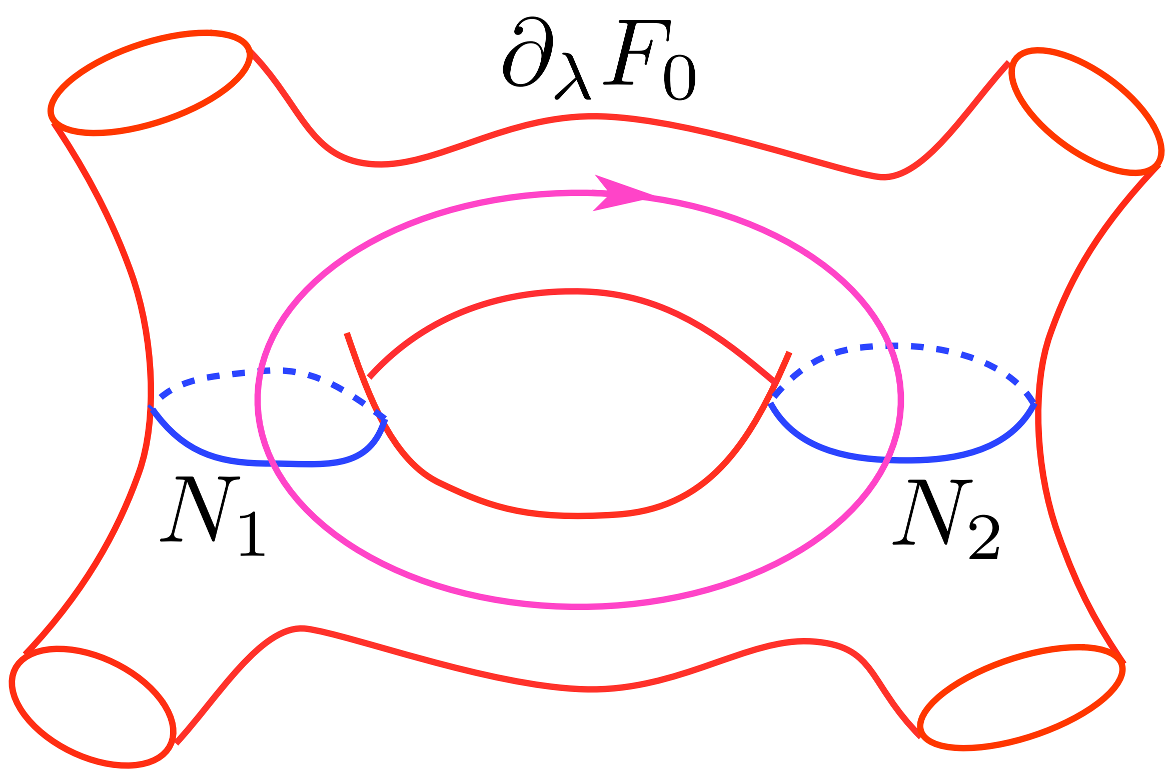
contribution 4 hypers

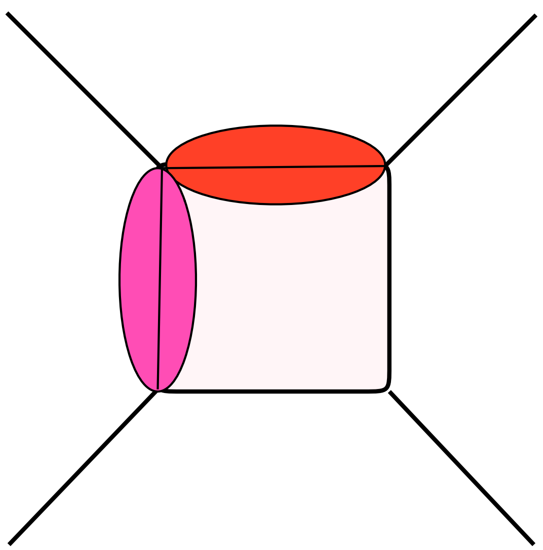
$$g_{\mathrm{top}} = \frac{2\pi \mathrm{i}}{k}$$

$$N_1 = N_2 = N \qquad F = \log Z_{\mathrm{ABJM}} = \sum_{g \geq 0} F_g(\lambda) g_{\mathrm{top}}^{2g-2}$$

Planar solution

The planar solution of the matrix model is encoded in a *spectral curve* and meromorphic differential, as in special geometry. It is closely related to CS matrix models [M.M.], local mirror symmetry, and large *N* dualities for topological strings [Gopakumar-Vafa, AKMV]

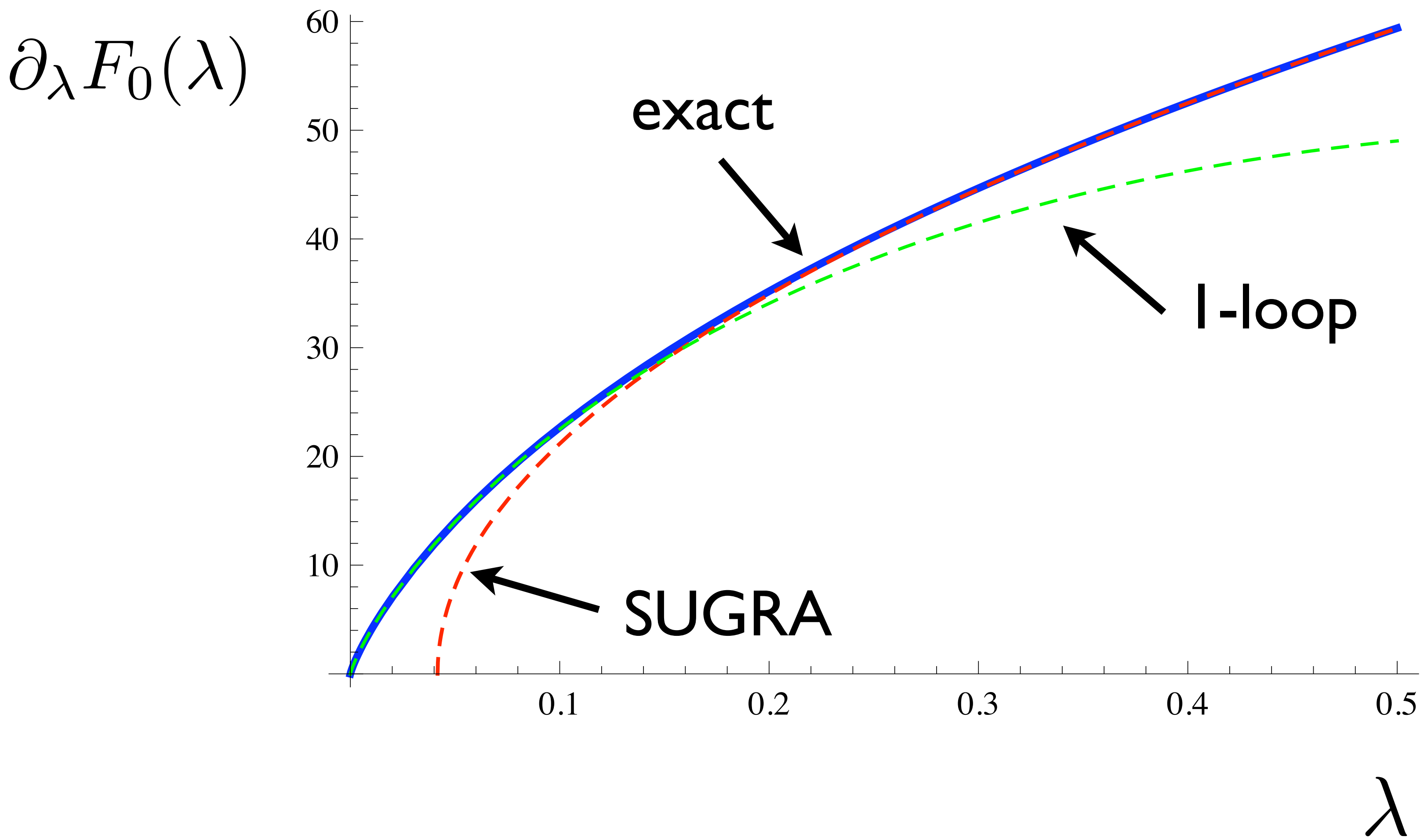




The solution to the matrix model involves an auxiliary real variable κ. The planar free energy is given by

$$\lambda(\kappa) = \frac{\kappa}{8\pi}\ {}_3F_2\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}; 1, \frac{3}{2}; -\frac{\kappa^2}{16}\right)$$

$$\partial_\lambda F_0(\lambda) = \frac{\kappa}{4} G^{2,3}_{3,3}\left(\begin{array}{ccc} \frac{1}{2}, & \frac{1}{2}, & \frac{1}{2} \\ 0, & 0, & -\frac{1}{2} \end{array} \middle| -\frac{\kappa^2}{16}\right) + 4\pi^3 \mathrm{i}\lambda(\kappa)$$



Nonperturbative effects in α'

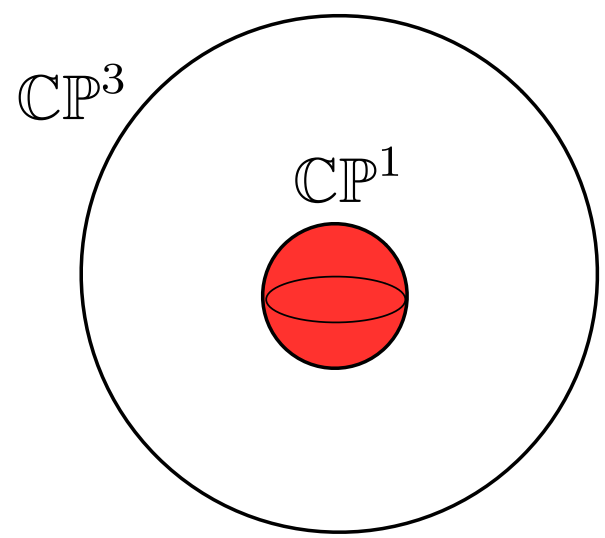
What is the nature of the corrections at strong coupling?

$$F_0(\hat{\lambda}) = \frac{4\pi^3\sqrt{2}}{3}\hat{\lambda}^{3/2} + \sum_{\ell \geq 1} \mathrm{e}^{-2\pi\ell\sqrt{2\hat{\lambda}}} f_\ell\left(\frac{1}{\pi\sqrt{2\hat{\lambda}}}\right)$$

SUGRA limit

worldsheet instantons

[cf. Cagnazzo-Sorokin-Wulff]



$$f_\ell(x) = \sum_{n=0}^{2\ell-3} f_n^{(\ell)} x^n$$

Like in *mirror symmetry*, special geometry leads to a calculable, infinite series of worldsheet instanton corrections!

Beyond the planar approximation

One can also compute in a systematic way the $1/N$ expansion of the free energy.

$1/N$ corrections in matrix models $\longrightarrow$ BCOV holomorphic anomaly equations $\longrightarrow$ solve via *direct integration* [Klemm+Haghighat, Huang, M.M., Rauch,...]

This leads to an *integrable recursion relation* for the genus g free energies, similar to what happens in matrix models for non-critical strings. The topological recursion plays here the role of the Painleve-type nonlinear ODEs.

$F_g(\lambda) =$ *quasi-modular forms* in the modular parameter of the (elliptic) spectral curve

M-theory free energy

From this genus expansion one obtains the *large radius expansion* of the M-theory free energy on $\mathrm{AdS}_4 \times \mathbb{S}^7/\mathbb{Z}_k$, for *any* k (no membrane instantons included)

use "renormalized" Planck length: $$\frac{\widehat{\ell}_p}{L} = \frac{\ell_p/L}{\left[1 - 12\pi^2 \left(\ell_p/L\right)^6\right]^{1/6}}$$

$$F = -\frac{1}{384\pi^2 k}\left(\frac{L}{\widehat{\ell}_p}\right)^9 + \frac{1}{6}\log\left[8\pi^3 k^3 \left(\frac{\widehat{\ell}_p}{L}\right)^9\right] + \sum_{n=1}^{\infty} d_{n+1}\pi^{2n}k^n \left(\frac{\widehat{\ell}_p}{L}\right)^{9n}$$

resummed to a *perturbatively exact M-theory partition function!*

[Fuji-Hirano-Moriyama]

$$Z_{\mathrm{ABJM}} \sim k^{1/3}\mathrm{Ai}(x) \qquad x \propto k^{-\frac{2}{3}}\left(\frac{L}{\widehat{\ell}_p}\right)^6$$

Exact Hartle-Hawking wavefunction? [Maldacena, Ooguri-Vafa-Verlinde, ...]

The nature of the genus expansion

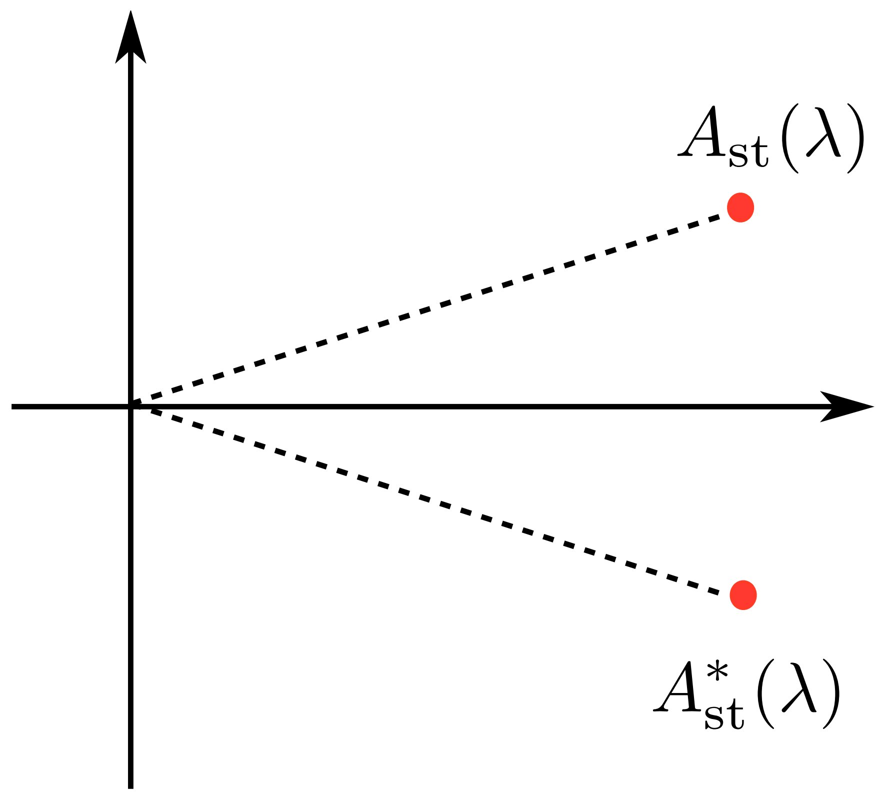
We can now study the large order behavior of the genus expansion to determine its nature and extract *spacetime instanton effects*

$$F_g(\lambda) \sim (2g)!\,(A_{\mathrm{st}}(\lambda))^{-2g}$$ [Shenker]

$$A_{\mathrm{st}}(\lambda) \propto \frac{1}{\pi}\partial_\lambda F_0(\lambda) + \pi^2 \mathrm{i}$$ instanton action

$$\longrightarrow \quad \mathcal{O}\left(\mathrm{e}^{-A_{\mathrm{st}}(\lambda)/g_{\mathrm{st}}}\right)$$ corrections

Since the instanton action is *complex*, the free energies at large genus have an oscillatory behavior. The superstring genus expansion for the free energy on $\mathrm{AdS}_4 \times \mathbb{CP}^3$ is then *Borel summable*!

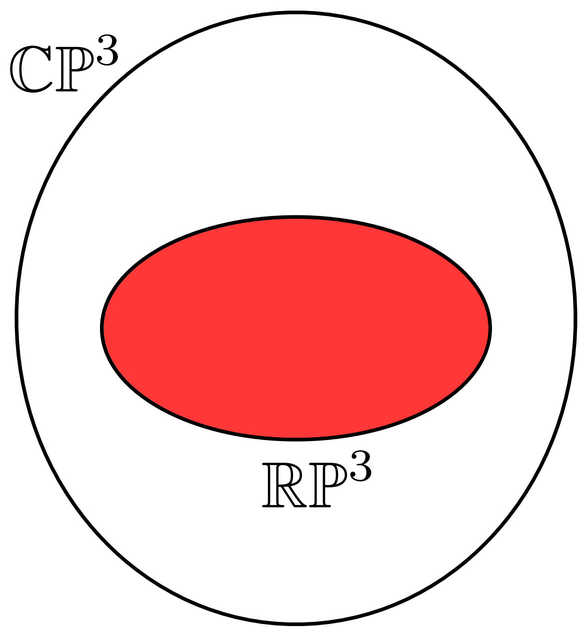


This is typical of stable quantum systems and has been observed before in non-critical, non-unitary string theories (like the Lee-Yang model coupled to 2d gravity)

Can we identify this spacetime instanton in the large *N* AdS dual?
It should come from a D-brane [Polchinski]

At strong coupling and in string units we find:

$$A_{\rm st} \approx \frac{1}{4}\left(\frac{L}{\ell_s}\right)^3\left(1+2\pi \mathrm{i}\frac{\ell_s^2}{L^2}\right)$$



The leading, real part is the action of an *Euclidean D2 brane wrapping* an $\mathbb{RP}^3$ inside $\mathbb{CP}^3$

The imaginary part -which is responsible for the Borel summability- is more puzzling. Notice that it is an α' correction, therefore it is invisible in the SUGRA limit

Conclusions

• SUGRA limit: new precision tests of AdS/CFT, which can be extended to many other 3d Chern-Simons-matter theories

[Couso-M.M.-Putrov, Klebanov et al., Martelli-Sparks, and many others]

•Matrix model predictions for *worldsheet instanton effects*, which should be decoded on the string side

• Matrix model predictions for *quantum gravity effects*: exact partition function in M-theory, also to be decoded. Relation to Hartle-Hawking wavefunction?

•Sheds light on the nature of the genus expansion (Borel summability!) and on *spacetime instanton effects*