Scattering Amplitudes +

The Positive Grassmannian

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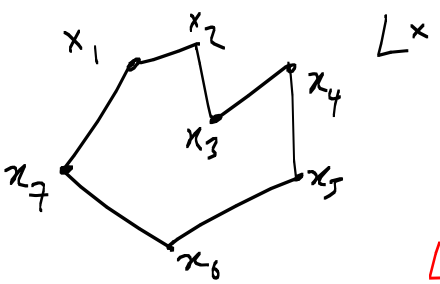
Conventional Picture of QFT

$$\mathcal{L} = -\frac{1}{4} \mathrm{tr}\, F_{\mu\nu}^2 + \cdots$$

- Gauge Redundancy needed to make locality and Unitarity manifest
- Obscures invariant content of theory — seen dramatically first @ strong coupling [Gauge/Gauge, Gauge/Gravity dualities]

Stunning Magic in Planar $\mathcal{N}=4$ SYM

e.g. Hidden Infinite-Dimensional Symmetries



$$p_a^\mu = x_a^\mu - x_{a-1}^\mu$$

Dual Superconformal Invariance

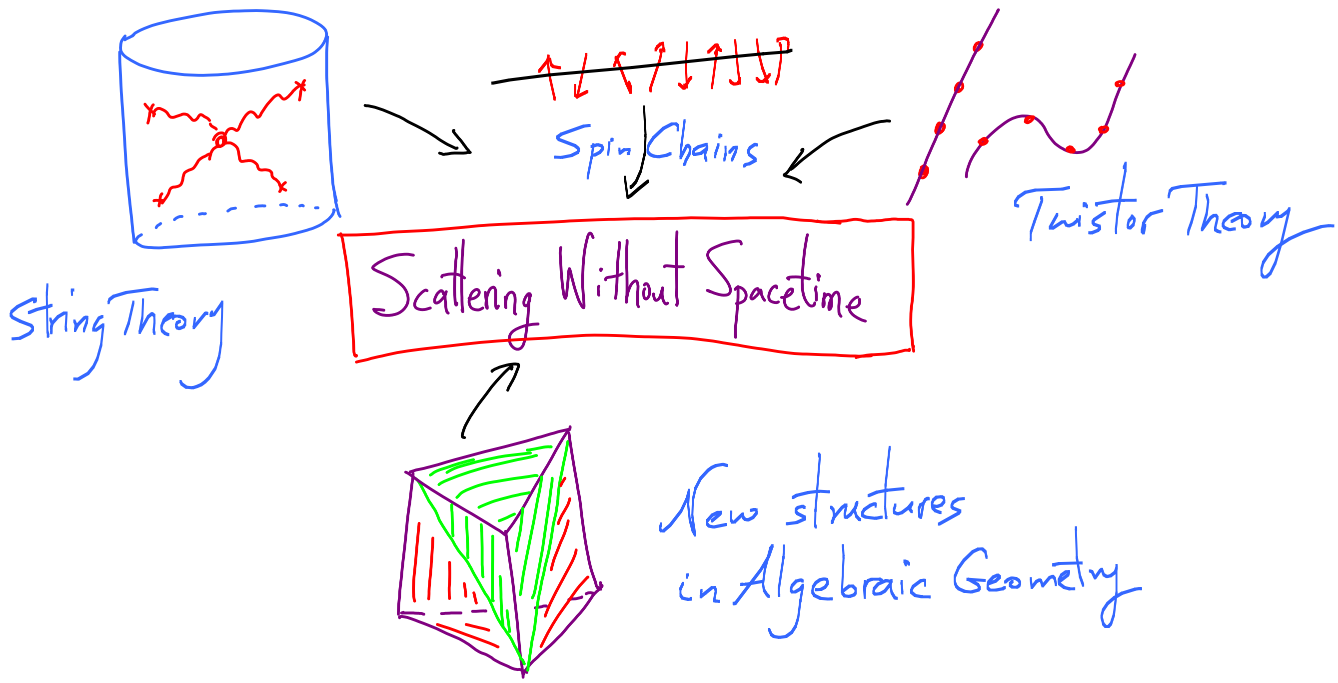
$\longrightarrow$ Yangian (Spin Chains, Integrability, ...)

We are after a theory for

$$M_{n,k}[\lambda_a, \tilde{\lambda}_a, \tilde{\eta}_a ; l_i]$$

Without Unitary evolution through Spacetime

Sitting Under our Noses for 60 yrs

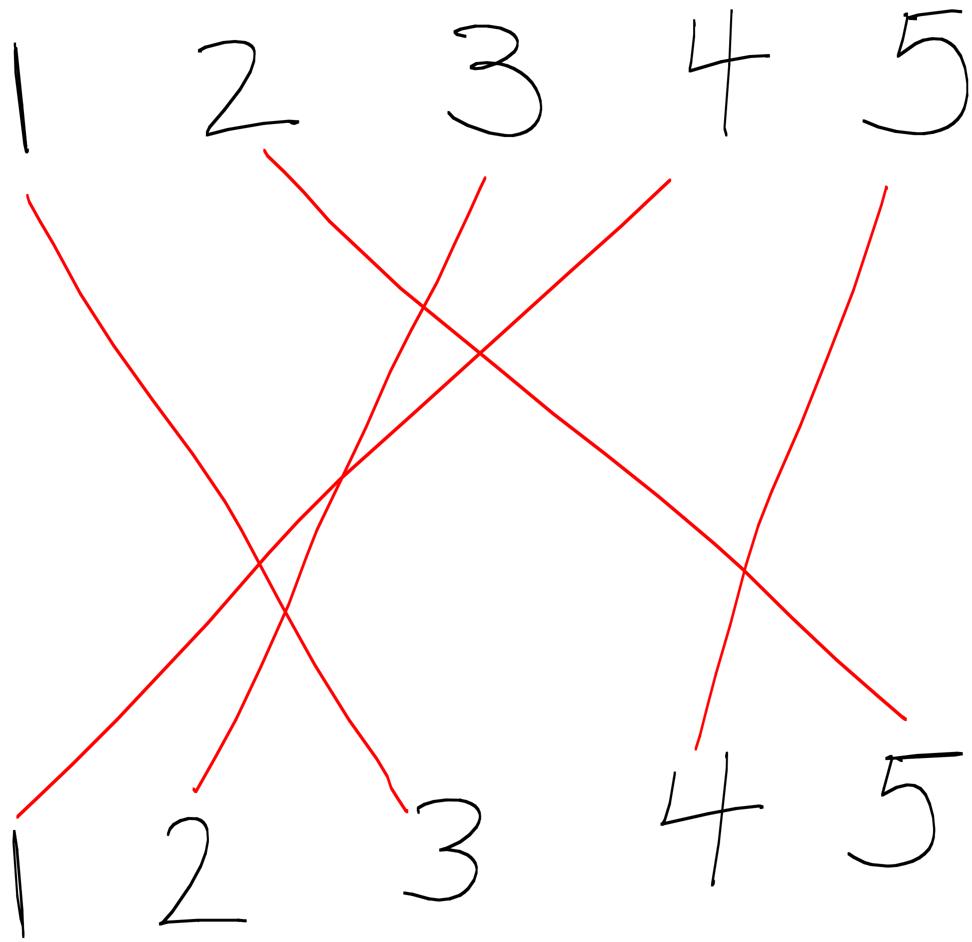


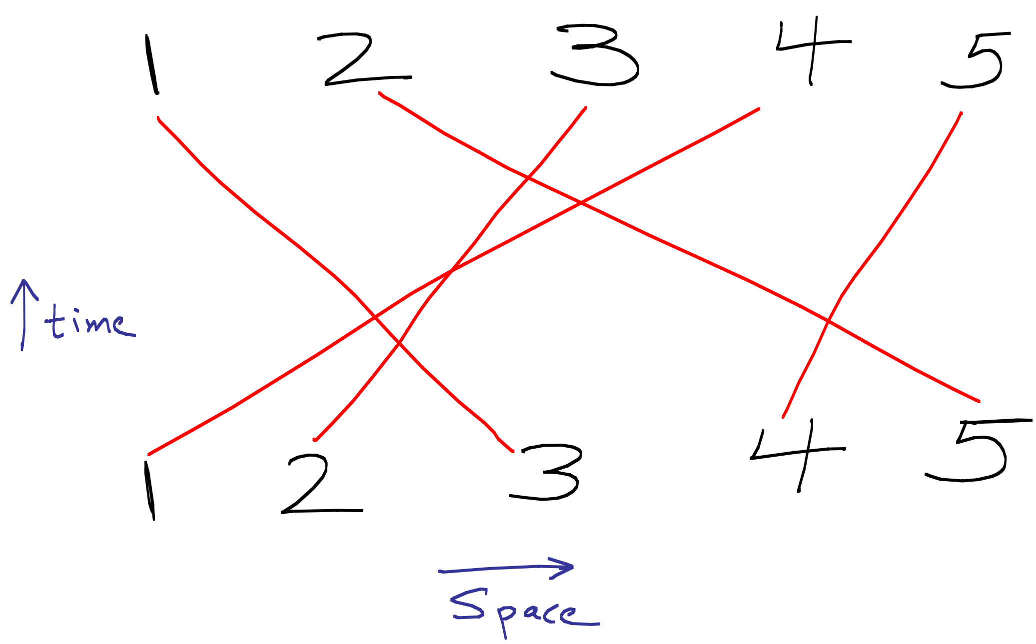
Drawings of Permutations

to

Pictures of Scattering

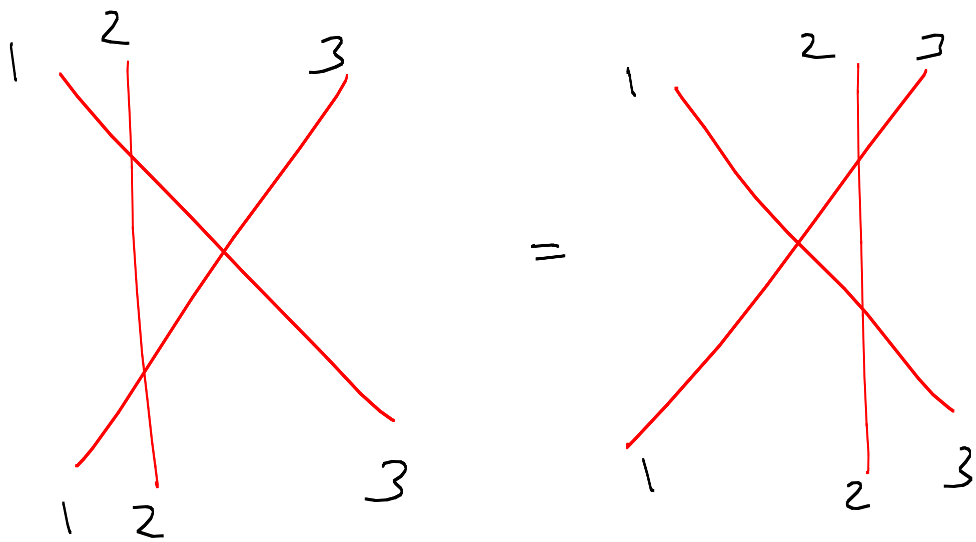
$12345 \rightarrow 35214$


1 2 3 4 5
1 2 3 4 5



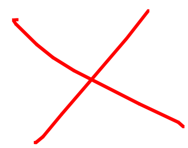
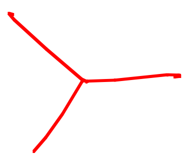
Breaking up permutation into product of adjacent transpositions [non-unique]

3	5	2	1	4
3	2	5	1	4
3	2	1	5	4
3	1	2	5	4
1	3	2	5	4
1	2	3	5	4
1	2	3	4	5

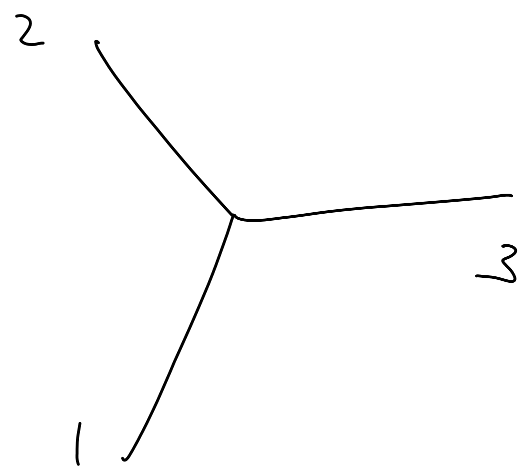


Yang-Baxter

Can't Apply to (3+1)-d

- No particle creation/destruction
- Fundamental interaction  ,
  not 

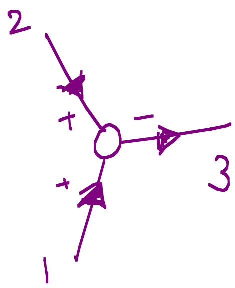
On-Shell Diagrams



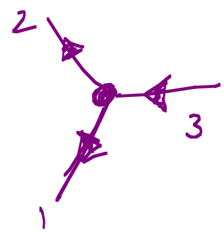
$$\lambda_1^A \tilde{\lambda}_1^{\dot{A}} + \lambda_2^A \tilde{\lambda}_2^{\dot{A}} + \lambda_3^A \tilde{\lambda}_3^{\dot{A}} = 0$$

$\Longrightarrow$ Either $\lambda_1 \propto \lambda_2 \propto \lambda_3$ or $\tilde{\lambda}_1 \propto \tilde{\lambda}_2 \propto \tilde{\lambda}_3$

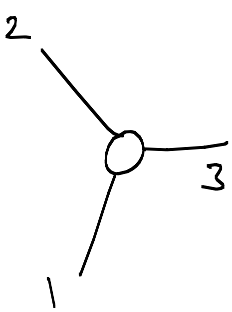
3-pt Vertices



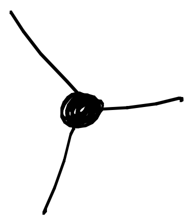
$$\frac{[12]^3}{[12][23]}\,\delta^4\Big(\sum_a p_a\Big)$$



$$\frac{\langle 12\rangle^3}{\langle 13\rangle\langle 23\rangle}\,\delta^4\Big(\sum_a p_a\Big)$$

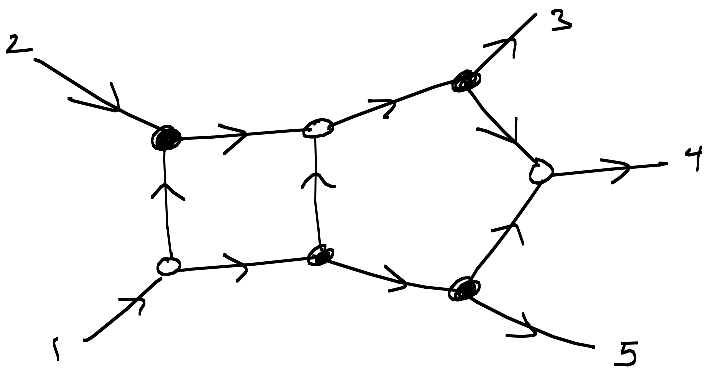


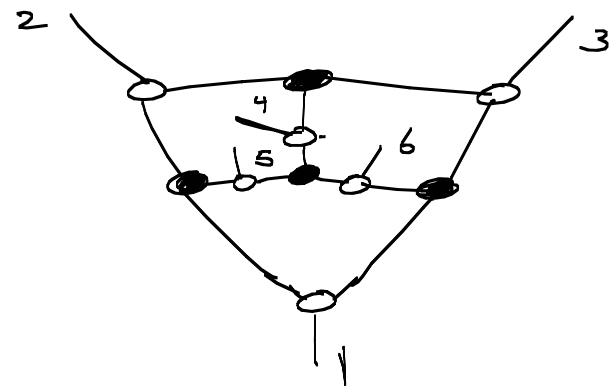
$$\frac{\delta^8\big(\Sigma\,\tilde{\lambda}_a\eta_a\big)}{[12][23][31]}\,\delta^4\Big(\sum_a p_a\Big)$$



$$\frac{\delta^8\big(\sum_a \lambda_a\tilde{\eta}_a\big)}{\langle 12\rangle\langle 23\rangle\langle 31\rangle}\,\delta^4\Big(\sum_a p_a\Big)$$

On-Shell Diagrams



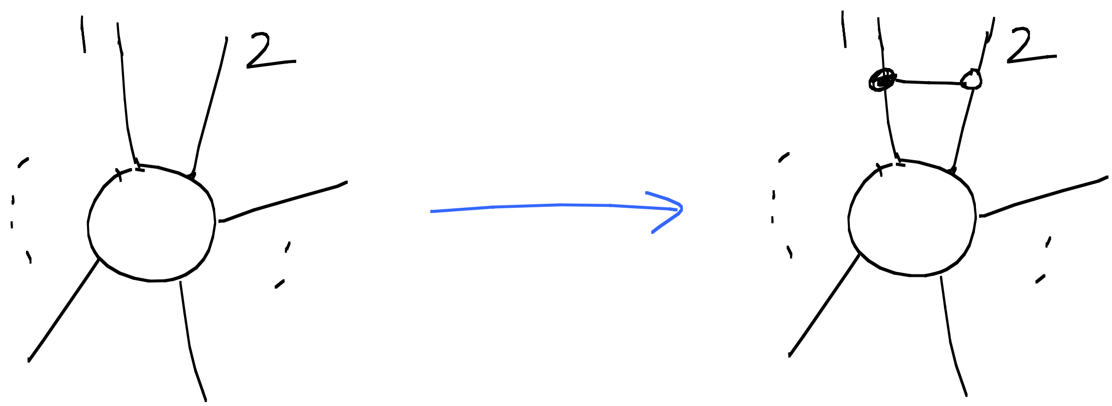


$\mathcal{N}=0$ $\qquad\qquad$ $\mathcal{N}=4$

$$\left(\int \frac{d^2\lambda\, d^2\tilde{\lambda}}{GL(1)}, \int \frac{d^4 W}{GL(1)}\right) \qquad \left(\int \frac{d^2\lambda\, d^2\tilde{\lambda}\, d^4\tilde{\eta}}{GL(1)}, \int d^{4|4} \mathcal{W}\right)$$

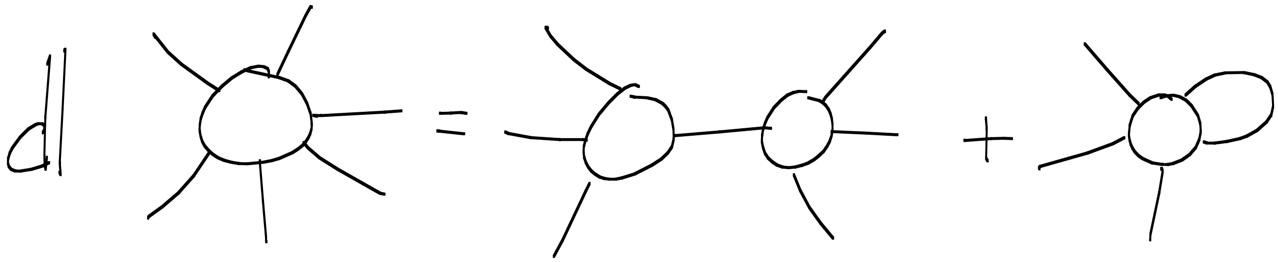
In general, a differential form

Ex: BCFW Deformation



$$\int [1, 2, \ldots n] \longrightarrow \frac{d\tau}{\tau} \int [\lambda_1 + \tau\lambda_2, \ldots, \tilde{\lambda}_2 - \tau\tilde{\lambda}_1, \ldots]$$

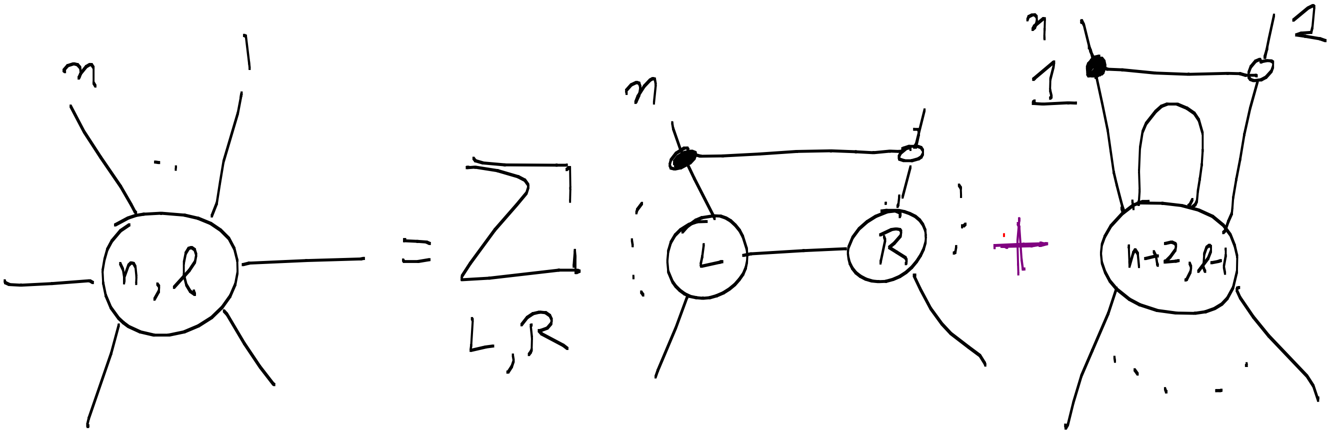
"Differential Equation" For Amps



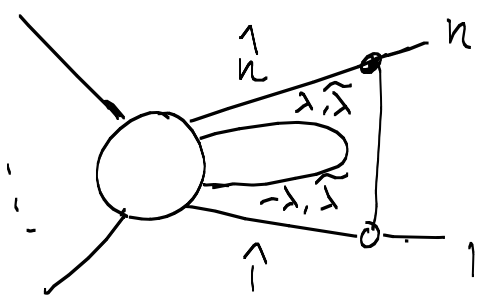
[Identical in Structure to Exact Wilsonian RG!
Not "integrating out modes" but "Hiding Particles"]

* BCFW : one way of solving this Diff Eq.

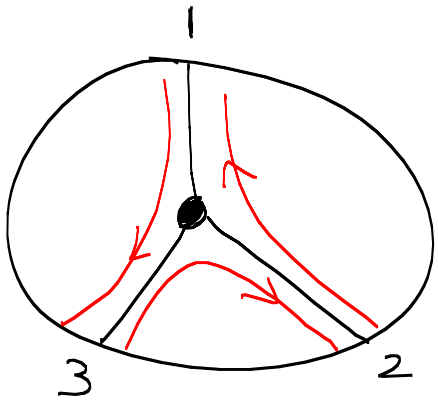
All-Loop Recursion for Planar $\mathcal{N}=4$

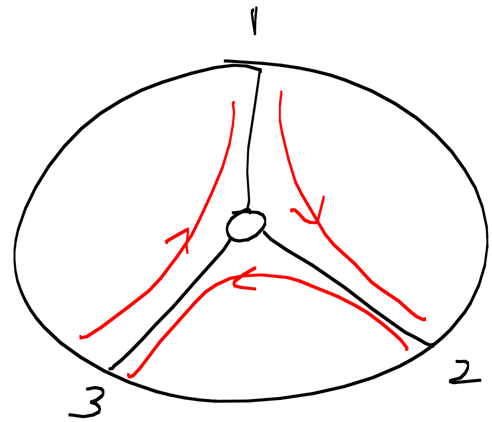


On-shell loops



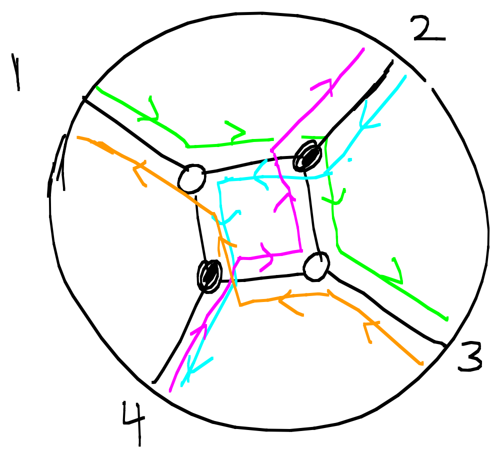
$$\underbrace{\int \frac{d^2\lambda\, d^2\tilde{\lambda}}{GL(1)} \frac{d\tau}{\tau}}_{\ell = \lambda\tilde{\lambda} + \tau \lambda_1 \tilde{\lambda}_2} \rightarrow \int d^4 \ell$$





$1 \to 3$
$2 \to 1$
$3 \to 2$

$1 \to 2$
$2 \to 3$
$3 \to 1$



$1 \to 3$
$2 \to 4$
$3 \to 1$
$4 \to 2$

"Affine" or "Decorated" Permutation

$1 \to 3$
$2 \to 4$
$3 \to 1+4=5$
$4 \to 2+4=6$

$a \to p(a)$, $a+n \geq p(a) \geq a$

$p(a) \bmod n$ is a perm.

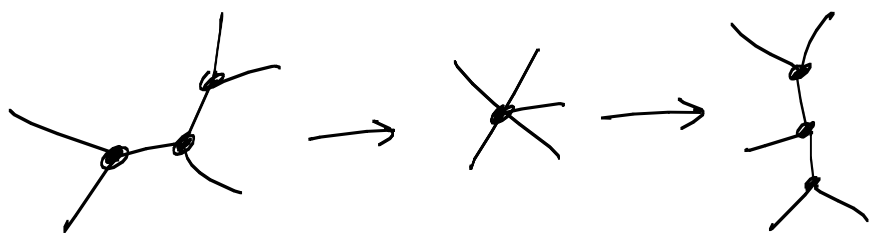
"K" = # that jump back $= \frac{1}{n}\sum_a (p(a)-a)$

So, a set of permutation for $\underline{\underline{(n,K)}}$

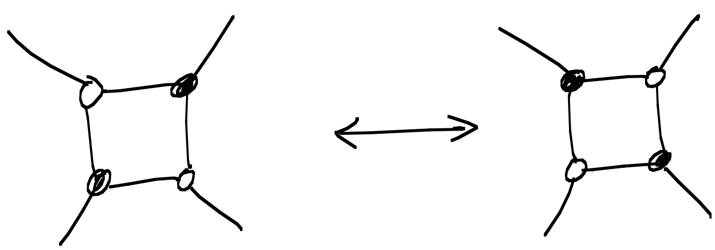
Enormous Redundancy

* First - can have infinitely many loops/faces! "Reduced" graphs → representing permutation with minimum # of faces.

* Even reduced graphs are far from unique; two "moves":

- 

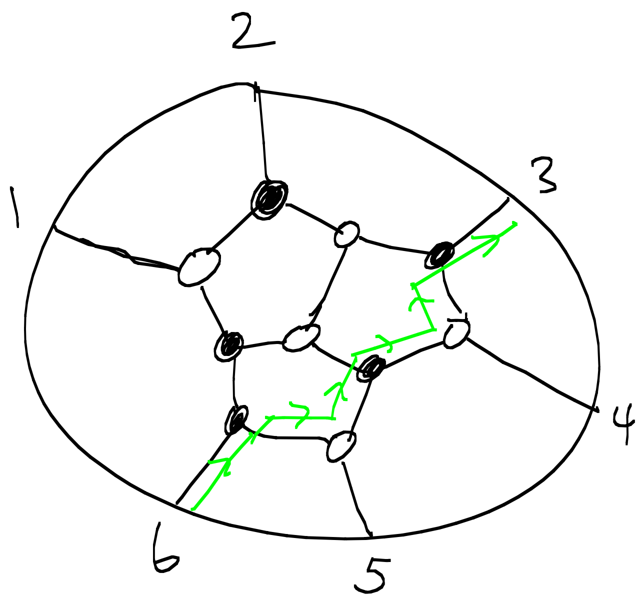
  Merge B's, W's

- 

  "Square Move"

* Every perm. can be represented like this

$1 \to 4$
$2 \to 6$
$3 \to 5$
$4 \to 7$
$5 \to 8$
$6 \to 9$

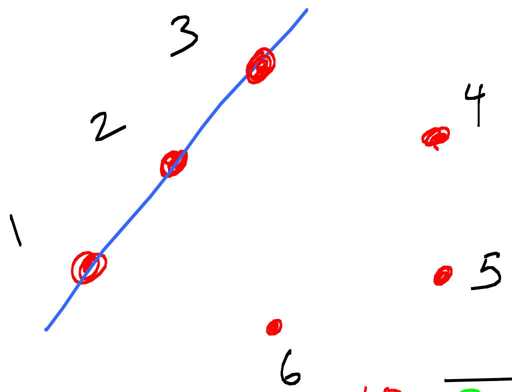


Permutations $\longleftrightarrow$ Config. of Vectors

$\updownarrow$

Positive Grassmannian

Conf. of Vectors w/ Consecutive Lin. Dep

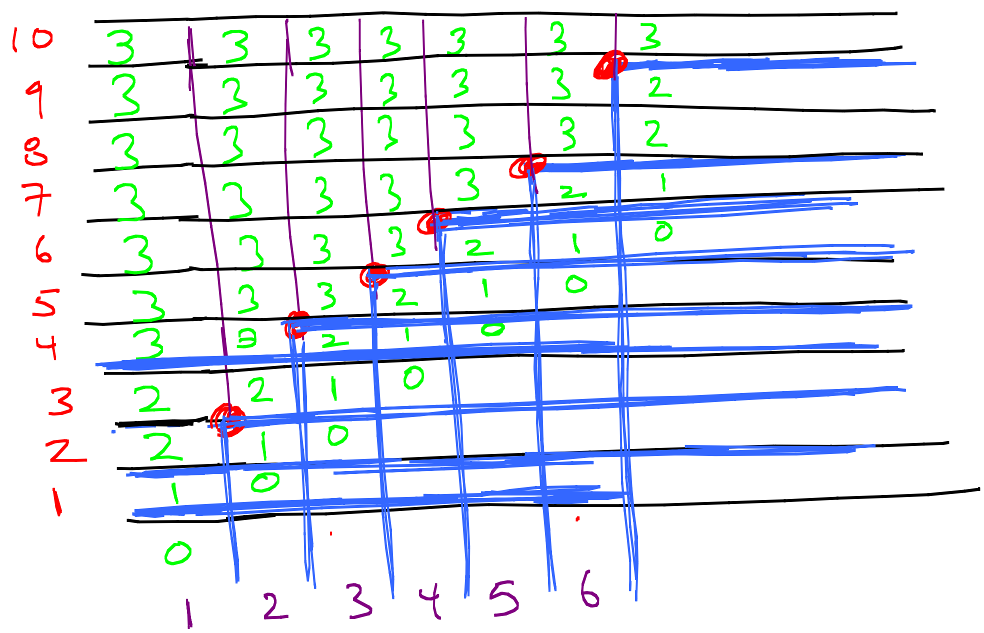


Fully Determined By

$a \to p(a)$

$a \in \mathrm{span}(a+1, \ldots, p(a))$

$1 \to 3$
$2 \to 5$
$3 \to 6$
$4 \to 7$
$5 \to 8$
$6 \to 10$



The Positive Grassmannian

$$C = \begin{array}{c} \overleftrightarrow{\quad n \quad} \\ \left[\begin{array}{cccc} c_1 & c_2 & \cdots & c_n \end{array}\right] \end{array} \updownarrow k \qquad n\ k\text{-vectors}.$$

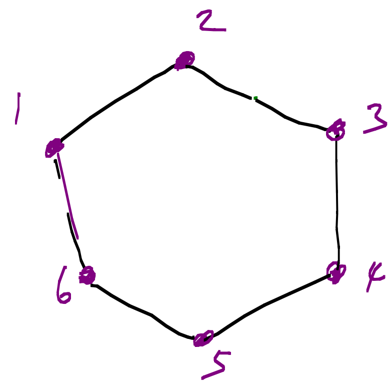
"Positive Part": $(c_{i_1} \cdots c_{i_k}) > 0$ for $i_k > \cdots > i_1$.

["All minors positive"].

Note: (twisted) cyclic structure

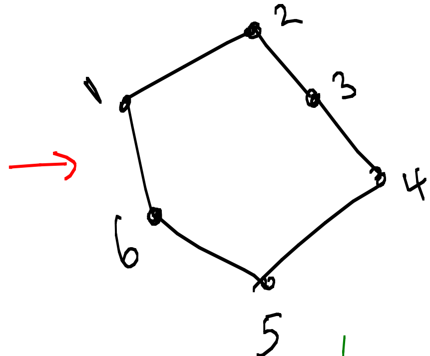
$c_1 \to c_2,\ c_2 \to c_3,\ \ldots,\ c_n \to (-1)^{k+1} c_1$

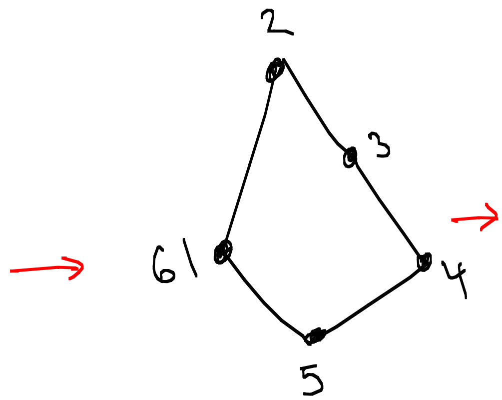
Positivity $\longleftrightarrow$ Convexity



$\longrightarrow$ Convex Polygon

Boundaries:





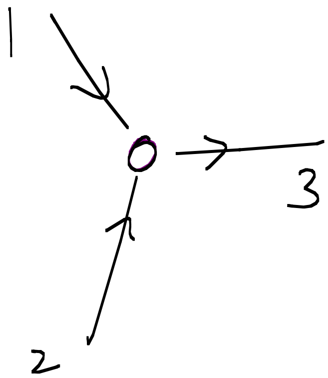
$\longrightarrow$ . . .

$\longrightarrow$ Consecutive linear dependencies $\longrightarrow$ Permutations

From On-Shell Diagrams

to

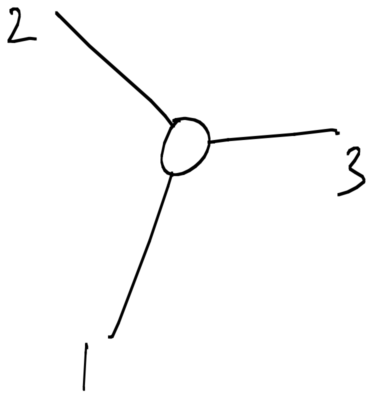
The Grassmannian



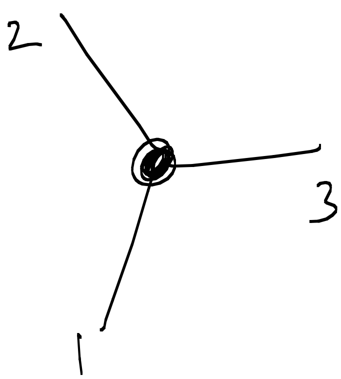
$$\frac{[12]^3}{[13][23]}\,\delta^4\Big(\sum_a p_a\Big)$$

$$=$$

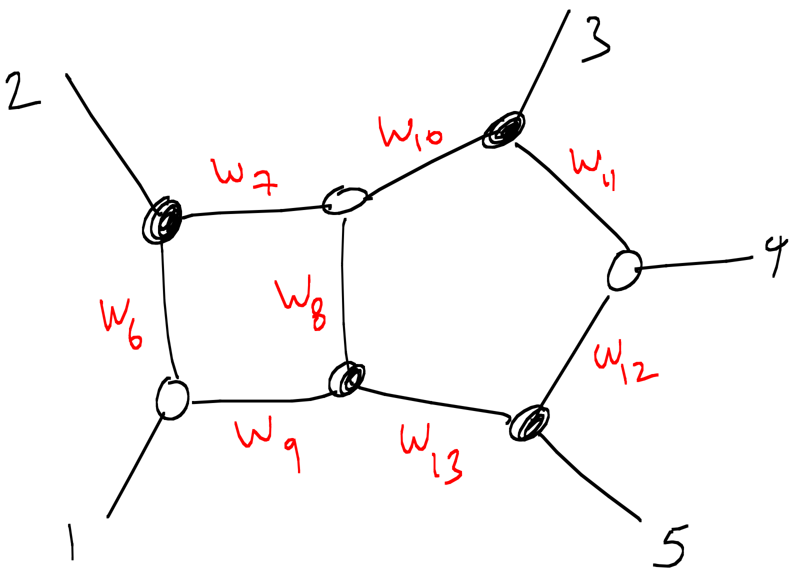
$$\int \frac{dt_1}{t_1}\frac{dt_2}{t_2}\;\delta^2[\tilde\lambda_3 + t_2\tilde\lambda_2 + t_1\tilde\lambda_1]\;\delta^2[\lambda_3 - t_1\lambda_1]\;\delta^2[\lambda_3 - t_2\lambda_2]$$



$$= \int \frac{dt_1}{t_1}\frac{dt_2}{t_2}\frac{dt_3}{t_3}\Big/_{GL(1)} \delta^{4|4}(t_i N_i)$$

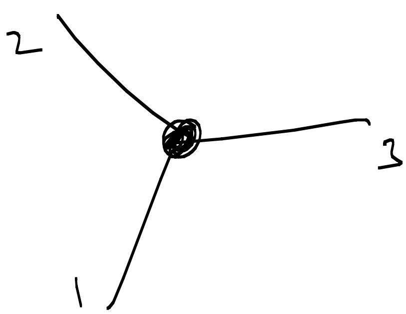


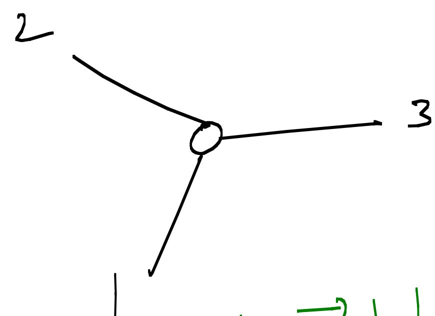
$$= \int d^2\vec{u}_1\, d^2\vec{u}_2\, d^2\vec{u}_3 \Big/_{GL(2)} \delta^{4|4}(\vec{u}_i N_i)$$



$$\int \frac{d^{4|4} \mathcal{W}_{\text{int}}}{GL(1)'s},$$

trivial





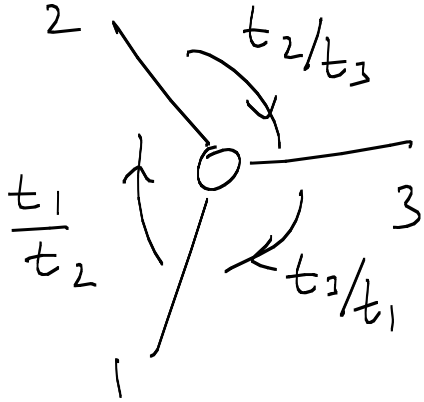
$t_1 W_1 + t_2 W_2 + t_3 W_3 = 0$

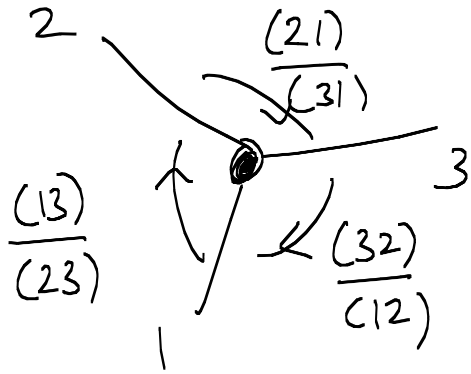
$\vec{u}_1 W_1 + \vec{u}_2 W_2 + \vec{u}_3 W_3 = 0$

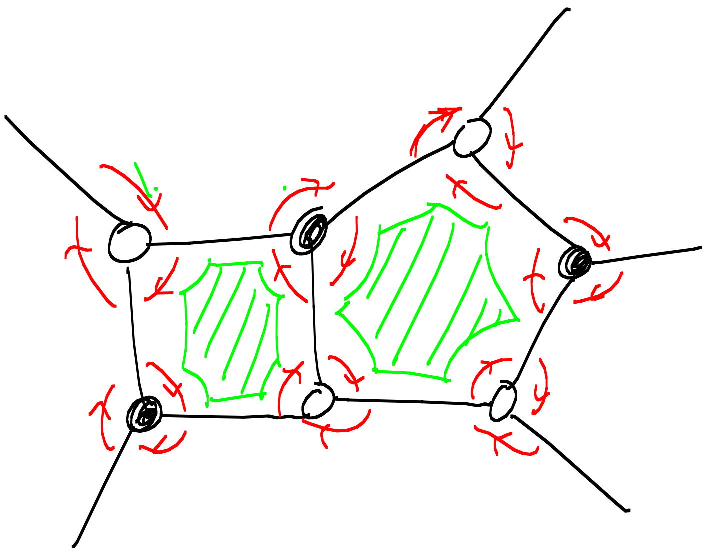
Now eliminate W_{internal}'s!

$$\longrightarrow \prod_{\alpha=1}^{k} \delta^{4|4}\left[C_{\alpha a}[t's, \vec{u}'s]\, W_a \right]$$

Point in $G(K,n)$

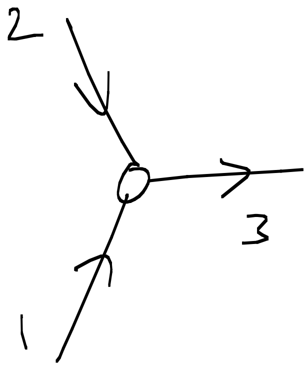




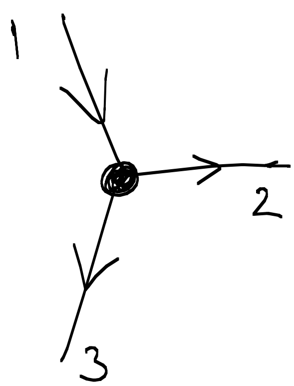


Variables associated with Faces

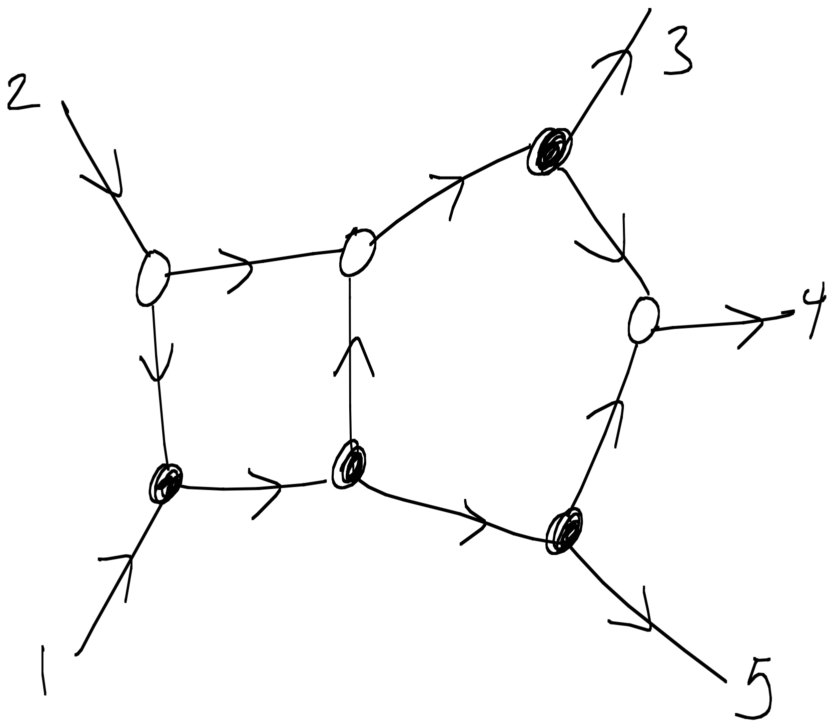
Convenient GFing + Explicit Co-ord.



$$x_3 W_3 = x_1 W_1 + x_2 W_2$$



$$y_1 W_1 = y_2 W_2 = y_3 W_3$$



* Edge Var.

$\longleftrightarrow$ U(1) Gauge Field on Graph

* Face Var: Flux through faces

$$C_{iJ} = \sum_{\substack{\text{all paths} \\ i \to J}} \prod_{\substack{\text{edges} \\ \text{along path}}} e_\alpha$$

So, we have explicitly

$$\prod_{\text{edges}} \frac{de_\alpha}{e_\alpha} \Big/ GL(1)'s$$

or

$$\prod_{\text{faces}} \frac{df_i}{f_i}$$

$$\prod_{\alpha=1} \delta^{4|4}\left[C_{\alpha a}[f]\, W_a \right]$$

The map from On-Shell-

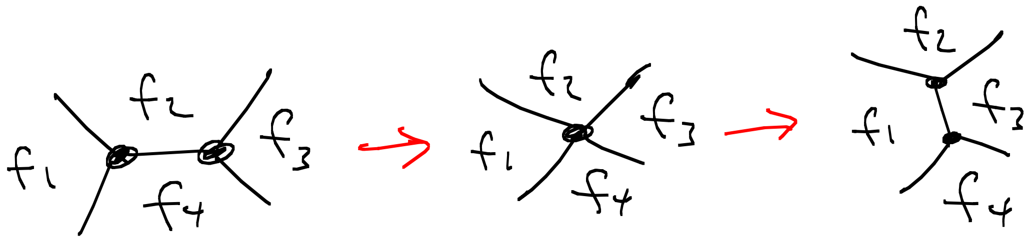
Diagrams $\longrightarrow$ Grassmannian

is natural + Universal.

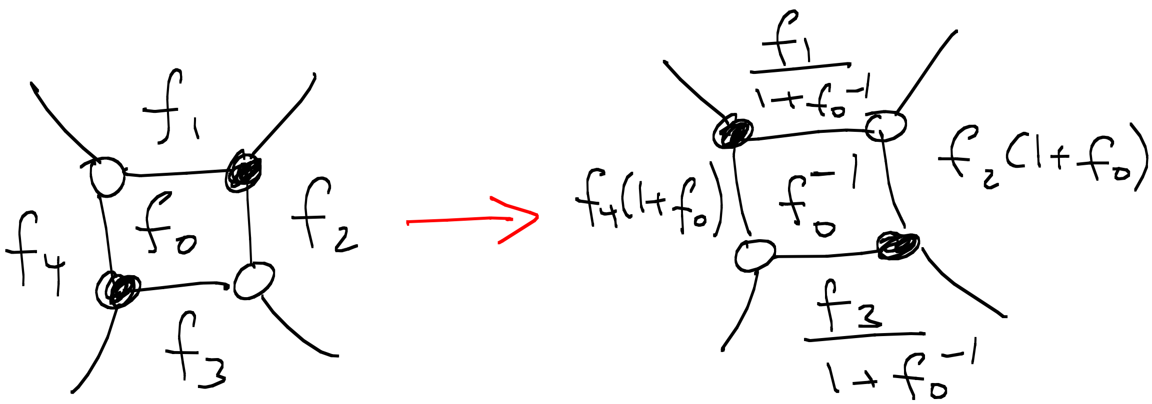
The measure depends on

the theory....

"Moves"

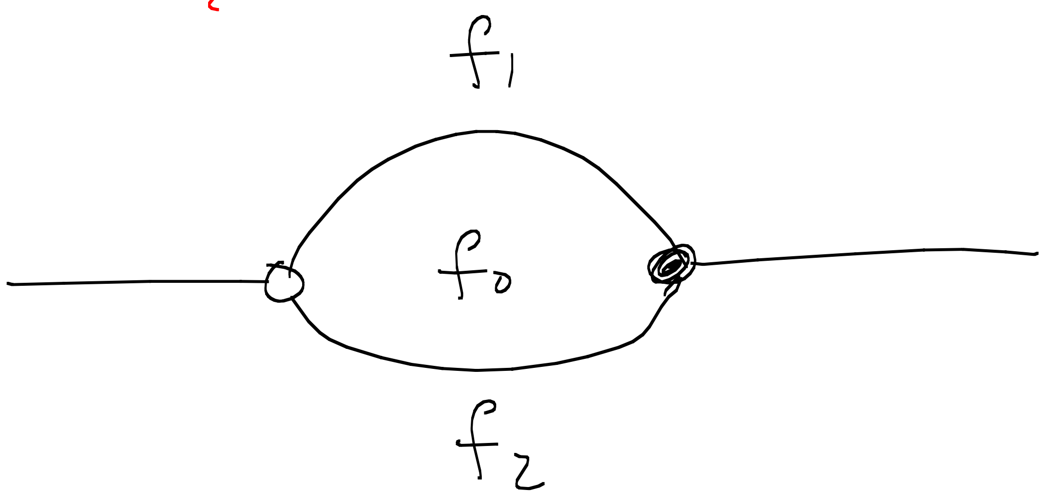


f_1
f_2
f_3
f_4
f_2
f_1
f_3
f_4
f_2
f_1
f_3
f_4



f_1
f_4
f_0
f_2
f_3
$\frac{f_1}{1+f_0^{-1}}$
$f_4(1+f_0)$
f_0^{-1}
$f_2(1+f_0)$
$\frac{f_3}{1+f_0^{-1}}$

"Reduction"



$$\downarrow$$

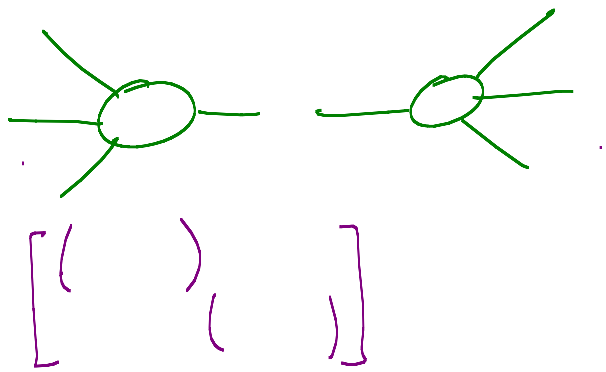
$$\frac{f_1(1+f_0)}{\dfrac{f_2}{1+f_0^{-1}}}$$

Planar $\mathcal{N}=4$ SYM

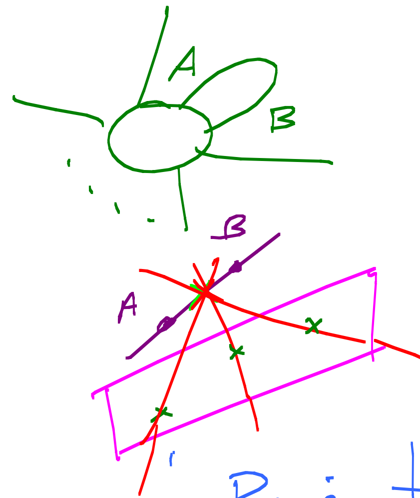
and

The Positive Grassmannian

* Basic operations:



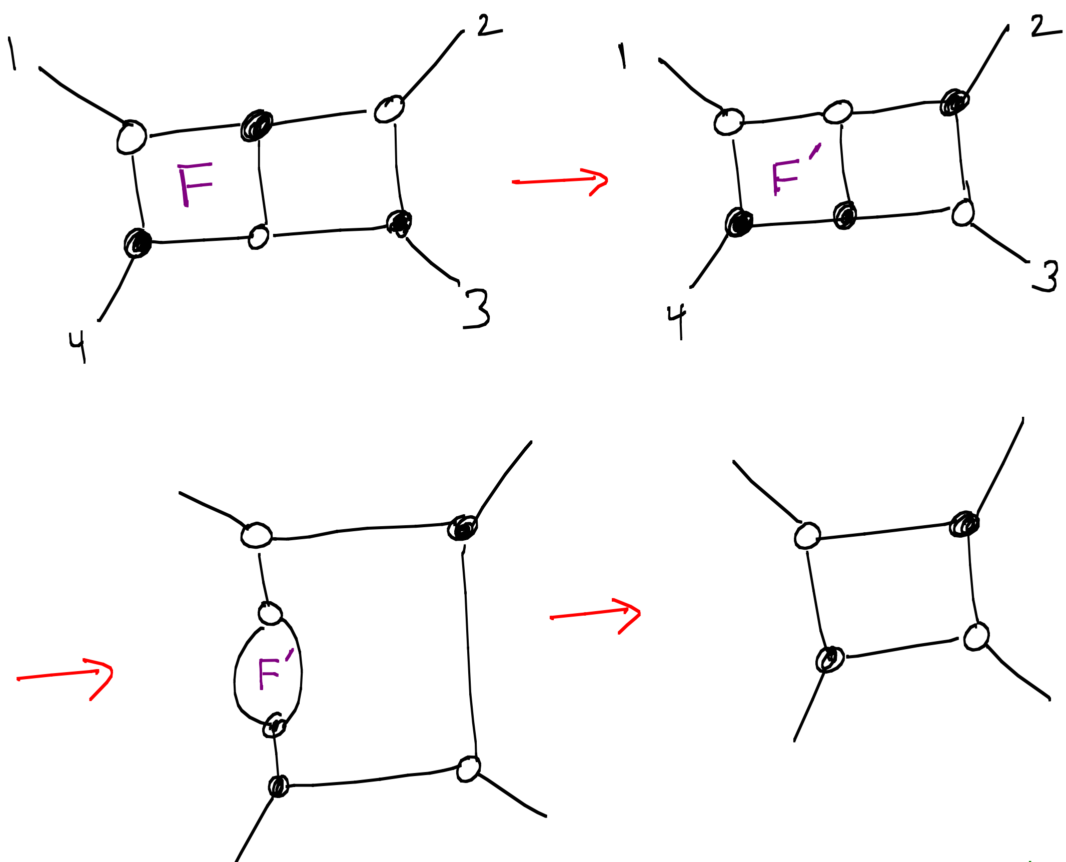
Direct Product



Projection

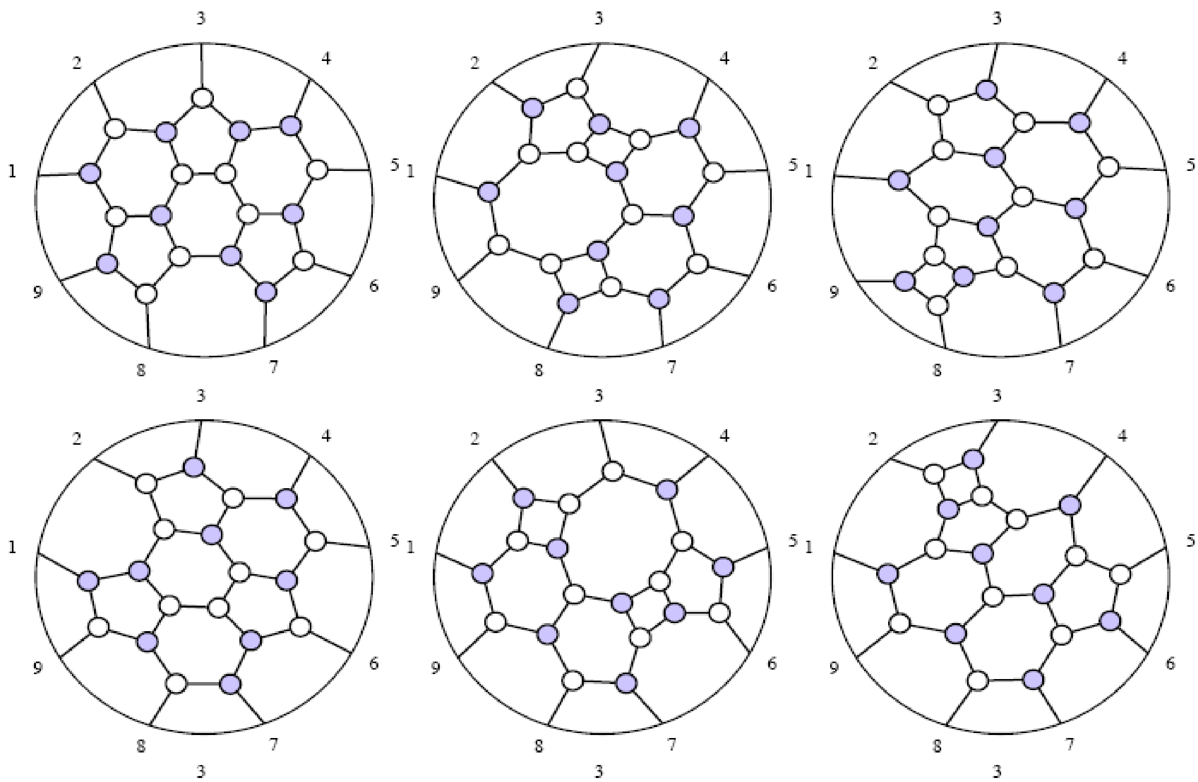
$\longrightarrow$ "Positive" Grassmannian Manifest for Planar Graphs from "local" picture

Reduction



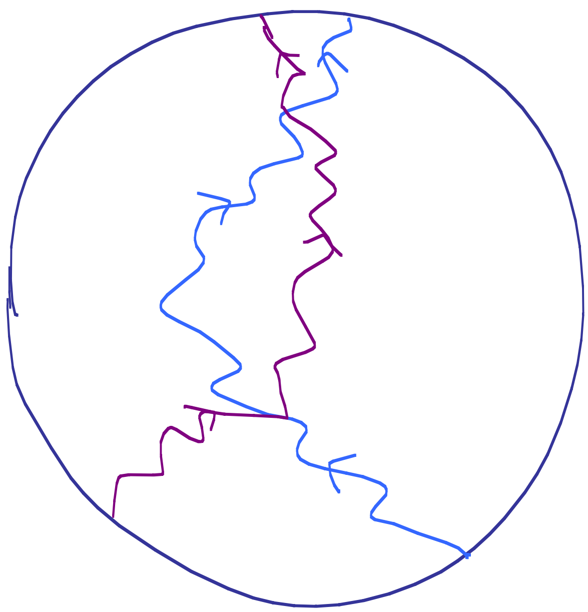
$$\frac{dF}{F}\prod_a \frac{df_a}{f_a} \longrightarrow \underbrace{\left(\frac{dF'}{F'}\right)}_{\text{factors out}} \prod_a \frac{df'_a}{f'_a}$$

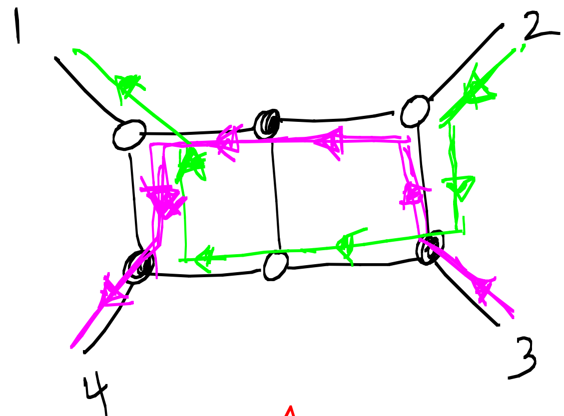
Invariants: Global Properties of L-R Paths!



Reduced? Same, different?

Reducible Graphs





"Bad double Crossing"

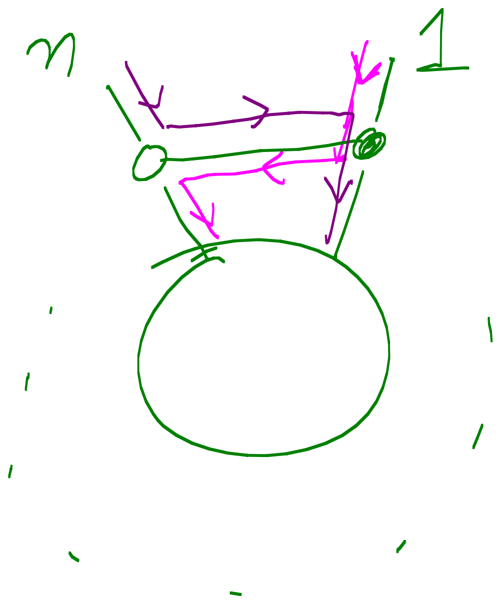
Once Reduced

* All content is in L-R path permutation

  → which gives lin dependencies of Grassmannian

  → And Specifies cell of Positive Gr.

BCFW = Adjacent Transp.



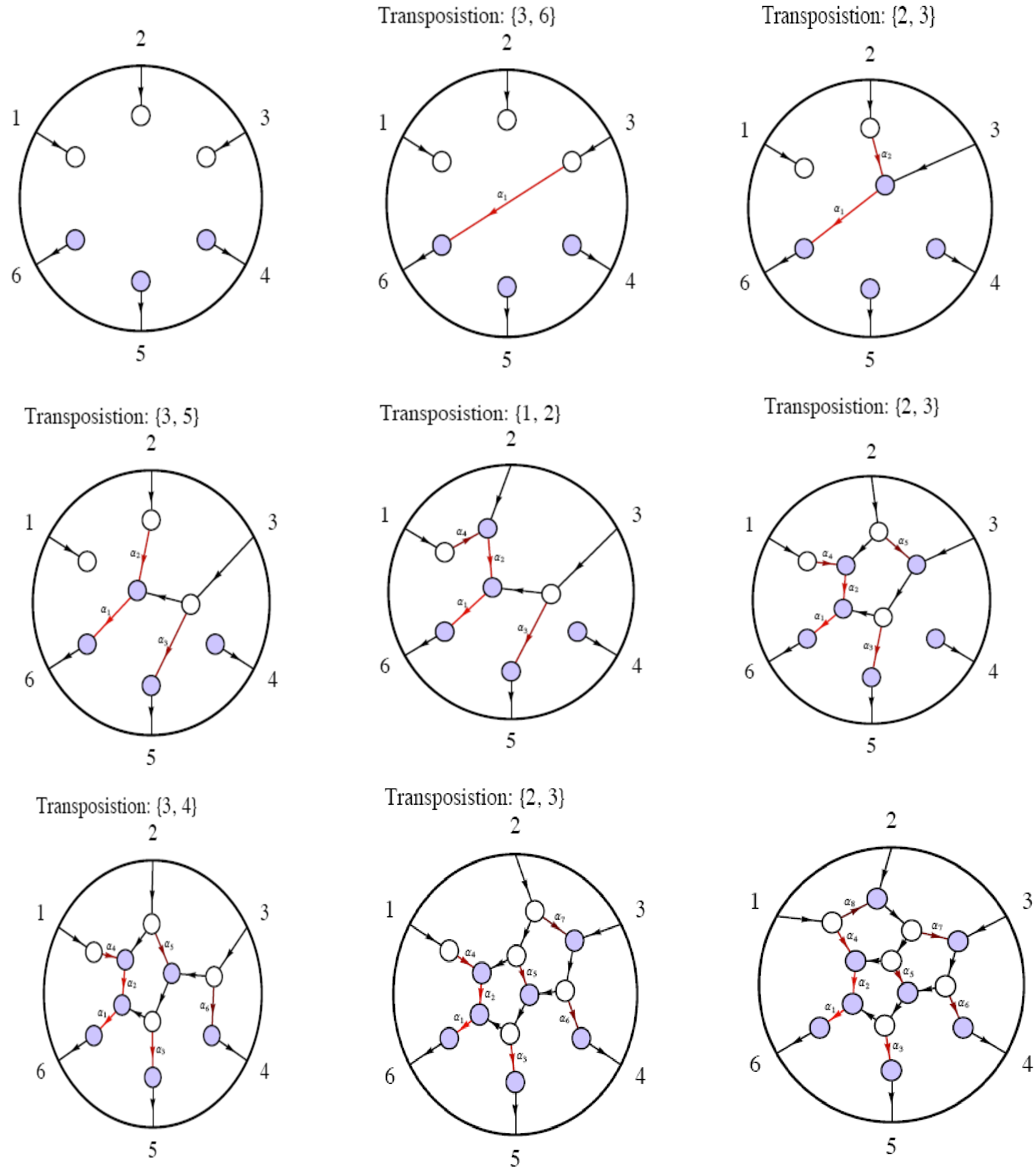
$n \longrightarrow p(1)$

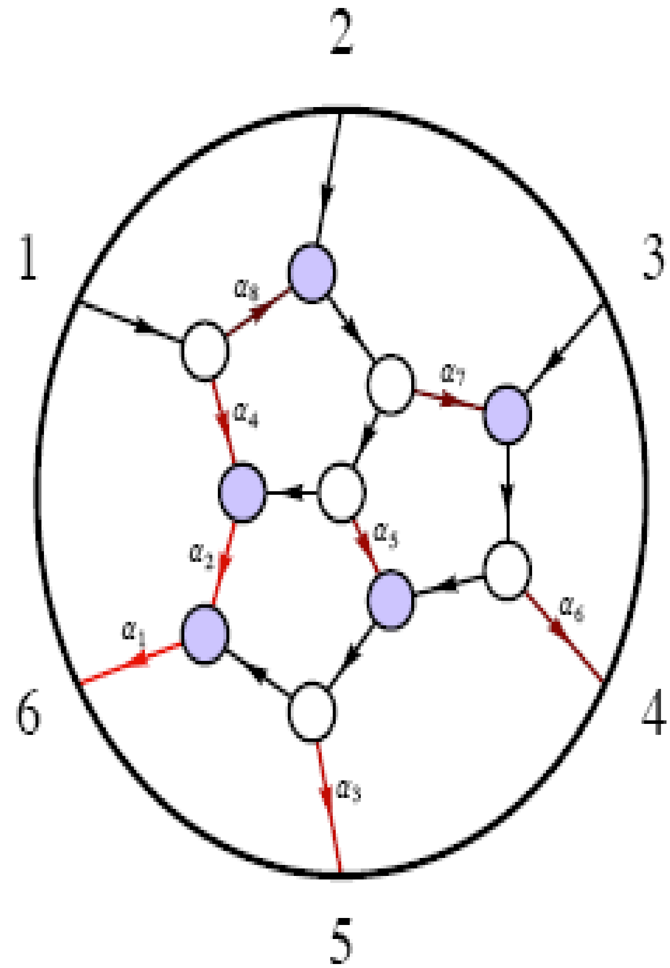
$1 \longrightarrow p(n)$

BCFW Perm. Decomp.

		(12)	(23)	(34)	(23)	(35)	(12)	(23)	(36)
1 →	4	6	6	6	6	6	7	7	7
2 →	6	4	5	5	7	7	6	8	8
3 →	5	5	4	7	5	8	8	6	9
4 →	7	7	7	4	4	4	4	4	4
5 →	8	8	8	8	8	5	5	5	5
6 →	9	9	9	9	9	9	9	9	6

Sequence of Adjacent Transpositions


Transposistion: {3, 6}
Transposistion: {2, 3}
Transposistion: {3, 5}
Transposistion: {1, 2}
Transposistion: {2, 3}
Transposistion: {3, 4}
Transposistion: {2, 3}
1
2
3
4
5
6



1
2
3
4
5
6
α_1
α_2
α_3
α_4
α_5
α_6
α_7
α_8

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix} \xrightarrow[6\to 6+z_1 3]{(36)} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & z_1 \end{bmatrix}$$

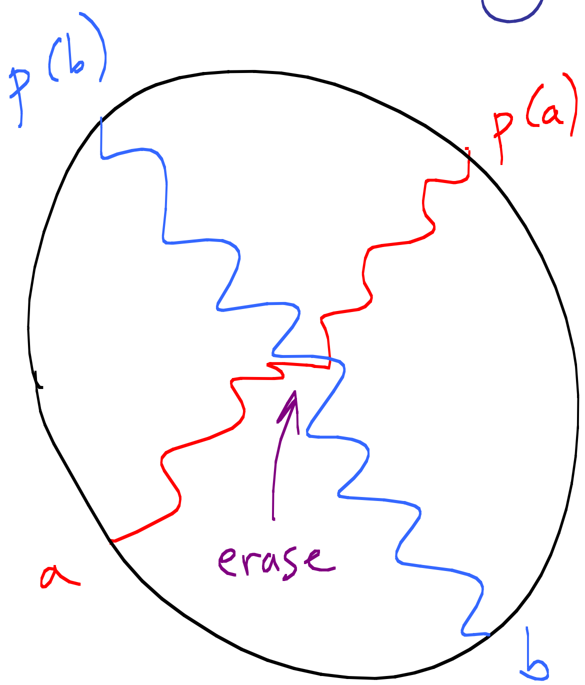
$$\xrightarrow[3\to 3+z_2 2]{(23)} \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & z_2 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & z_1 \end{bmatrix} \quad \cdots\cdots$$

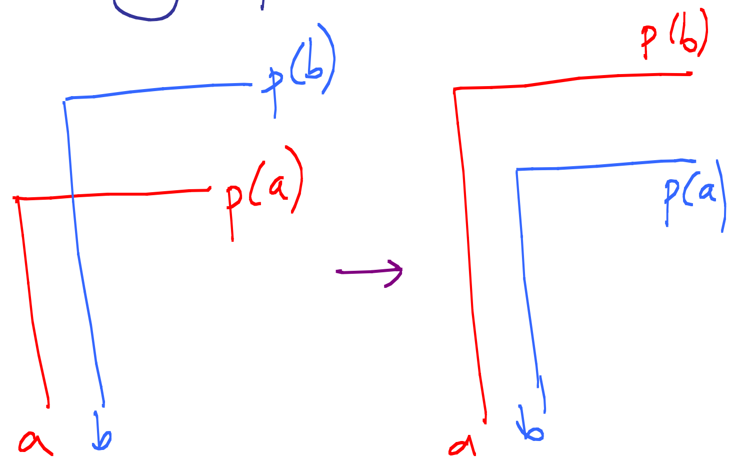
$$\longrightarrow \begin{bmatrix} 1 & z_4+z_8 & z_4z_5+z_4z_7 & z_4z_5z_6 & 0 & 0 \\ 0 & 1 & z_2+z_5+z_7 & (z_2+z_5)z_6 & z_2z_5 & 0 \\ 0 & 0 & 1 & z_6 & z_3 & z_1 \end{bmatrix}$$

(Manifestly Positive)

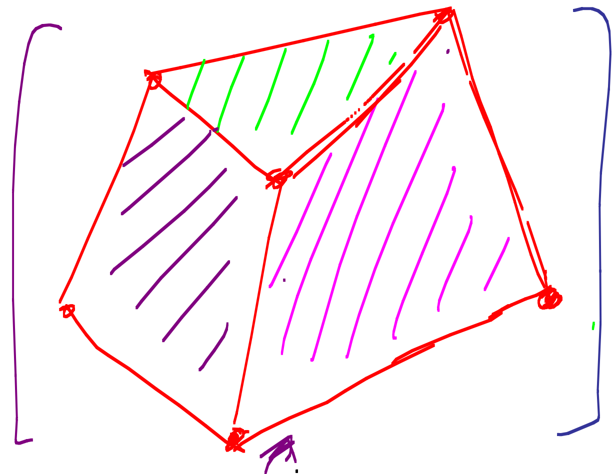
Singularities / Residues

Erase an edge [so graph still reducible!]

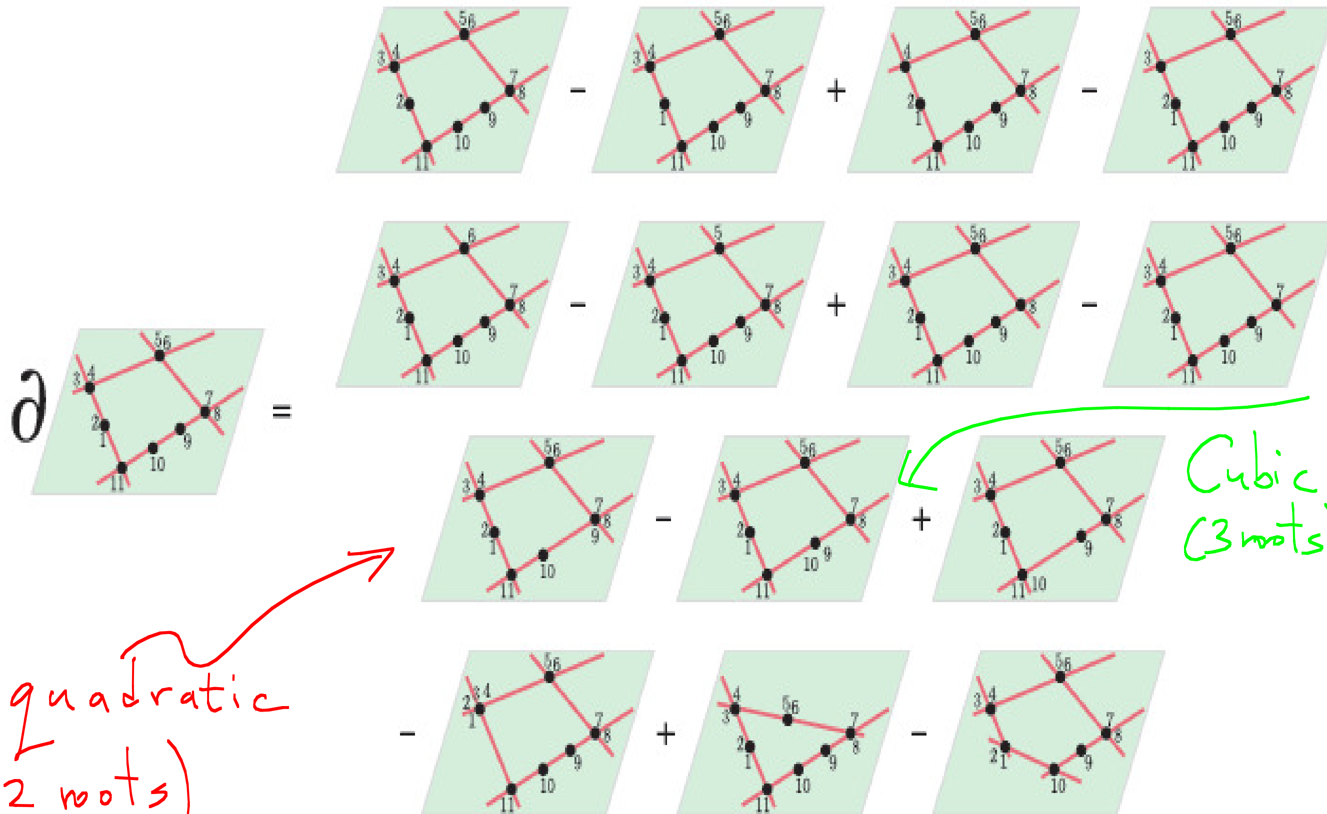




Relations

∂  $= 0$

Ex: 14-term identity involving rationals, $\sqrt{\ }$'s, $\sqrt[3]{\ }$'s:

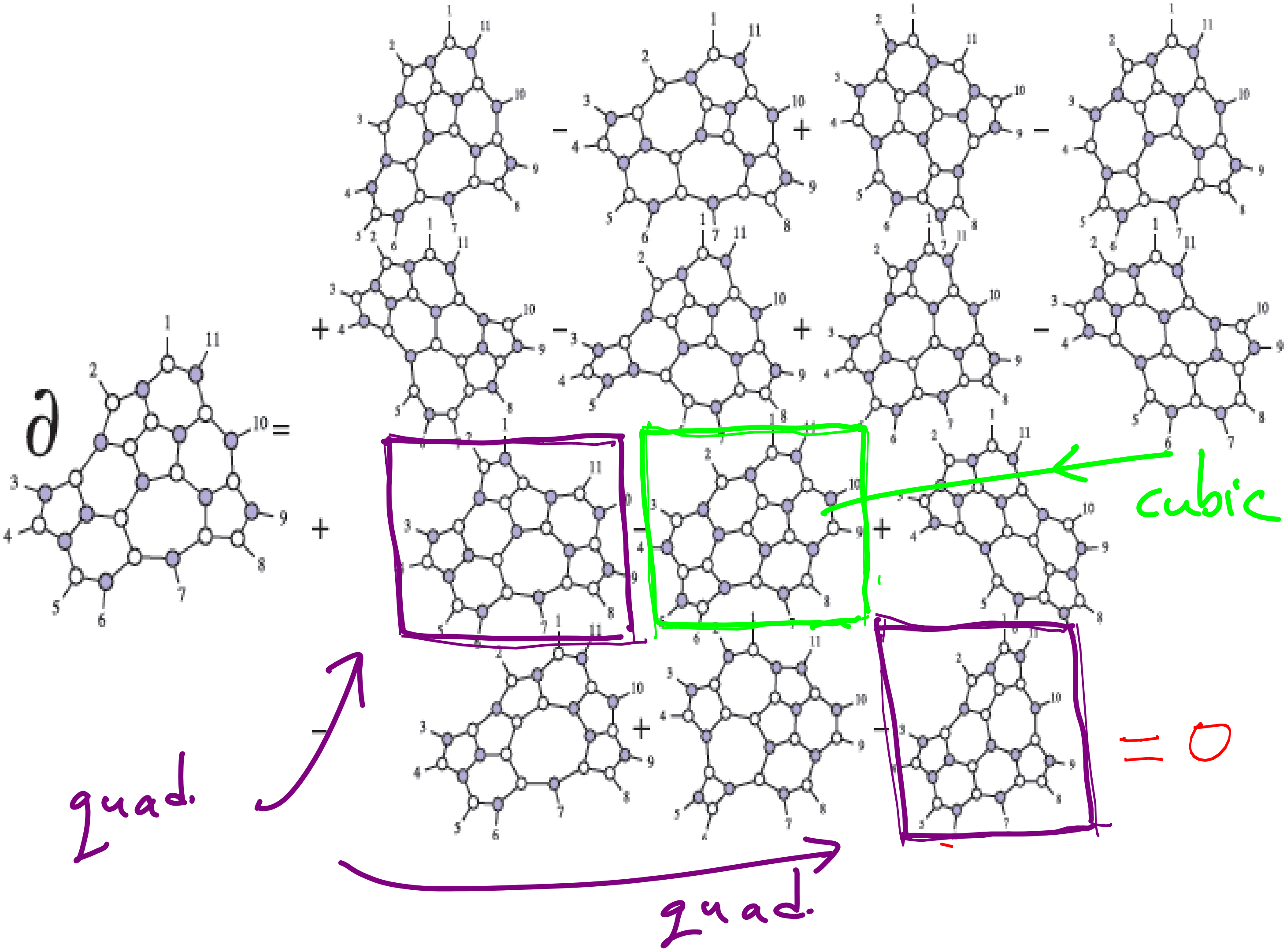


Cubic (3 roots)

quadratic (2 roots)

quadratic (2 roots)

$$= 0$$



∂
cubic
quad.
quad.
= 0

Invariant Top Form

$$\frac{d^{k\times n}C}{(12..k)..(n1..k-1)} \Big/ GL(K) = \frac{df_1}{f_1} \cdots \frac{df_{k(n-k)}}{f_{k(n-k)}}$$

Logarithmic singularities
only on boundary of positive part!

Purpose in life: Makes

Dual. Conf. Inv. Manifest

At level of permutation

$a \rightarrow p(a)$

Original space

$a \rightarrow p(a) - 2$

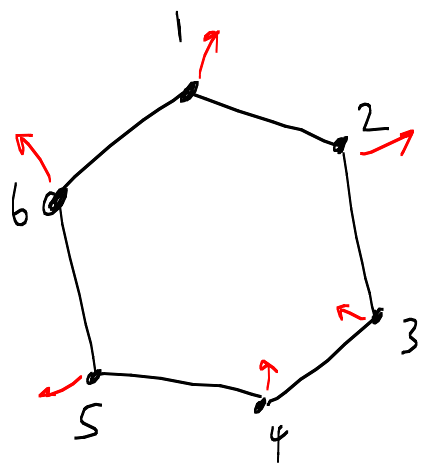
$(k \rightarrow k-2)$

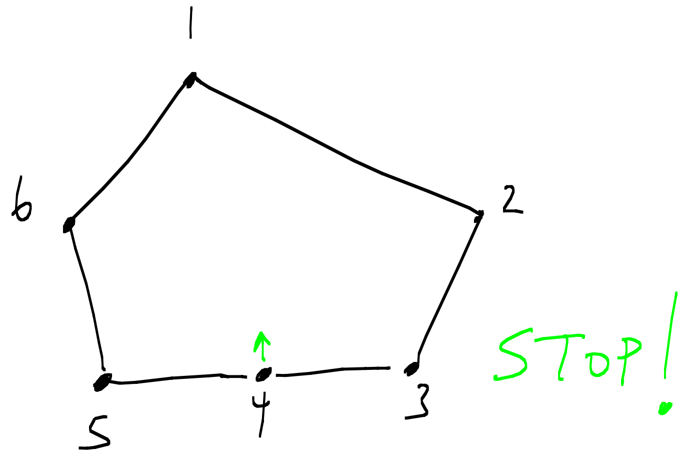
Dual Space

Yangian = Diffs on Positive Part

Obvious symmetry of Geometry

Beautiful action of BCFW vars.





Must have (at least!),

$\delta(i\ i{+}1 \ldots i{+}k{-}1) \to 0$ as $(i\ i{+}1 \ldots i{+}k{-}1) \to 0$

• First non-trivial variations quadratic in C's:

$$\delta \vec{C}_a = (\vec{C}_a \cdot \vec{\xi})\, \Omega^a_{\ b}\, \vec{C}_b$$

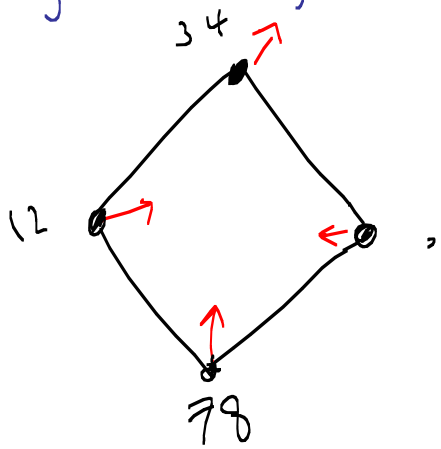
k-vector   parameters

• Demanding $\delta(12\ldots k) \to 0$ when $(12\ldots k) \to 0$

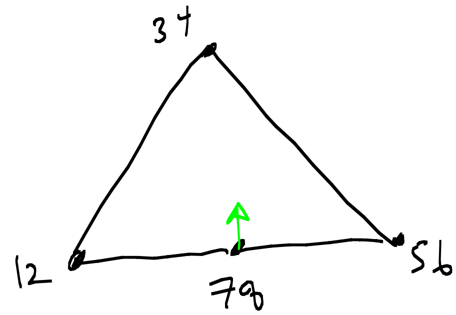
fixes

$$\delta \vec{C}_a = (\vec{C}_a \cdot \vec{\xi}) \sum_{a<b} W_b \vec{C}_b \,; \quad W_c \vec{C}_c = 0.$$

Remarkably, this is enough to guarantee that these diffs act nicely on all faces of the positive part!



,



STOPS!

i.e. in this config, $\delta(571) \to 0$ as $(571) \to 0$.

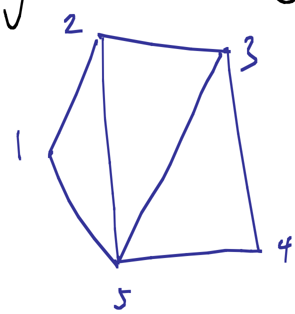
Of course this is exactly how the level-one Yangian generator $\sum_{a<b} \left[\mathcal{W}^A \frac{\partial}{\partial \mathcal{W}^C} \mathcal{W}^C \frac{\partial}{\partial \mathcal{W}^B} \right]$

acts on $\int d^{k \times n} C \; f(C) \; \delta^{4|4}(C \cdot \mathcal{W})$

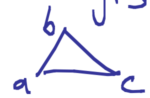
"Grassmannian rep. of Yangian".

{The Positive Grassmannian is an example of a Cluster Variety. In addition to this natural top form, there is a natural 2-form [Symplectic Structure]

e.g. $G(2,n)$



$$\Omega^{(2)} = \sum_{\text{triangles}} \left[d\log(ab)\, d\log(bc) + \text{cyclic} \right]$$

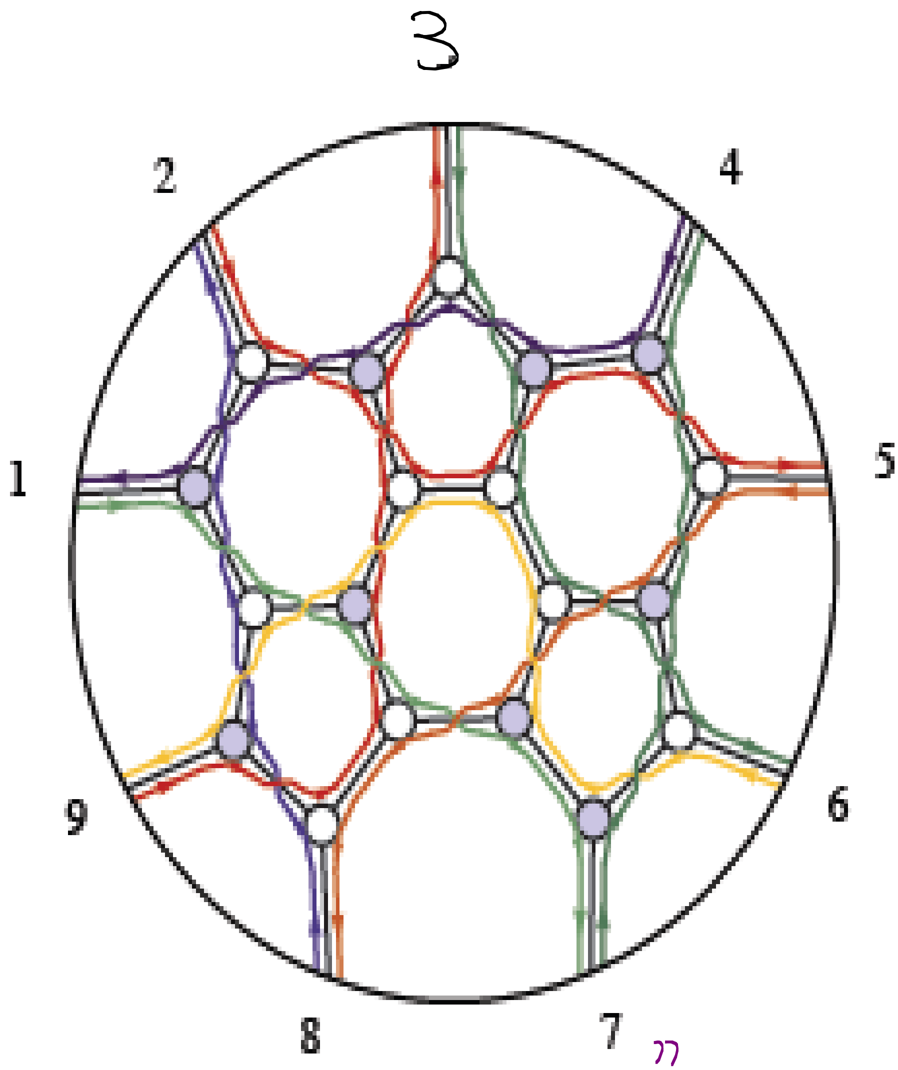


}

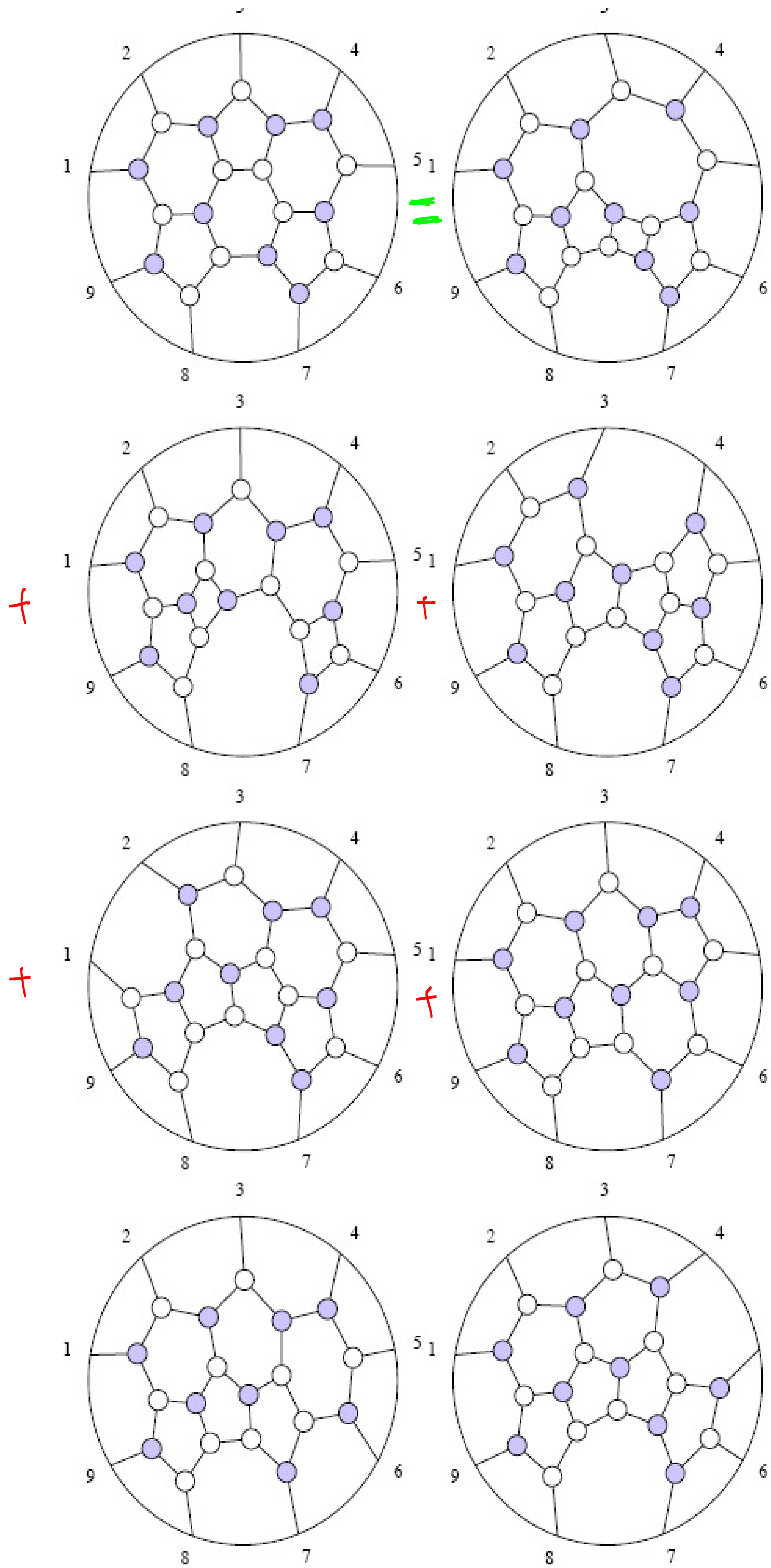
Easy to see, for LS, and generic external data:

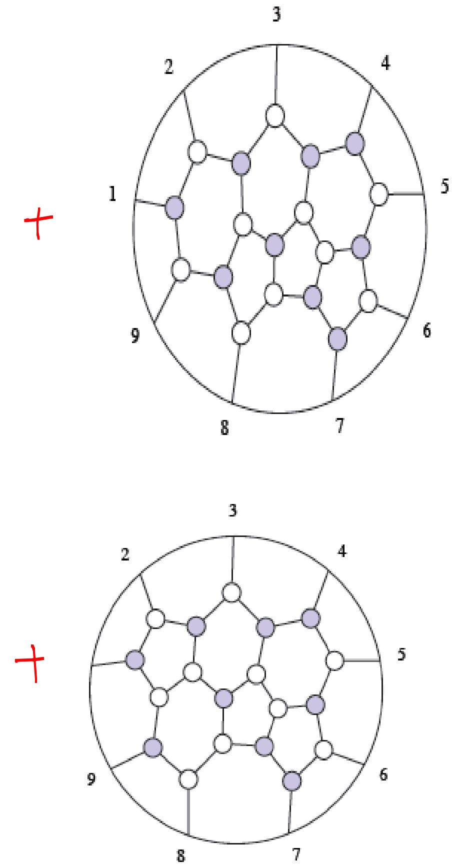
No new objects for $n \geq 5(K-2)$

e.g. for $K=2$: BCFW terms, 4 m box and ....



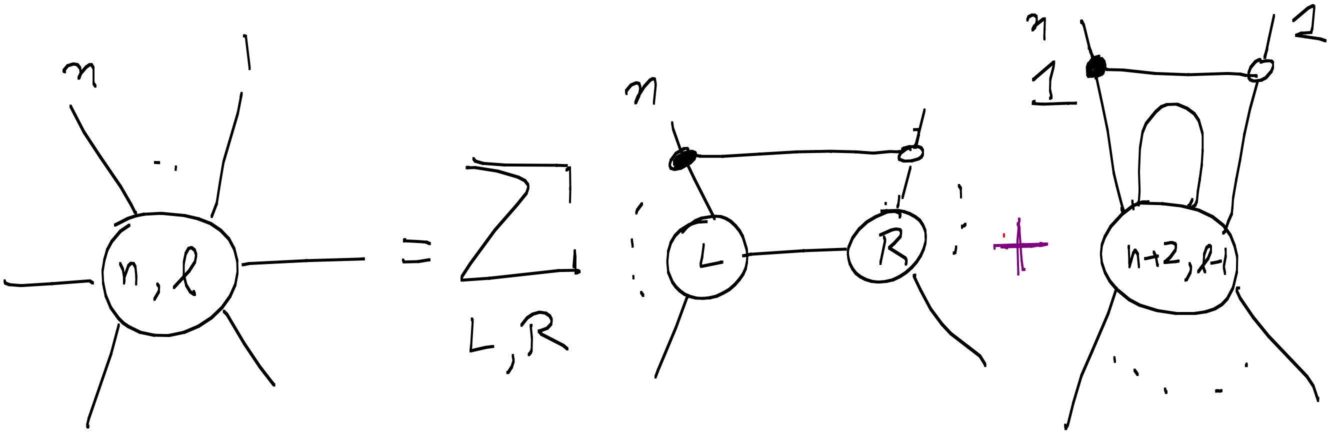
“Spurion”





[All on RHS are BCFW terms]

All-Loop Integrand in Grassm.



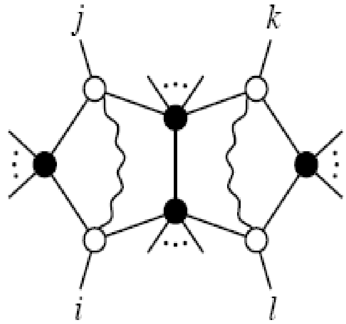
$\longrightarrow$ Explicitly, Σ's of dlog's!

Complete definition, where

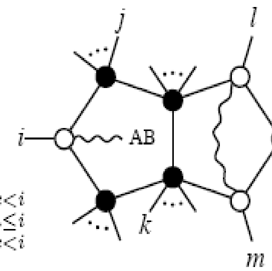
The words "spacetime", "Lagrangian", "Path Integral", "Gauge Symmetry"...

make no appearance.

$$\mathcal{A}^{2-\mathrm{loop}}_{\mathrm{MHV}} = \frac{1}{2}\sum_{i<j<k<l<i}$$

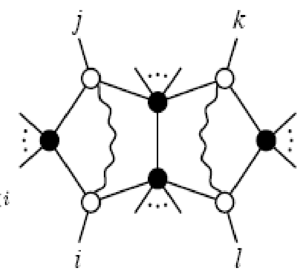


$$\mathcal{A}^{2-\mathrm{loop}}_{\mathrm{NMHV}} = \sum_{\substack{i<j<l<m\leq k<i\\ i<j<k<l<m\leq i\\ i\leq l<m\leq j<k<i}}$$



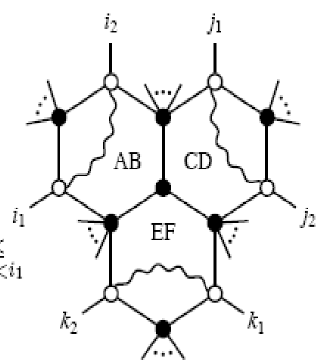
$$\times\,[i, j, j+1, k, k+1]$$

$$+\frac{1}{2}\sum_{i<j<k<l<i}$$

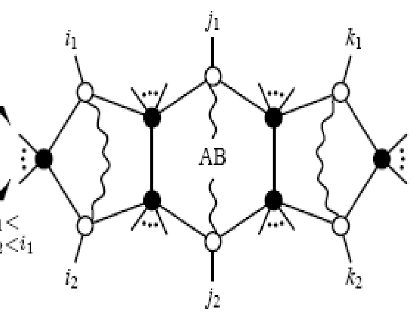


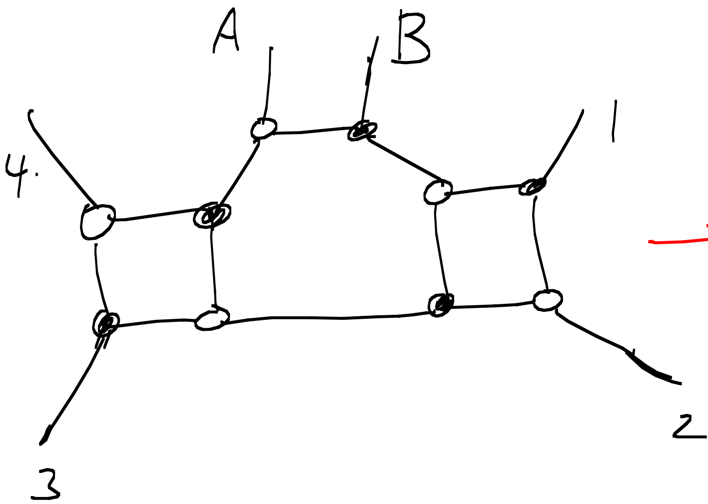
$$\times\left\{\begin{array}{c} \mathcal{A}^{\mathrm{tree}}_{\mathrm{NMHV}}(j,\ldots,k;\, l,\ldots,i) \\ +\mathcal{A}^{\mathrm{tree}}_{\mathrm{NMHV}}(i,\ldots,j) \\ +\mathcal{A}^{\mathrm{tree}}_{\mathrm{NMHV}}(k,\ldots,l) \end{array}\right\}$$

$$\mathcal{A}^{3-\mathrm{loop}}_{\mathrm{MHV}} = \frac{1}{3}\sum_{\substack{i_1\leq i_2<j_1\leq\\ \leq j_2<k_1\leq k_2<i_1}}$$

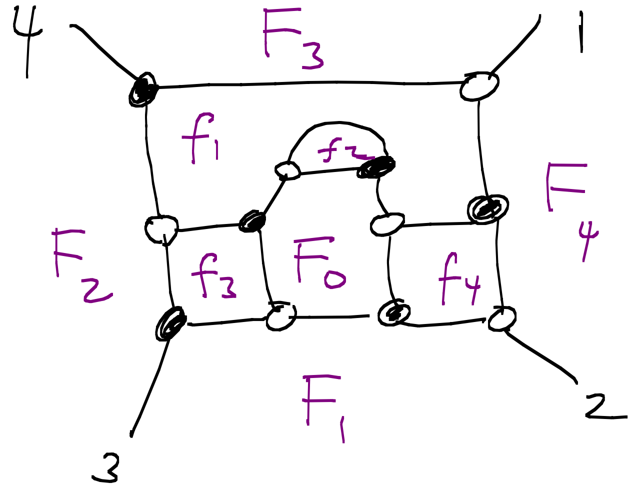


$$+\frac{1}{2}\sum_{\substack{i_1\leq j_1<k_1<\\ <k_2\leq j_2<i_2<i_1}}$$

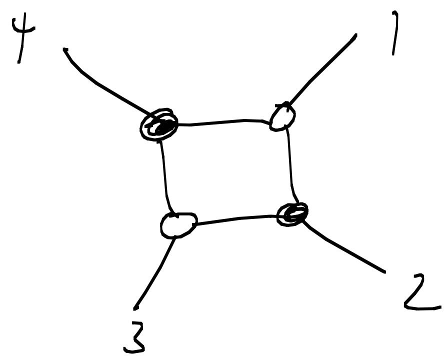




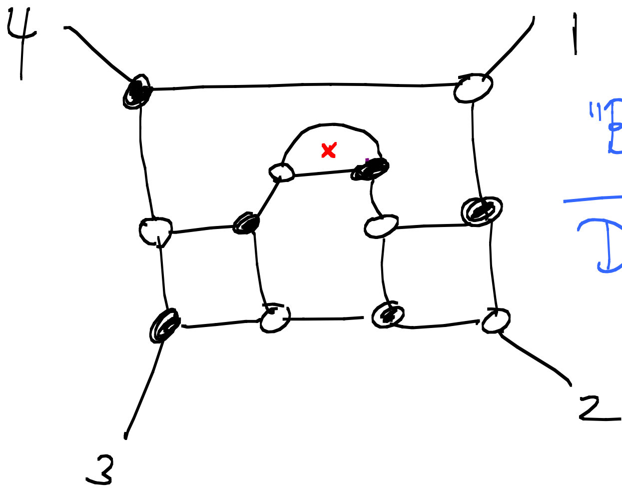
$\longrightarrow$



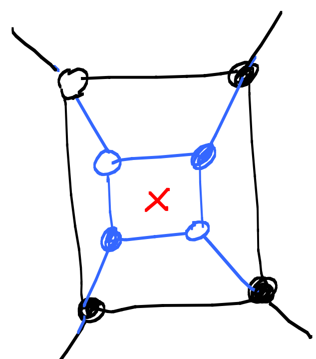
$$\longrightarrow \quad \underbrace{d\log f_1 \cdots d\log f_4}_{\text{Loop Integrand}}$$



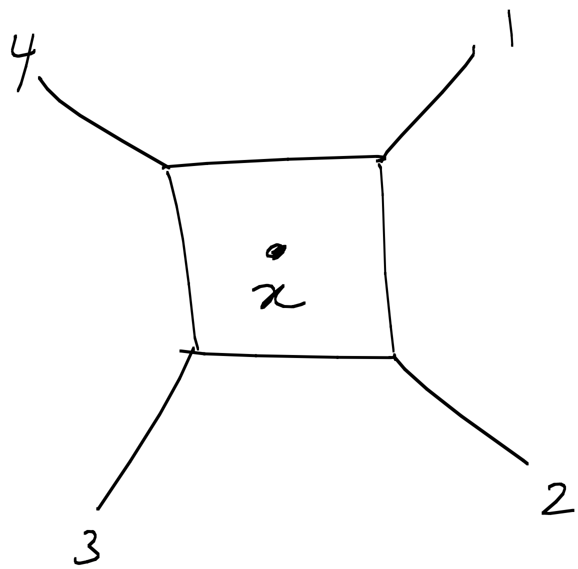
New Picture For Loops



"Bridge Decomp"



Completely On Shell



$$\frac{\mathcal{N}\, d^4 x}{(x-x_1)^2(x-x_2)^2(x-x_3)^2(x-x_4)^2}$$

$$= d\log\frac{(x-x_1)^2}{(x-y)^2}\; d\log\frac{(x-x_2)^2}{(x-y)^2}\; d\log\frac{(x-x_3)^2}{(x-y)^2}\; d\log\frac{(x-x_4)^2}{(x-y)^2}$$

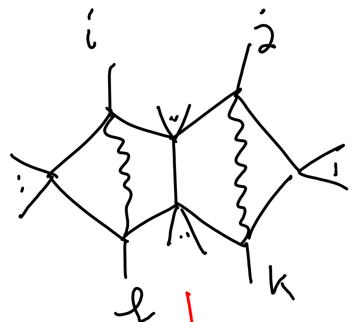
y, y^* : Null-separated from $x_{1,2,3,4}$

$$\mathcal{M}^{1\text{-loop}}_{MHV} = \Omega \wedge \Omega \,, \quad \Omega = \sum_i d\log \frac{\langle AB\, l\, i\rangle}{\langle AB\, i\, i{+}1\rangle}\, d\log \frac{\langle AB\, l\, i{+}1\rangle}{\langle AB\, i\, i{+}1\rangle}$$

$\shortparallel$

Fock-Goncharov 2-form!

$$\mathcal{M}^{2\text{-loop}}_{MHV} = \sum_{i,j,k,l}$$



$$d\log \frac{\langle AB\, i{-}1\, i\rangle}{\langle ABCD\rangle}\; d\log \frac{\langle AB\, i\, i{+}1\rangle}{\langle ABCD\rangle}\; d\log \frac{\langle AB\, l{-}1\, l\rangle}{\langle AB \cap (l{-}1\, l\, l{+}1)\, i\, CD\rangle}\; d\log \frac{\langle AB\, l\, l{+}1\rangle}{\langle AB \cap (l{-}1\, l\, l{+}1)\, i\, CD\rangle}$$

$$d\log \frac{\langle CD\, j{-}1\, j\rangle}{\langle CD\, i\, l\rangle}\; d\log \frac{\langle CD\, j\, j{+}1\rangle}{\langle CD\, i\, l\rangle}\; d\log \frac{\langle CD\, k{-}1\, k\rangle}{\langle CD \cap (k{-}1\, k\, k{+}1)\, j\, i\, l\rangle}\; d\log \frac{\langle CD\, k\, k{+}1\rangle}{\langle CD \cap (k{-}1\, k\, k{+}1)\, j\, i\, l\rangle}$$

• The Fact that the Integrand can be decomposed into dlogs is extremely difficult to see from any space-time picture — instead is handed to us on a platter by the Combinatorial/Grassmannian structure.

Loops From Spacetime

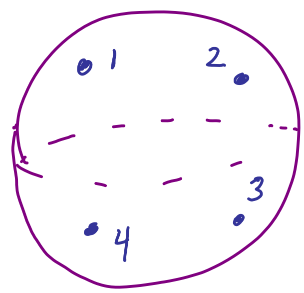
"Why are amplitudes [not] Polylogs?"

"Can we see string worldsheet picture?"

$$I = \int d\log\frac{P}{P^*}\; d\log\frac{P_1}{P_2}\; d\log\frac{P_2}{P_3}\; d\log\frac{P_3}{P_4}$$

$$\downarrow$$

$$\int_{S^2} \log\frac{P_1}{P_2}\; d\log\frac{P_2}{P_3}\; d\log\frac{P_3}{P_4}$$



$$dI = d\log \frac{P_3(12)\,P_4(12^*)}{P_3(12^*)\,P_4(12)}$$

$$\times \int_{S^2} d\log \frac{P_1}{P_2}\; d\log \frac{P(12)}{P(12^*)}$$

$\longrightarrow$ Recursive [+ gives "symbol" directly]

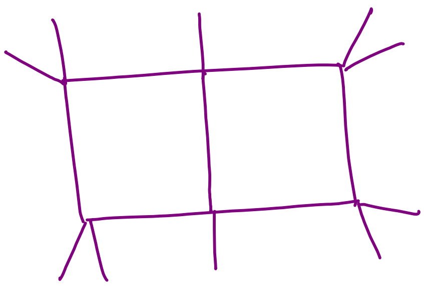
$$d\, I_D = \sum_{i,j} d\log\left[\frac{P_i(\ell_{ij})\, P_j(\ell_{ij}^*)}{P_i(\ell_{ij}^*)\, P_j(\ell_{ij})}\right]$$

$$\times\, I_{\hat{i}\hat{j}}^{D-2}$$

$\Big\{$ c.f. Goncharov for Even D $\Big\}$

Extends to Multi-loops ....

4 ext. points $\longrightarrow$ Localized to a Sphere.

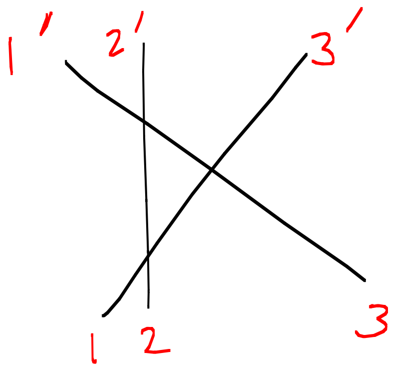


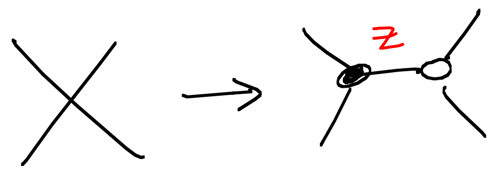
$\longrightarrow$ dlog $\wedge \dots \wedge$ dlog $\wedge$ d Elliptic

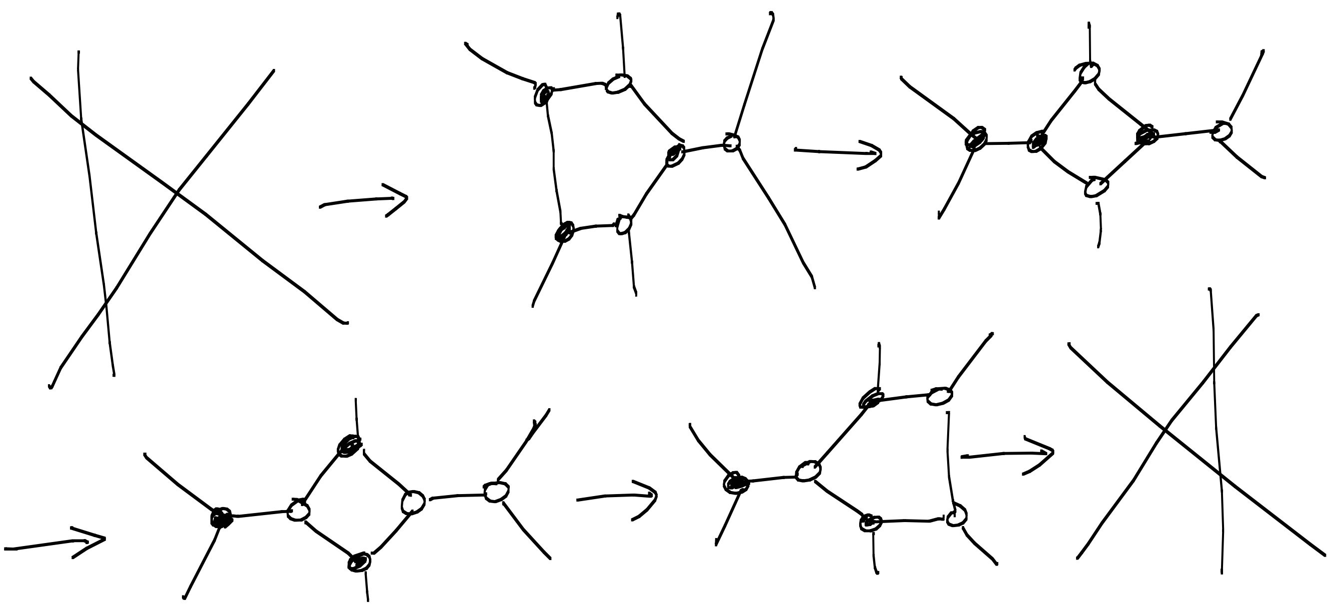
Yang-Baxter +

ABJM

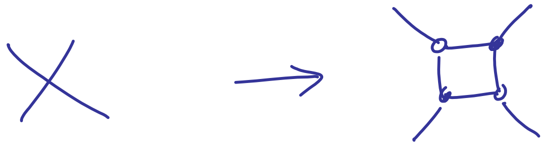
Old Perm. $\subset$ New Perm.

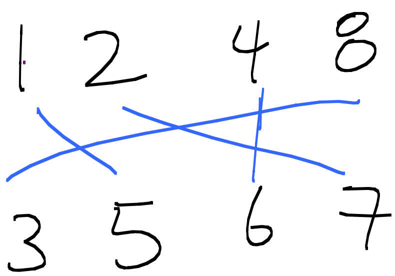


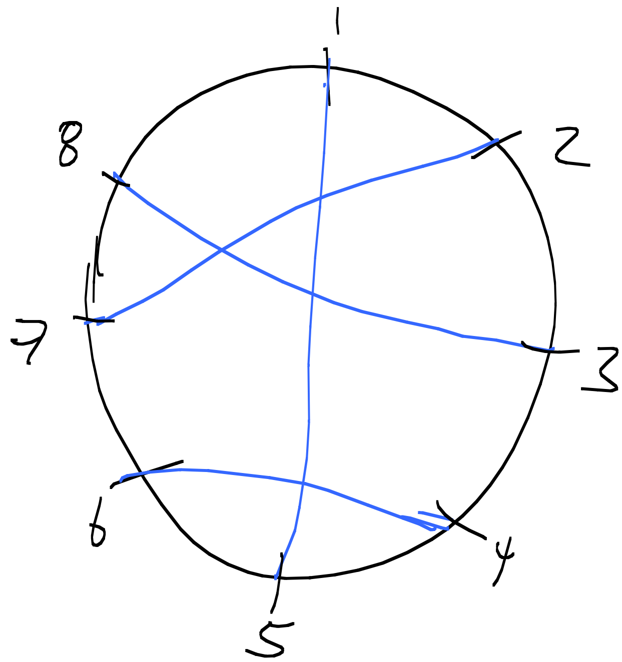




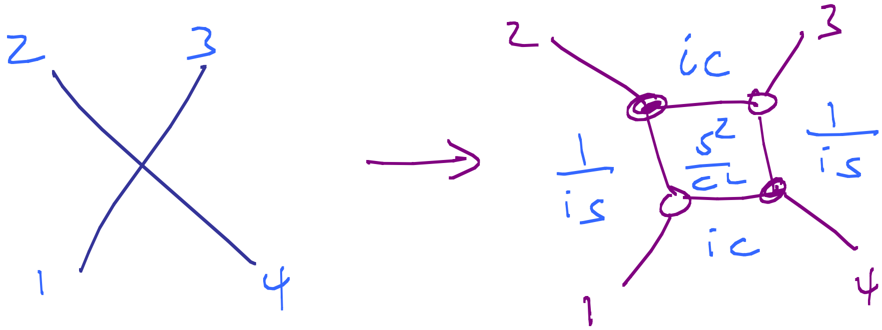
Yang-Baxter consequence of more "atomic" square move





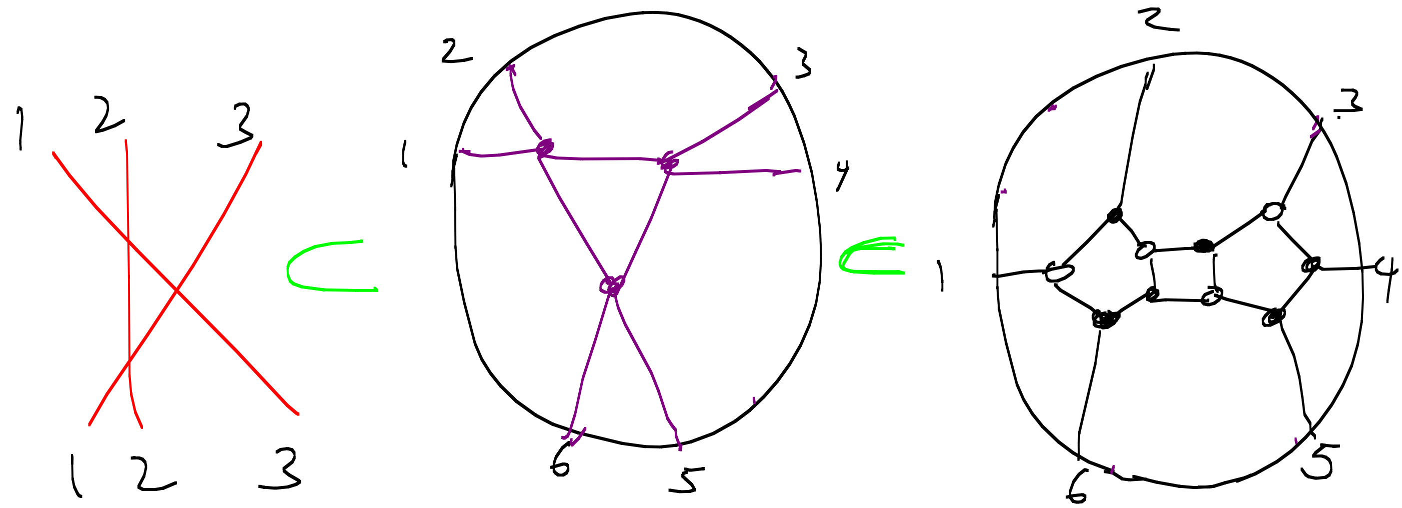


On-shell diagrams for ABJM



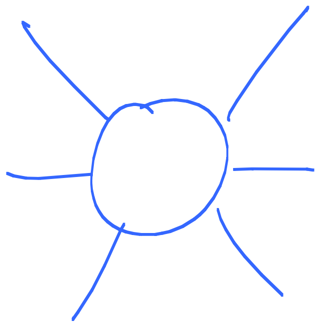
$$\begin{array}{cccc} 1 & 2 & 3 & 4 \\ \end{array}$$
$$\begin{bmatrix} 1 & 0 & ic & is \\ 0 & 1 & -is & ic \end{bmatrix}$$

"Orthogonal"
Grassmannian

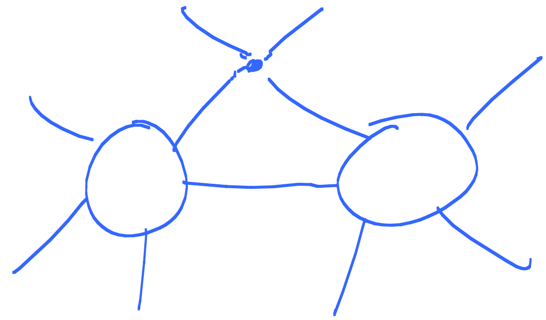


$1+1-d \subset 2+1-d \subset 3+1-d$

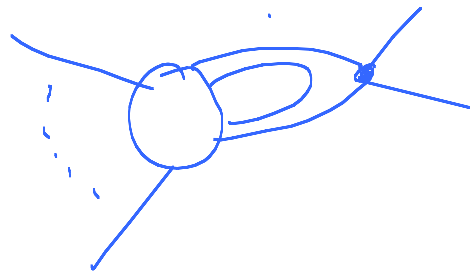
Y-B $\subset$ ABJM $\subset$ $\mathcal{N}=4$ SYM



$= \sum$

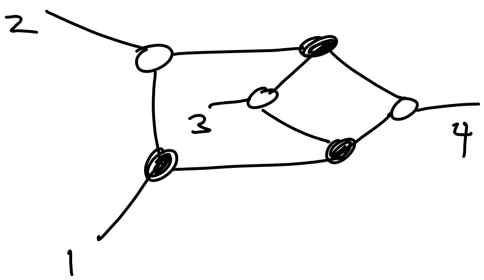


$+$



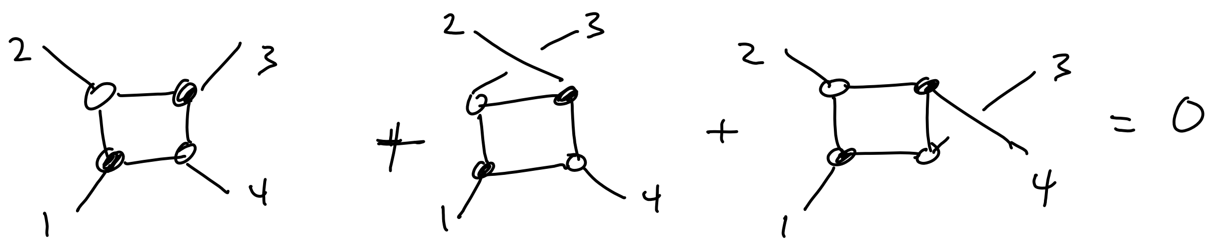
Beyond $\left\{ \begin{array}{l} \text{Planarity} \\ \mathcal{N}=4 \text{ SYM} \\ \text{Amplitudes} \end{array} \right.$

Non-Planar Diagrams

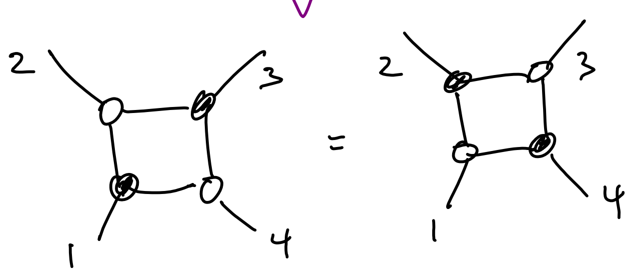


Can't be reduced according to usual rules

Most Basic Identity

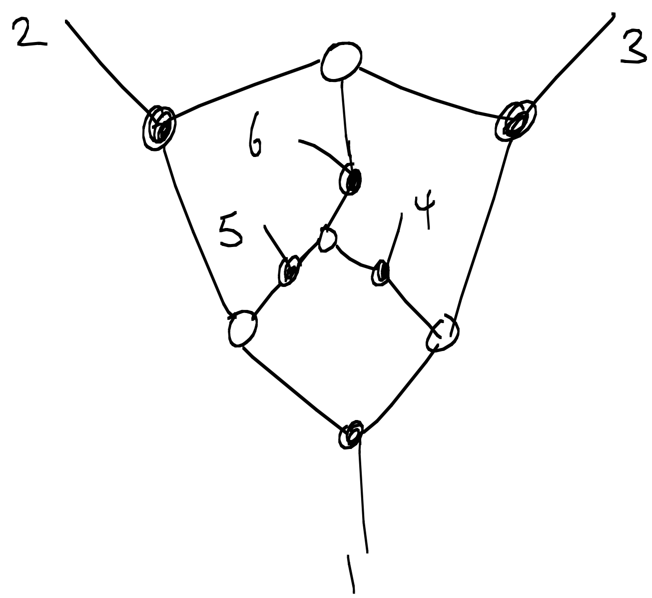


Twice



On-shell diagrams for Non-planar amplitudes

* Solved for MHV:
  - Reduction
  - Computing reduced graphs



$$= \frac{\left(\langle 14\rangle\langle 36\rangle\langle 25\rangle - \langle 15\rangle\langle 26\rangle\langle 34\rangle\right)^2}{\langle 12\rangle\langle 13\rangle\langle 14\rangle\langle 15\rangle\langle 23\rangle\langle 25\rangle\langle 26\rangle\langle 34\rangle\langle 36\rangle\langle 45\rangle\langle 46\rangle\langle 56\rangle}$$

= Sum of PT factors !

On-Shell Diagrams for $\mathcal{N}<4$

$$\left(\prod \frac{de_i}{e_i}\right) J^{\mathcal{N}-4} \prod_{\alpha=1}^{K} \delta^{4|\mathcal{N}}\left[C_{\alpha a}[e]\, W_a\right]$$

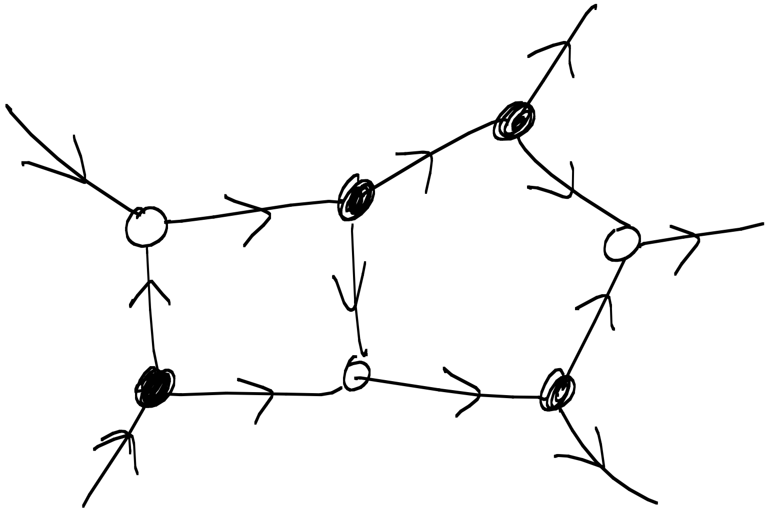
New

Universal

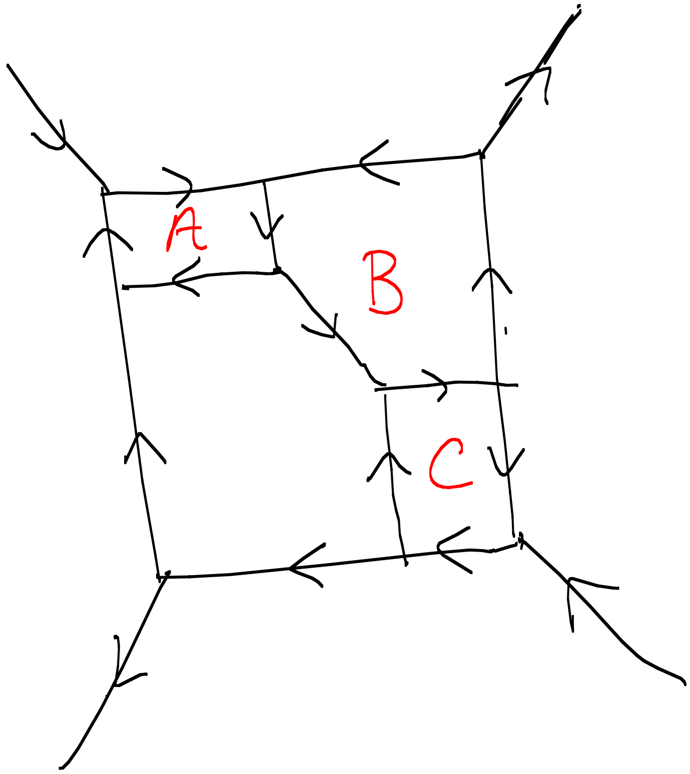
$$J = \det(\mathbb{1} - A)$$

$$A_{ij} = \begin{cases} 0 & \text{if no } i \to j \\ e_{ij} & \text{if } i \to j \end{cases}$$

"Spectrum" of Graph



$J = 1$ if no "orientation loops"



$$J = 1 + A + B + C + AC$$

* Some residues

$$\prod_i \underbrace{\frac{de_i}{e_i}}_{\text{residues here}} \delta^4(C\cdot W)\, J^{\mathcal{N}-4}$$

multiplicative factor on $\mathcal{N}=4$ answer

* More interesting

$$\prod \frac{de_i}{e_i} \; \underset{\sim\sim\sim\sim\sim}{J^{N-4}} \qquad \delta^{4|N}(C\cdot W)$$

also use these singularities!

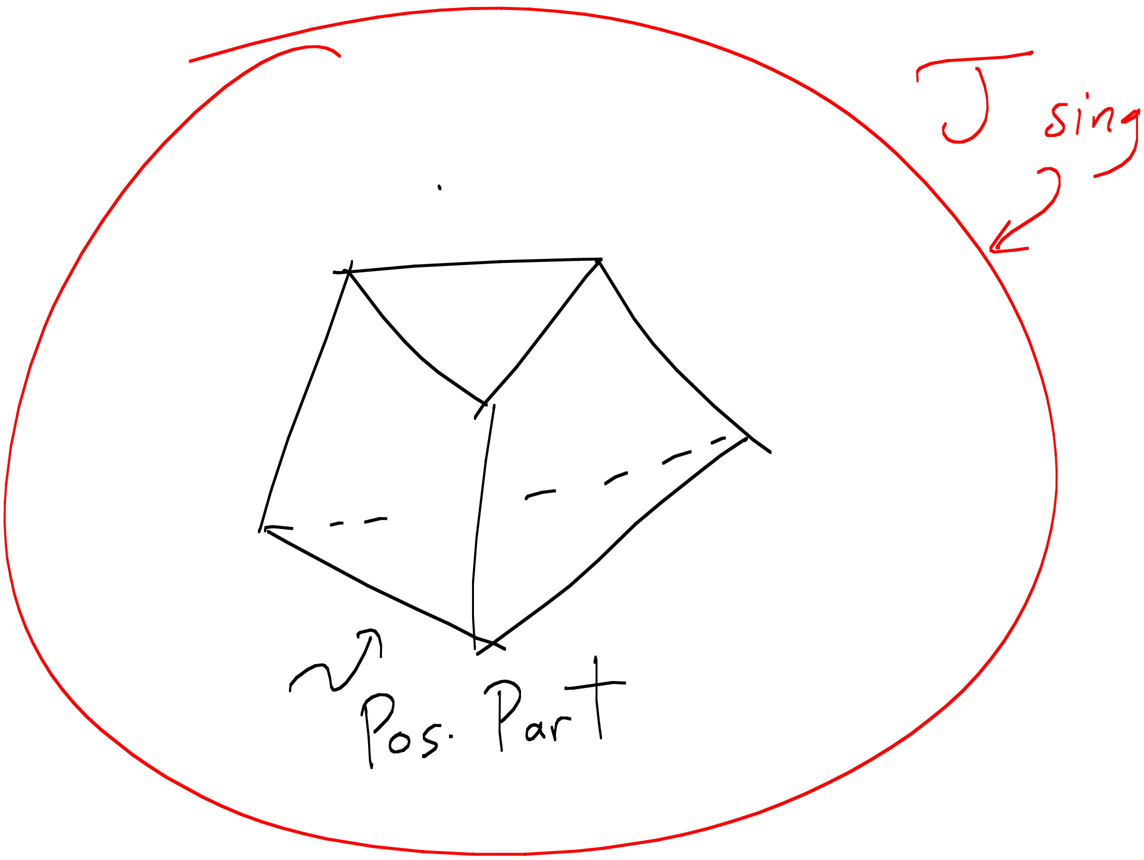
These Correspond to "Poles at ∞"

* Triangles ....
* Bubbles + β-functions
* BCFW residues @ ∞ for:

"bad shifts"

⋮

Remarkably



Locus of $\mathcal{J} = 0$ "UV"

Positive Grassmannian "IR"

Naturally Paired

* Amazingly — exactly the same mathematical structure shows up in $\mathcal{N}=1$ Quiver Gauge Theories,

[also cluster structures in wall-crossing, Amplitudes @ strong coupling, …]

- We are seeing, very concretely, more primitive ideas underlying scattering amplitudes. Locality, Unitarity are not the Stars Of The Show.

Much of This Magic Clearly descends from (2,0) theory — can we see structure in 6D?

Next Frontier In This Business

~~Gauge Red.~~

Find a Dual
Formulation of
String Scattering Amplitudes.