Gravity localization & AdS4 / CFT3

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STRINGS 2012 (Munich)

based on:

CB, J. Estes, *arXiv:1103.2800 [hep-th]*

B. Assel, CB, J. Estes, J. Gomis, *1106.4253 [hep-th]*

1207.xxxx [hep-th]

Standard Hypothesis:

4D Graviton

→ is **massless**

→ couples **universally**

to all forms of matter & radiation

These properties are automatic in any **compactification**

of string theory *(or higher-Dim theory of gravity)* ....

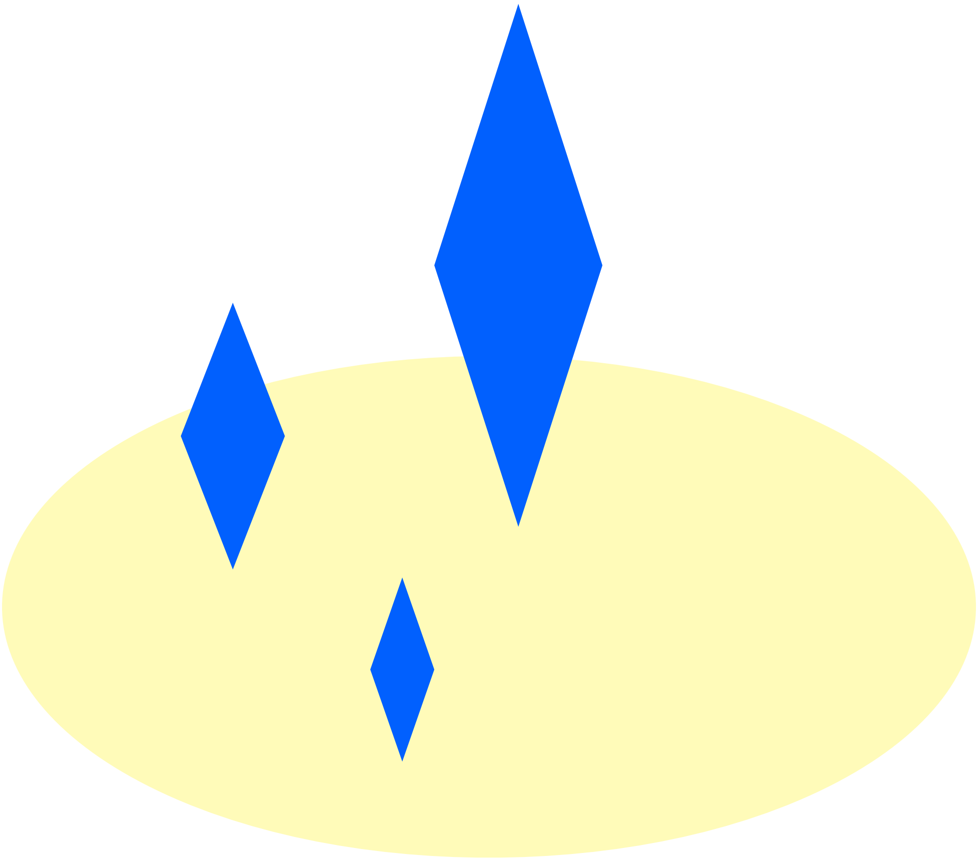
[Note: I will stick with classical 2-derivative supergravities]

KK reduction for spin 2:

Consider *warped*-(A)dS vacuum,

$$\widehat{ds^2} = e^{2A(y)} \boxed{\bar{g}_{\mu\nu}(x)\, dx^\mu dx^\nu} + \boxed{\hat{g}_{ab}(y)\, dy^a dy^b}$$

$$\boxed{\begin{gathered}\bar{\mathcal{M}}_4 = \mathrm{AdS}_4,\ \mathbb{M}_4,\ \mathrm{dS}_4 \\ k = -1, 0, 1\end{gathered}}$$



Consistent reduction of (spin-2) metric perturbations:

$$ds^2 = e^{2A}\,(\bar{g}_{\mu\nu} + h_{\mu\nu})\,dx^\mu dx^\nu + \hat{g}_{ab}\,dy^a dy^b \;,$$

with $\quad h_{\mu\nu}(x,y) \;=\; h^{[\mathrm{tt}]}_{\mu\nu}(x)\,\psi(y)$

where $\quad \boxed{(\bar{\Box}^{(2)}_x - \lambda)\,h^{[\mathrm{tt}]}_{\mu\nu} = 0 \qquad \text{and} \qquad \bar{\nabla}^\mu h^{[\mathrm{tt}]}_{\mu\nu} = \bar{g}^{\mu\nu} h^{[\mathrm{tt}]}_{\mu\nu} = 0\;.}$

Pauli-Fierz equations $\quad (\lambda = m^2 + 2k)$

Linearizing the Einstein equations $R_{MN} - \frac{1}{2} g_{MN} R = T_{MN}$

leads to a **universal Schrödinger problem** in the 6D transverse space:

depends only on geometry,
not on "matter" fields

Brandhuber, Sfetsos;
Csaki, Erlich, Hollowood, Shirman;
CB, Estes

$$\boxed{-\frac{e^{-2A}}{\sqrt{[\hat{g}]}} \left(\partial_a \sqrt{[\hat{g}]}\, \hat{g}^{ab} e^{4A} \partial_b\right) \psi = m^2 \psi}$$

mass operator $\mathcal{M}$

Using the standard norm $\|\psi\|^2 = \int [dy] e^{2A} \psi^* \psi$ one finds :

$$\langle \psi, \mathcal{M}^2 \psi \rangle = \int [dy] e^{4A} \partial_a \psi^* \partial^a \psi$$

$$\boxed{\mathcal{M}^2 \geq 0 \quad \text{and} \quad \mathcal{M}^2 = 0 \longrightarrow \psi_0 = constant}$$

- A massless 4D graviton requires $\int [dy]\, e^{2A} < \infty$ (automatic for smooth compactification)

- Its couplings are universal: $\int [dy]\, e^{2A}\, \psi_0 \phi_i^* \phi_j = \psi_0\, \delta_{ij}$

Can it be otherwise?

- **4D graviton must be massive**
- **transverse space non-compact**

A model for this was proposed ten years ago by **Karch & Randall:**

Thin AdS4 brane in AdS5 bulk

Randall, Sundrum;
Dvali, Gabadadze, Porrati; .....

$$I_{\rm KR} = -\frac{1}{2\kappa_5^2}\int d^4x\, dy\, \sqrt{g}\left(R + \frac{12}{L^2}\right) + \lambda \int d^4x\, \sqrt{[g]_4}\ ,$$

AdS5 radius

brane tension

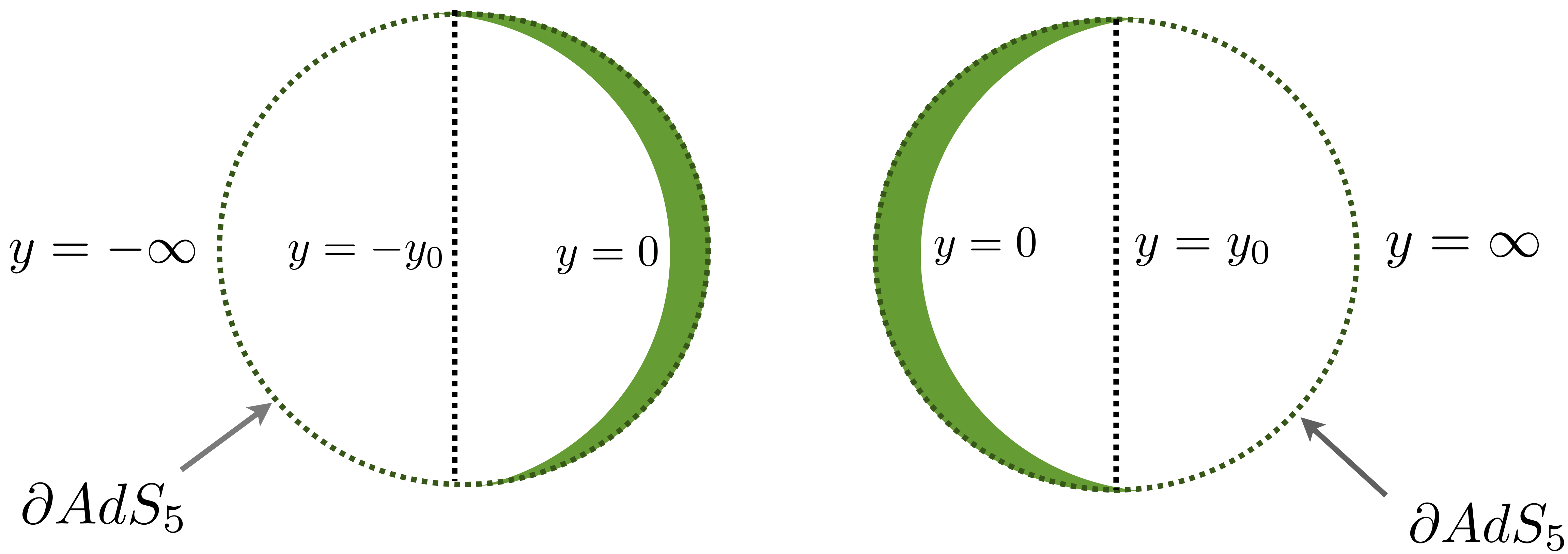
$$ds^2 = L^2 \cosh^2\left(\frac{y_0 - |y|}{L}\right)\, \bar{g}_{\mu\nu} dx^\mu dx^\nu + dy^2$$

AdS4 foliation

Brane radius of curvature: $\ell = L\cosh(y_0/L) = \dfrac{L}{\sqrt{1-\kappa_5^2\lambda L/6}} \gg L$

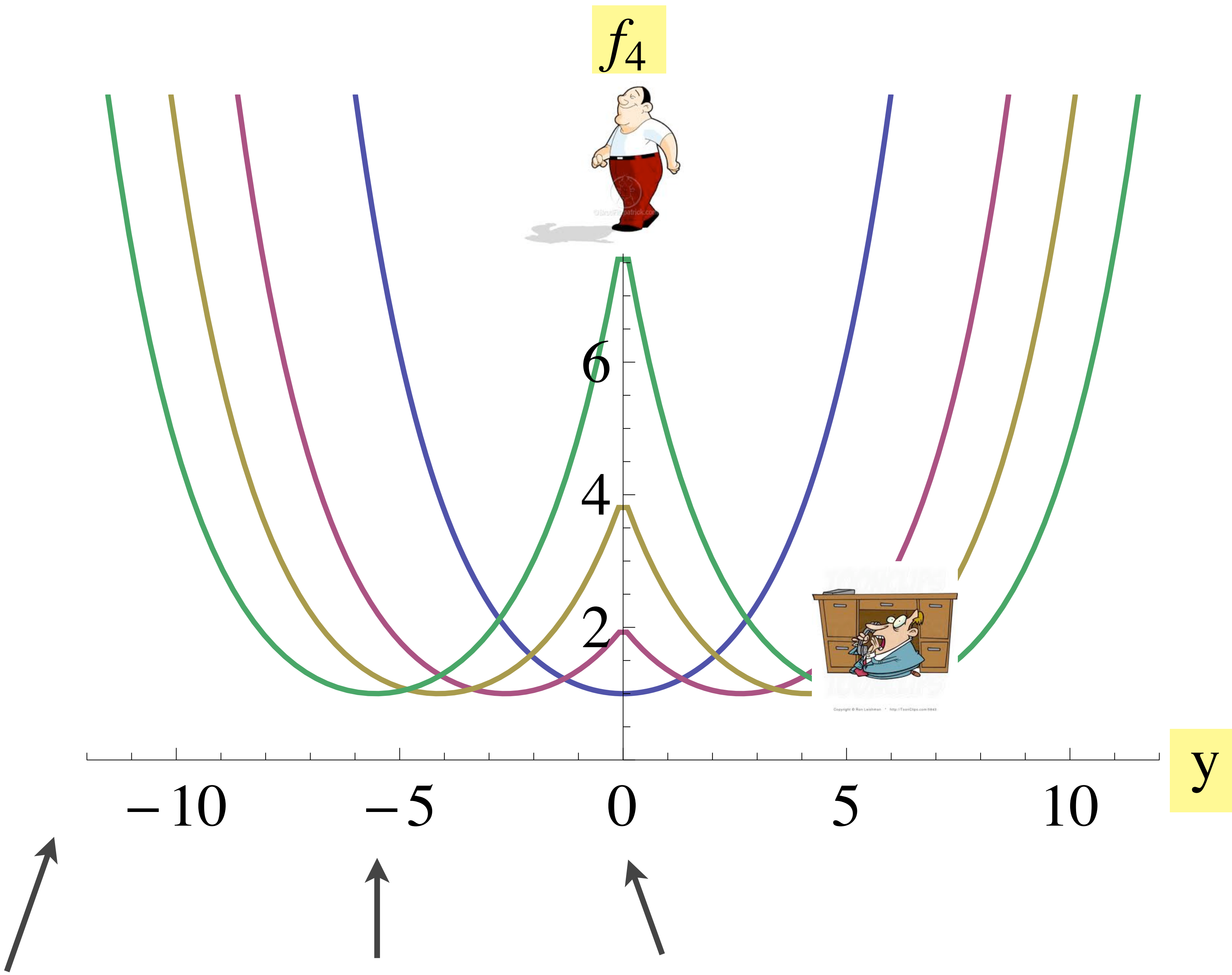
can be tuned so that

Two slices of AdS5 glued along a AdS4 brane:



(cut away green slices and glue)

Warp factor as function of transverse coordinate:



AdS5 boundary

“would be horizon”

3-brane

(Randall-Sundrum II)

Spectrum :

one nearly-constant, **nearly massless** mode

vanishes near bottom of warp factor wells

$$\ell^2 m_0^2 \simeq \frac{3L^2}{2\ell^2}$$

two infinite towers of nearly AdS5 modes

$$\ell^2 m^2 \simeq (2n+1)(2n+4) \qquad n = 0, 1, \cdots$$

supported at bottom of wells

Their wavefunctions are **exponentially suppressed** at the brane position, so they are **hidden from 4D gravity:**

$$\int [dy] e^{2A}\, \psi_0 \phi_h^* \phi_h \ll \psi_0$$

No conventional effective 4D theory, but physics does look 4D at

$$L \ll r \ll \ell$$

$$8\pi G_N \simeq \kappa_5^2/L$$

as in usual KK

Newton's law:

$$V_{\rm Newton} + \Delta V \simeq -\frac{G_N m_1 m_2}{r}\,(1 + \gamma\frac{L^2}{r^2} + \cdots)$$

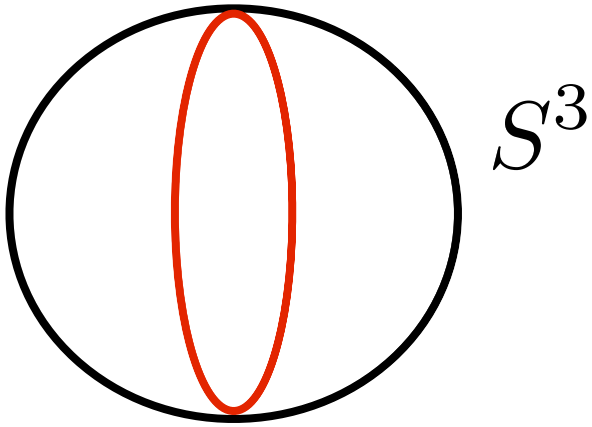
unlike standard KK

Warning: this is not a model for the Universe, since $\Lambda_{\rm eff} < 0$

But **any** long-distance modification of Einstein gravity is (potentially) interesting

Can the KR model be embedded in string theory ?

(holographic duals to conformal domain walls of CFT4)



Boundary in global coordinates

Two beautiful developments got us started:

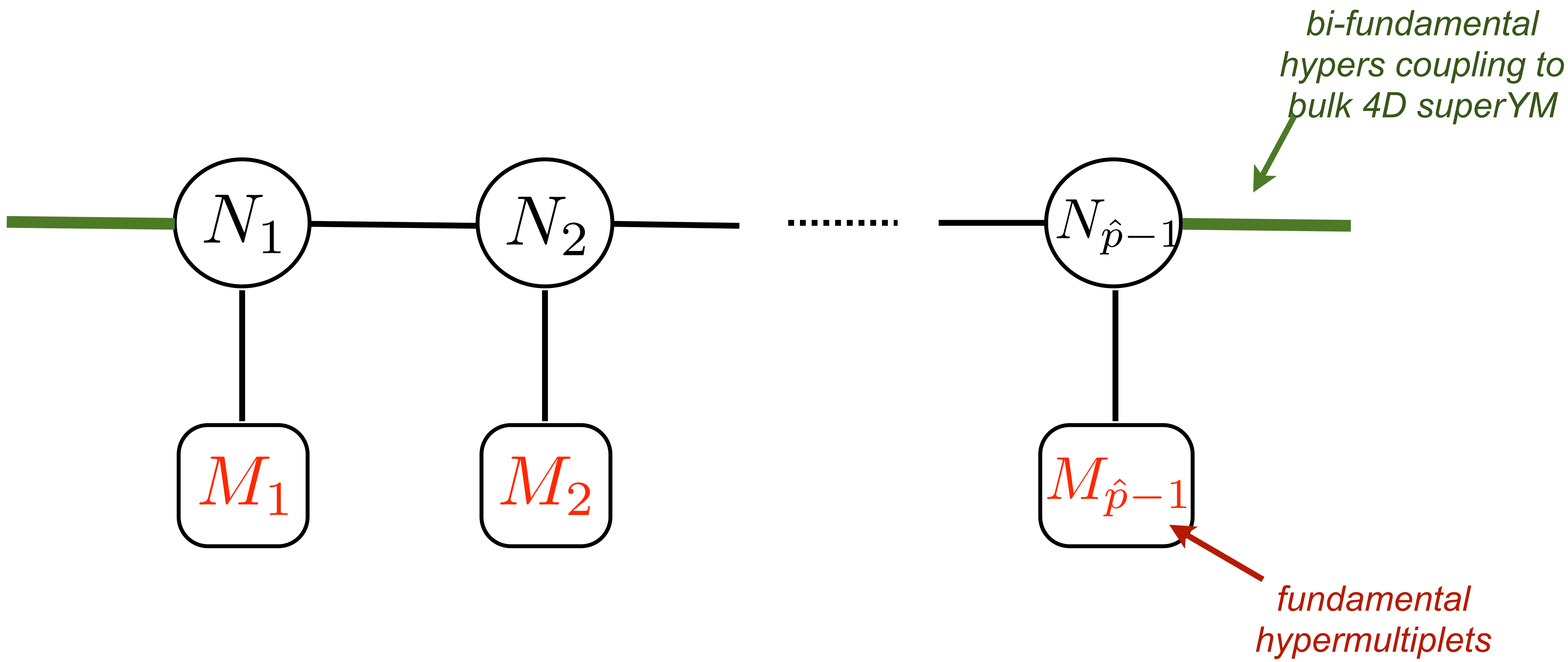
D'Hoker, Estes & Gutperle '07 found the **general local form** of type-IIB supergravity solutions with $OSp(2,2|4) \supset SO(2,3) \times SO(3) \times SO(3)$ supersymmetry.

This is the symmetry of conformal 1/2-BPS domain walls in N=4 D=4 super-Yang-Mills theory, which were classified by **Gaiotto & Witten '08**

DeWolfe, Freedman, Ooguri; .....

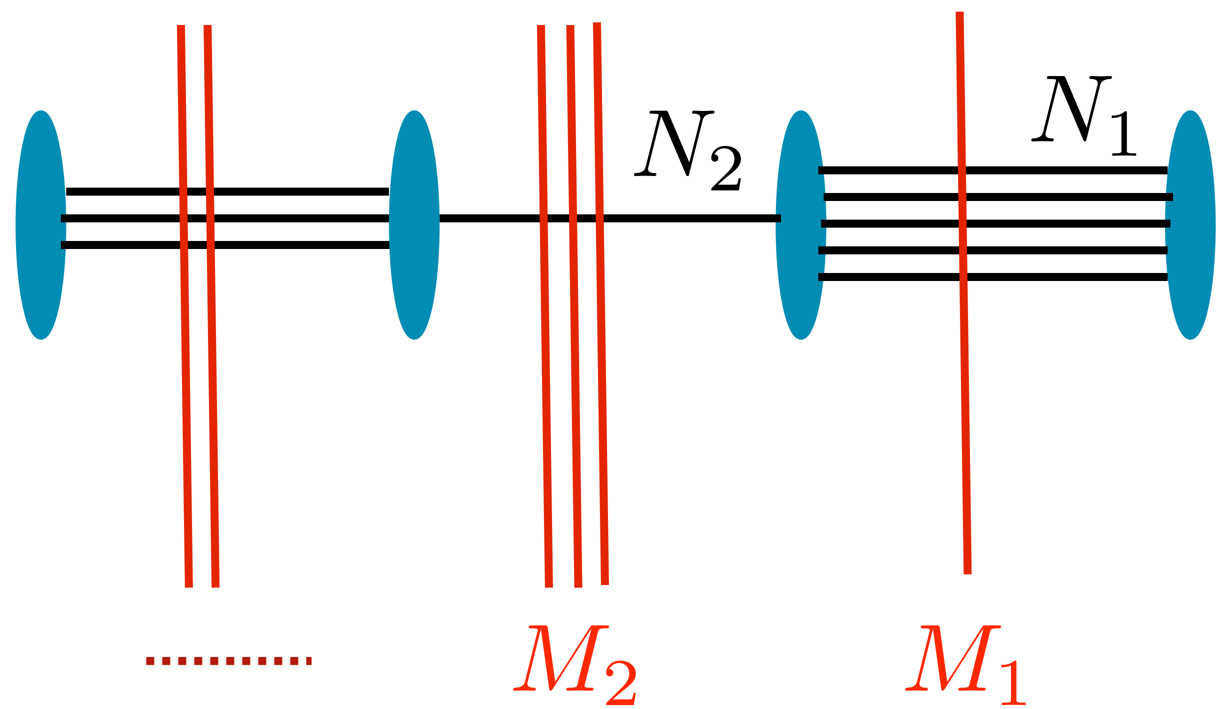
To address our question, we established the precise dictionary between these two works. Bonus: some new lessons for AdS4/CFT3.

The Gaiotto-Witten domain walls support **linear-quiver N=4 CFT3s**



$U(N_1) \times U(N_2) \times \cdots \times U(N_{\hat{p}-1})$	gauge group
$U(M_1) \times U(M_2) \times \cdots \times U(M_{\hat{p}-1})$	**manifest** global symmetry
$U(\hat{M}_1) \times U(\hat{M}_2) \times \cdots \times U(\hat{M}_{p-1})$	**hidden** global symmetry

Engineered on D3-branes suspended/intersecting NS5- and D5- branes



Hanany-Witten

Mirror symmetry exchanges NS5 with D5 (Intriligator, Seiberg; de Boer, Hori, Ooguri, Oz; .....)

(Almost) invariant description in terms of **linking numbers**:

i th D5	ℓ_i	= (net # of D3-branes ending on it from left) + (# of NS5 to its right)
j th NS5	$\hat{\ell}_j$	= (net # of D3-branes ending on it from right) + (# of D5 to its left)

Focus on 3D gauge theories (coupling to 4D YM: slight complication)

Gaiotto-Witten conjectured that these **flow to non-trivial SCFT** whenever

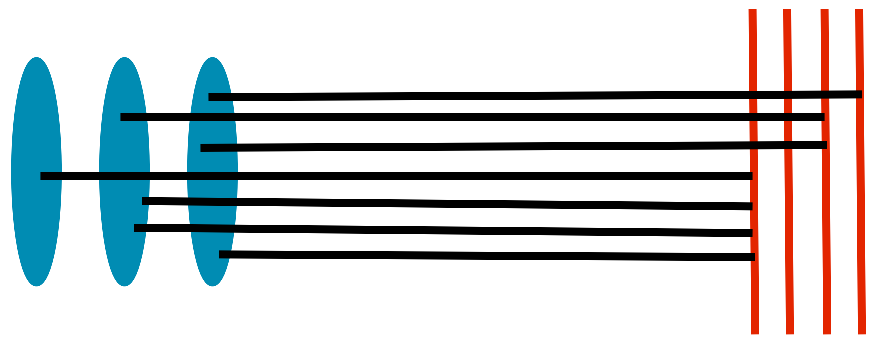
$$\# \ hypers \geq 2N_a$$ for each gauge group

(complete Higgsing)

These conditions translate to $\hat{\ell}_1 \geq \hat{\ell}_2 \cdots \geq \hat{\ell}_{\hat{p}} > 0$ (labelled from right to left)

while also $\ell_1 \geq \ell_2 \cdots \geq \ell_p > 0$ (labelled from left to right)

Move all NS5 to the left of all D5 $\Longrightarrow$ $\boxed{\sum \ell_i = \sum \hat{\ell}_j = N}$



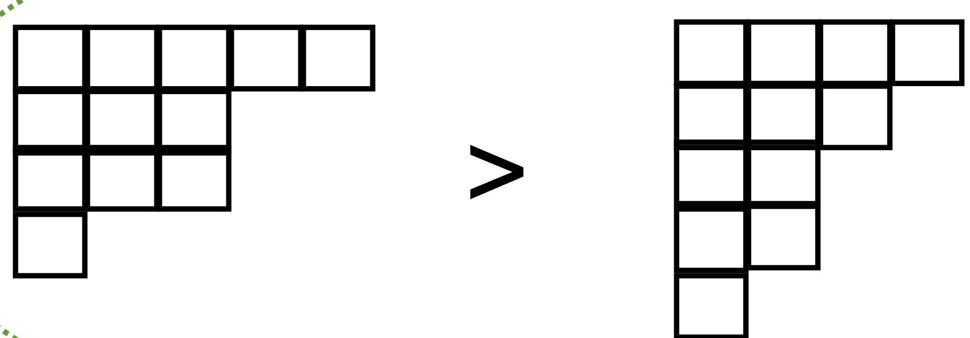
two partitions ρ & $\hat{\rho}$

Supersymmetry (s-rule) implies the (partial ordering) constraints

$$\rho^T > \hat{\rho}$$

(# of boxes in first n columns of ρ **>** # boxes in first n rows of $\hat{\rho}$ $\forall\ n$)

e.g.



arise in many contexts related
to Nahm's equations

Nakajima; Kronheimer;
CB, Hoppe, Pioline; ....

<u>Summarize</u>: **non-trivial N=4 SCFT3** $T_\rho^{\hat{\rho}}(SU(N))$

conjectured for any pair of partitions obeying these constraints

The IIB solutions are **AdS4 x S2 x S2** fibrations over a Riemann surface basis Σ . Local solutions are determined by two **harmonic functions** h_1 and h_2 . Modulo an **SL(2,R) rotation:**

metric :
$$ds^2 = f_4^2 ds^2_{\mathrm{AdS}_4} + f_1^2 ds^2_{\mathrm{S}_1^2} + f_2^2 ds^2_{\mathrm{S}_2^2} + 4\rho^2 dz d\bar{z} \ ,$$

$$f_4^8 = 16\,\frac{N_1 N_2}{W^2}\ , \quad f_1^8 = 16\, h_1^8 \frac{N_2 W^2}{N_1^3}\ , \quad f_2^8 = 16\, h_2^8 \frac{N_1 W^2}{N_2^3}$$

$$\rho^8 = \frac{N_1 N_2 W^2}{h_1^4 h_2^4}$$

dilaton :
$$e^{4\phi} = \frac{N_2}{N_1}$$

where
$$W = \partial h_1 \bar{\partial} h_2 + \bar{\partial} h_1 \partial h_2 = \partial\bar{\partial}(h_1 h_2)\ ,$$
$$N_1 = 2 h_1 h_2 |\partial h_1|^2 - h_1^2 W\ , \qquad N_2 = 2 h_1 h_2 |\partial h_2|^2 - h_2^2 W\ .$$

There are also p-form backgrounds: $F_5, \; H_3, \; F_3$

expressed in terms of the dual harmonic functions:

$$h_1 = -i(\mathcal{A}_1 - \bar{\mathcal{A}}_1) \quad \rightarrow \quad h_1^D = \mathcal{A}_1 + \bar{\mathcal{A}}_1$$

$$h_2 = \mathcal{A}_2 + \bar{\mathcal{A}}_2 \quad \rightarrow \quad h_2^D = i(\mathcal{A}_2 - \bar{\mathcal{A}}_2)$$

and the (normalized) volume forms of the (pseudo)spheres:

ω^{0123}	AdS_4
ω^{45}	S_1^2
ω^{67}	S_2^2

<u>Note</u>: the constant parts of $Re(\mathcal{A}_1)$ and $Im(\mathcal{A}_2)$ are pure gauge.

3-forms :

$$H_{(3)} + iF_{(3)} = \omega^{45} \wedge db_1 + i\,\omega^{67} \wedge db_2$$

where:

$$b_1 = 2ih_1 \frac{h_1 h_2(\partial h_1 \bar{\partial} h_2 - \bar{\partial} h_1 \partial h_2)}{N_1} + 2h_2^D$$

$$b_2 = 2ih_2 \frac{h_1 h_2(\partial h_1 \bar{\partial} h_2 - \bar{\partial} h_1 \partial h_2)}{N_2} - 2h_1^D$$

5-form :

$$F_{(5)} = -4\, f_4^4\, \omega^{0123} \wedge \mathcal{F} + 4\, f_1^2 f_2^2\, \omega^{45} \wedge \omega^{67} \wedge (*_2 \mathcal{F})\ ,$$

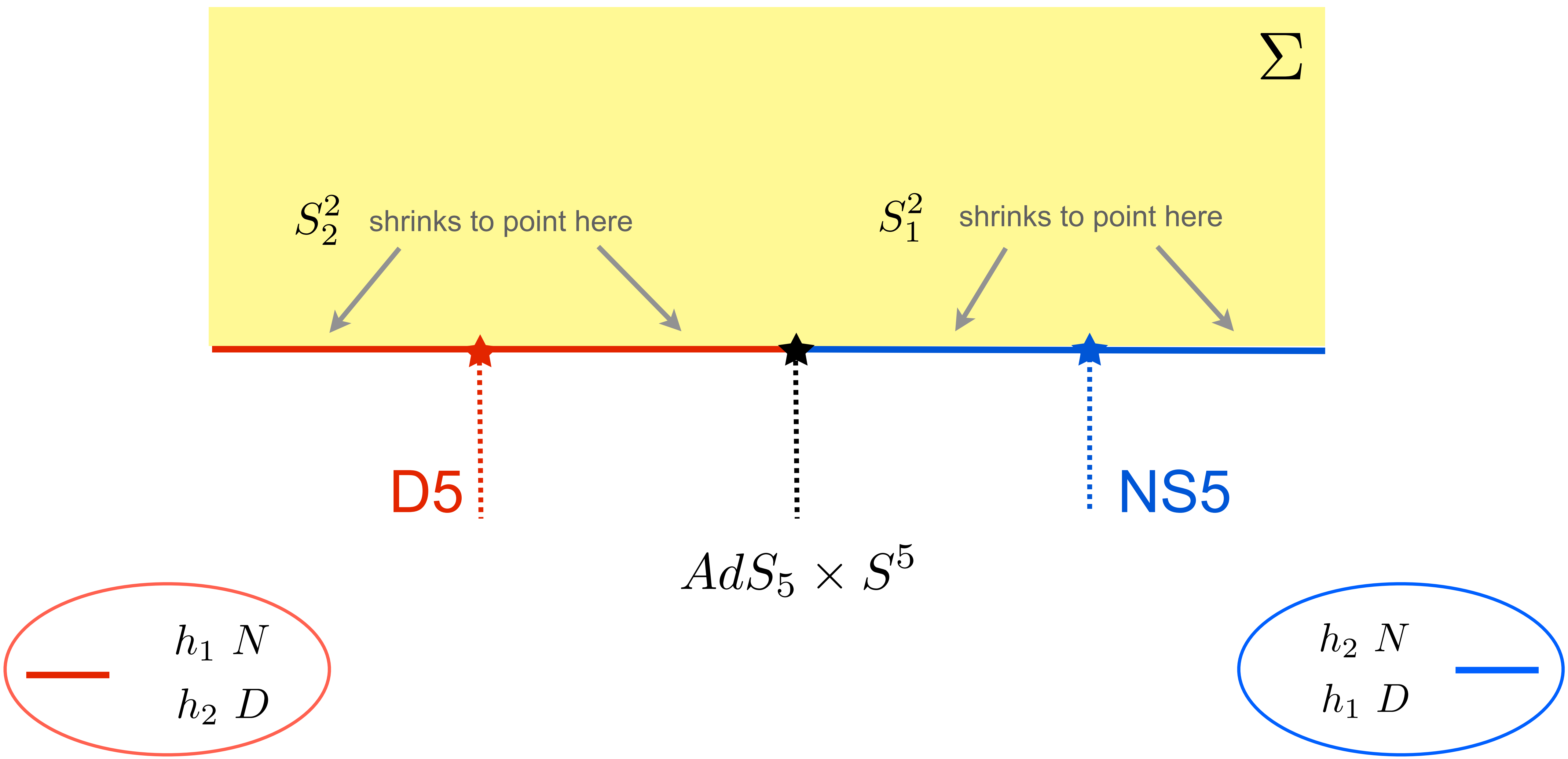
where

$$f_4^4 \mathcal{F} = dj_1 \qquad \text{with} \qquad j_1 = 3\mathcal{C} + 3\bar{\mathcal{C}} - 3\mathcal{D} + i\frac{h_1 h_2}{W}(\partial h_1 \bar{\partial} h_2 - \bar{\partial} h_1 \partial h_2)$$

$$\partial \mathcal{C} = \mathcal{A}_1 \partial \mathcal{A}_2 - \mathcal{A}_2 \partial \mathcal{A}_1 \qquad\qquad \mathcal{D} = \bar{\mathcal{A}}_1 \mathcal{A}_2 + \mathcal{A}_1 \bar{\mathcal{A}}_2$$

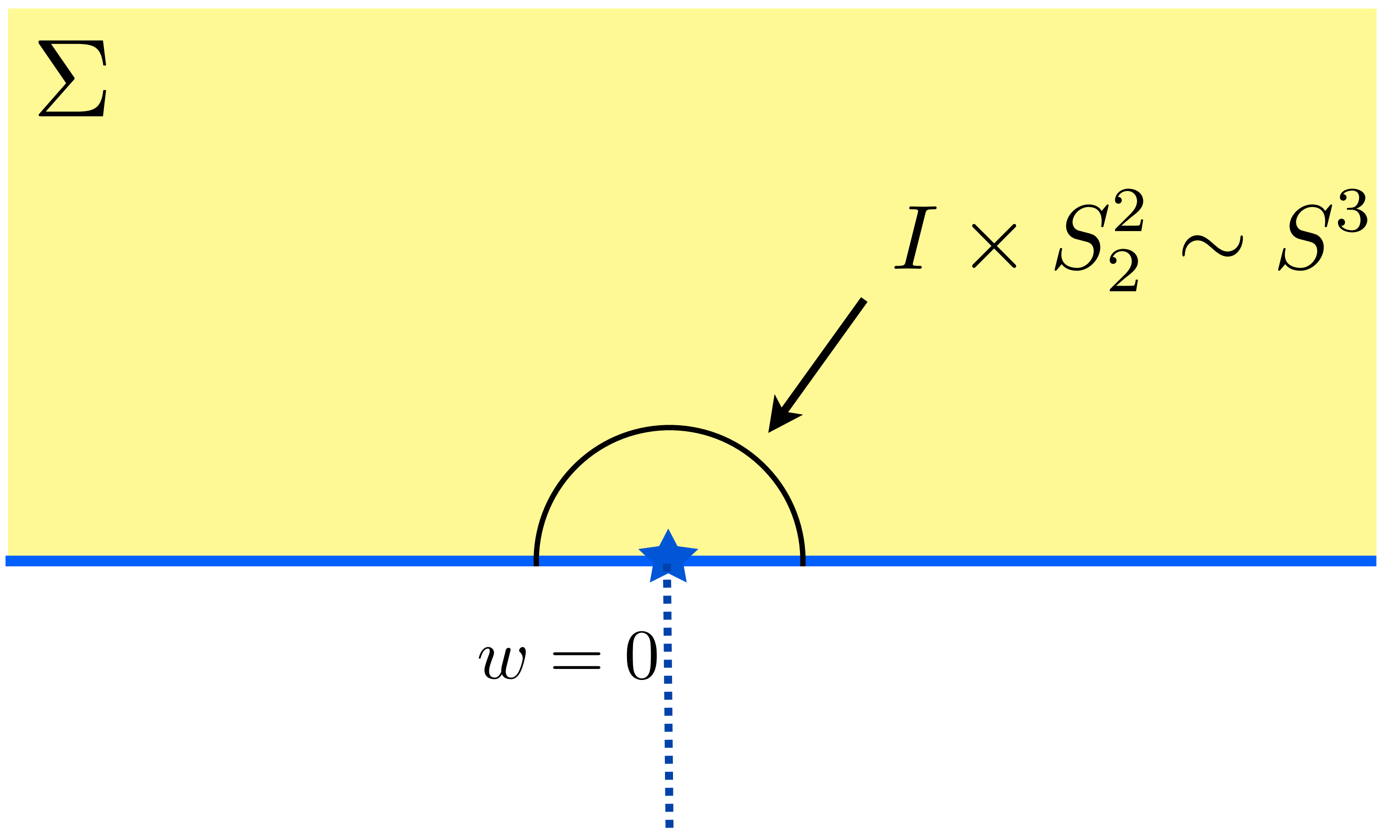
The surface Σ can have **boundaries** which are **interior points** of the 10D geometry. For this, require either h_1 to have a **Dirichlet** condition and h_2 a **Neumann** condition, or vice versa. One 2-sphere then shrinks to zero, and the geometry is (locally) $\sim AdS_4 \times S^2 \times \mathcal{D}^2$

Asymptotic regions arise as singularities on boundary:



Local form of **NS5 singularity**:

(For D5-brane: exchange roles of $\mathcal{A}_1$ and $\mathcal{A}_2$)

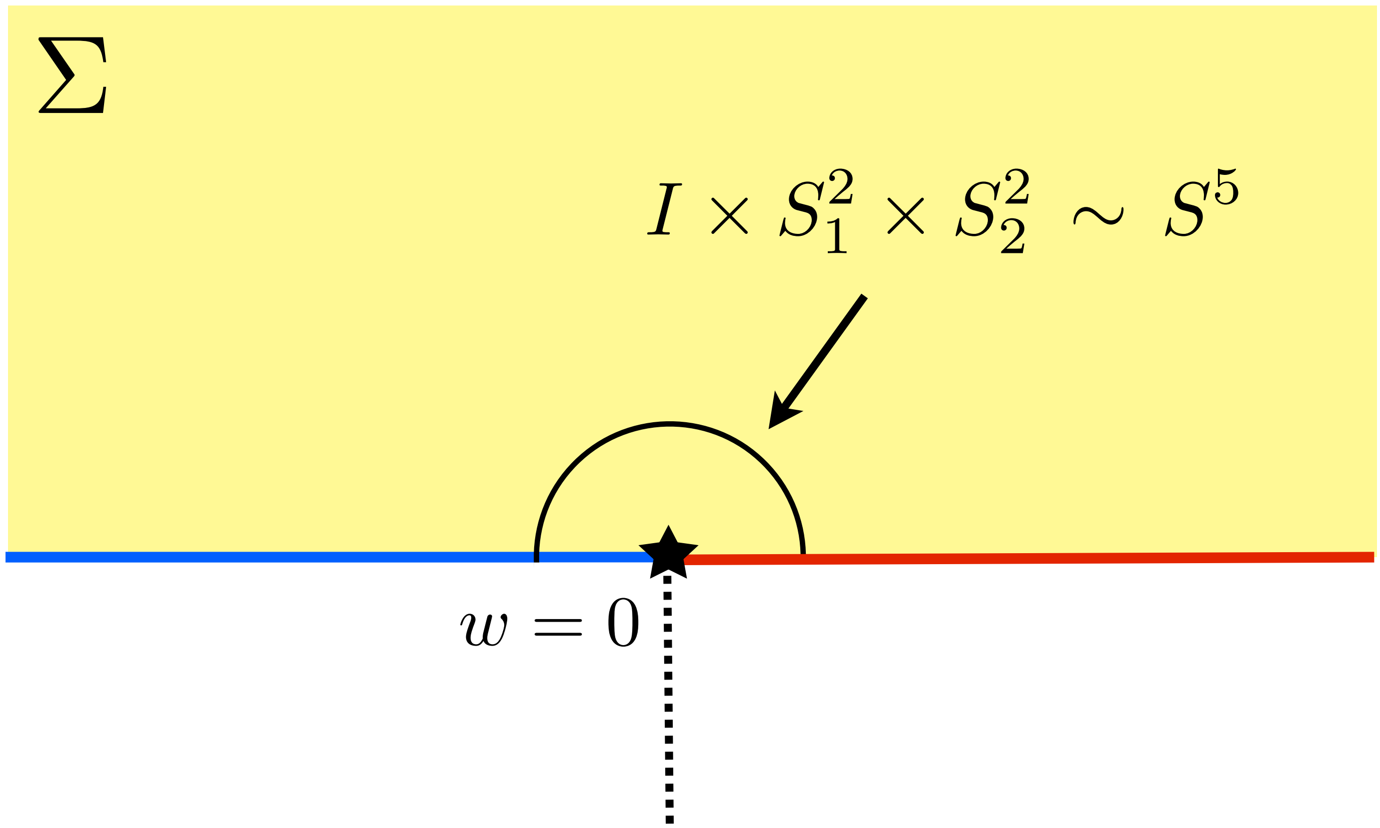


$$\mathcal{A}_1 \simeq \pi \hat{\ell} + \cdots$$
$$\mathcal{A}_2 \simeq -\hat{M} \log w + \cdots$$

$$\hat{M} = \frac{1}{16\pi^2} \int_{S^3} H_3$$ ⟵ **# NS5 branes**

$$\hat{M} \hat{\ell} = \frac{1}{(4\pi)^4} \int_{S^3 \times S_1^2} (F_5 + C_2 \wedge H_3)$$ ⟵ **D3-brane Page charge**

Local form of $AdS_5 \times S^5$:



$$\mathcal{A}_1 = \frac{1}{\sqrt{w}}(a_1 + b_1 w + \cdots)$$

$$\mathcal{A}_2 = \frac{1}{\sqrt{w}}(a_2 + b_2 w + \cdots)$$

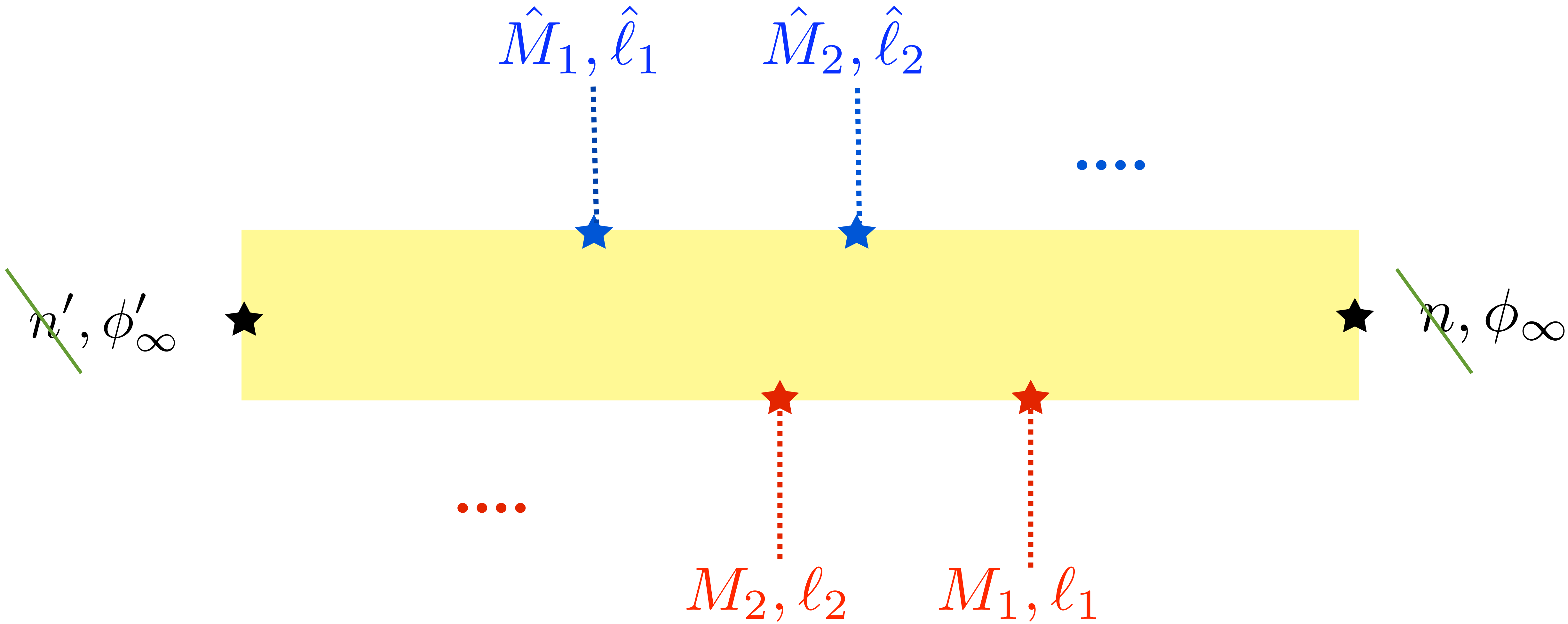
$$n = \frac{1}{(4\pi)^4}\int_{S^5} F_5 = \frac{(a_2 b_1 - a_1 b_2)}{2\pi} \quad \longleftarrow \text{ D3 charge}$$

Limit $\quad a_1, a_2 \to 0 \quad$ gives a smooth capping-off of geometry

Aharony, Berdichevsky, Berkooz, Shamir; ABEG

Solution for linear quivers

$$
\begin{aligned}
h_1 &= \left[\cancel{-i\alpha\sinh(z-\beta)} - \sum_{a=1}^{q}\gamma_a \ln\left(\tanh\left(\frac{i\pi}{4}-\frac{z-\delta_a}{2}\right)\right)\right] + c.c. \\
h_2 &= \left[\cancel{\hat{\alpha}\cosh(z-\hat{\beta})} - \sum_{b=1}^{\hat{q}}\hat{\gamma}_b \ln\left(\tanh\left(\frac{z-\hat{\delta}_b}{2}\right)\right)\right] + c.c.
\end{aligned}
$$



Parameter count: $\{M_a, \ell_a, \hat{M}_b, \hat{\ell}_b\} \longleftrightarrow \{\gamma_a, \delta_a, \hat{\gamma}_b, \hat{\delta}_b\}$

May have **zero, one** or **two** asymptotic $AdS_5 \times S^5$ regions,
more leads to conical singularities.

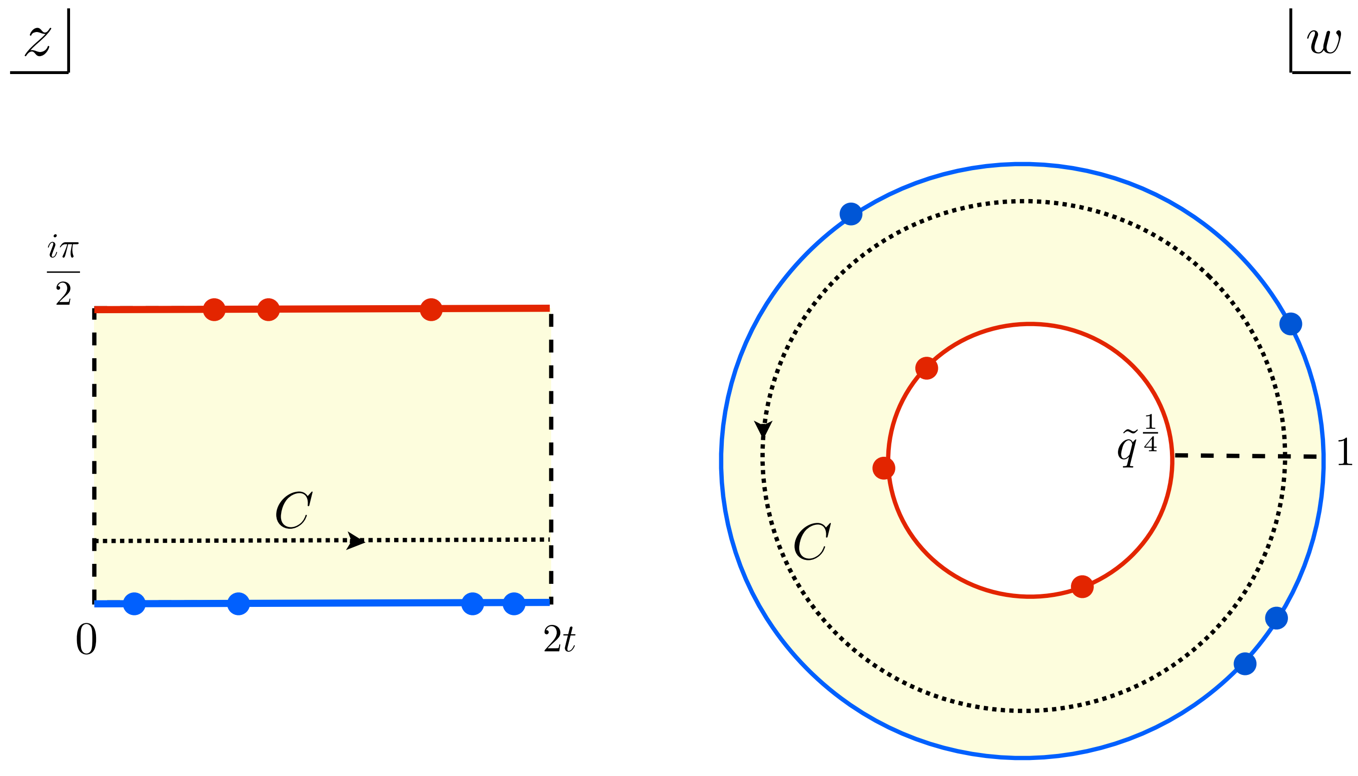
All parameters in first case are discrete: no N=4 moduli .

Global symmetries of CFT3 realized as gauge symmetries on 5-branes

Can show that linking numbers define two partitions obeying the
Young-tableau inequalities

One-to-one correspondence with Gaiotto-Witten SCFT3s

Solution for **circular quivers**



$$\mathcal{A}_1 = -i \sum_{a=1}^{p} \gamma_a \ln\left(\frac{\vartheta_1\left(q|\nu_a\right)}{\vartheta_2\left(q|\nu_a\right)}\right) + \varphi_1 \qquad i\,\nu_a = -\frac{z-\delta_a}{2\pi} + \frac{i}{4}$$

$$\mathcal{A}_2 = -\sum_{b=1}^{\hat{p}} \hat{\gamma}_b \ln\left(\frac{\vartheta_1\left(q|\hat{\nu}_b\right)}{\vartheta_2\left(q|\hat{\nu}_b\right)}\right) + i\varphi_2 \qquad i\,\hat{\nu}_b = \frac{z-\hat{\delta}_b}{2\pi}$$

Annulus modulus $\longleftrightarrow$ K = # of winding D3 branes

Cannot couple to 4D sYM, i.e. add asymptotic $AdS_5 \times S^5$ regions

Several calculations/observations concerning this rich set of holographic dualities, that I have no time to discuss here. Just mention some:

- Free energy on S3 from sugra = from CFT:

checked that for $T(SU(N))$ and some related theories

$$F = \frac{1}{2} N^2 \ln N + \mathcal{O}(N^2)$$

non-trivial

Assel, Estes, Yamazaki

using results of Benvenuti & Pasquetti; Nishioka,Tachikawa, Yamazaki

- Large K corresponds to $t \longrightarrow 0$; T-dual to ABJM-like theories.

- SL(2, Q) transformations generalize mirror symmetry in classical limit

....

Let’s return now to the original question

To create the hierarchy $\boxed{\ell \gg L}$ must look at degeneration limits

AdS_4 AdS_5

- Factorize 5-brane singularities
  Pinch annulus $t \rightarrow \infty$ $\Big\}$ **"Worm-branes"** (Janus throats)

- $\boxed{\text{Cap-off } AdS_5 \times S^5 \text{ (small rank for bulk D=4 SYM group)}}$

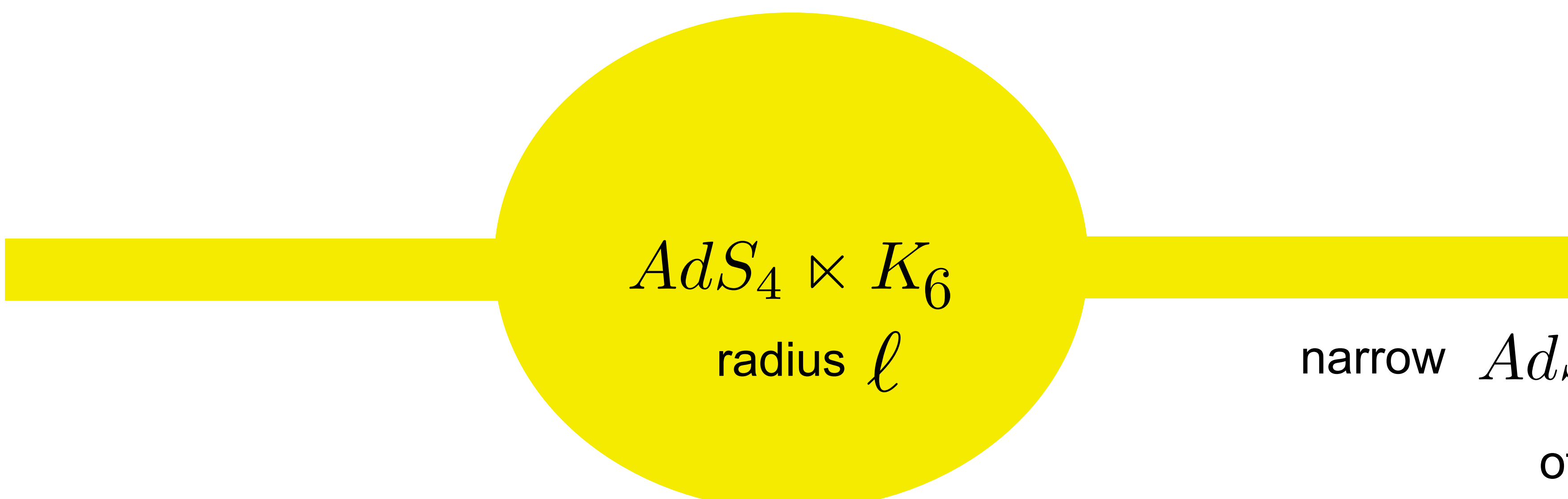
  $\alpha, \hat{\alpha} \rightarrow 0$

This is possible, provided there are both NS5s and D5s *(to stabilize dilaton) ;*

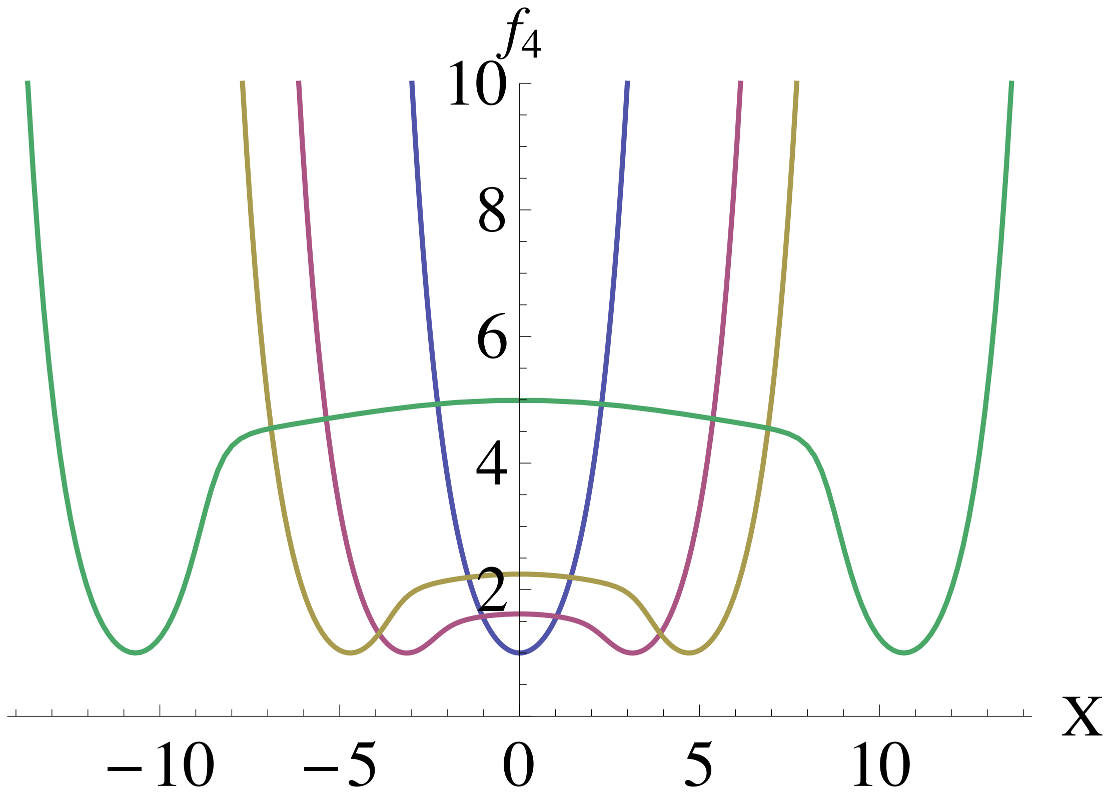
The **lowest** AdS4 **spin-2 mode** has a (tiny) mass and non-universal couplings.

(Assel, CB, unpublished)

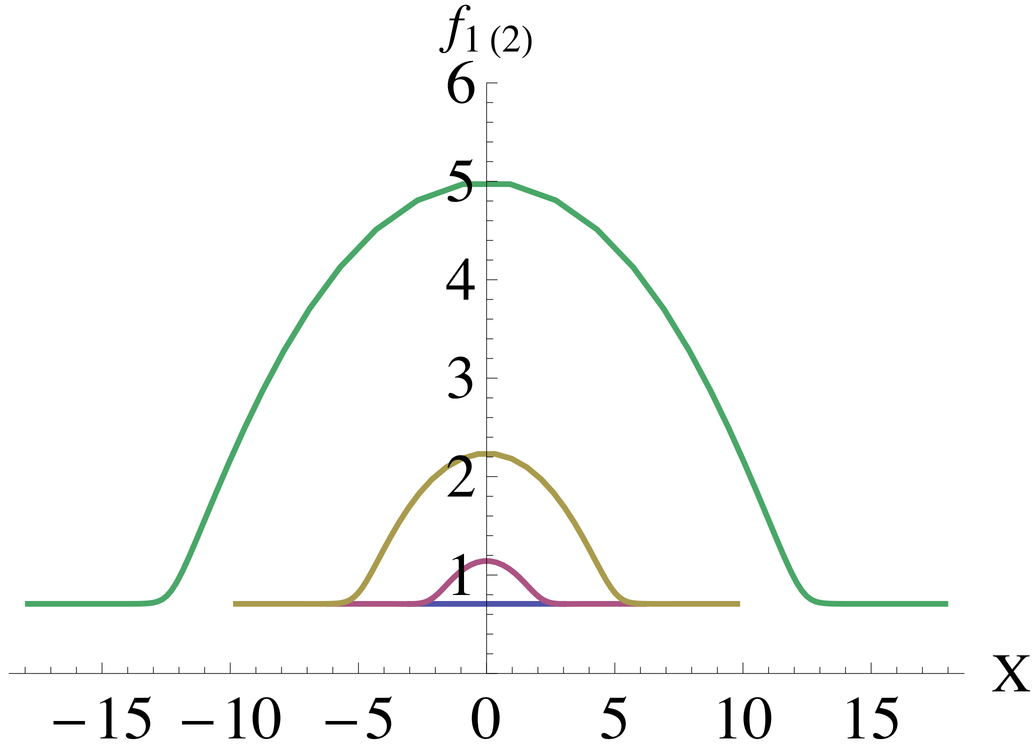
But, as in other efforts to create classical hierarchies, **spacetime decompactifies**



Gravity is never 4-dimensional !



warp factor



sphere radii

What we need:

- An AdS4/CFT3 pair for which the **compact space stays small** ($T_{\mu\nu}$ and few other operators, then **large gap**)

interesting, and hard problem by itself

- Couple CFT3 to SYM4 **without destabilizing fixed point**

need understanding of geometric singularities

Summary

Found a **large class of type-IIB backgrounds**, that are duals of the Gaiotto-Witten N=4 SCFT3s , and of their circular-quiver extensions.

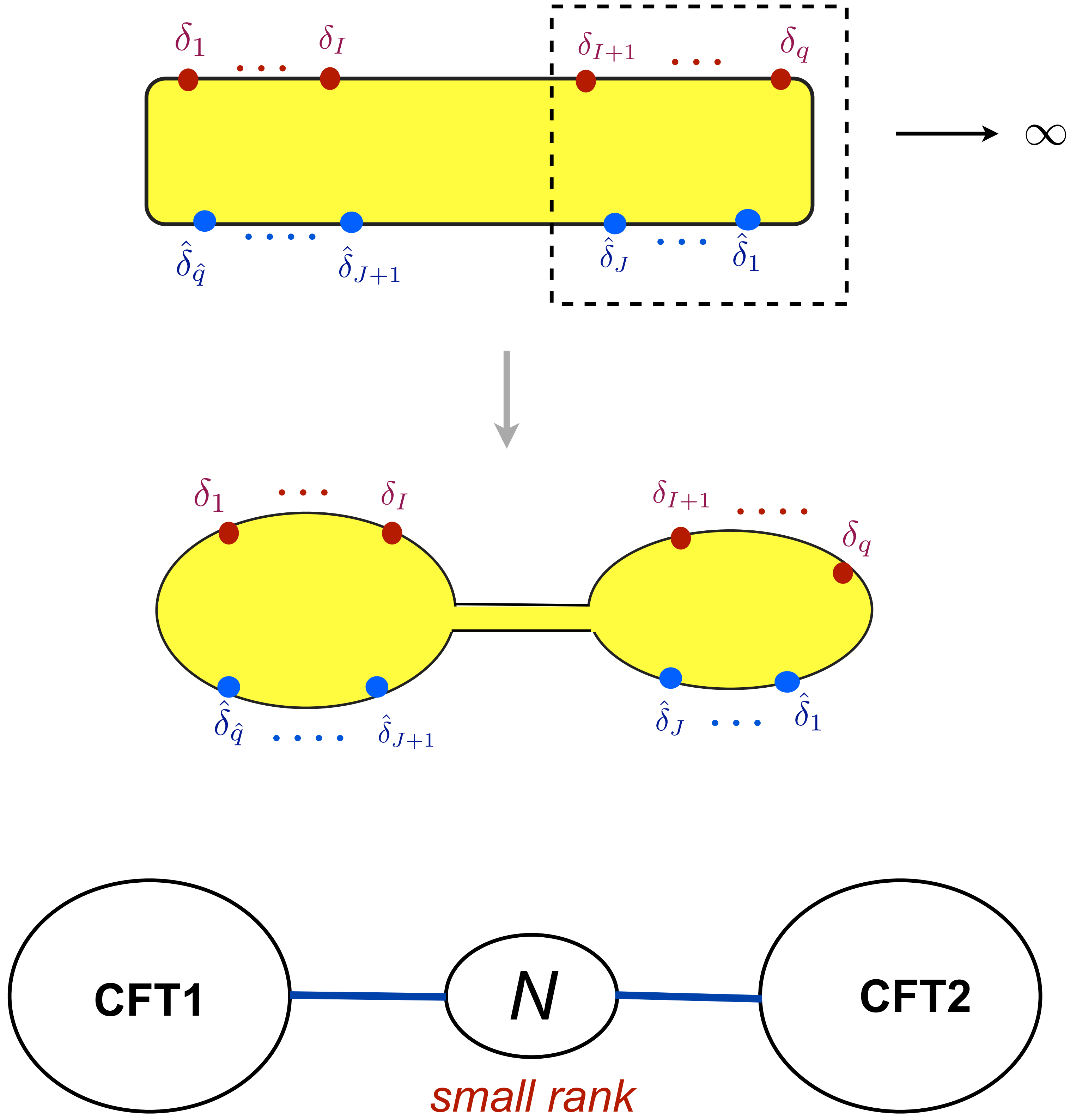
[These are AdS4 compactifications with localized 5-branes]

Degeneration limits correspond to **couplings via small-rank** gauge groups.

Failed to localize 4D gravity, because the brane fattens to 10D spacetime.

Given the difficulty in modifying gravity in IR, worth pursuing......

as opposed to local multitrace couplings,e.g.
Kiritsis, Niarchos; Aharony, Clark, Karch



δ_1
δ_I
δ_{I+1}
δ_q
$\hat{\delta}_{\hat{q}}$
$\hat{\delta}_{J+1}$
$\hat{\delta}_J$
$\hat{\delta}_1$
∞
δ_1
δ_I
δ_{I+1}
δ_q
$\hat{\delta}_{\hat{q}}$
$\hat{\delta}_{J+1}$
$\hat{\delta}_J$
$\hat{\delta}_1$
CFT1
N
CFT2
small rank

ΑΚΤΙΣ ΑΕΛΙΟΥ ΚΑΛΛΙΣΤΟΝ ΦΑΟΣ

A ray of sun is the best of lights

(chorus in Antigone of Sophocles)