The Wave Function of the Universe in Higher Spin Gravity

Daniel Harlow (with Anninos, Denef)

Stanford University - SITP, Princeton University - PCTS

July 25, 2012

dS/CFT

dS/CFT relates the Hartle Hawking wave function and the partition function of some CFT:

$$\Psi_{HH}[\epsilon^{d-\Delta}\sigma, \epsilon^{-2}g_{ij}] \sim e^{-S_{local}[g,\sigma,\epsilon]} \int \mathcal{D}\mathcal{M}\, e^{-S_{CFT}[g_{ij},\mathcal{M}]+\int d^d x \sqrt{g}\sigma(x)\mathcal{O}[\mathcal{M}]} \tag{1}$$

Typically the local divergences are pure phase so $|\Psi_{HH}|^2$ has a smooth limit as $\epsilon \to 0$.

The Sp(N) Theory

I'll describe some computations of $|\Psi_{HH}|^2$ using this dictionary in the $Sp(N)$ model Andy described:

$$S_{CFT} = \frac{1}{2}\int d^3x\sqrt{g}\Omega_{ab}\left[\partial_\mu\chi^a\partial^\mu\chi^b + \frac{1}{8}R[g]\chi^a\chi^b + \sigma(x)\chi^a\chi^b\right] \quad (2)$$

Here Ω_{ab} is some symplectic $N \times N$ matrix.

The Spectrum

- $\Omega_{ab}\chi^a\chi^b$ is dual to a massive bulk scalar and $T_{\mu\nu}$ is dual to the bulk metric. σ and g are finite deformations of the action by these operators.

The Spectrum

- $\Omega_{ab}\chi^a\chi^b$ is dual to a massive bulk scalar and $T_{\mu\nu}$ is dual to the bulk metric. σ and g are finite deformations of the action by these operators.
- There are also a set of higher spin "single-trace" $Sp(N)$ singlet operators

$$J^{(s)}_{\mu_1\ldots\mu_s} = \Omega_{ab}\chi^a \overleftrightarrow{\partial_{\mu_1}} \ldots \overleftrightarrow{\partial_{\mu_s}} \chi^b + \ldots \tag{3}$$

  which are symmetric, traceless, and conserved. For $s > 2$ these are dual to massless higher-spin bulk fields.

The Spectrum

- $\Omega_{ab}\chi^a\chi^b$ is dual to a massive bulk scalar and $T_{\mu\nu}$ is dual to the bulk metric. σ and g are finite deformations of the action by these operators.
- There are also a set of higher spin "single-trace" $Sp(N)$ singlet operators
$$J^{(s)}_{\mu_1\ldots\mu_s} = \Omega_{ab}\chi^a\overleftrightarrow{\partial}_{\mu_1}\ldots\overleftrightarrow{\partial}_{\mu_s}\chi^b + \ldots \quad (3)$$
which are symmetric, traceless, and conserved. For $s > 2$ these are dual to massless higher-spin bulk fields.
- Today I will only turn on sources for $s = 0, 2$.

Boundary Conditions and the Singlet Constraint

- Since the dimension of $J^{(0)}$ is $1 < \frac{3}{2}$, the "alternate quantization" is in play here so
$$\phi - \epsilon^3 \Pi = \frac{1}{4}\sqrt{N}\epsilon^2\sigma \tag{4}$$
at the future boundary in the bulk.

Boundary Conditions and the Singlet Constraint

- Since the dimension of $J^{(0)}$ is $1 < \frac{3}{2}$, the "alternate quantization" is in play here so
$$\phi - \epsilon^3 \Pi = \frac{1}{4}\sqrt{N}\epsilon^2\sigma \tag{4}$$
at the future boundary in the bulk.
- In FG gauge $\epsilon = |T|$ and $\Pi = T^{-2}\partial_T \phi$.

Boundary Conditions and the Singlet Constraint

- Since the dimension of $J^{(0)}$ is $1 < \frac{3}{2}$, the "alternate quantization" is in play here so
$$\phi - \epsilon^3 \Pi = \frac{1}{4}\sqrt{N}\epsilon^2\sigma \tag{4}$$
at the future boundary in the bulk.
- In FG gauge $\epsilon = |T|$ and $\Pi = T^{-2}\partial_T \phi$.
- Note in dS/CFT there is only one quantization; the "alternate quantization" is just looking at the same wave function in a different basis.

Boundary Conditions and the Singlet Constraint

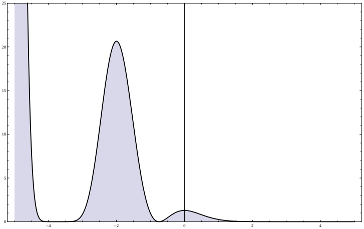
- Since the dimension of $J^{(0)}$ is $1 < \frac{3}{2}$, the "alternate quantization" is in play here so
$$\phi - \epsilon^3 \Pi = \frac{1}{4}\sqrt{N}\epsilon^2\sigma \tag{4}$$
at the future boundary in the bulk.
- In FG gauge $\epsilon = |T|$ and $\Pi = T^{-2}\partial_T \phi$.
- Note in dS/CFT there is only one quantization; the "alternate quantization" is just looking at the same wave function in a different basis.
- To impose the singlet constraint we in general need to gauge the $Sp(N)$ symmetry; we will do this by coupling to Chern-Simons and taking the limit $k \to \infty$. This factors out when the topology is S^3

A Constant Mass deformation on S^3

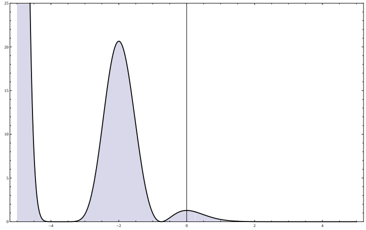
We want to compute

$$\log Z = \frac{N}{2}\log\det(-\Delta^2 + \frac{1}{8}R + \sigma_0)$$
$$= -S_{ct} + \frac{N}{2}\sum_{\ell=0}^{\infty}(\ell+1)^2 \log\left(\ell(\ell+2) + \frac{3}{4} + \sigma_0\right). \quad (5)$$

This can be done analytically, here is a plot of $|\Psi_{HH}|^2$ as a function of σ_0:

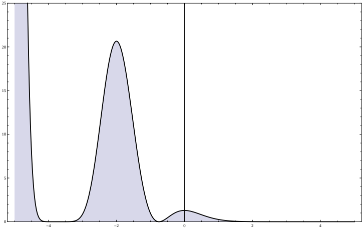


This can be done analytically, here is a plot of $|\Psi_{HH}|^2$ as a function of σ_0:



- It is locally peaked at the dS-invariant point $\sigma = 0$, and has zeros in the expected places.

This can be done analytically, here is a plot of $|\Psi_{HH}|^2$ as a function of σ_0:



- It is locally peaked at the dS-invariant point $\sigma = 0$, and has zeros in the expected places.
- It grows exponentially at large negative σ_0! This suggests a non-perturbative instability of dS space, recall that $\sigma \sim 1$ means $\phi \sim \sqrt{N}$. More comments below.

The wave function on a squashed sphere

We can study this partition function, with $\sigma = 0$, on a squashed three sphere as a function of the squashing parameter α.

$$ds^2 = \frac{r^2}{4}\left(d\theta^2 + \cos^2\theta d\phi^2 + \frac{1}{(1+\alpha)}(d\psi + \sin\theta d\phi)^2\right) \ , \qquad (6)$$

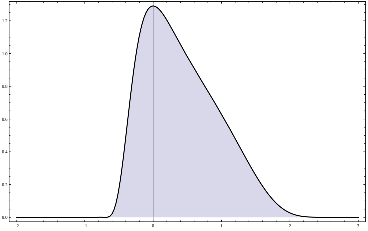
We do this numerically using a heat-kernel regulator and Euler-Maclaurin methods, here is the result:

The wave function on a squashed sphere

We can study this partition function, with $\sigma = 0$, on a squashed three sphere as a function of the squashing parameter α.

$$ds^2 = \frac{r^2}{4}\left(d\theta^2 + \cos^2\theta d\phi^2 + \frac{1}{(1+\alpha)}\left(d\psi + \sin\theta d\phi\right)^2\right) , \qquad (6)$$

We do this numerically using a heat-kernel regulator and Euler-Maclaurin methods, here is the result:



Here I we have plotted $|\Psi_{HH}|^2$ as a function of ρ, defined by $\alpha = e^{2\rho} - 1$. It is globally peaked at the round S^3.

S2 times S1

The Chern-Simons no longer decouples. We are interested in the partition function as a function of $\beta = 1/T$, the relative size of S^1 and S^2. (Shenker/Yin, Aharony/Marsano, et. al.)

S2 times S1

The Chern-Simons no longer decouples. We are interested in the partition function as a function of $\beta = 1/T$, the relative size of S^1 and S^2. (Shenker/Yin, Aharony/Marsano, et. al.)

$$Z_{CFT} = \frac{1}{N!} \int \prod_n (d\alpha_n) \exp\Big[\sum_{n<m} \log \sin^2 \left(\frac{\alpha_n - \alpha_m}{2} \right) - 2 \sum_{m=1}^{\infty} \frac{1}{m} z_S(e^{-\beta m}) \sum_{n=1}^{N} \cos(m\alpha_n) \Big] , \quad (7)$$

S2 times S1

The Chern-Simons no longer decouples. We are interested in the partition function as a function of $\beta = 1/T$, the relative size of S^1 and S^2. (Shenker/Yin, Aharony/Marsano, et. al.)

$$Z_{CFT} = \frac{1}{N!} \int \prod_n (d\alpha_n) \exp\Big[\sum_{n<m} \log \sin^2 \left(\frac{\alpha_n - \alpha_m}{2} \right) - 2 \sum_{m=1}^{\infty} \frac{1}{m} z_S(e^{-\beta m}) \sum_{n=1}^{N} \cos(m\alpha_n) \Big] \ , \quad (7)$$

At large enough "temperature" , meaning $T \gg \sqrt{N}$, the eigenvalue potential $\cos(\alpha_i)$ pushes all the eigenvalues to π (Gross/Witten).

S2 times S1

The Chern-Simons no longer decouples. We are interested in the partition function as a function of $\beta = 1/T$, the relative size of S^1 and S^2. (Shenker/Yin, Aharony/Marsano, et. al.)

$$Z_{CFT} = \frac{1}{N!} \int \prod_n (d\alpha_n) \exp \Big[\sum_{n<m} \log \sin^2 \left(\frac{\alpha_n - \alpha_m}{2} \right) - 2 \sum_{m=1}^{\infty} \frac{1}{m} z_S(e^{-\beta m}) \sum_{n=1}^{N} \cos(m\alpha_n) \Big] , \quad (7)$$

At large enough "temperature" , meaning $T \gg \sqrt{N}$, the eigenvalue potential $\cos(\alpha_i)$ pushes all the eigenvalues to π (Gross/Witten). We then have

$$\log Z_{CFT} = 3\zeta(3) N T^2. \quad (8)$$

Apparently the wave function squared diverges at small S^1.

Final Comments

- The divergence at small S^1 also happens in Einstein gravity, if one includes certain complex solutions in a dS version of the standard Hawking Page analysis.

Final Comments

- The divergence at small S^1 also happens in Einstein gravity, if one includes certain complex solutions in a dS version of the standard Hawking Page analysis.
- The divergence at negative σ_0 actually brings about a non-perturbative inconsistency in the *critical* interacting $Sp(N)$ model on S^3. Nonperturbatively in N there seems to be no reasonable path integral that produces the expected $1/N$ perturbation theory. This does not happen in the $O(N)$ model (it had better not!), we study both in our paper.

- Our interpretation of the scalar divergence is that the probability distribution at late times is not concentrated around asymptotically de Sitter configurations; in other words de Sitter in Vasiliev theory is unstable!* This is consistent with general arguments about de Sitter space.

- Our interpretation of the scalar divergence is that the probability distribution at late times is not concentrated around asymptotically de Sitter configurations; in other words de Sitter in Vasiliev theory is unstable!* This is consistent with general arguments about de Sitter space.
- If it is unstable, where does it go? Chang, Minwalla, Sharma, and Yin have recently argued that Vasiliev's theories can be understood as limits of string theories, perhaps these decays dynamically connect Vasiliev theory in dS space to the rest of the string landscape?

- Our interpretation of the scalar divergence is that the probability distribution at late times is not concentrated around asymptotically de Sitter configurations; in other words de Sitter in Vasiliev theory is unstable!* This is consistent with general arguments about de Sitter space.
- If it is unstable, where does it go? Chang, Minwalla, Sharma, and Yin have recently argued that Vasiliev's theories can be understood as limits of string theories, perhaps these decays dynamically connect Vasiliev theory in dS space to the rest of the string landscape?

Thanks for listening!