Domain walls for RG flows

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Definition

In a D-dimensional CFT
a d-dimensional conformal defect
preserves SO(D-d) x SO(d,2) in SO(D,2)

- BPS Conformal defects are ubiquitous in SCFTs and 2d CFTs
- Few non-supersymmetric examples are known in 3d

Extended defects are powerful tools for SCFT calculations

Today's examples

- Index of N=2 4d SCFTs from surface defect insertions L.Rastelli, S.Razmat, D.G.
- Index of N=2 3d SCFTs and duality walls tested by line defects T.Dimofte, S.Gukov, D.G.

How do you pick a defect?

Local operators are an intrinsic or defining property of a CFT.

- How do we know if a generic CFT has any conformal defects?
- Which defects are useful?

Too many defects?

Any 3d N=4 SCFT with flavor symmetry G gives a boundary condition for 4d G N=4 SYM

- The space of boundary conditions for a D-dimensional CFT may be ``larger'' than the space of (D-1)-dimensional field theories.
- We should focus on defects which encode useful properties of the CFT.

Janus domain walls

CFTs with exactly marginal deformations

- Interpolate between CFT at different values of the marginal coupling
- Very useful in SCFTs:

  S-dualities for N=4 SYM, N=2 class S theories

  BPS wall-crossing in 2d and 2d/4d systems

RG defects

Dynamical, massive objects become defects in IR

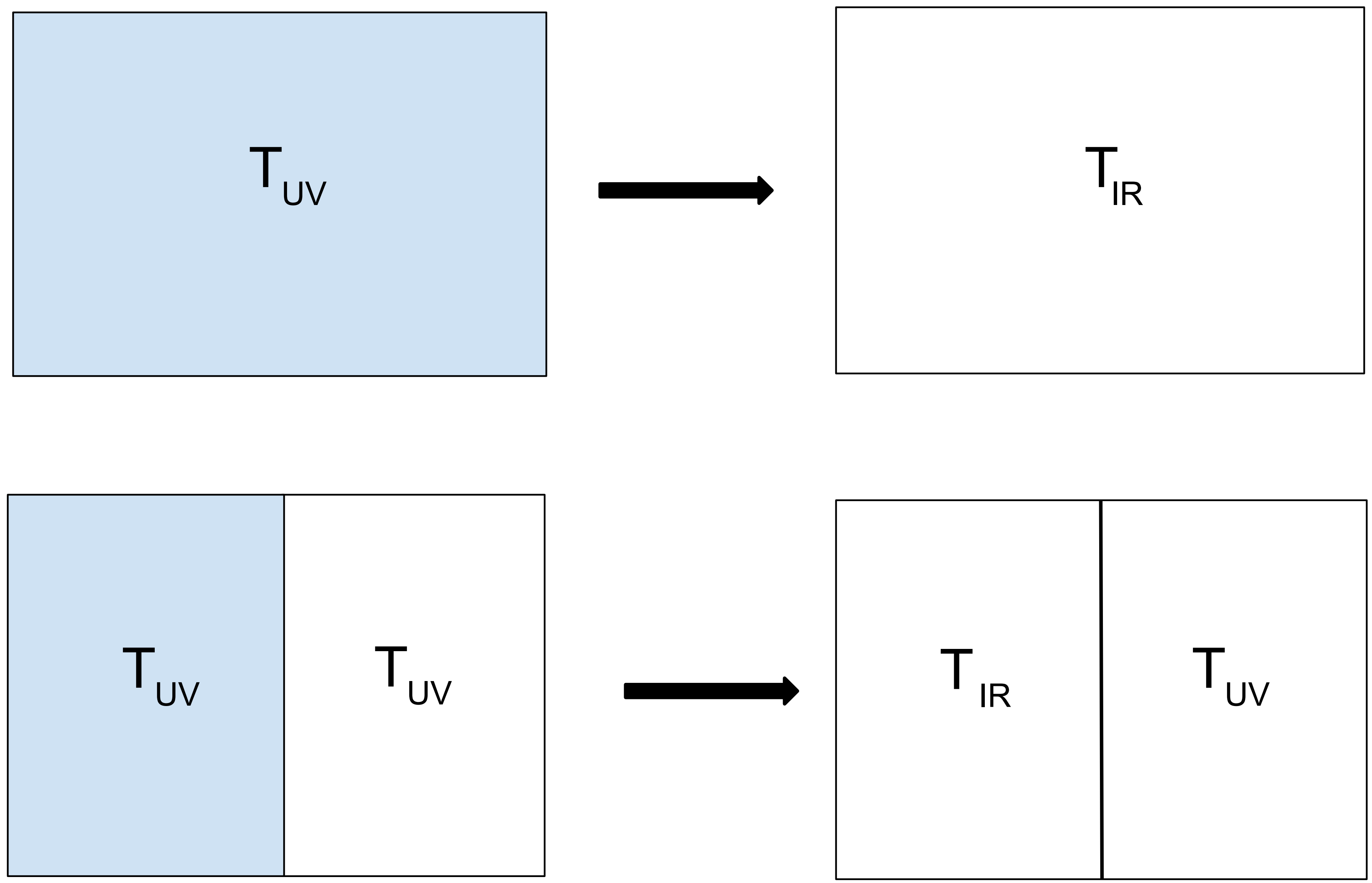
- Classical example: Wilson loop from massive particle
- Codimension 2 defects from dynamical vortices $\mathcal{N}=2$ examples, see Rastelli talk

RG flow from T_{UV} to T_{IR} implies the existence of many defects in T_{IR}

RG domain walls

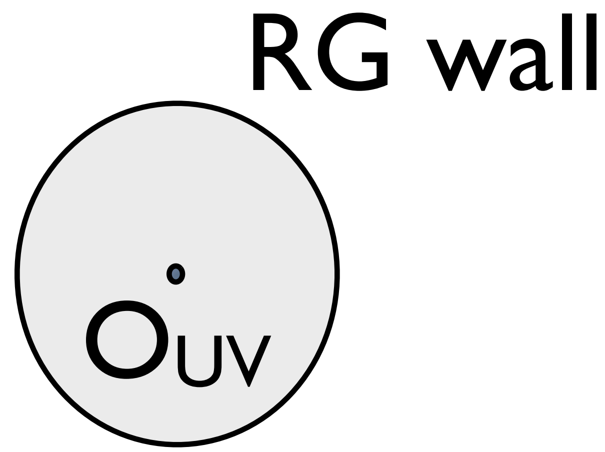
Interpolate between T_{UV} and T_{IR}

- Assume T_{UV} deformed by O flows to T_{IR}
- Turn on O only for $x>0$. Flow to IR
- T_{UV} for $x<0$, T_{IR} for $x>0$, RG domain wall at $x=0$
- Alternative: vary gauge coupling in asymptotically free gauge theory N=2 examples, upcoming work with T.Dimofte


T_{UV}
T_{IR}
T_{UV}
T_{UV}
T_{IR}
T_{UV}

RG domain walls properties

Conjecture: RG domain wall encodes the mixing of UV operators to give IR operators.

$O_{IR} =$ 

Conformal perturbation theory

- T_{UV} deformed by barely relevant O
  - [O] = D - ε
  - beta function zero at g* of order ε
- At leading order, operators mix if difference between dimensions is of order ε
  - Diagonalize dilatation at leading order in ε

RG domain wall should reproduce mixing matrix

RG flows between minimal models

Classical example of conformal perturbation theory

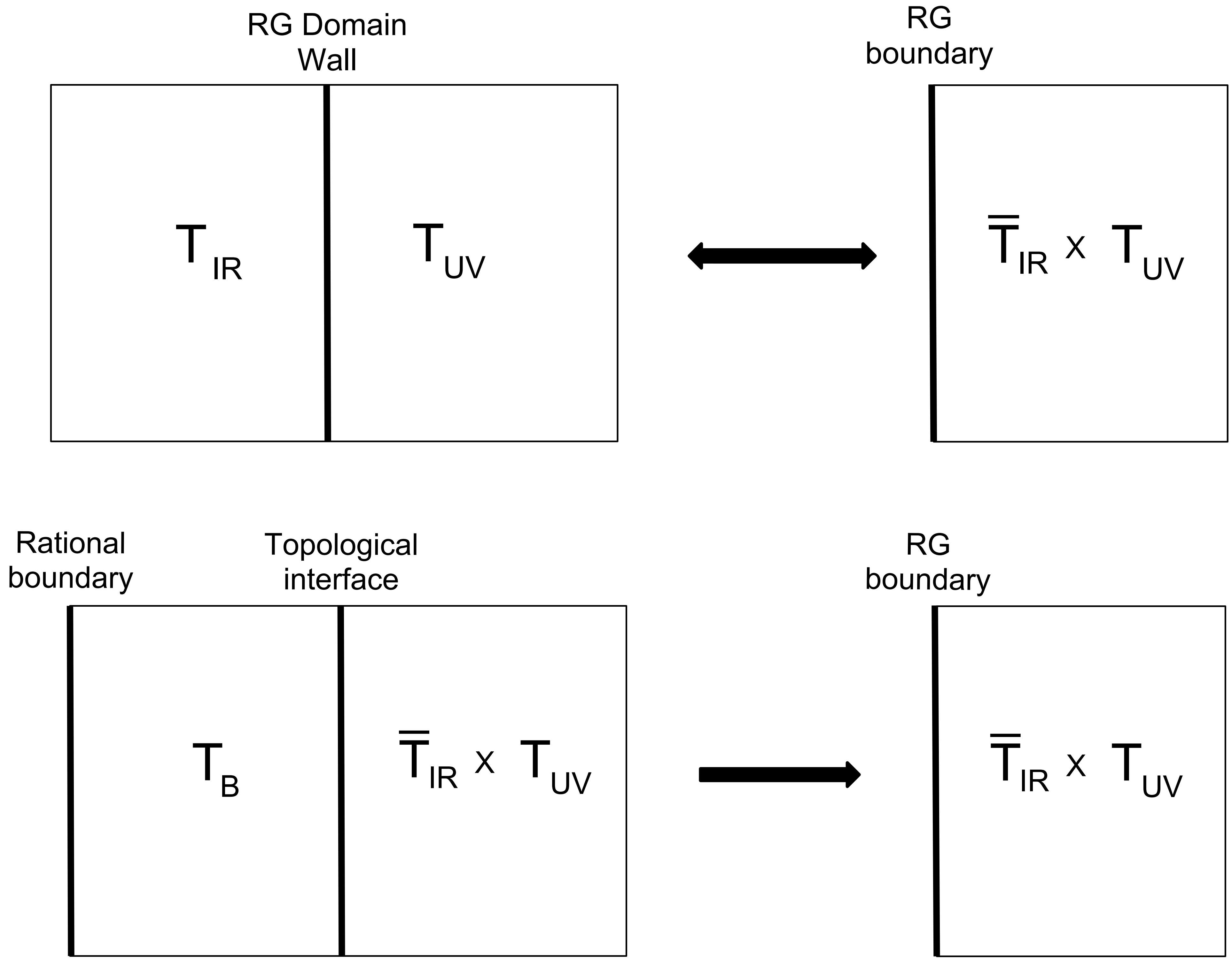
- Start from unitary minimal model $M_{p,p+1}$
- Turn on $O_{1,3}$: flow to $M_{p-1,p}$
- Mixing only among $[O_{r,s}]$ with same r give IR $[O_{t,r}]$
- Mixing matrices computed explicitly for many operators by Zamolodchikov

Can we identify the RG domain wall between consecutive minimal models?

- RG domain wall should be a conformal interface
- Reflection trick: a brane for $M_{p-1,p} \times M_{p,p+1}$
- **Problem:** $M_{p-1,p} \times M_{p,p+1}$ is a rational CFT for the product of two Virasoro
    - Cardy branes for $M_{p-1,p} \times M_{p,p+1}$ are the product of branes of $M_{p-1,p}$ and $M_{p,p+1}$: completely reflective interface.
    - RG domain wall should be almost transparent!

Can we identify the RG domain wall between consecutive minimal models?

- Building non-Cardy interfaces is notoriously hard.
- Conformal interfaces can be produced sometimes through ``generalized orbifold" from boundary conditions in another theory



RG Domain Wall
T_{IR}
T_{UV}
RG boundary
$\bar{T}_{IR} \times T_{UV}$
Rational boundary
Topological interface
T_B
$\bar{T}_{IR} \times T_{UV}$
RG boundary
$\bar{T}_{IR} \times T_{UV}$

RG interface is a brane in

$$\mathcal{M}_{k+2,k+1} \times \mathcal{M}_{k+3,k+2} = \frac{\hat{su}(2)_{k-1} \times \hat{su}(2)_1}{\hat{su}(2)_k} \times \frac{\hat{su}(2)_k \times \hat{su}(2)_1}{\hat{su}(2)_{k+1}}$$

Boundary state only involves operators

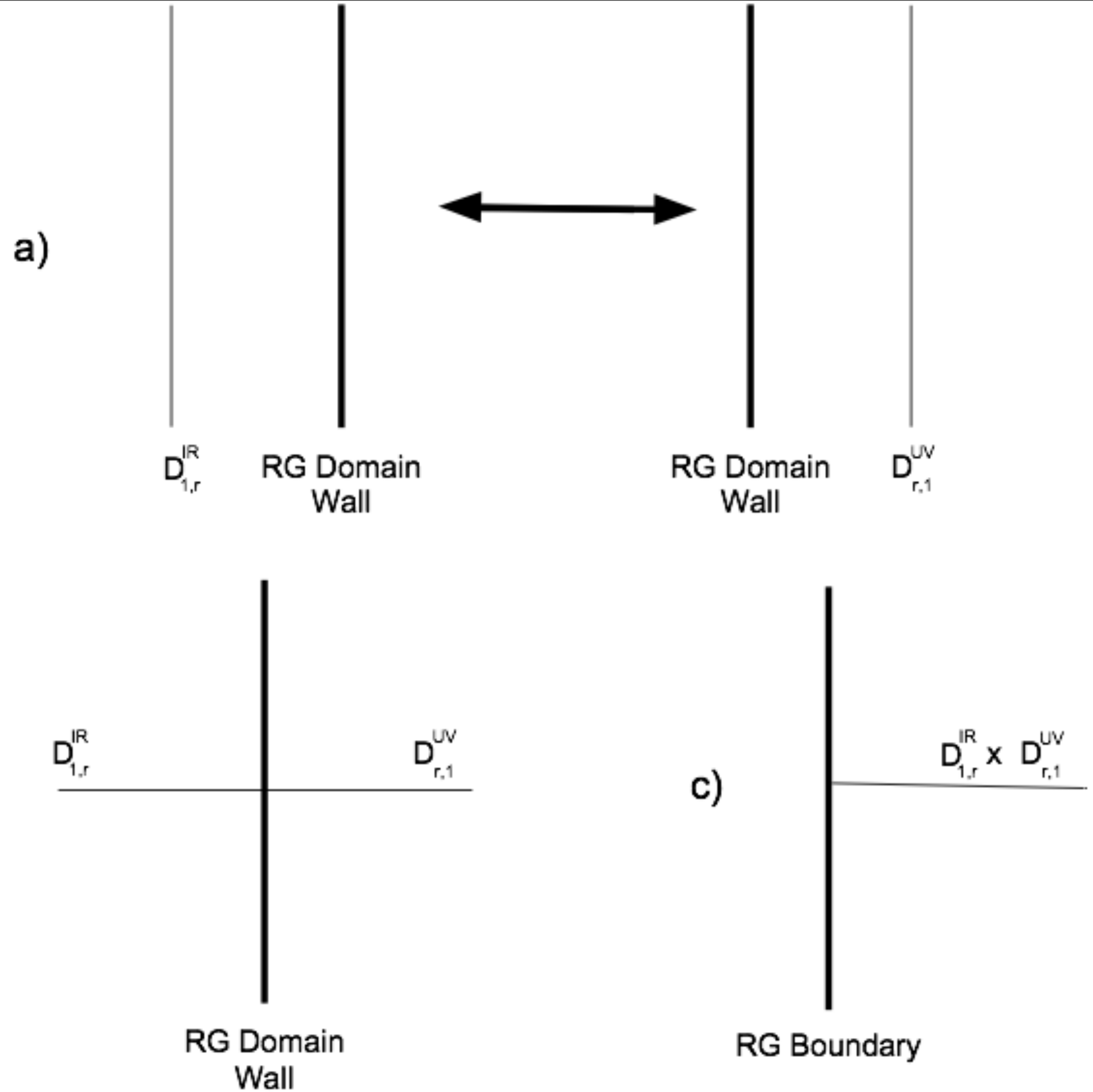
${}^{IR}O_{t,r}$ ${}^{UV}O_{r,s}$

which secretly belong to the "simplified coset"

$$\mathcal{B} = \frac{\hat{su}(2)_{k-1} \times \hat{su}(2)_1 \times \hat{su}(2)_1}{\hat{su}(2)_{k+1}}$$

Can we identify the RG domain wall between consecutive minimal models?

- Observation: the RG interface has a hidden symmetry
  - $O_{1,3}$ commutes with topological interfaces $D_{r,1}$
  - RG interface must commute with the $D_{r,1}$ too!


a)
$D^{IR}_{1,r}$
RG Domain Wall
RG Domain Wall
$D^{UV}_{r,1}$
b)
$D^{IR}_{1,r}$
$D^{UV}_{r,1}$
RG Domain Wall
c)
$D^{IR}_{1,r}$ x $D^{UV}_{r,1}$
RG Boundary

Final result

RG interface boundary state is computable

- Mixing matrices can be computed exactly

  Full agreement with Zamolodchikov calculations at the leading order!

- Method applies to most general RG flow of this type, for cosets of the form $\frac{\hat{g}_l \times \hat{g}_m}{\hat{g}_{l+m}}$