Strings 2011, Uppsala July 1st, 2011

The Hidden Phase Space of Our Universe

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What is string theory?

We like to believe that:

- String theory is based on general principles.
- String theory applies to the real world.

Or better yet:

- String theory follows from general principles, that apply to the real world.

What is gravity?

Gravity is a macroscopic force that dominates in IR.

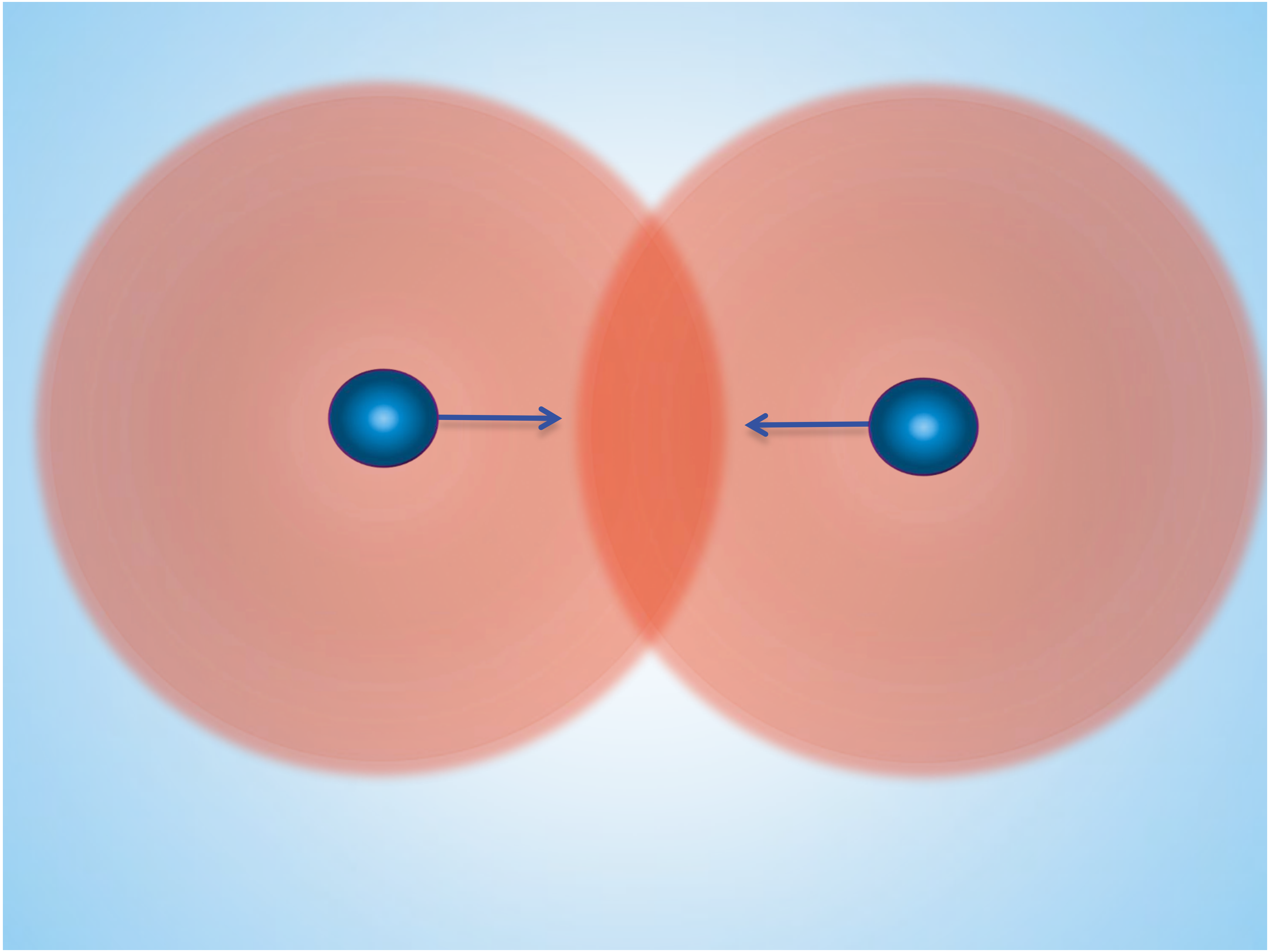
Yet it knows about microscopic entropy

(= phase space, quantum states, or information)

associated with the UV degrees of freedom.

But why? Is it a UV/IR conspiracy?

Or is it due to a principle?



Outline:

- Motivation: BH’s + String-/matrix theory.
- Adiabatic reaction forces.
- Hidden phase space: proposal.
- Inertia/gravity as adiabatic reaction force.
- “Speculations” about dark energy/dark matter.

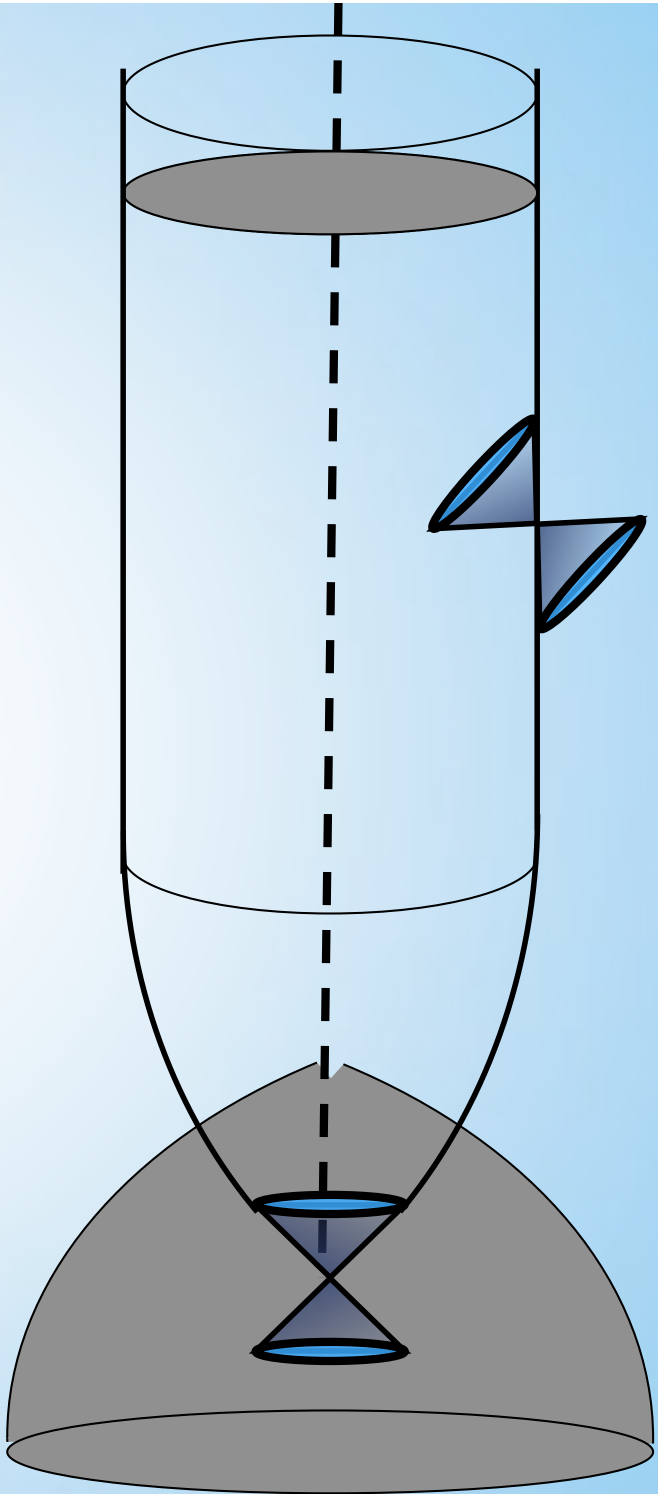
GRAVITATIONAL COLLAPSE:

What happens to the phase space occupied by the fermions?

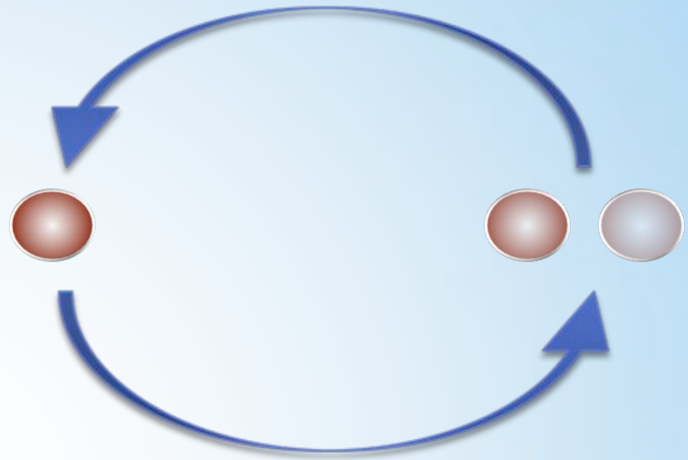
String theory/ AdS-CFT:

$$\Psi(x) \leftrightarrow \mathrm{tr}(X^n \lambda)$$

Bulk fields correspond to (short) traces of boundary fields.



FD/BE statistics => "D-brane" statistics



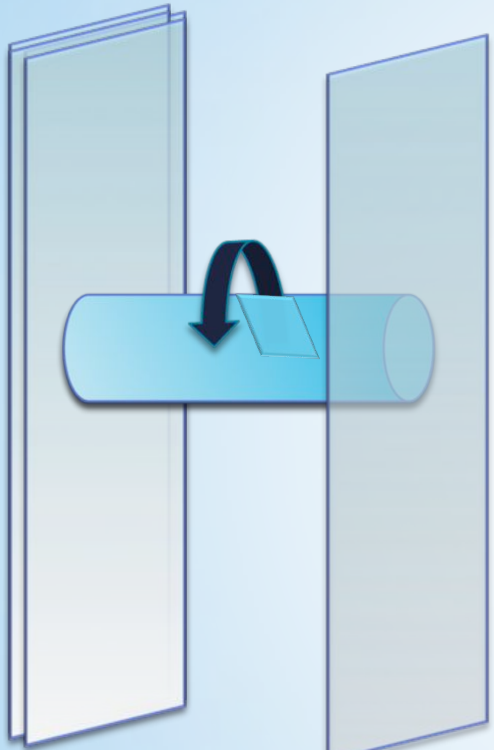
$$Z_2 \times U(1)^2 \rightarrow U(2)$$

$$x_1, x_2 \Rightarrow \begin{pmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{pmatrix}$$

$$S_N \times U(1)^N \rightarrow U(N)$$

$$x_{ij} = \langle i | \hat{x} | j \rangle$$

Gravity results from integrating out “off diagonal” degrees of freedom.



Coordinates as matrices

$$X = \begin{pmatrix} x_{11} & x_{12} & \cdots & x_{1N} \\ x_{21} & x_{22} & \vdots & \vdots \\ \vdots & \vdots & \vdots & x_{N-1N} \\ x_{N1} & \cdots & x_{NN-1} & x_{NN} \end{pmatrix}$$

$$\mathrm{H} = \mathrm{tr}\left(P_I^2 + [X_I, X_J]^2 + \lambda^* X_I \Gamma^I \lambda \right)$$

Eigenvalues: describe position of matter

Space, matter and forces emerge together

GRAVITATIONAL COLLAPSE:

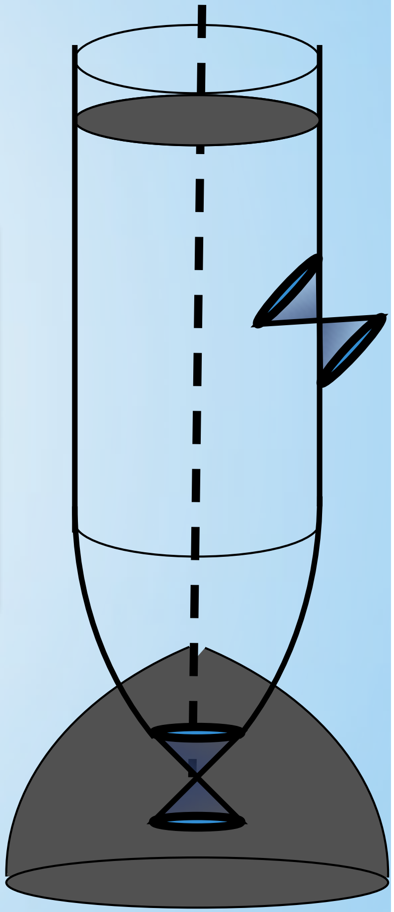
The phase space occupied by the fermions goes into the "off diagonal" modes.

Eigenvalues and off diagonal modes equilibrate and together form a thermal state describing the black hole.

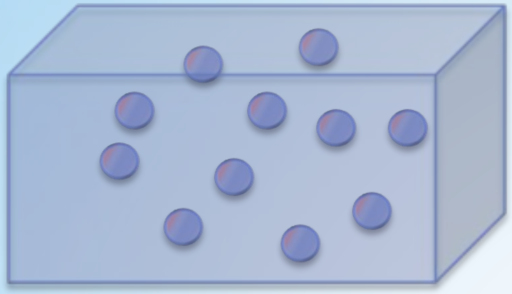
Space-time in its usual sense ceases to exist.

$$X = \begin{pmatrix} x_{11} & x_{12} & .. & x_{1N} \\ x_{21} & x_{22} & :: & : \\ : & :: & :: & x_{N-1N} \\ x_{N1} & .. & x_{NN-1} & x_{NN} \end{pmatrix}$$

$$X = \begin{pmatrix} x_1 & & & \\ & x_2 & & \\ & & :: & \\ & & & x_N \end{pmatrix}$$



Degenerate Fermions



$$X = \begin{pmatrix} x_{11} & x_{12} & \cdot\cdot & x_{1N} \\ x_{21} & x_{22} & \vdots & \vdots \\ \vdots & \vdots & \vdots & x_{N-1N} \\ x_{N1} & \cdot\cdot & x_{NN-1} & x_{NN} \end{pmatrix}$$

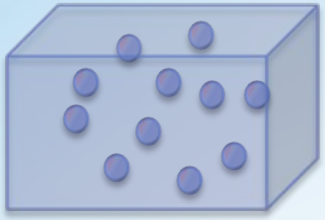
The off diagonal modes carry a **<u>positive</u>** gravitational energy

$$\boxed{E_g = \frac{1}{8\pi G}\int \left|\nabla\Phi\right|^2}$$

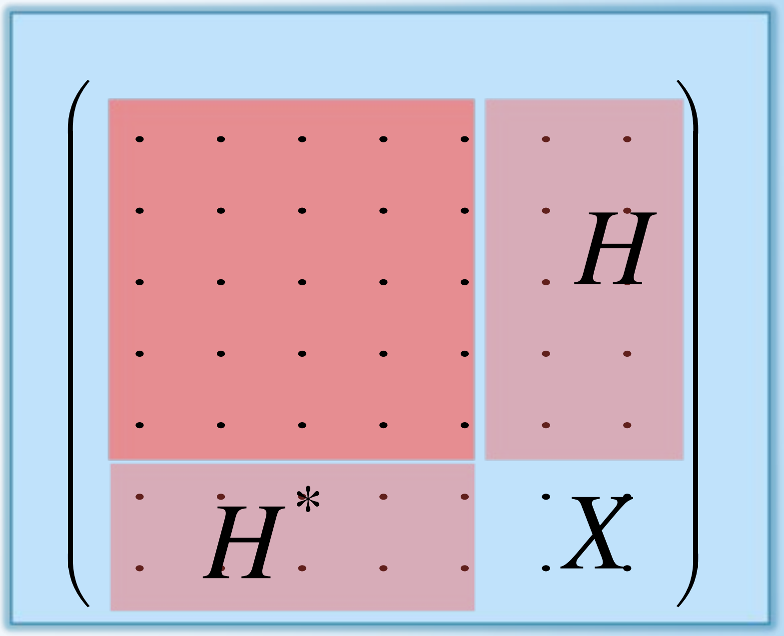
$$E_g = \frac{1}{8\pi G}\int \left|\nabla\Phi\right|^2$$

$$M_1 - \Phi_2 M_1 \qquad\qquad M_2 - \Phi_1 M_2$$

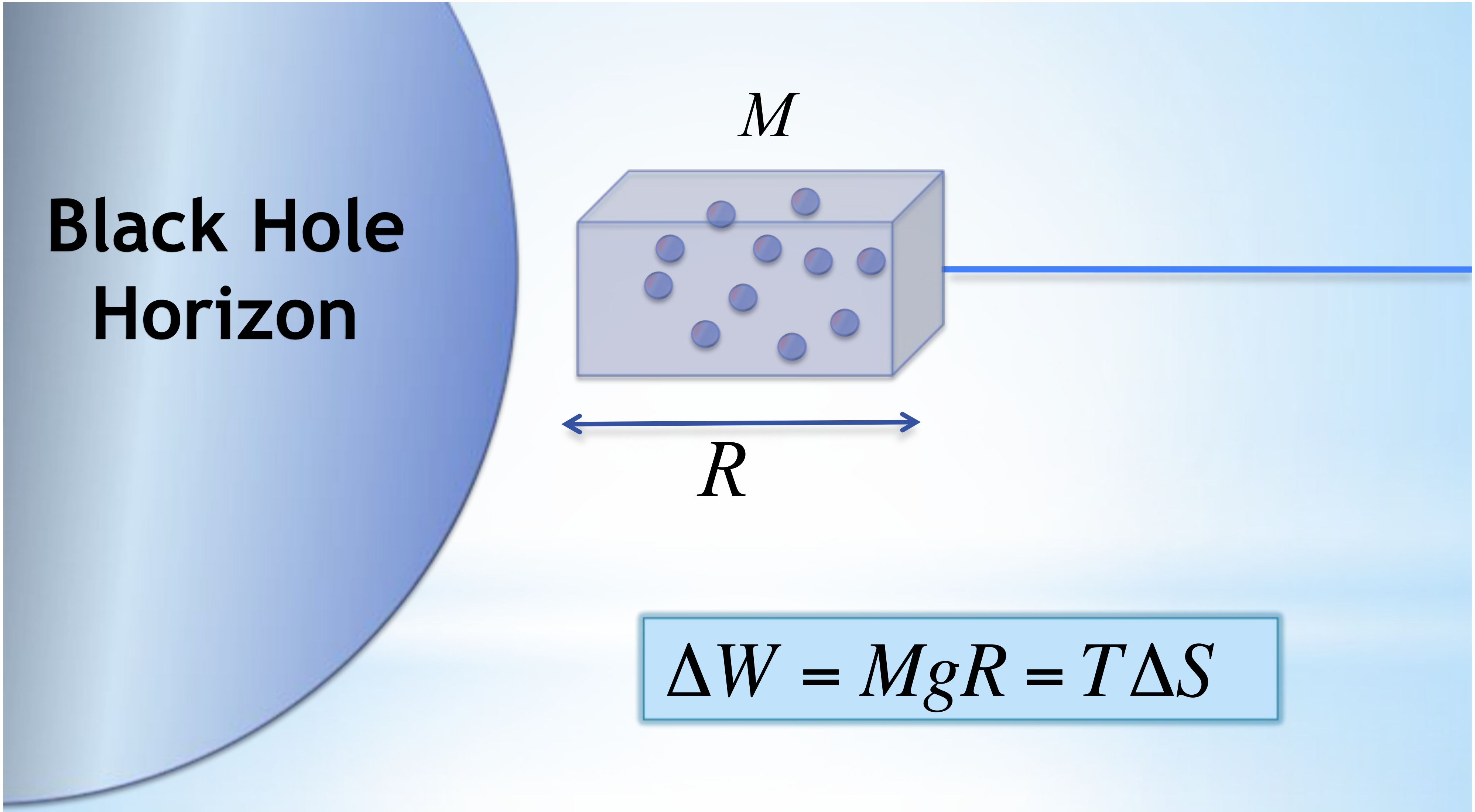
Black Hole



$$| X \cdot H |^2$$



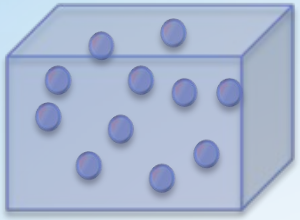
Matrices gets "Higgsed"
Eigenvalues disappear as flat directions
Coulomb branch goes over into Higgs branch



$$\Delta W = MgR = T\Delta S$$

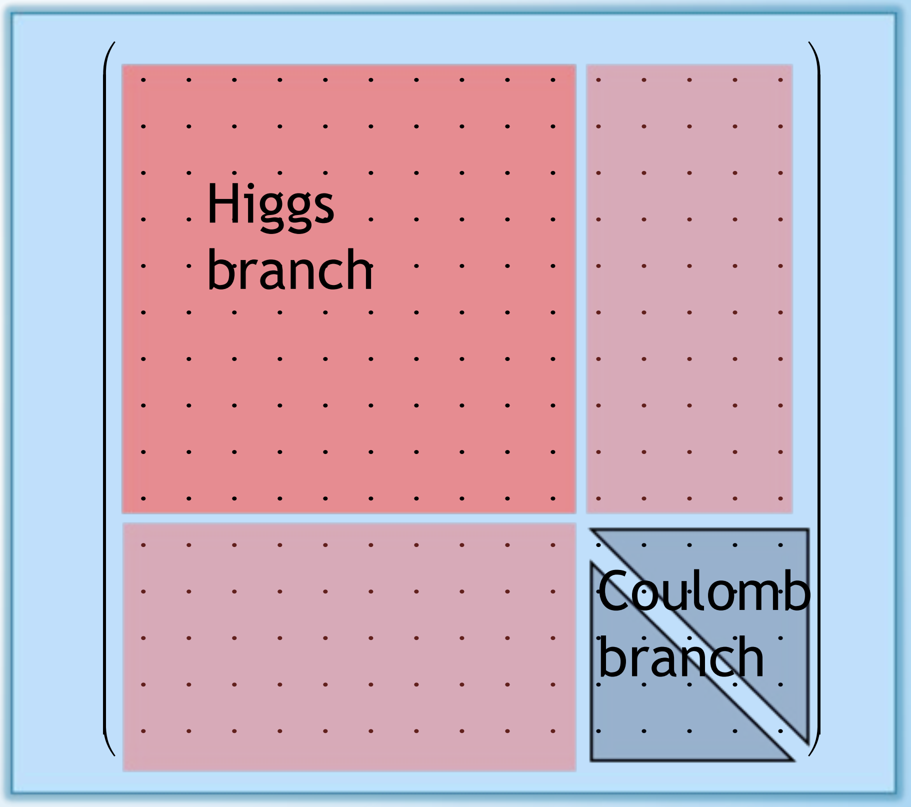
$$T = \frac{g}{2\pi} \quad \Rightarrow \quad \Delta S = 2\pi MR$$

Black Hole

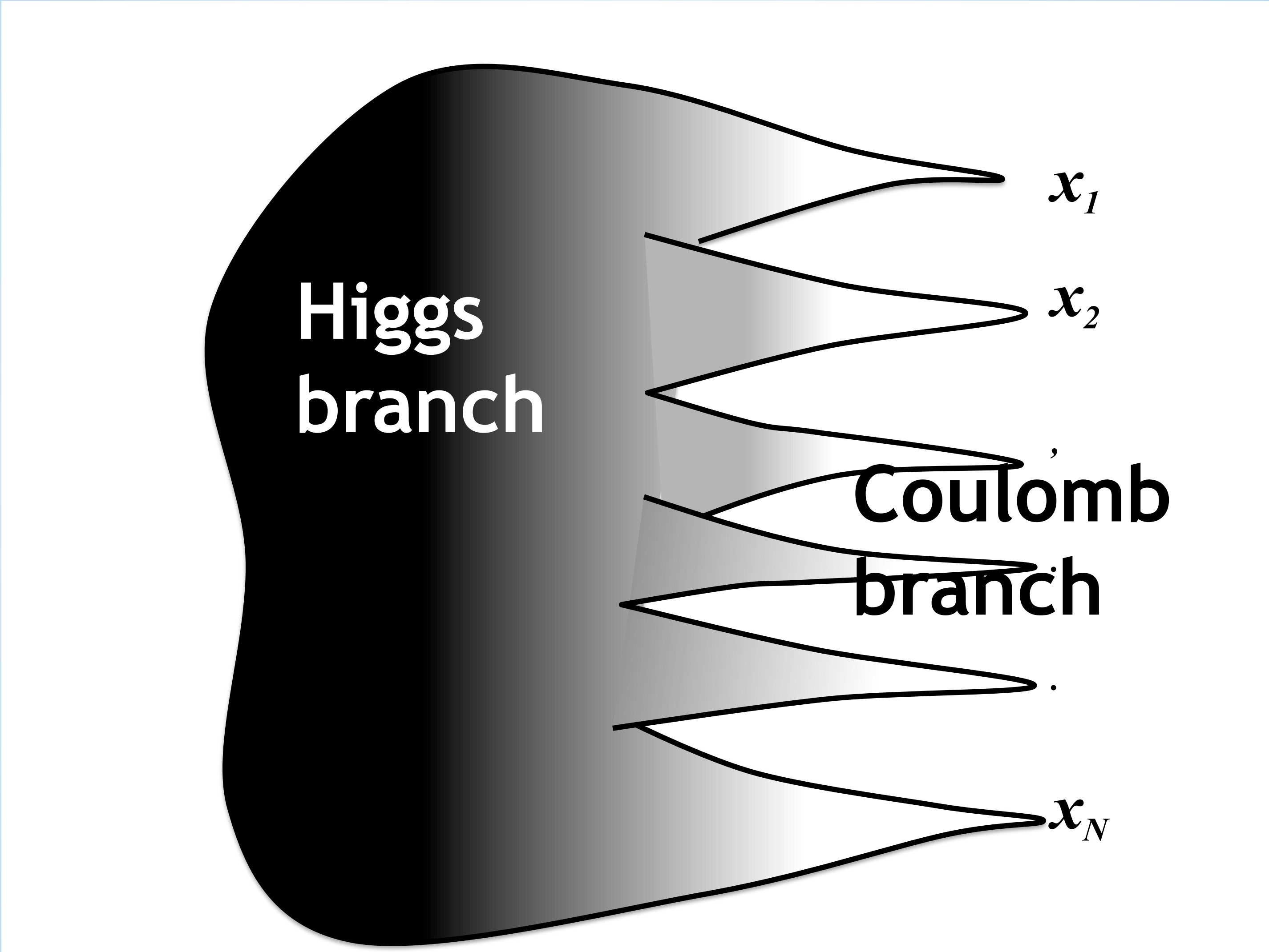


$$N = \frac{McR}{\hbar}$$

Number of "eigenvalues" associated with a region with size *R* and containing mass *M*



PHASE SPACE



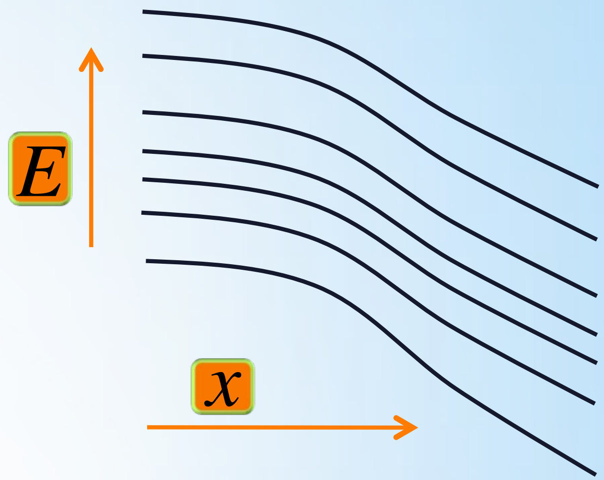
Adiabatic reaction forces:

When a fast dynamical system is driven by a slow system the fast reacts back on the slow and creates a reaction force.

When the time scales are widely separated the force is determined by the principle that the phase space volume of the fast system is preserved.

In quantum mechanics this is a consequence of the Born-Oppenheimer approximation.

Adiabatic Reaction Force



Harmonic oscillator with slowly varying frequency

$$H(p,q;x) = \tfrac{1}{2}\left(p^2 + \omega^2(x)q^2\right)$$

Bohr-Sommerfeld action integral = adiab

$$J \equiv \frac{1}{2\pi}\oint p\,dq$$

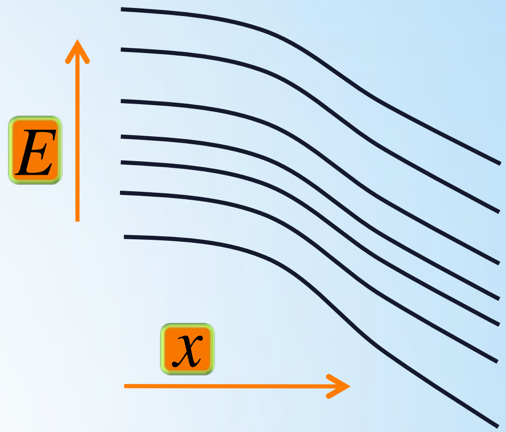
$$dE = \omega\; dJ - F dx$$

Adiabatic reaction force:

$$F = -\left(\frac{\partial E}{\partial x}\right)_J = \omega\left(\frac{\partial J}{\partial x}\right)_E$$

$$\frac{1}{\omega} = \left(\frac{\partial J}{\partial E}\right)_x$$

Adiabatic Reaction Force



For a (chaotic/ergodic) system with many DOF

$$\Omega(E,x) = \int d^N p \, d^N q \, \Big|_{H(p,q;x) \leq E}$$

the phase space volume is an adiabatic invariant. Formally it defines an entropy

$$S = k_B \log \Omega$$

Born Oppenheimer force:

$$dE = T\,dS - F\,dx$$

$$F = -\left(\frac{\partial E}{\partial x}\right)_S = T\left(\frac{\partial S}{\partial x}\right)_E$$

$$\frac{1}{T} = \left(\frac{\partial S}{\partial E}\right)_x$$

Berry Phase and Crossing Eigenvalues

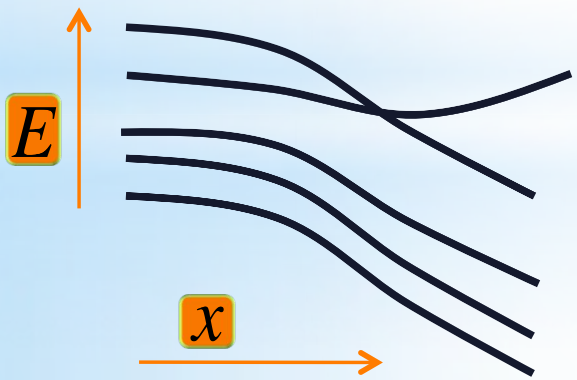
$$F = F^{(0)} + \varepsilon F^{(1)} + ..$$

Systematic expansion of the reaction force in terms of a “slowness” parameter.

$$F^{(0)} = -\partial_x \langle \psi | H | \psi \rangle$$

$$F^{(1)} = \dot{x} \wedge d \langle \psi | d | \psi \rangle$$

magnetic force due to **Berry phase**.



$$H = \begin{pmatrix} z & x + iy \\ x - iy & -z \end{pmatrix} = \vec{x} \cdot \vec{\sigma}$$

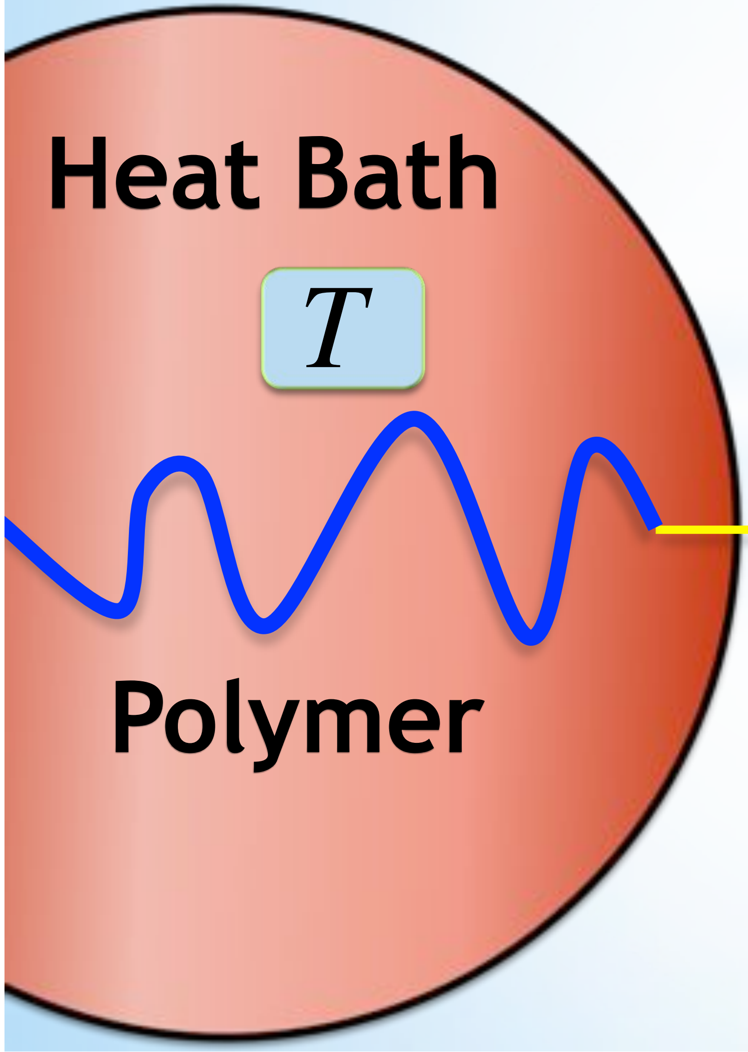
$$\vec{B} = \frac{\hat{x}}{4\pi |\vec{x}|^2}$$

Dirac monopool

At the locus of coinciding eigenvalues exists a **non-abelian Berry connection**.

$$A_{ij} = \langle \psi_i | d | \psi_j \rangle$$

$$Z(T,F) = \int dE dx\, \Omega(E,x)\, e^{-(E+Fx)/kT}$$



$$F/T = \partial_x S(E,x)$$
$$1/T = \partial_E S(E,x)$$

An **entropic force** does not lead to a change in entropy, despite its name.

It acts adiabatically.

Replace all degrees of freedom by just one

$$\boxed{\int dE\, \Omega(E,x)\; e^{-E/T} = \int [dp\, dq] e^{I[p,q;x]/\hbar}}$$

$$\boxed{I[p,q;x] = \oint \left(p\,dq - H(p,q;x)\,dt \right)}$$

Action/Entropy correspondence

$$\boxed{S = 2\pi \frac{J}{\hbar}} \qquad \boxed{T = \frac{\hbar\omega}{2\pi}}$$

=> Adiabatic reaction forces agree

Conjecture:

Gravity is an adiabatic reaction force

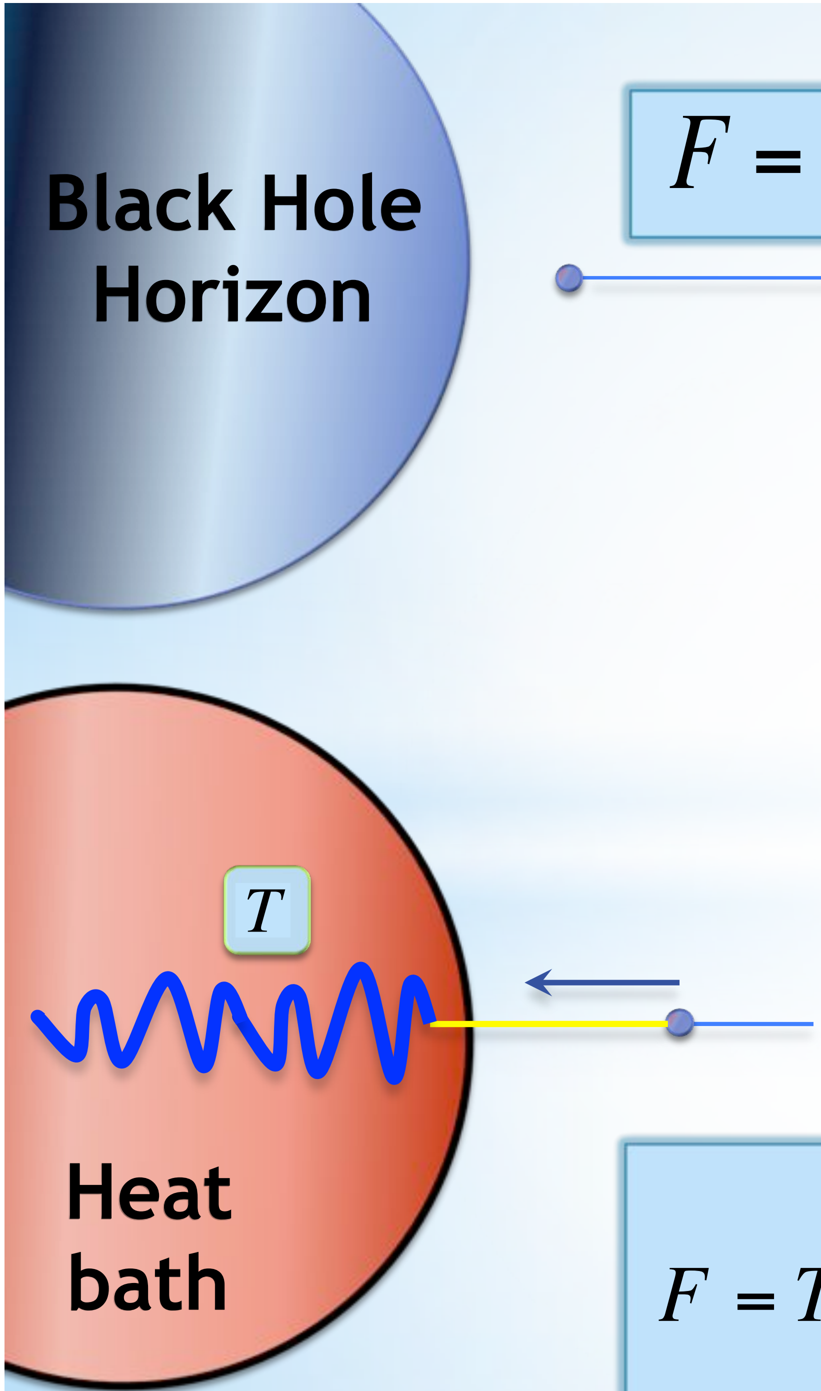
Due to a fast dynamical system underlying our universe.

Inertia (gravity) is the leading order reaction force.

The separation of time scales breaks down at horizons.

The other forces arise from higher order corrections

and are related to the crossing of eigenvalues.



$$F = mg$$

$$X = \begin{pmatrix} x_{11} & .. & x_{1N} & z_1 \\ \vdots & \vdots\vdots & \vdots & \vdots \\ x_{N1} & .. & x_{NN} & z_N \\ z_1^* & .. & z_N^* & xI \end{pmatrix}$$

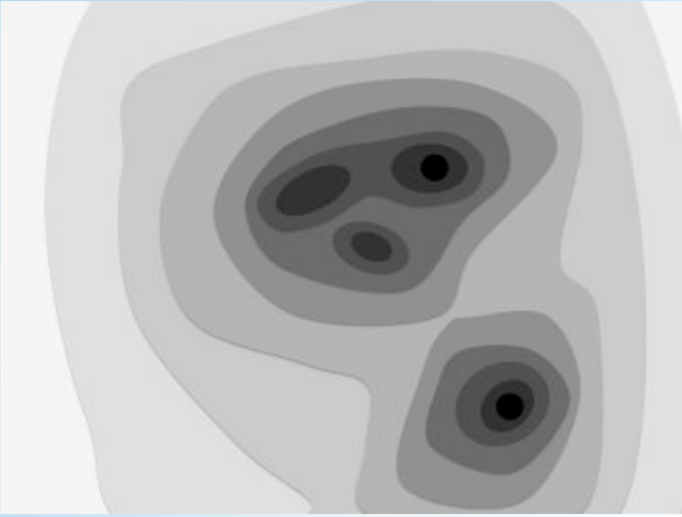
$$F = T\frac{\partial S}{\partial x}$$

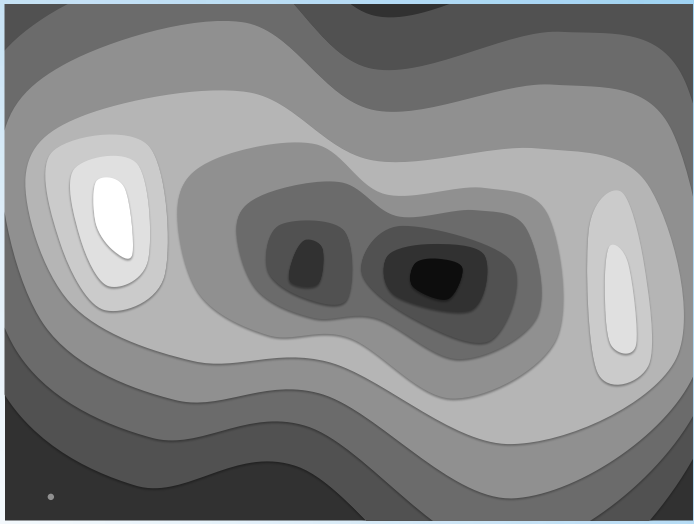
$$T = \frac{\hbar g}{2\pi} \quad \Rightarrow \quad \frac{\partial S}{\partial x} = 2\pi\frac{mc}{\hbar}$$

$$F = m\nabla\Phi$$

Basic idea: far away from horizons gravity/inertia is a Born-Oppenheimer force.

The inertial force follows from the response of the phase space volume of the underlying dynamical system under virtual displacements.





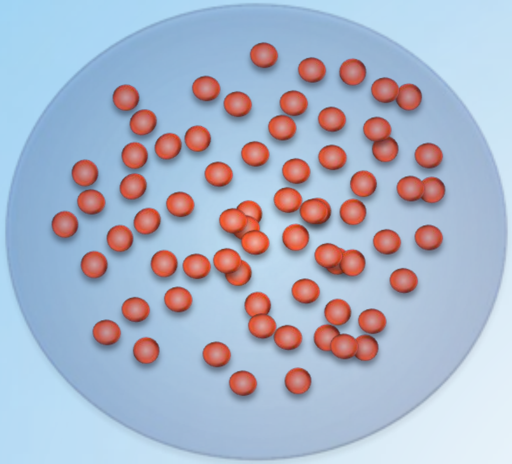
What is the total phase space volume associated with a region of space contained within a certain surface given the amount of energy inside of it?

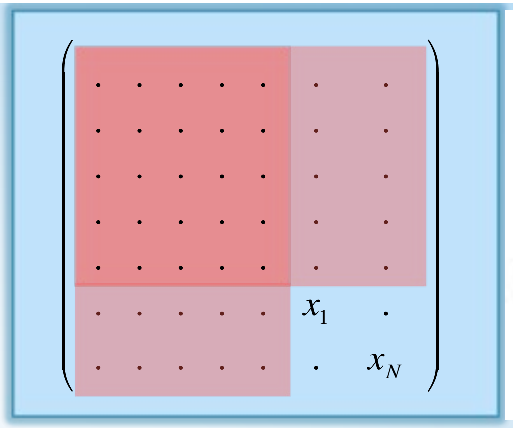
Conjecture

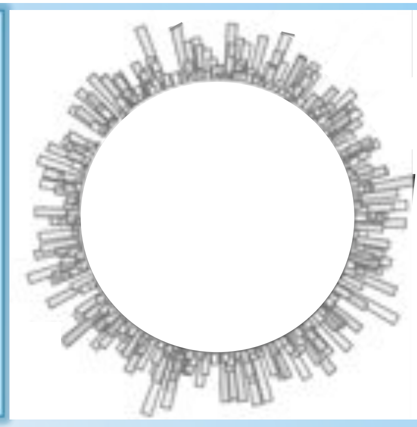
$$S = \frac{A}{4G} v$$

$$\Phi = \tfrac{1}{2} v^2$$

$v =$ escape velocity







Matter disassociates into *N* entangled "quantum energy bits".

$$\boxed{N = MR}$$

To disentangle one inserts a complete set of states on the boundary.

$$\boxed{C = \frac{A}{8\pi G}}$$

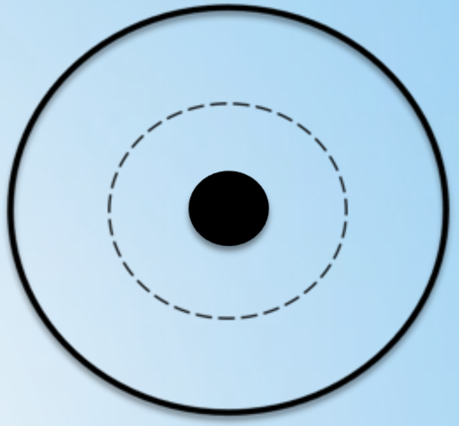
The micrsocopic phase space is obtained by partitions the *N* quanta over these states.

$$\boxed{S = 2\pi\sqrt{C \cdot N}}$$

For a black hole: *N*=*C*

For metrics of Schwartschild / de Sitter type

$$ds^2 = -(1-v^2)dt^2 + \frac{dr^2}{1-v^2} + r^2 d\Omega^2$$



we have

$$F = \gamma m v \frac{dv}{dr}$$

$$T = \frac{1}{2\pi} \frac{dv}{dr}$$

$$v = H_0 r$$

and

$$F = T \frac{dS}{dr}$$

$$\frac{dS}{dr} = 2\pi\gamma m v$$

$$S = \frac{A}{4G} v$$

The phase space volume of the dynamical system that is underlying our universe saturates the holographic FSB bound

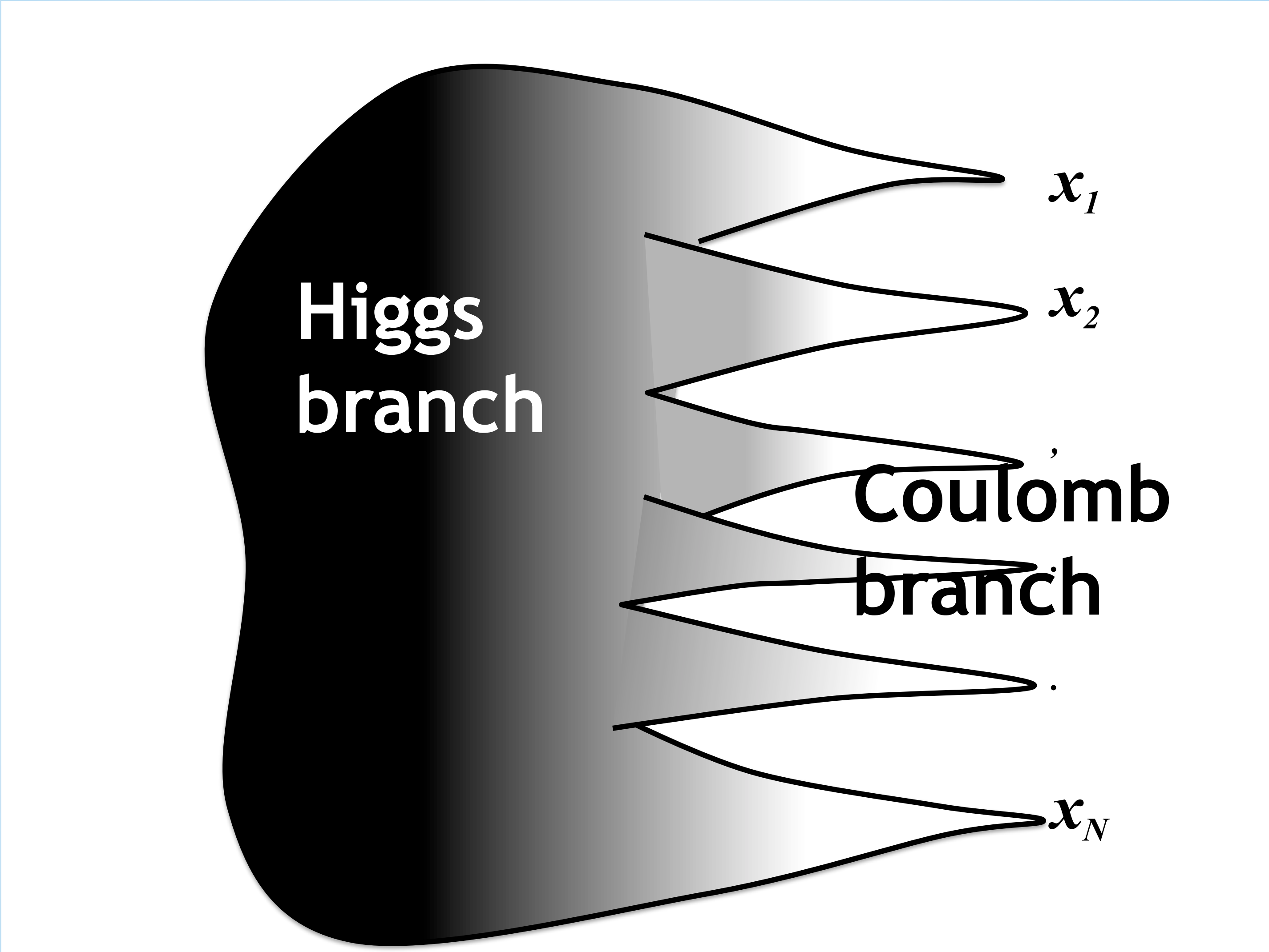
$$S = \frac{H_0 V}{4G(d-1)}$$

$$T = \frac{H_0}{2\pi}$$

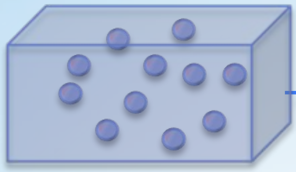
Inertia is a reaction force due to this system. The energy of the system is (dominated by) the (observed) dark energy in our universe

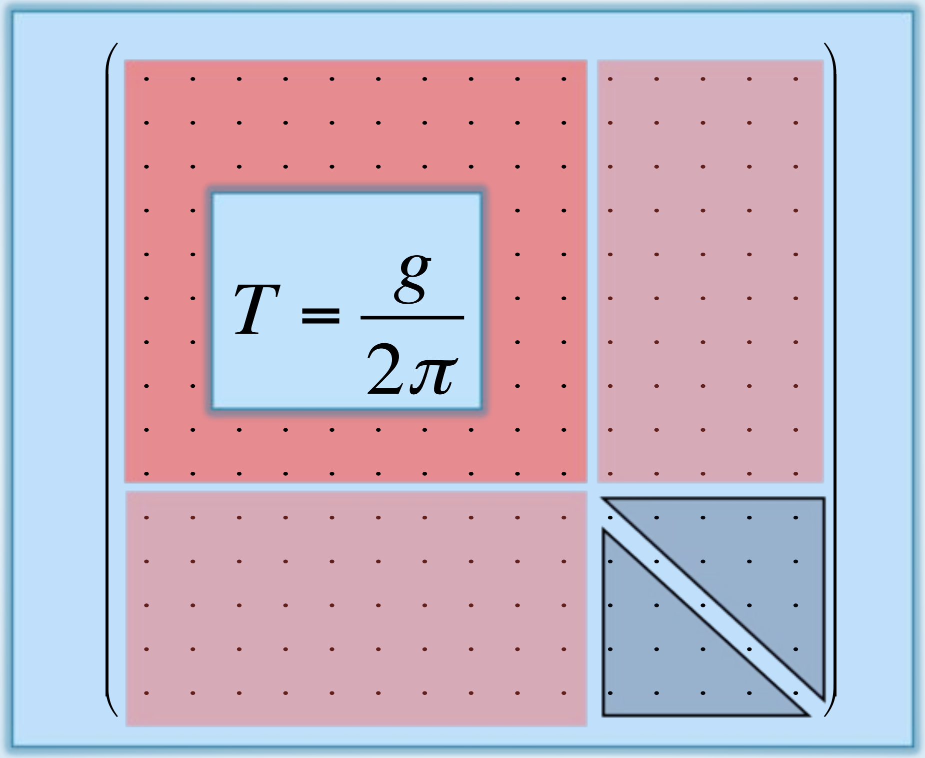
$$H_0^2 = \frac{16\pi G}{n(n+1)} \frac{E}{V}$$

PHASE SPACE



Black Hole

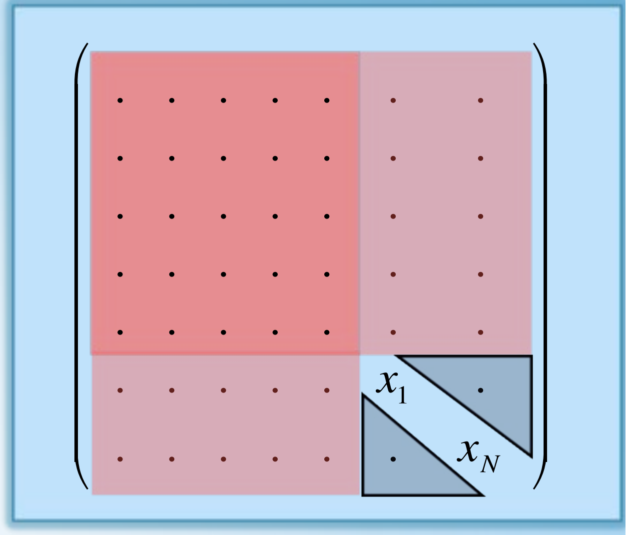




$$N = MR$$

$$E_g = \frac{1}{8\pi G}\int |\nabla\Phi|^2$$

Fluctuations in the inertial field



$$\left\langle |\nabla\Phi|^2 \right\rangle = \int [d\Phi]\, \mathrm{e}^{-\frac{1}{8\pi G k_B T}\int |\nabla\Phi|^2} |\nabla\Phi|^2$$

A short distance cut off

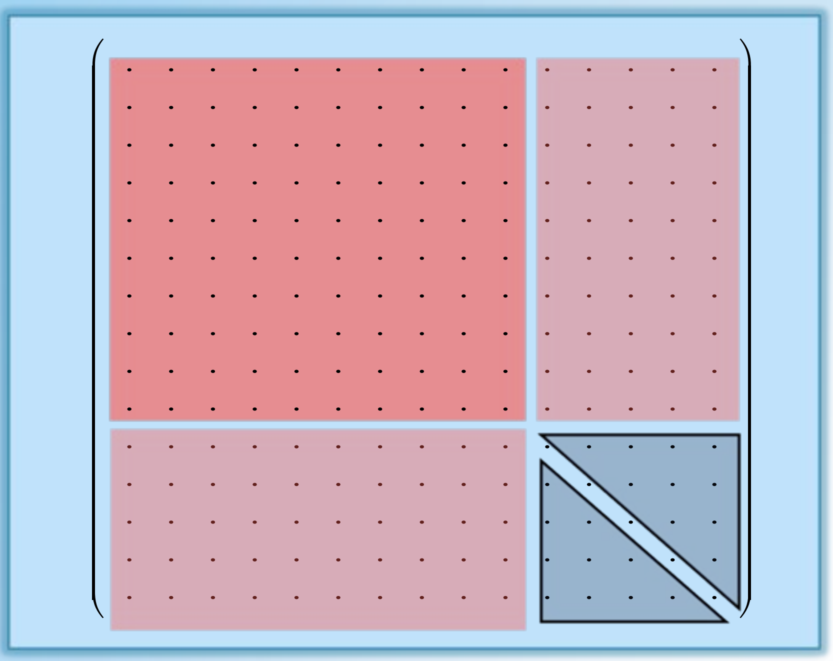
$$\left\langle \nabla\Phi(x)\nabla\Phi(0) \right\rangle = \frac{G k_B T}{x^3}$$

$$x^3 = \frac{R^3}{N}$$

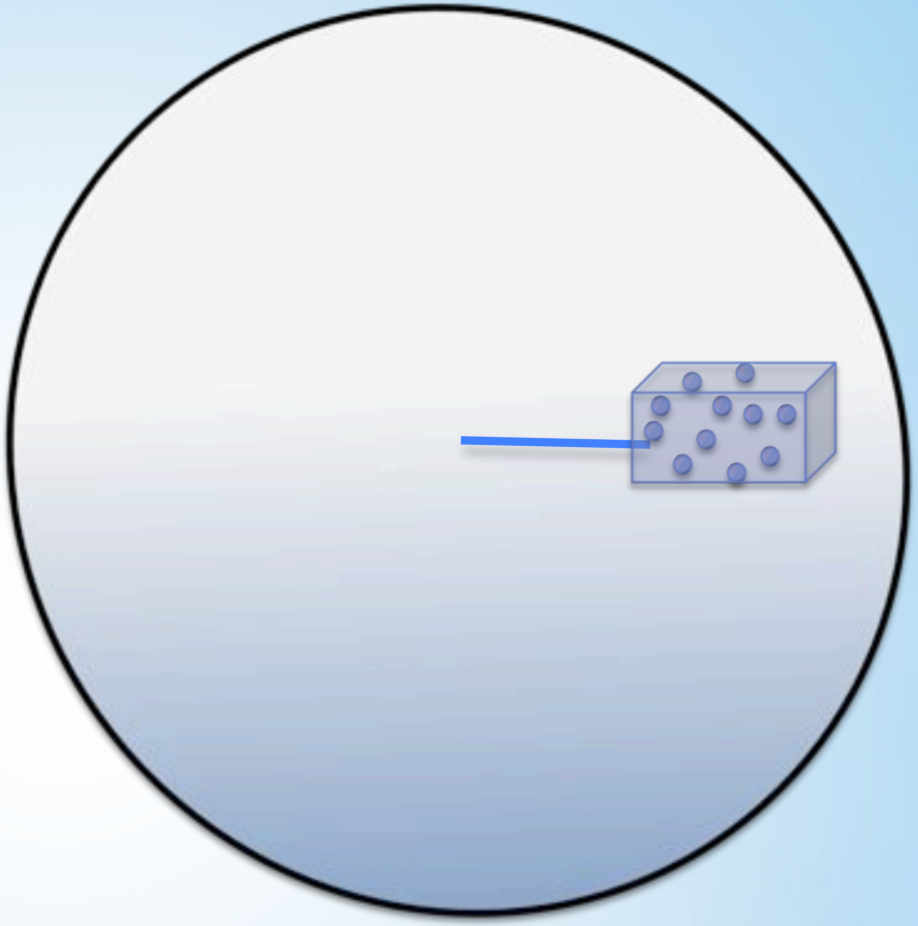
or a mode cut off. Both give

$$E_g = \frac{1}{8\pi G}\int |\nabla\Phi|^2 = N k_B T$$

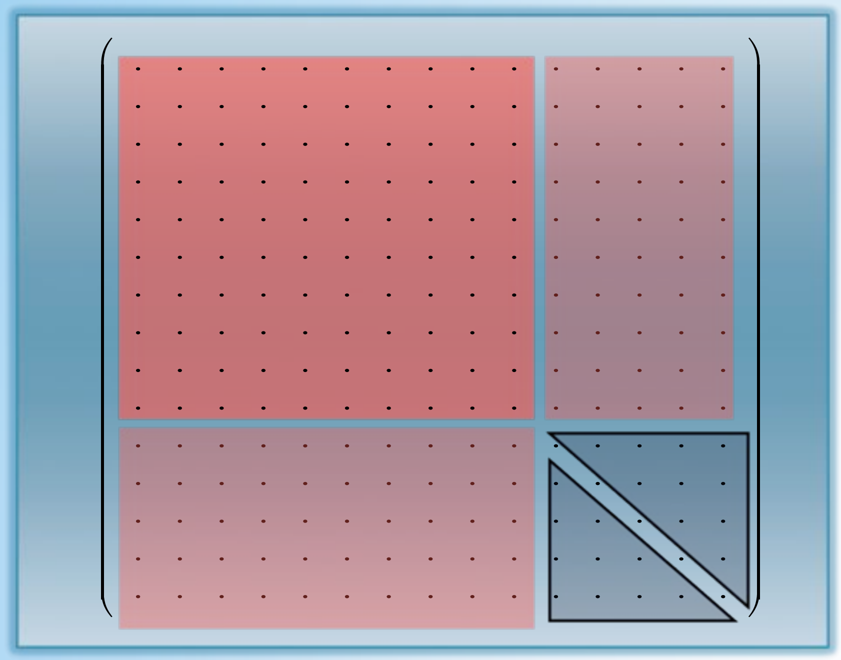
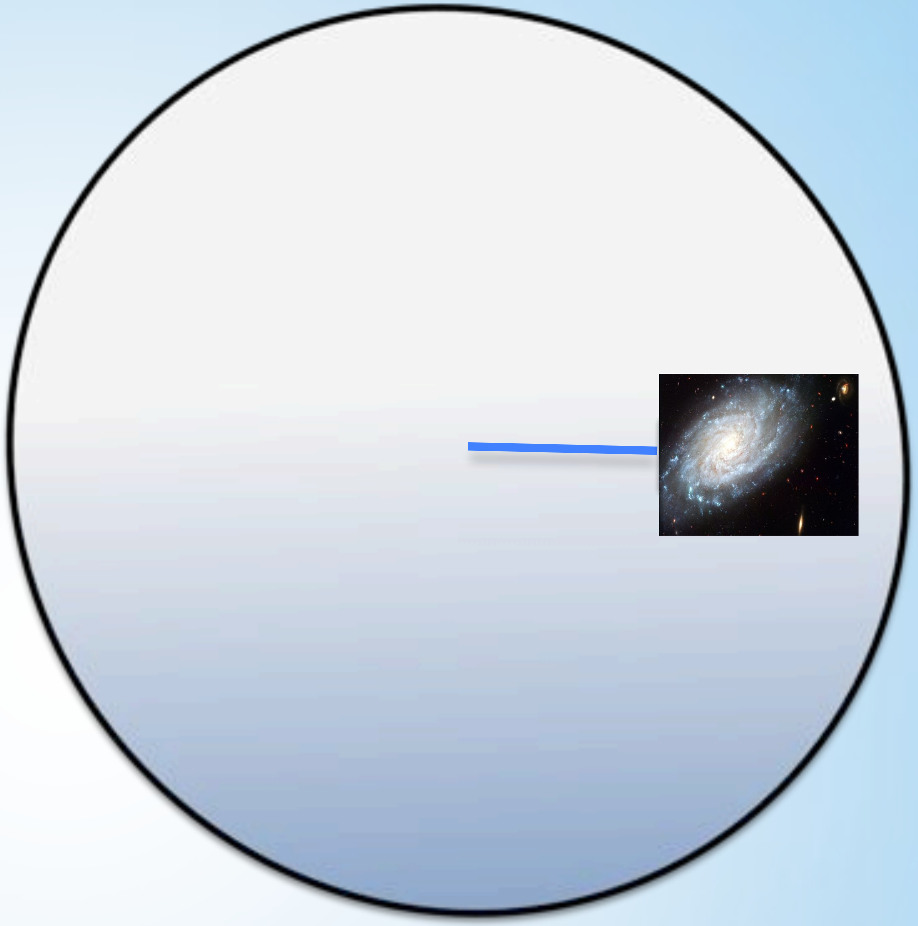
$$|\nabla\Phi|^2 = \frac{G N k_B T}{R^3}$$



$$N = MR$$

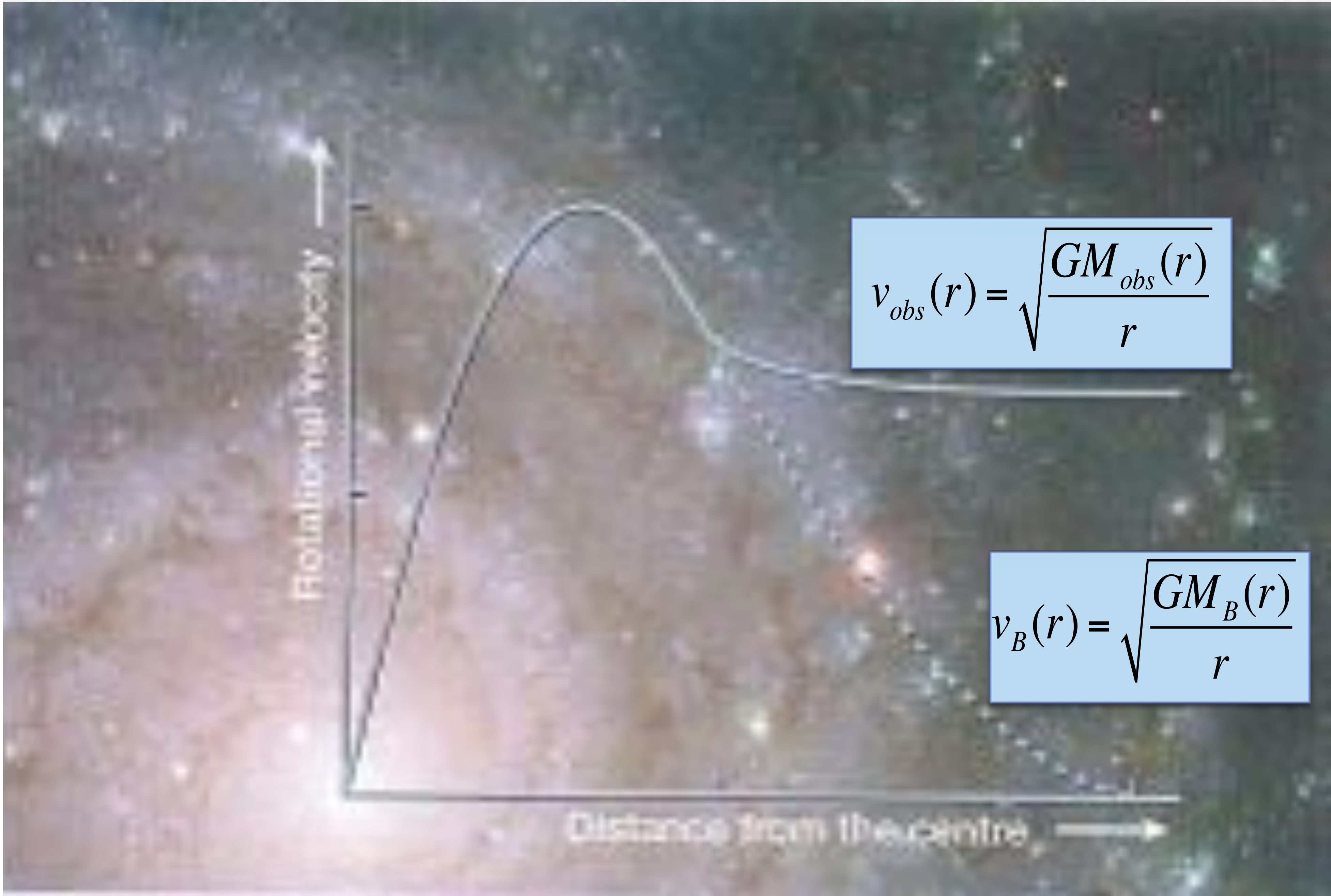


$$T = \frac{H_0}{2\pi}$$

Galaxy rotation curves

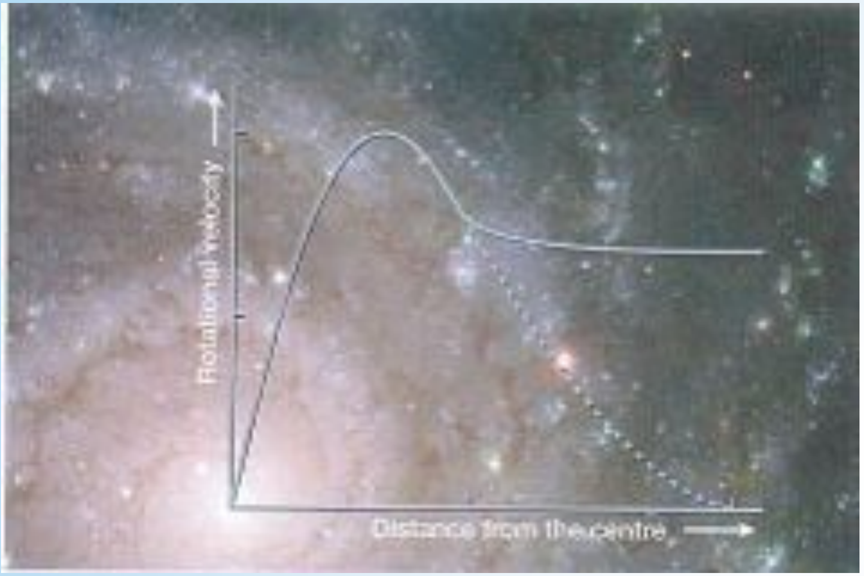
$$\frac{v(r)^2}{r} = \frac{GM(r)}{r^2}$$

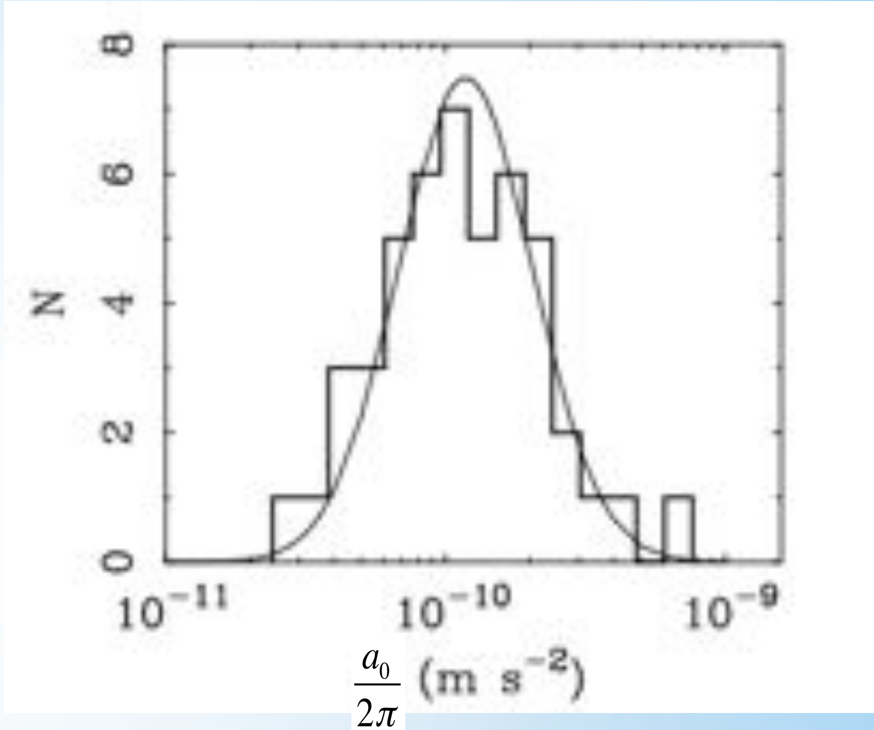


$$v_{obs}(r) = \sqrt{\frac{GM_{obs}(r)}{r}}$$

$$v_B(r) = \sqrt{\frac{GM_B(r)}{r}}$$

Baryonic Tully-Fisher relation



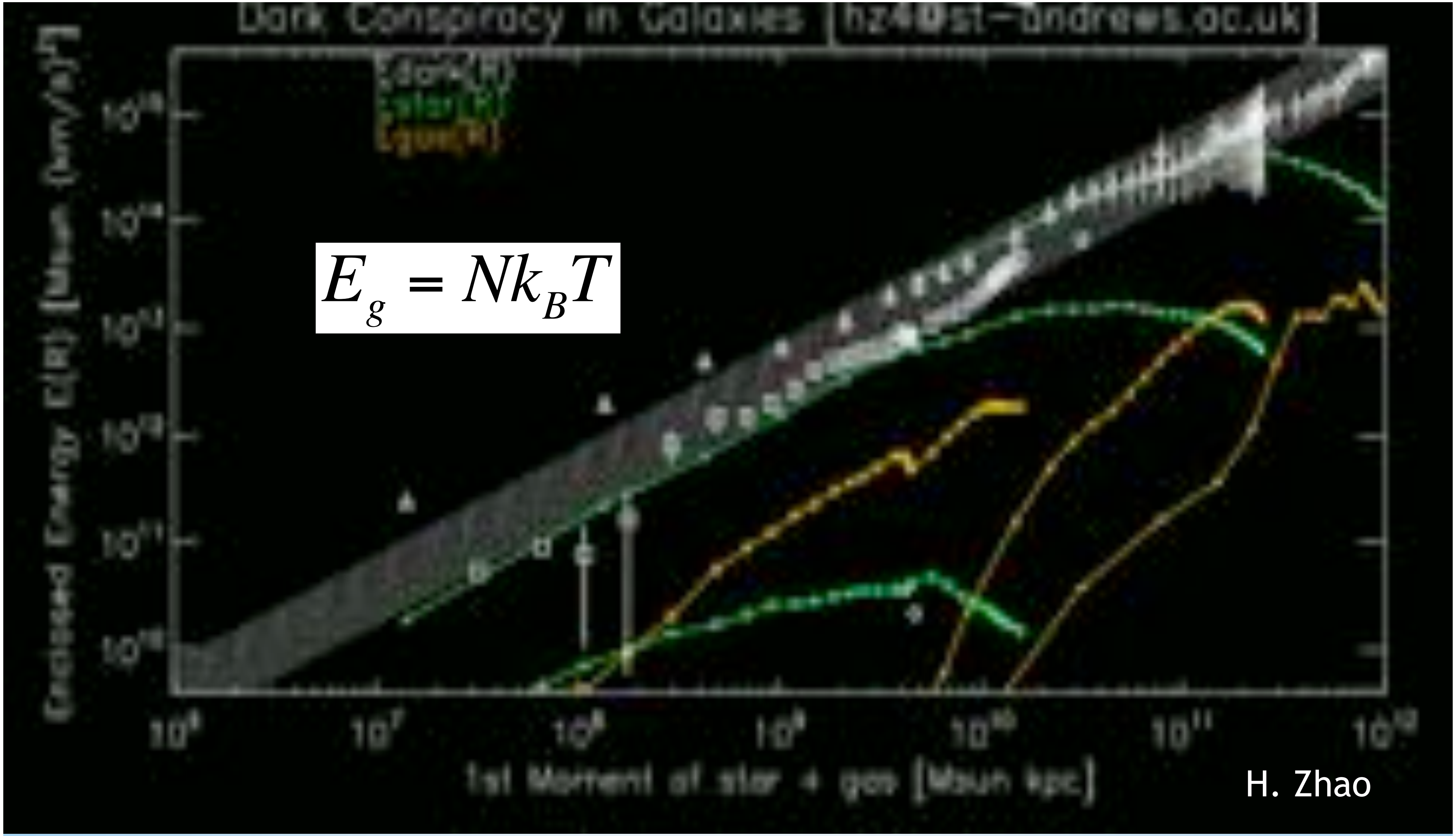


$$\frac{a_0}{2\pi} = 1.24 \pm 0.14 \cdot 10^{-10} \text{ m/s}^2$$

$$V_{obs}^4 = GM_B \frac{a_0}{2\pi}$$

$$a_0 \approx cH_0$$

Why?



$$E_g = Nk_BT$$

H. Zhao

$$E(R) = \frac{1}{8\pi G}\int_0^R \delta\vec{g}^2\, 4\pi r^2\, dr$$

$$N(R) = \int_0^R \rho(r) r\, 4\pi r^2\, dr$$

At horizons space and time dissappear.

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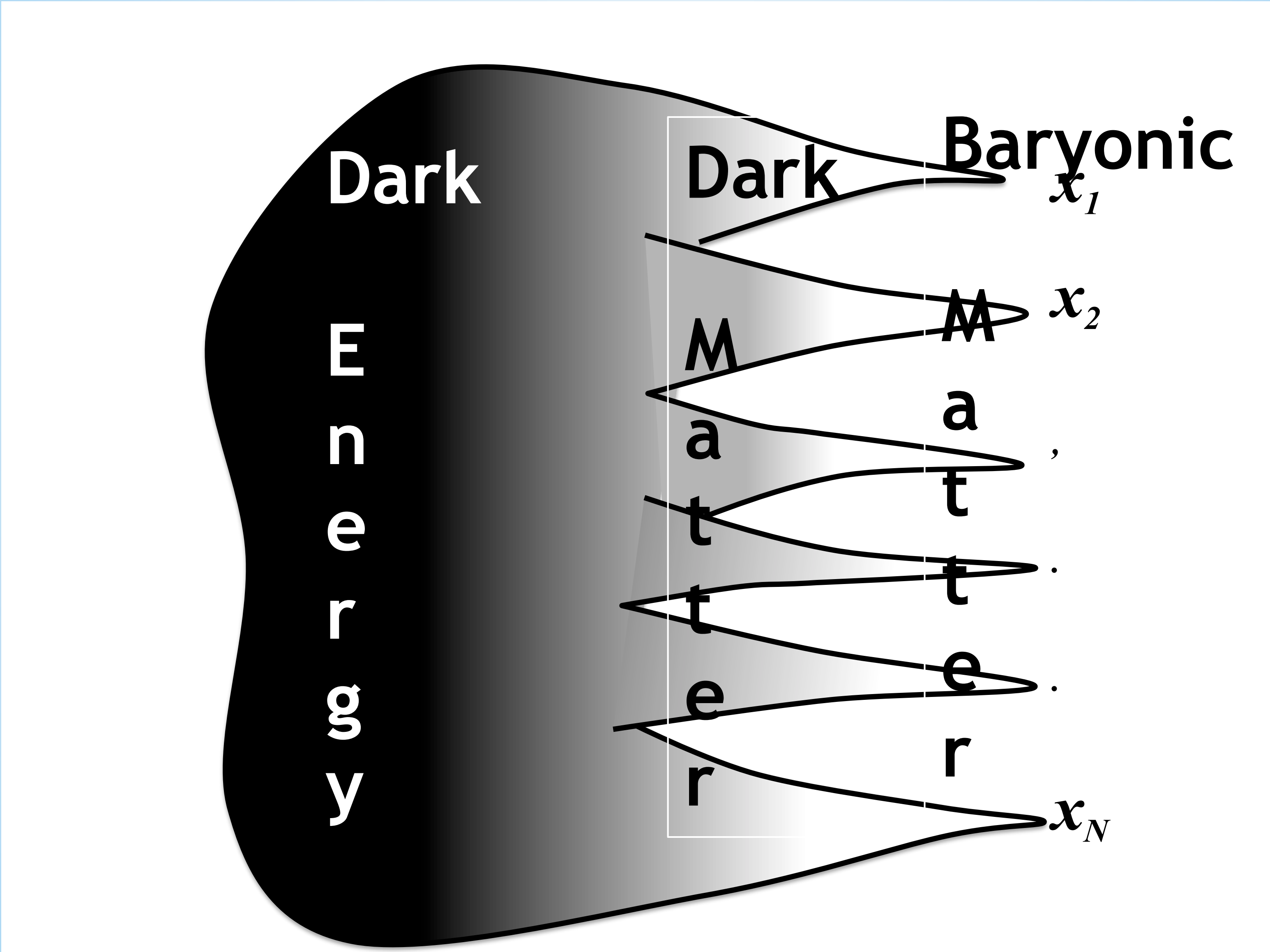
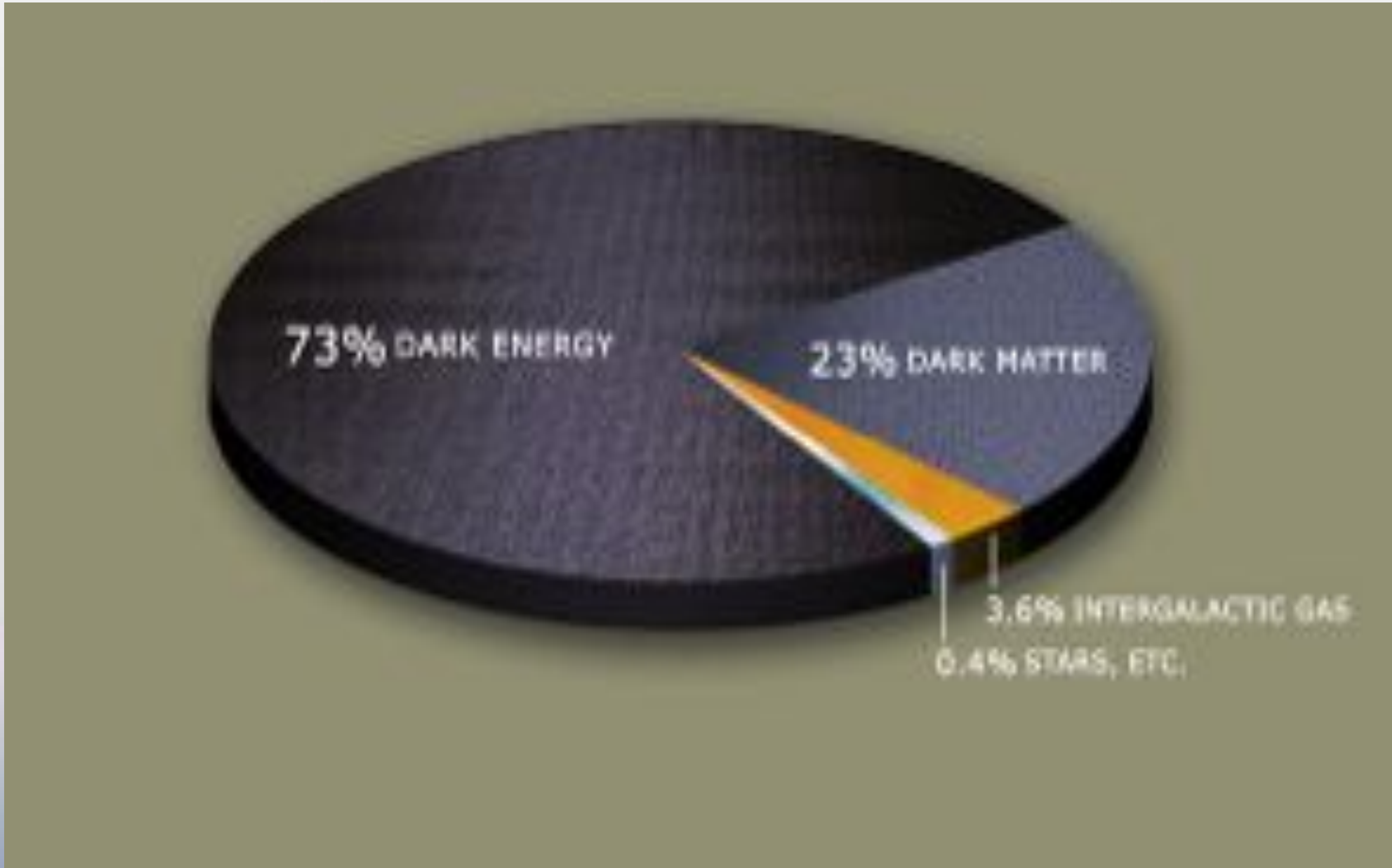
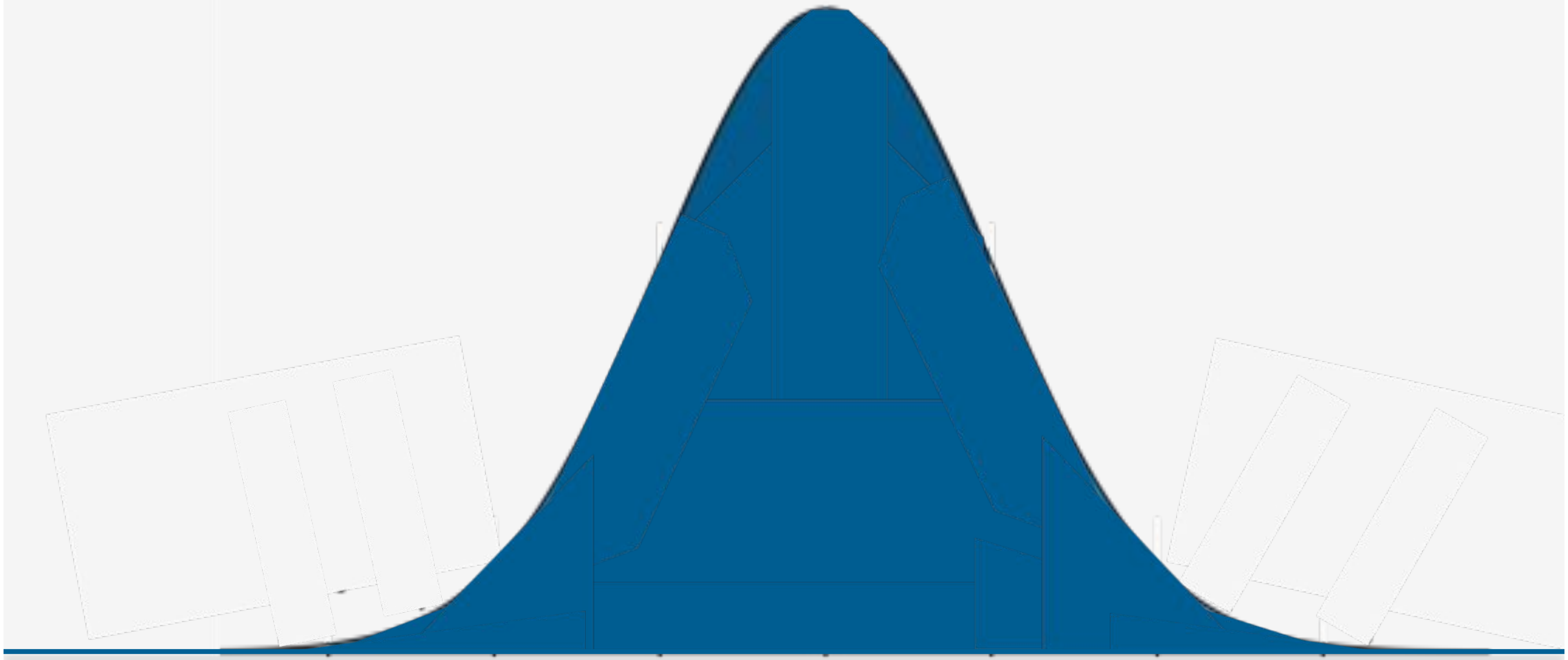
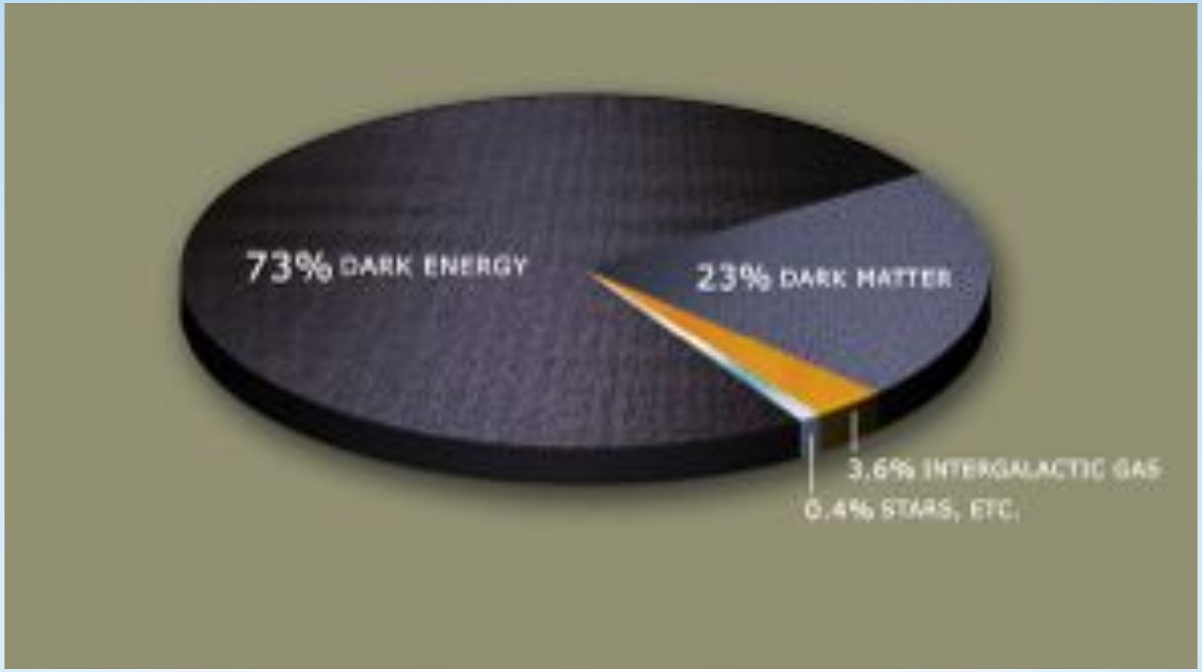



73% DARK ENERGY
23% DARK MATTER
3.6% INTERGALACTIC GAS
0.4% STARS, ETC.




73% DARK ENERGY
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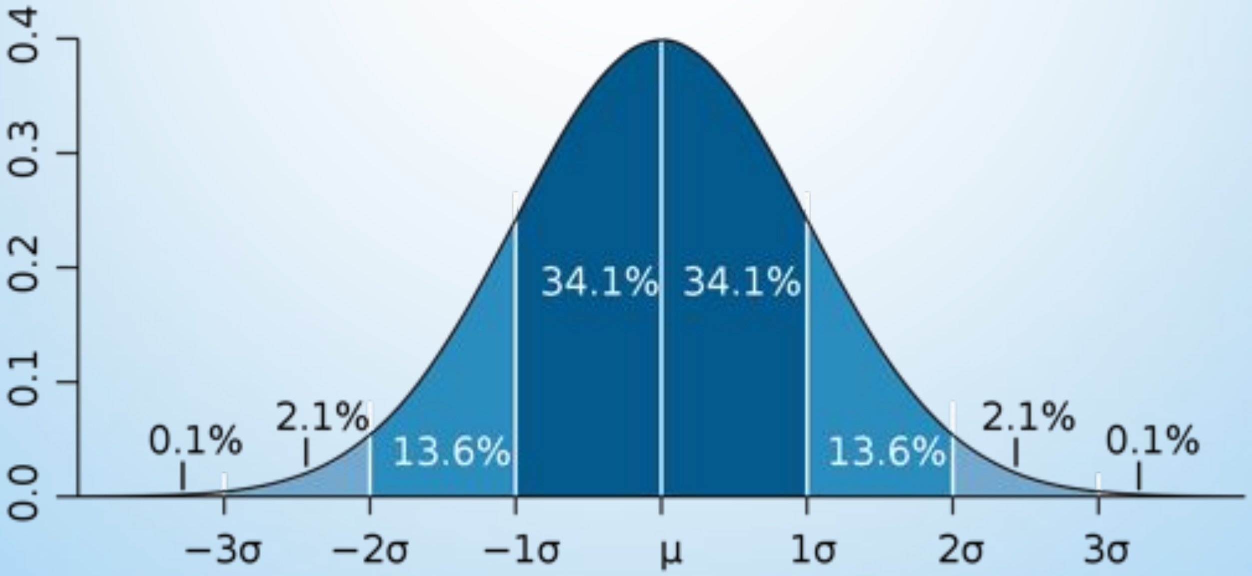
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2σ
3σ

Basic idea:

To arrive at our paradigm in terms of space time, matter, and forces we ignore (non-local) UV DOF.

This requires a separation of time scales:
breaks down near horizons and in cosmology.

The distinction between matter and space time will disappear and forces should be seen as emergent.

Gravity, and the other forces, are reaction forces due to the fast microscopic dynamics of the underlying dynamical system.