

Black Holes as a Quantum Phase Transition

Work Done with:

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Two Interconnected Claims:

- ① • The quantum Theory of Einsteinian gravitons
($S=2$, $m=0$ with Einstein inter. vertices in Minkowski)
is: UV-complete.
- The Key mechanism of SELF-UV completion is:
Classicalization.
- ② • BH's in asymptotically flat space are:
Self-sustained Bose-Einstein condensates
of weakly interacting soft gravitons.
- Moreover: BH's are at the critical point
of a: Quantum Phase Transition

The two basic problems of quantum Gravity:

P.1/ To unitarize scattering amplitudes at ultra-Planck energies:

$$\underline{s \gg M_{\text{Pl}}^2} \quad \text{[diagram of a black dot with four red arrows crossing it]} \quad \sim \underline{s L_{\text{Pl}}^2}$$

P.2/ To account for the microscopic origin of BH entropy:

$$S_{\text{BH}} \sim \frac{1}{4} A$$

The Wilsonian recipe:

- To integrate in extra degrees of freedom
- To use them:
 - * To solve perturbative unitarity
 - * To account for the BH-entropy.

* What is classicalization?

Quantization:

$$\begin{array}{ccc} a & \longrightarrow & \hat{a} \\ \text{c \#} & & \text{operator} \\ \hbar=0 & & \hbar \neq 0 \end{array}$$

Classicalization: Is the inverse map BUT Keeping $\hbar \neq 0$.

It happens whenever the occupation # N is $\gg 1$.
In many body physics this is known as
Bogoliubov approximation:

$$\begin{array}{ll} \hat{a}|N\rangle \rightarrow |N-1\rangle & \text{For } N \gg 1 \quad \hat{a}, \hat{a}^+ \rightarrow \text{c \#'s.} \\ \hat{a}^+|N\rangle \rightarrow |N+1\rangle & \hbar \neq 0 \end{array}$$

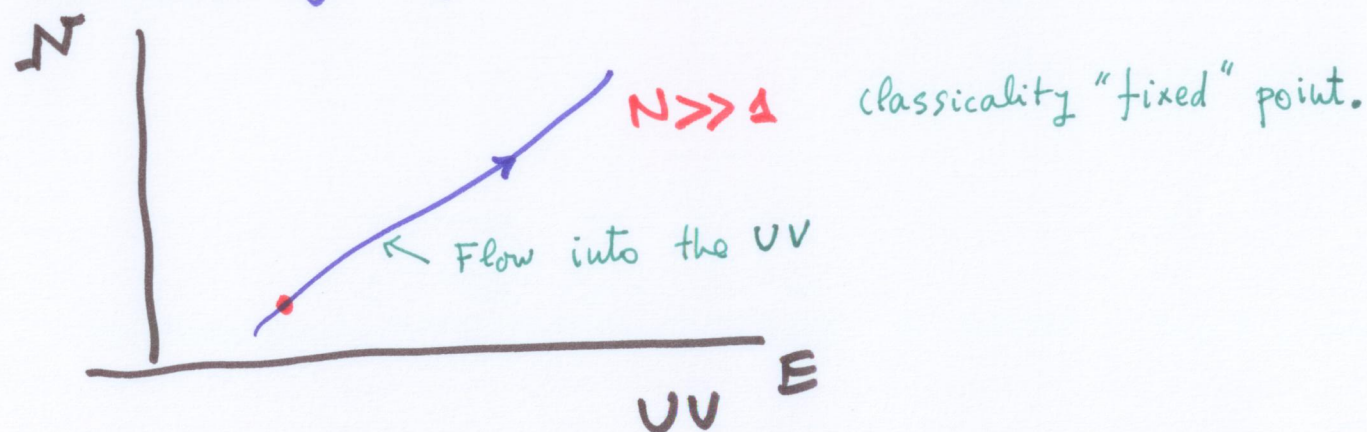
In other words:

Classicalization is Large N Quantum Physics.

* Under what conditions can we use classicalization to solve the problem of UV-completion?

* We can iff increasing E unavoidably leads to increase the occupation number N

i.e. If the Theory by itself Flows in the DEEP UV into classicality (i.e. into large N)



Note that underlying this FLOW there exists a

UV-IR correspondence

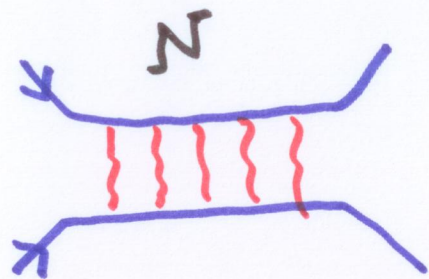
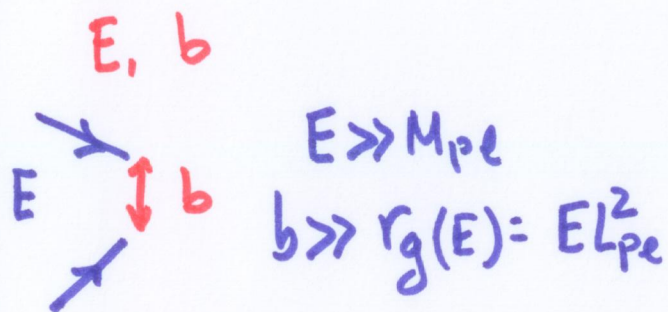
* Mapping states with **Low** occupation # of
HARD (strongly interacting) quanta into states
of **LARGE** occupation # of (weakly interacting)
Soft quanta. "S-duality"

Self Completeness simply means :

UV-classicalization

Let us consider $|in\rangle$ -state of two hard quanta with energy E and impact parameter b .

Track the corresponding **gravitational picture**:



Eikonal Approximation

Gravitational Energy: $E_{gr} \sim \frac{E \cdot r_g(E)}{b}$

$E_{gr} \ll E$

Graviton #: $N(E) \sim \frac{E \cdot r_g(E)}{\hbar}$

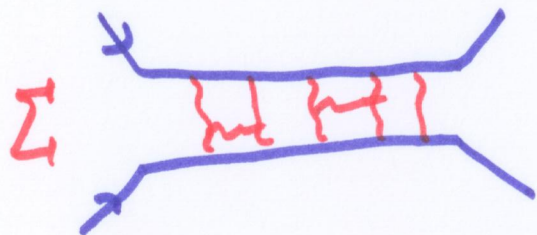
of typical wave length $\sim b$

Graviton interaction: $\alpha_{gr} \sim \frac{L_{pe}^2}{b^2} \ll \frac{1}{N(E)}$

For $E \gg M_{pe}$ $N(E) \gg 1$

Bose condensate of $N(E)$ -gravitons: The BEC order parameter solves the Gross Pitaeuskii eq $\sim$ gravitational potential.

Keep E fixed and reduce b .



From the BEC what we are doing is changing the strength of the interaction.

E_{gr} increases.

- What happens to the BEC when $b \sim r_g(E)$?

- At this point:

$$E_{gr} \sim E$$

$$\alpha_{gr} = \frac{1}{N}$$

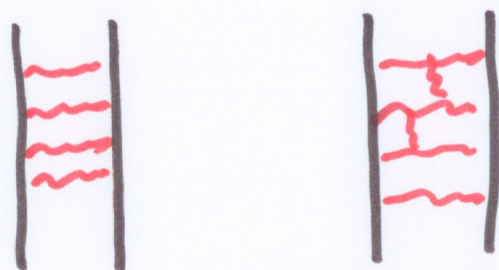
The BEC becomes self sustained i.e

The Quantum Pressure $\approx$ Gravit attraction

The BEC undergoes a QUANTUM PHASE TRANSITION

(Black Hole Formation)

Imaginary
Contribution to
the scatt amplitude



Imaginary
part.
(Bound state)

weak coupling
uniform BEC.

$1/N$

Quantum Phase Transition point
(BH) (Symmetry Breaking).

$\alpha_{gr.}$

b

The Quantum Phase Transition Unitarizes Grav Amplitudes.

The BH as a Quantum Phase Transition (Entropy and Hawking Radiation)

① The Q. Phase Transition.

(1+1) Toy model. Gas of N interacting bosons in S_R^1

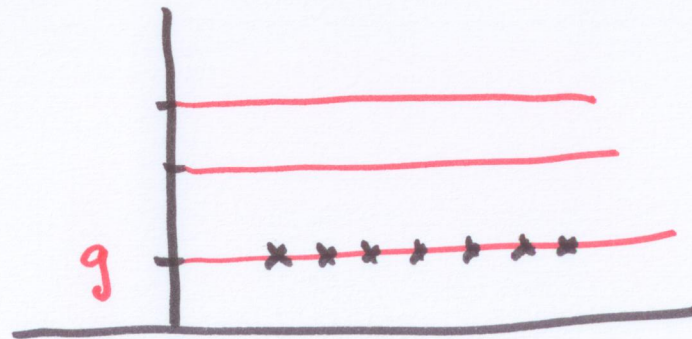
$$H = \int \psi^\dagger \frac{\partial^2}{\partial \theta^2} \psi - g(\psi^\dagger \psi)^2 \quad (\hat{a}_k, \hat{a}_k^\dagger)$$

weak coupling: $\Psi_0 = \sqrt{N} \psi_0 \quad \int |\psi_0|^2 = 1$

$$i\hbar \frac{\partial \psi_0}{\partial t} = \left(\frac{\partial^2}{\partial \theta^2} - g|\psi_0|^2 \right) \psi_0$$

Gross-Pitaevskii equation.

Ground state = Homogeneous Bose condensate.



- Low Lying Bogoliubov quasi-particles.

Bogoliubov - De Gennes equation.

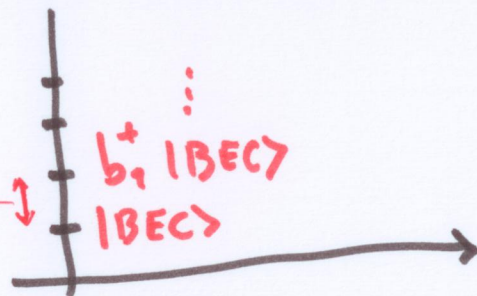
Determined by Bogoliubov Transformation

$$\hat{a}_k = u_k \hat{b}_k + v_k \hat{b}_k^\dagger$$

$$\begin{aligned} \hat{b}_k |BEC\rangle &= 0 \\ \hat{b}_k^\dagger |BEC\rangle & \end{aligned}$$

Spectrum:

$\Delta(g)$
(mass-gap)



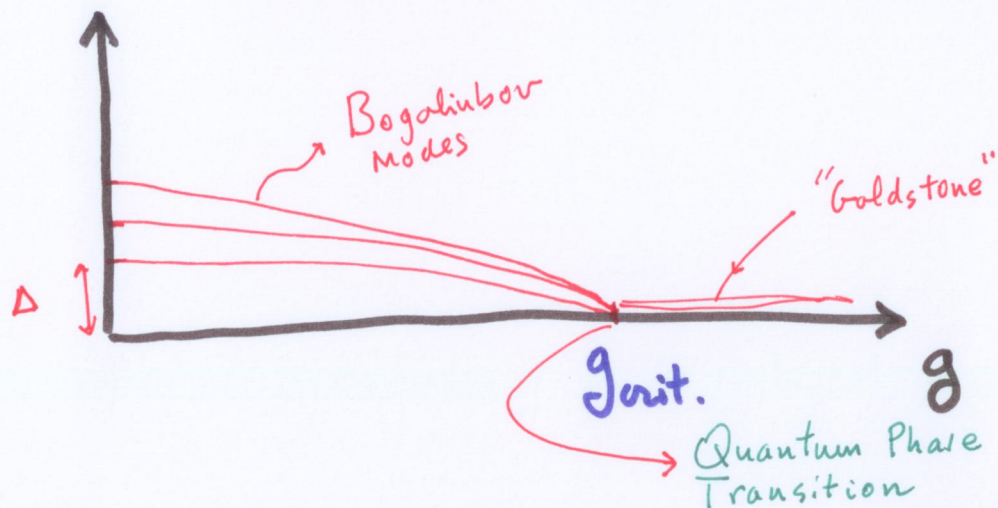
- Quantum Depletion.

$$\Delta N_k \sim \langle BEC | \hat{a}_k^\dagger \hat{a}_k | BEC \rangle$$

determined by
Bogoliubov Transform.

$$\Delta N_1(g)$$

Spectral Flow in coupling g .

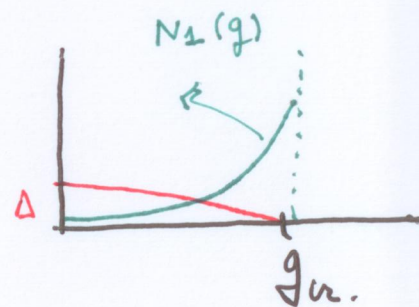


In (1+1) the phase $g > g_c$ is a solitonic phase.

At the critical point low lying Bogalubov modes become gapless.

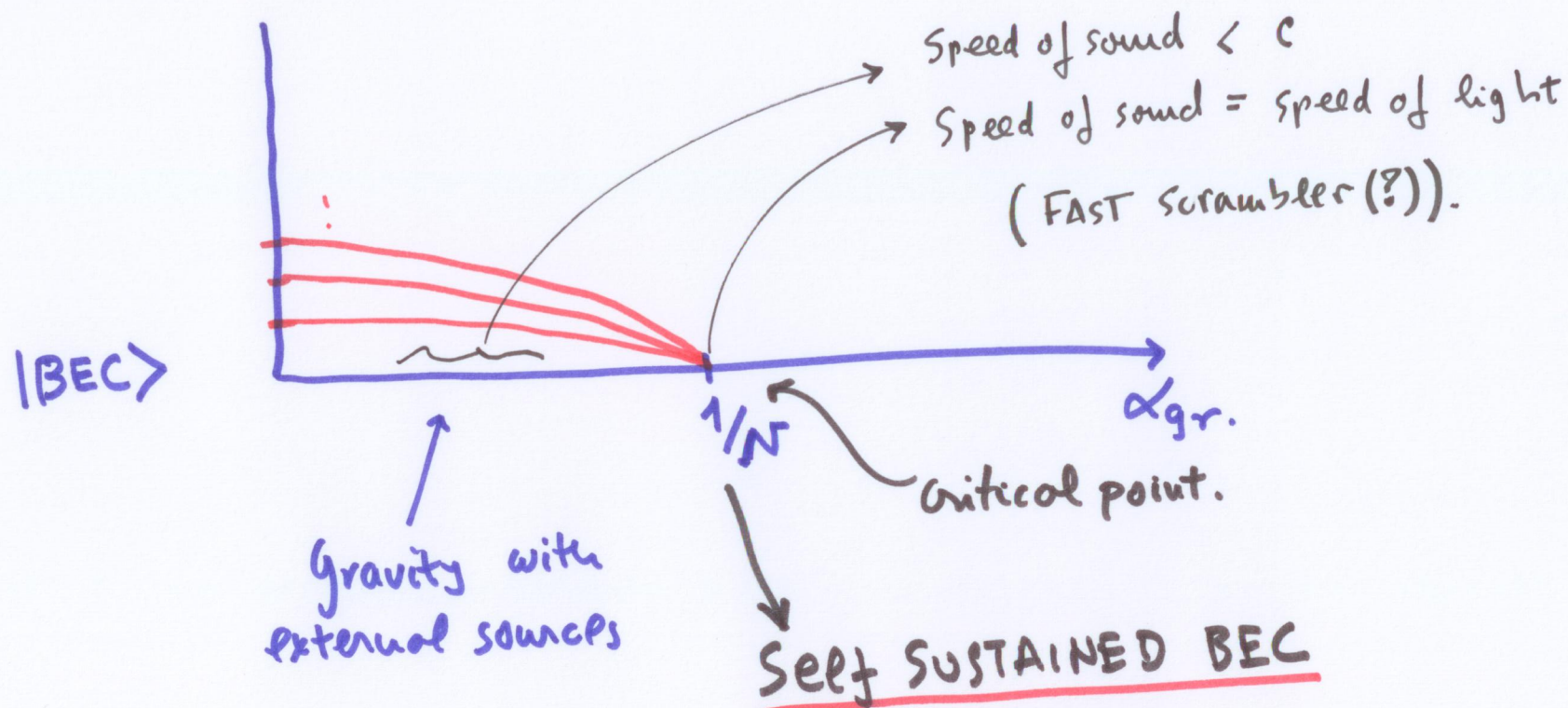
Critical Exponents:

$$\left\{ \begin{array}{l} \Delta(g) \sim (g - g_c)^{\delta_1} \\ N_1(g) \sim (g - g_c)^{-\delta_2} \end{array} \right.$$



3+1 Gravitational BEC

Bogalubov Spectrum:

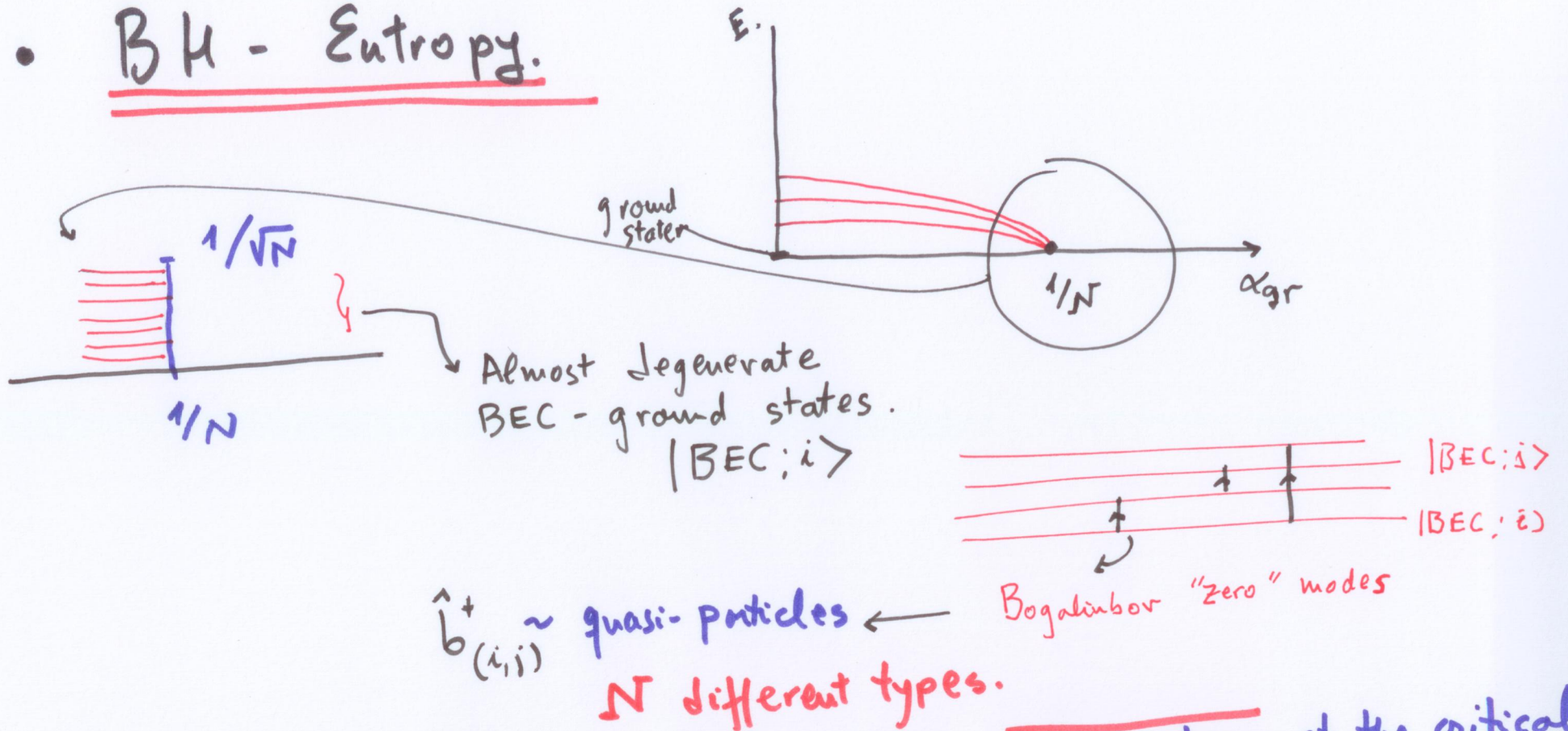


$$M = \sqrt{N} M_{pe}$$

$$R = \sqrt{N} L_{pe}$$

Black Hole mass-size relation.

- BH - Entropy.



Microcanonical Entropy: Ground state Degeneration at the critical point.

$$S \sim N$$

Emergent Picture:

Holographic degrees of freedom $\leftrightarrow$
Almost gapless Bogoliubov quasi-particles.

N -different B. Modes $\leftrightarrow$ C.F.T ($c_{eff} \sim N$)

Quantum Correction

Denoting $|BEC; i\rangle$ the "Bogoliubov" vacuum at $\alpha_{gr} \sim \frac{1}{N}$

$$|BH\rangle = \sum c_i |BEC; i\rangle$$

$$\rho = |BH\rangle\langle BH| =$$

$$\begin{pmatrix} \ddots & & c_i c_j^* \\ & c_i^2 & \\ & & \ddots \end{pmatrix}$$

↓

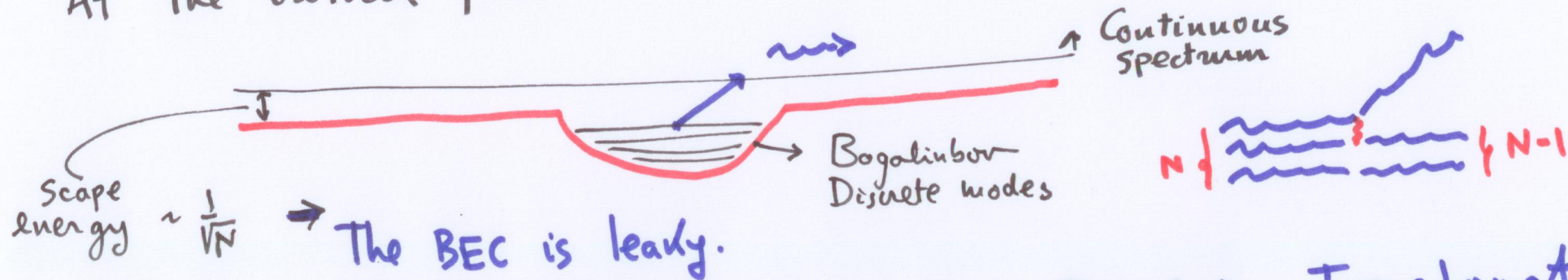
$$\begin{pmatrix} \ddots & & 0 \\ & c_i^2 & \\ & & \ddots \end{pmatrix}$$

Von Neuman Entropy:

$$S_{v.n} \sim + \ln N$$

Depletion and Hawking Radiation

At the critical point:



Quantum Depletion (Determined by BEC Bogolubov Transform)

Leads to radiation:

$$\boxed{\frac{dN}{dt} = \frac{1}{\sqrt{N}L_{pe}} + O\left(\frac{1}{N}\right)} \quad (*)$$

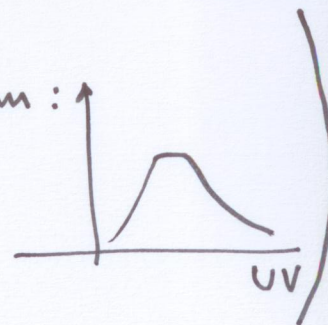
In the semi classical limit:

$$\boxed{N \rightarrow \infty \quad L_{pe} \neq 0 \quad \sqrt{N}L_{pe} = r_g \text{ (finite)}}$$

(*) = Hawking-Stefan-Boltzmann Law for BH's

$$\boxed{T_H = \frac{1}{\sqrt{N}L_{pe}}}$$

(*) Thermal spectrum:
Due to combinatorial suppression in the UV

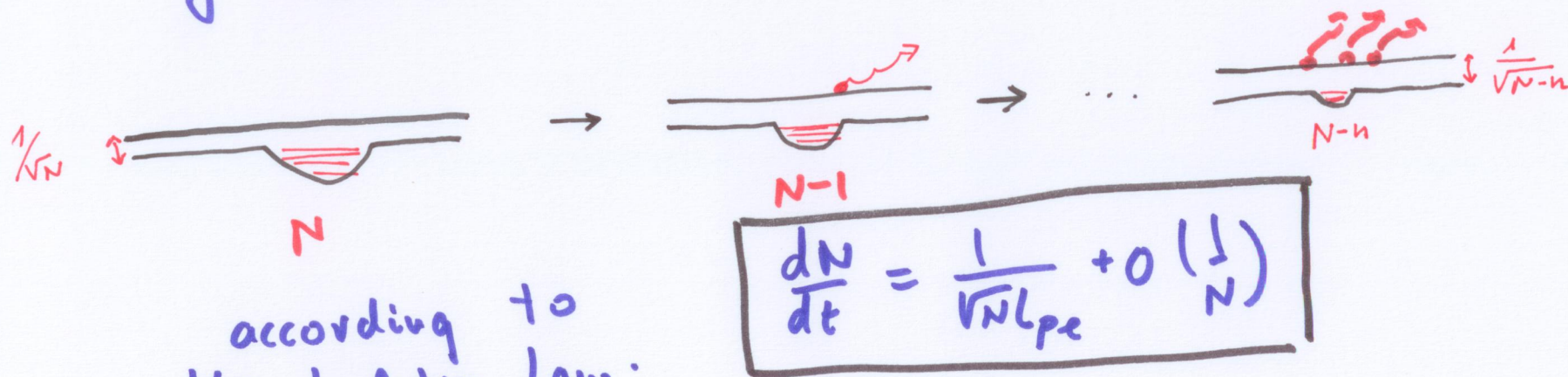


Hawking Radiation and Self Similar "Collapse".

In (3+1) we cannot define stable "solitons".

(Low Lying Bogalubov modes don't create enough Quantum Pressure to compensate attraction).

However: Due to the fact that the BEC is leaky $E_{sc} \sim \frac{1}{\sqrt{N}}$ the "collapse" takes place through a self similar cascade of Depletions.



according to
the depletion law:

$$\boxed{\frac{dN}{dt} = \frac{1}{\sqrt{N} L_{pe}} + O\left(\frac{1}{N}\right)}$$

$1/N$ - Effects (Hair)

Black Holes carry hair w.r.t global symmetries.

The rate of emission:

$$\frac{dB}{dt} = \frac{1}{\sqrt{N} L_{pe}} \frac{B}{N} + \dots$$

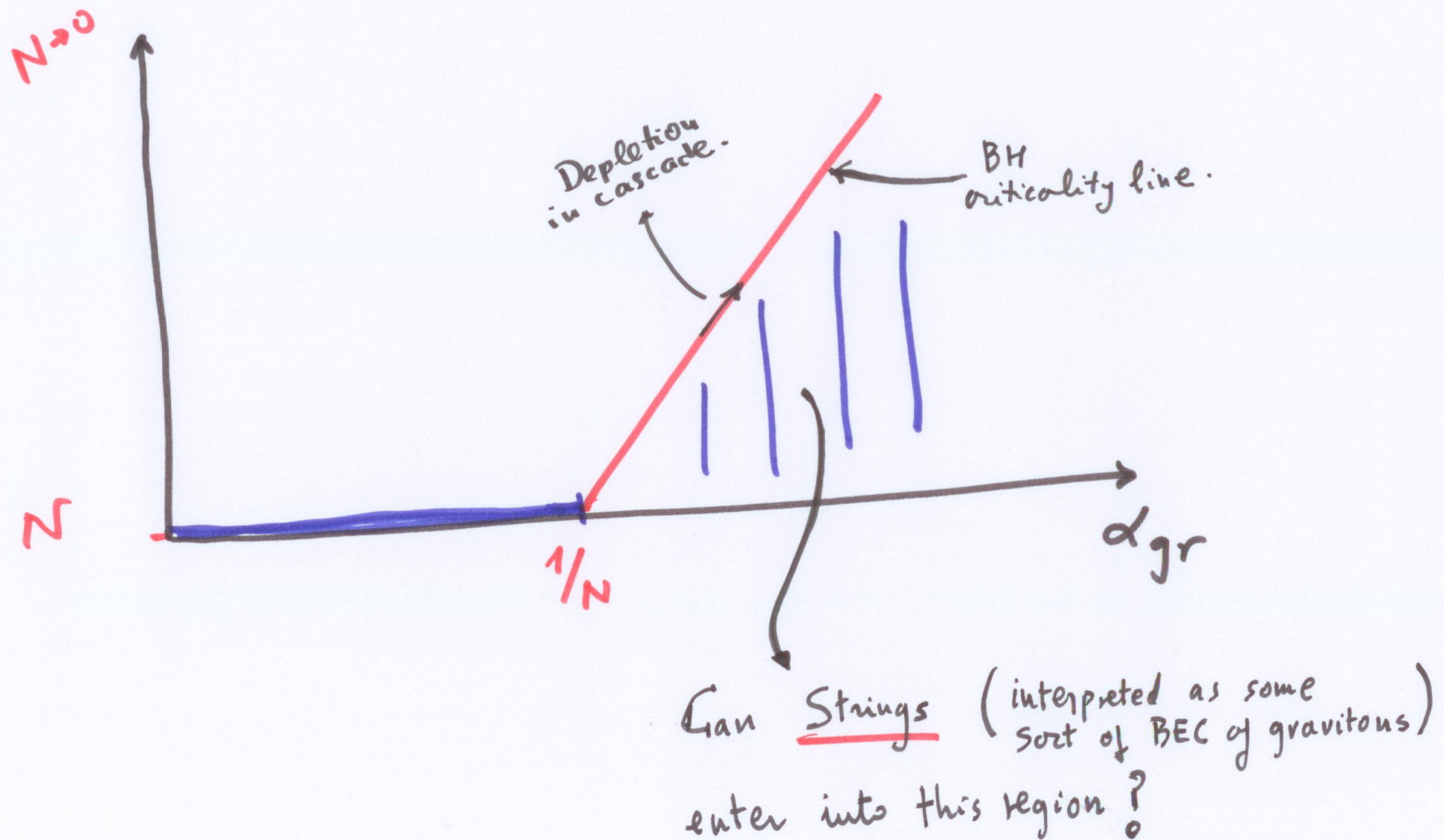
In the semi-classical limit $N \rightarrow \infty$ $\sqrt{N} L_{pe} = r$

$$\frac{dB}{dt} = 0$$

i.e. no Hair.

Note: These $1/N$ effects have astro. consequences.

The Gravitational BEC phase Diagram.



Thank - You

