Black Holes: Complementarity vs. Firewalls

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The Question

Complementarity

The AMPS Gedankenexperiment

Complementarity resolves the AMPS paradox

Conclusion
What happens when you fall into a large old black hole?

(1) You die at the horizon.
   - Almheiri, Marolf, Polchinski, Sully, arXiv:1207.3123: You hit a firewall at the horizon if \( t > O(R \log R) \) (scrambling time).
   - Susskind, arXiv:1207.4090: The usual singularity moves out to the apparent horizon if \( t > O(R^3) \) (Page-time).

Either way, the equivalence principle is badly violated. This is necessary to preserve unitarity of the S-matrix.
What happens when you fall into a large old black hole?

(2) You freely fall through the horizon.
   ▶ RB, arXiv:1207.5192
   ▶ Harlow, arXiv:1207.6243

Complementarity upholds the equivalence principle and unitarity.
▶ None of the (1)-authors is here.
▶ Both of the (2)-authors are here.
▶ Beware of selection bias.
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For earlier firewall-related ideas (minus the AMPS argument):
Giddings et al. (bit-models), Mathur et al. (fuzzballs), . . .

For complementarity:
Preskill; ’t Hooft, Stephens, Whiting; Susskind; Thorlacius, Uglum; Hayden & Preskill; Banks & Fischler; . . .
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The idea of complementarity arose 20 years ago (Preskill, Susskind, . . . ), from an apparent conflict between two important principles

- **Unitarity**, which is built into quantum mechanics
- **Equivalence Principle**, which is built in to GR
Consider the formation and complete Hawking evaporation of a black hole in asymptotically flat space. I will assume throughout the talk that this process is described by a unitary S-matrix.

To a particle theorist, this is natural—why should the appearance of black holes in the path integral invalidate a fundamental principle of quantum mechanics?
The equivalence principle dictates that experiments on scales much smaller than the spacetime curvature radii should behave like in flat space.

In particular, an observer falling into a much larger black hole should see nothing special while crossing the horizon.
Are they compatible?

I will first give an intuitive example, then a precise example of a contradiction between unitarity and the equivalence principle.
According to the outside observer, the black hole is an object like any other. It behaves as a warm membrane located at the stretched horizon (a timelike hypersurface located $l_P$ away from the mathematical event horizon). The membrane has specific electrical and mechanical properties that can be measured by lowering probes. (This is closely related to AdS/Hydrodynamics duality.) The membrane absorbs and thermalizes infalling objects, and later returns them as Hawking radiation. Objects do not cross the horizon. The infalling observer must see a violation of the equivalence principle.
According to the infalling observer, matter and its information content can be carried across the horizon. By causality, this information cannot return to the outside observer. The outside observer should see information loss. This is consistent with Hawking’s 1974 result that the out-state is a thermal density matrix.
Membrane vs. Free Fall

This conflict arises between physics at infinity and physics near the horizon, but far from the singularity. Regions of high curvature are not involved, so our approximations should be valid.
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A sharper contradiction is obtained by considering evolution of the quantum state, while assuming that both principles hold.
A black hole forms by the collapse of an object in the pure state $|\psi\rangle$. Adopt the Heisenberg picture.
Xeroxing paradox

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After the black hole has evaporated, the Hawking cloud is in the quantum state $|\psi\rangle$, by unitarity.

At the same time inside the black hole, the object is still collapsing and remains in the quantum state $|\psi\rangle$, by the equivalence principle.
The black hole acts as a quantum xeroxing machine:

$$|\psi\rangle \rightarrow |\psi\rangle \otimes |\psi\rangle$$

This violates the linearity of quantum mechanics.
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However, no observer can see both copies!
Strategy 1 (Susskind Thorlacius 1993)

Alice falls in with the star, sends a bit to Bob right after crossing the horizon. Bob waits until he can recover the bit in the Hawking radiation and then jumps in to receive Alice’s message. But: Bob has to wait for $O(R^3)$ (Page 1993) until the first bit of information comes out. Alice would need energy $O(\exp R^2)$ to send a signal that reaches Bob before he hits the singularity. Bob fails to see both copies, by a lot.
Strategy 2 (Hayden Preskill 2007)

Bob already controls more than half of the Hawking radiation, then Alice throws in a bit. Since Bob’s radiation is maximally entangled with the black hole, Alice’s bit can be recovered as soon as the black hole has completely thermalized it. Speculative arguments suggest that the scrambling time can be as fast as $O(R \log R)$. In this case, Bob fails to see both copies, but only barely.
It appears that the following statements cannot all be true:

1. **Unitarity**: Hawking radiation contains the information about the state that formed the black hole

2. **Equivalence Principle**: Nothing special happens to an infalling observer while crossing the horizon

Let us sacrifice (3) in order to rescue (1) and (2). This leads to the principle of complementarity.
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Let us sacrifice (3) in order to rescue (1) and (2). This leads to the principle of complementarity.
A fundamental description of Nature need only describe experiments that are consistent with causality. This principle can be applied to arbitrary spacetimes. The regions that can be probed are the *causal diamonds*: the intersection of the past and future of an arbitrary worldline.
Complementarity implies that there must be a theory for every causal diamond, but not necessarily for spacetime regions that are too large to be contained in any causal diamond. If we attempt to describe such regions, we may encounter contradictions, but such contradictions cannot be verified in any experiment.
Complementarity

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But recently, a deep and subtle new gedankenexperiment was proposed by Almheiri, Marolf, Polchinski, Sully, arXiv:1207.3123
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The AMPS experiment

A black hole is allowed to evaporate more than half of its mass

The resulting “early Hawking radiation” $A$ is maximally entangled with what remains of the black hole, by unitarity (Page 1993).

Equivalently, the early radiation is maximally entangled with the remaining “late” Hawking radiation.
The AMPS experiment

Consider one of the quanta of the late radiation, $B$, a wavepacket of size $O(R)$.

The early radiation $A$ must in particular be maximally entangled with $B$. 
Now consider an observer Alice who falls into the black hole.

She crosses the horizon when $B$ is still close to the horizon and has size $1 \ll \lambda \ll R$. 
The AMPS experiment

By the equivalence principle, Alice should see flat space behavior on such scales.

But in the Minkowski vacuum, modes with support on one side of a surface are maximally entangled with modes on the other side. (To see this, write $|0\rangle_M$ as an entangled product of states on the left and right Rindler wedge.)

This implies that Alice must see maximal entanglement between $B$ and a mode $C$ inside the horizon.
Figure 1. The causal diamond of an infalling (outside) observer is shown in red (blue); the overlap region is shown purple. Observer complementarity is the statement that the description of each causal diamond must be self-consistent; but the (operationally meaningless) combination of results from different diamonds can lead to contradictions. (a) Unitarity of the Hawking process implies that the original pure state is present in the final Hawking radiation. The equivalence principle implies that it is present inside the black hole. If we consider both diamonds simultaneously, then these arguments would lead to quantum xeroxing. However, no observer sees both copies, so no contradiction arises at the level of any one causal diamond. (b) Unitarity implies that the late Hawking particle is maximally entangled with the early radiation (see text for details). At the earlier time when the mode is near the horizon, the equivalence principle implies that it is maximally entangled with a mode inside the black hole. Since can be maximally entangled only with one other system, this constitutes a paradox. However, no observer can verify both entanglements, so no contradiction arises in any single causal diamond. Therefore, it is not necessary to posit a violation of the equivalence principle for the infalling observer.

Consider a causally nontrivial geometry, such as a black hole or cosmological space–time. Observer complementarity implies that there must be a theory for every causal diamond, but not necessarily for spacetime regions that are contained in no causal diamond.

This is a contradiction. By the strong subadditivity of entanglement entropy, cannot be maximally entangled with two different systems (both with A and with C).
We must give up one of these:

1. **Unitarity**  
   (entanglement of $B$ with $A$)

2. **QFT**  
   (evolution of the mode $B$ from the near to the far region)

3. **Equivalence Principle**  
   (entanglement of $B$ with $C$)

AMPS argue that (3) is the most conservative choice.
The lack of entanglement of short distance modes $B$ with modes $C$ inside the horizon implies a divergent stress tensor at the stretched horizon—a “firewall” or singularity. This is similar to the Rindler vacuum.

Alice will hit the firewall and will not enter the black hole.
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The upshot (RB; Harlow)

Xeroxing paradox: each observer sees only one copy

Firewall paradox: each observer sees only one entanglement
The infalling observer

Alice will find that the early Hawking radiation is consistent with unitarity. However, she cannot measure the full S-matrix and she cannot verify that the late radiation purifies the quantum state of the Hawking radiation. She can consistently assume that $B$ is entangled only with $C$. 
This implies that there is a separate theory for different infalling observers.

This is consistent with the version of complementarity advocated earlier.

It is not consistent with alternate statements of complementarity that refer to the black hole interior in its entirety.
Bob can measure the full S-matrix and finds it to be unitary. He cannot probe the horizon under free fall. To him, Alice seems to be absorbed by a membrane, and Alice can do nothing to prove him wrong. By the time she has verified that $B$ is entangled with $C$, it is too late to send Bob a message, so Bob can consistently assume that $B$ is entangled only with $A$. 
Quantitative checks (Harlow)

Alice and Bob have to agree on all observations performed by Alice while Alice still has a chance to

- send a message to Bob with sub-Planck frequency, or
- turn around and change her mind about entering the black hole, with sub-Planck acceleration

These lead to the same condition: experiments up to one Planck time before horizon crossing must agree.
Quantitative checks (Harlow)

This means Alice must find outgoing Hawking quanta $B$ entangled with the early radiation while she is more than one Planck time away from the horizon.
Harlow shows that she is (just barely) unable to receive a signal from an earlier infalling observer containing the (supposedly entangled) state of the inside partner of such quanta.
Quantitative checks \textit{(Harlow)}

Alice can verify the entanglement of $B$ with the inside mode, if and only if she is unable to verify the entanglement of $B$ with the early Hawking radiation.
AMPS give additional arguments that QFT may be modified near the horizon, involving careful measurements of the early radiation (quantum steering)

There should indeed be deviations from flat space suppressed by powers of the curvature radius, so the question is whether there is more.

More thought experiments should and will be studied (Alice hovering near the horizon, then jumping in, . . . )

The discussion is sure to continue.
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For now, I believe that the equivalence principle is safe. The AMPS thought-experiment is a beautiful diagnostic for what complementarity means and for what it can accomplish. No matter how this plays out, we will have learned something deep.