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Universality in all-order α' corrections to BPS/non-BPS brane world volume theories.

Ehsan Hatefi

ICTP,Trieste

based on :

1205.5079 [E.H,I.Park ,NPB],

1003.0314 [E.H,JHEP],

1203.1329 [E.H ,PRD],

1203.5553 [E.H, I.Park,PRD],

1204.2711, [E.H, A.Nurmagambetov, I.Park,NPB],

1204.6303[E.H,A.Nurmagambetov,I.Y.Park]

Motivations

1) Discovering Universality in all-order α' corrections to BPS/non-BPS brane world volume theories.

2) It seems that the description of world volume dynamics of D-brane is still lacking at some fundamental level .

3) We are able to produce the string theory corrections to the field theory perturbatively in α' by means of scattering amplitude argument.

4) Another more ambitious direction would be to make progress in the complete form of the non-abelian DBI and tachyonic effective actions.

5) Discovering closed form of higher derivative corrections for all BPS/non-BPS amplitudes.

6) Given that a close interplay between an open string and a closed string must be behind AdS/CFT, amplitudes involving a mixture of open string states and closed string states should be especially worth studying.

7) Discovering new Myers terms

8) I believe that the result of these works will provide the basis for future research on, e.g., next-to-leading order dielectric effect and other related topics in string theory.

9) Study of higher derivative corrections might shed new light in understanding properties of string theory in time-dependent background ,in particular for Tachyons.

10) higher derivative terms are not included in BSFT formalism so the only way to find hdcs with exact coefficients is indeed S-Matrix computations in super string theory.

Note that the derivatives of the gauge field strength, and the derivatives of the scalars are not included in DBI action.

Applying S-Matrix method , we embed them in DBI.

Let us start by giving an example

- Suppose , we want to produce the amplitude of one RR, and three scalar fields in II superstring theory:

$$\mathcal{A}'_3 \sim 2^{-1/2} L'_1 \Big\{ \Big[\xi_{1i} \mathrm{Tr}\,(P_- \not{H}_{(n)} M_p \gamma^i)(ts\xi_3.\xi_2) \Big] + \Big[1 \leftrightarrow 2 \Big] + \Big[1 \leftrightarrow 3 \Big] \Big\}$$

$$L'_1 = (2)^{-2(t+s+u)} \pi \frac{\Gamma(-u+\frac{1}{2})\Gamma(-s+\frac{1}{2})\Gamma(-t+\frac{1}{2})\Gamma(-t-s-u)}{\Gamma(-u-t+1)\Gamma(-t-s+1)\Gamma(-s-u+1)}$$

$$
\begin{aligned}
stL_1' &= -\pi^{5/2}\Bigg(\sum_{n=0}^{\infty} c_n(s+t+u)^n + \frac{\sum_{n,m=0}^{\infty} c_{n,m}[s^n t^m + s^m t^n]}{(t+s+u)} \\
&\quad + \sum_{p,n,m=0}^{\infty} f_{p,n,m}(s+t+u)^p[(s+t)^n(st)^m]\Bigg),
\end{aligned}
$$

Massless scalar poles for $n = p+2$ case

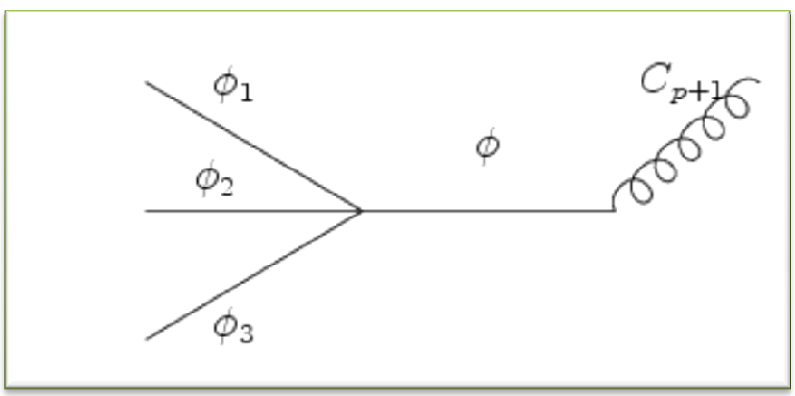
Working out the trace and using special expansions in the previous section, the massless scalar poles of the string amplitude take

$$
\begin{aligned}
&16\pi^3\mu_p\frac{\epsilon^{a_0\cdots a_p}H^{i(p+2)}_{a_0\cdots a_p}}{(p+1)!(s+t+u)}\mathrm{Tr}\,(\lambda_1\lambda_2\lambda_3)\sum_{n,m=0}^{\infty} c_{n,m}\Big(us\xi_3^i\xi_1.\xi_2[s^m u^n + s^n u^m] \\
&+ut\xi_2^i\xi_1.\xi_3[t^m u^n + t^n u^m] + ts\xi_1^i\xi_3.\xi_2[s^m t^n + s^n t^m]\Big)
\end{aligned}
$$

The first simple massless scalar pole is reproduced by the **non-abelian kinetic terms of the scalar field ,**

$$-T_p(2\pi\alpha')^4\mathrm{STr}\left(-\frac{1}{4}D_a\phi^i D_b\phi_i D^b\phi^j D^a\phi_j + \frac{1}{8}(D_a\phi^i D^a\phi_i)^2\right)$$

In order to produce all massless scalar poles one has to find the higher derivative corrections of BI action to all orders of α'



$$-T_p(2\pi\alpha')^4 \mathrm{STr}\left(-\frac{1}{4}D_a\phi^i D_b\phi_i D^b\phi^j D^a\phi_j + \frac{1}{8}(D_a\phi^i D^a\phi_i)^2\right)$$

Applying our prescription, one can easily determine their higher derivative forms by noting universality property that was present in the previous works as follows

$$(2\pi\alpha')^4 \frac{1}{4\pi^2} T_p\,(\alpha')^{n+m} \sum_{m,n=0}^{\infty} (\mathcal{L}_{11}^{nm} + \mathcal{L}_{12}^{nm} + \mathcal{L}_{13}^{nm})$$

$$\mathcal{L}_{11}^{nm} = -\mathrm{Tr}\left(a_{n,m}\mathcal{D}_{nm}[D_a\phi^i D_b\phi_i D^b\phi^j D^a\phi_j] + \ b_{n,m}\mathcal{D}'_{nm}[D_a\phi^i D^b\phi^j D_b\phi_i D^a\phi_j] + h.c. \right)$$
$$\mathcal{L}_{12}^{nm} = -\mathrm{Tr}\left(a_{n,m}\mathcal{D}_{nm}[D_a\phi^i D_b\phi_i D^a\phi^j D^b\phi_j] + \ b_{n,m}\mathcal{D}'_{nm}[D_b\phi^i D^b\phi^j D_a\phi_i D^a\phi_j] + h.c. \right)$$
$$\mathcal{L}_{13}^{nm} = \mathrm{Tr}\left(a_{n,m}\mathcal{D}_{nm}[D_a\phi^i D^a\phi_i D_b\phi^j D^b\phi_j] + \ b_{n,m}\mathcal{D}'_{nm}[D_a\phi^i D_b\phi^j D^a\phi_i D^b\phi_j] + h.c \right)$$

where

$$\mathcal{D}_{nm}(EFGH) \equiv D_{b_1}\cdots D_{b_m}D_{a_1}\cdots D_{a_n}EFD^{a_1}\cdots D^{a_n}GD^{b_1}\cdots D^{b_m}H,$$
$$\mathcal{D}'_{nm}(EFGH) \equiv D_{b_1}\cdots D_{b_m}D_{a_1}\cdots D_{a_n}ED^{a_1}\cdots D^{a_n}FGD^{b_1}\cdots D^{b_m}H.$$

The crucial step seems to extract the symmetric trace in term of ordinary trace and applying the higher derivative corrections $\mathcal{D}_{nm}, \mathcal{D}'_{nm}$ on them.

Universality in all-order α' higher derivative corrections of non-BPS and BPS branes

Several amplitudes suggest that there exists a regularity in the higher derivative expansions. One can formulate a prescription based on them

In order to find all infinite higher derivative corrections we must find S-matrix element of desired amplitudes which are either non-BPS or BPS amplitudes.

The next step is, using the relation between Mandelstam variables. We must rewrite the amplitudes such that all poles can be seen in a clear way.

The third step is finding leading couplings from tachyonic DBI or DBI action.

The last step is to express the symmetric trace in term of ordinary trace and applying the higher derivative corrections D_{nm}, D'_{nm} on them.

2^{nd} example

four-gauge field couplings

$$-T_p(2\pi\alpha')^4 S\mathrm{Tr}\left(-\frac{1}{8}F_{bd}F^{df}F_{fh}F^{hb}+\frac{1}{32}(F_{ab}F^{ba})^2\right).$$

The closed form of the higher derivative corrections of four gauge fields to all orders of α' (which must be added to DBI) was shown to be

$$(2\pi\alpha')^4\frac{1}{8\pi^2}T_p\,(\alpha')^{n+m}\sum_{m,n=0}^{\infty}(\mathcal{L}_5^{nm}+\mathcal{L}_6^{nm}+\mathcal{L}_7^{nm}),$$

with

$$\mathcal{L}_5^{nm} = -\mathrm{Tr}\left(a_{n,m}\mathcal{D}_{nm}[F_{bd}F^{df}F_{fh}F^{hb}]+\ b_{n,m}\mathcal{D}'_{nm}[F_{bd}F_{fh}F^{df}F^{hb}]+h.c.\right),$$
$$\mathcal{L}_6^{nm} = -\mathrm{Tr}\left(a_{n,m}\mathcal{D}_{nm}[F_{bd}F^{df}F^{hb}F_{fh}]+\ b_{n,m}\mathcal{D}'_{nm}[F_{bd}F^{hb}F^{df}F_{fh}]+h.c.\right),$$
$$\mathcal{L}_7^{nm} = \frac{1}{2}\mathrm{Tr}\left(a_{n,m}\mathcal{D}_{nm}[F_{ab}F^{ab}F_{cd}F^{cd}]+\ b_{n,m}\mathcal{D}'_{nm}[F_{ab}F^{cd}F^{ab}F_{cd}]+h.c\right),$$

where the higher derivative operators D_{nm} and D'_{nm} are defined

These couplings are exact up to total derivative terms and these corrections have been checked by explicit computations of the amplitude of one RR and **three gauge fields** [E.H,1003.0314,JHEP].

The 3rd example is **two scalar and two gauge field** couplings [E.H,I.Park 1203.5553,PRD](**CAA\phi**)

$$-\frac{T_p(2\pi\alpha')^4}{2}\mathrm{STr}\left(D_a\phi^i D^b\phi_i F^{ac}F_{bc}-\frac{1}{4}(D_a\phi^i D^a\phi_i F^{bc}F_{bc})\right).$$

Implementing the crucial step mentioned above, the closed form of the higher derivative corrections of two scalars and two gauge fields (which must be added to DBI) turned out to be

$$(2\pi\alpha')^4\frac{1}{2\pi^2}T_p\,(\alpha')^{n+m}\sum_{m,n=0}^{\infty}(\mathcal{L}_8^{nm}+\mathcal{L}_9^{nm}+\mathcal{L}_{10}^{nm}),$$

$$\begin{aligned}
\mathcal{L}_8^{nm} &= -\mathrm{Tr}\left(a_{n,m}\mathcal{D}_{nm}[D_a\phi^i D^b\phi_i F^{ac}F_{bc}]+b_{n,m}\mathcal{D}'_{nm}[D_a\phi^i F^{ac}D^b\phi_i F_{bc}]+h.c.\right),\\
\mathcal{L}_9^{nm} &= -\mathrm{Tr}\left(a_{n,m}\mathcal{D}_{nm}[D_a\phi^i D^b\phi_i F_{bc}F^{ac}]+\ b_{n,m}\mathcal{D}'_{nm}[D_a\phi^i F_{bc}D^b\phi_i F^{ac}]+h.c.\right),\\
\mathcal{L}_{10}^{nm} &= \frac{1}{2}\mathrm{Tr}\left(a_{n,m}\mathcal{D}_{nm}[D_a\phi^i D^a\phi_i F^{bc}F_{bc}]+\ b_{n,m}\mathcal{D}'_{nm}[D_a\phi^i F_{bc}D^a\phi_i F^{bc}]+h.c\right),
\end{aligned}$$

As usual, the above couplings are valid up to total derivative terms and terms such as $\partial_a\partial^a FFD\phi D\phi$ that vanish on-shell.

4th example :

Given the leading order vertices as follows, let us apply the

prescription to the higher derivatives vertices of **two tachyons**

and two scalar fields on the world volume of N non-BPS D-branes [E.H,1203.1329,PRD].

$$2T_p(\pi\alpha')^3\mathrm{STr}\left(m^2T^2(D_a\phi^iD^a\phi_i) + D^\alpha TD_\alpha TD_a\phi^iD^a\phi_i - 2D_a\phi^iD_b\phi_iD^bTD^aT\right)$$

$$2T_p(\pi\alpha')^3\mathrm{STr}\bigg(m^2T^2(D_a\phi^iD^a\phi_i)+\ D^\alpha TD_\alpha TD_a\phi^iD^a\phi_i-2D_a\phi^iD_b\phi_iD^bTD^aT\bigg)$$

the all-order vertices turned out to be

$$\mathcal{L} \;=\; -2T_p(\pi\alpha')(\alpha')^{2+n+m}\sum_{n,m=0}^{\infty}(\mathcal{L}_1^{nm}+\mathcal{L}_2^{nm}+\mathcal{L}_3^{nm}+\mathcal{L}_4^{nm})$$

where

$$\begin{aligned}
\mathcal{L}_1^{nm} &= m^2\mathrm{Tr}\bigg(a_{n,m}[\mathcal{D}_{nm}(T^2D_a\phi^iD^a\phi_i)+\mathcal{D}_{nm}(D_a\phi^iD^a\phi_iT^2)]\\
&\quad +b_{n,m}[\mathcal{D}'_{nm}(TD_a\phi^iTD^a\phi_i)+\mathcal{D}'_{nm}(D_a\phi^iTD^a\phi_iT)]+h.c.\bigg)\\
\mathcal{L}_2^{nm} &= \mathrm{Tr}\bigg(a_{n,m}[\mathcal{D}_{nm}(D^\alpha TD_\alpha TD_a\phi^iD^a\phi_i)+\mathcal{D}_{nm}(D_a\phi^iD^a\phi_iD^\alpha TD_\alpha T)]\\
&\quad +b_{n,m}[\mathcal{D}'_{nm}(D^\alpha TD_a\phi^iD_\alpha TD^a\phi_i)+\mathcal{D}'_{nm}(D_a\phi^iD_\alpha TD^a\phi_iD^\alpha T)]+h.c.\bigg)\\
\mathcal{L}_3^{nm} &= -\mathrm{Tr}\bigg(a_{n,m}[\mathcal{D}_{nm}(D^\beta TD_\mu TD^\mu\phi^iD_\beta\phi_i)+\mathcal{D}_{nm}(D^\mu\phi^iD_\beta\phi_iD^\beta TD_\mu T)]\\
&\quad +b_{n,m}[\mathcal{D}'_{nm}(D^\beta TD^\mu\phi^iD_\mu TD_\beta\phi_i)+\mathcal{D}'_{nm}(D^\mu\phi^iD_\mu TD_\beta\phi_iD^\beta T)]+h.c.\bigg)\\
\mathcal{L}_4^{nm} &= -\mathrm{Tr}\bigg(a_{n,m}[\mathcal{D}_{nm}(D^\beta TD^\mu TD_\beta\phi^iD_\mu\phi_i)+\mathcal{D}_{nm}(D^\beta\phi^iD^\mu\phi_iD_\beta TD_\mu T)]\\
&\quad +b_{n,m}[\mathcal{D}'_{nm}(D^\beta TD_\beta\phi^iD^\mu TD_\mu\phi_i)+\mathcal{D}'_{nm}(D_\beta\phi^iD_\mu TD^\mu\phi_iD^\beta T)]+h.c.\bigg)
\end{aligned}$$

This phenomenon **seems quite universal** and must have deep origins going back to the relation of **an open string and a closed string**.

It should originate from the composite nature of a closed string state in terms of open string states.

The amplitude of $< V_C V_\phi V_\phi V_\phi >$ has some extra terms that are absent in $< V_C V_A V_A V_A >$
to find the field theory vertices that reproduce all contact terms of the amplitude.

As a comment we were not able to reproduce all contact terms of four point amplitude $< V_C V_\phi V_\phi V_\phi >$
with usual pull-back. Thus it is a hint shows up that **pull-back may need modification.**

Several remarks on T-duality are in order.

T-duality can be straightforwardly employed to deduce a pure open string tree amplitude of scalar vertex operators from gauge amplitudes.

Once one considers an amplitude of a mixture of open and closed strings, direct computation is necessary because of the subtleties associated with T-duality.

Two subtleties exist in the very construction of RR C vertex operator .

First, the construction of the C vertex operator was such that one set of oscillators was used instead of two.

The second issue is that the C vertex operator does not contain winding modes, this must be related to the fact that we have pointed out where the terms that contain p^i are absent in (**CAAA**) amplitude.

In fact those new terms could have been obtained provided C vertex contains **winding modes** as well.

Above we have successfully determined the field theory amplitudes that reproduce all of the poles of the string amplitude. **We attempt** to achieve the same for the **contact terms**.

We have succeeded ***just*** in the case of $p + 4 = n$. However, as for the cases of $n = p + 2$, $n = p$, **we could not find the field theory vertices** that reproduce the leading order contact terms nor the infinite extension.

Perhaps this is a **hint** that **pull-back may need modification**.

Note that a supersymmetric generalization of DBI action is still unknown.

Thank you for your attention

The final form of the amplitude

$$\mathcal{A}'_1 \sim 2^{-1/2} 2\xi_{1i}\xi_{2j}\xi_{3k}(t+s+u)L'_1\Big[k_{3b}k_{2a}\mathrm{Tr}\,(P_-\not{H}_{(n)}M_p\Gamma^{kbjai}) - k_{3b}p^j\mathrm{Tr}\,(P_-\not{H}_{(n)}M_p\Gamma^{kbi})$$
$$-k_{2a}p^k\mathrm{Tr}\,(P_-\not{H}_{(n)}M_p\Gamma^{jai}) + p^jp^k\mathrm{Tr}\,(P_-\not{H}_{(n)}M_p\gamma^i)\Big]$$

$$\mathcal{A}'_2 \sim 2^{-1/2}L'_2\Big\{2us\xi_1.\xi_2k_{2a}k_{3b}\xi_{3k}\mathrm{Tr}\,(P_-\not{H}_{(n)}M_p\Gamma^{kba}) - ust\xi_{1i}\xi_{2j}\xi_{3k}\mathrm{Tr}\,(P_-\not{H}_{(n)}M_p\Gamma^{kji})$$
$$-2ut\xi_3.\xi_1k_{2a}k_{3b}\xi_{2j}\mathrm{Tr}\,(P_-\not{H}_{(n)}M_p\Gamma^{bja}) + 2st\xi_3.\xi_2k_{2a}k_{3b}\xi_{1i}\mathrm{Tr}\,(P_-\not{H}_{(n)}M_p\Gamma^{bai})$$
$$-2usk_{2a}p^k\xi_1.\xi_2\xi_{3k}\mathrm{Tr}\,(P_-\not{H}_{(n)}M_p\gamma^a) + 2utk_{3b}p^j\xi_1.\xi_3\xi_{2j}\mathrm{Tr}\,(P_-\not{H}_{(n)}M_p\gamma^b)\Big\}$$

$$\mathcal{A}'_3 \sim 2^{-1/2} L'_1 \Big\{ \Big[\xi_{1i} \mathrm{Tr}\,(P_- \not{H}_{(n)} M_p \gamma^i)(ts\xi_3.\xi_2) \Big] + \Big[1 \leftrightarrow 2 \Big] + \Big[1 \leftrightarrow 3 \Big] \Big\}.$$

The functions L'_1, L'_2 are given as follows:

$$\begin{aligned}
L'_1 &= (2)^{-2(t+s+u)} \pi \frac{\Gamma(-u+\frac{1}{2})\Gamma(-s+\frac{1}{2})\Gamma(-t+\frac{1}{2})\Gamma(-t-s-u)}{\Gamma(-u-t+1)\Gamma(-t-s+1)\Gamma(-s-u+1)}, \\
L'_2 &= (2)^{-2(t+s+u)} \pi \frac{\Gamma(-u)\Gamma(-s)\Gamma(-t)\Gamma(-t-s-u+\frac{1}{2})}{\Gamma(-u-t+1)\Gamma(-t-s+1)\Gamma(-s-u+1)}
\end{aligned}$$

These scalars appear in the action in three different ways:

1st: there is the explicit appearance in the exponential in Wess-Zumino action .

$$i_\Phi i_\Phi C^{(n)} = \frac{1}{2(n-2)!}[\Phi^i, \Phi^j]\, C^{(n)}_{ji\mu_3\cdots\mu_n} dx^{\mu_3}\cdots dx^{\mu_n},$$

$$\boxed{C^{(n)} = \frac{1}{n!} C^{(n)}_{\mu_1\cdots\mu_n} dx^{\mu_1}\cdots dx^{\mu_n}}$$

2^{nd},covariant derivatives of the non abelian scalars appear in the pull-back.That is,

$$P[E]_{ab} = E_{ab} + \lambda\, E_{ai}\, D_b\Phi^i + \lambda\, E_{ib}\, D_a\Phi^i + \lambda^2\, E_{ij} D_a\Phi^i D_b\Phi^j,$$

3rd, the action includes the transverse derivatives of the closed string fields through the Taylor expansion of these fields

$$\begin{aligned} G_{\mu\nu} &= \exp\left[\lambda\Phi^i\,\partial_{x^i}\right] G^0_{\mu\nu}(\sigma^a, x^i)|_{x^i=0} \\ &= \sum_{n=0}^{\infty} \frac{\lambda^n}{n!} \Phi^{i_1}\cdots\Phi^{i_n}\,(\partial_{x^{i_1}}\cdots\partial_{x^{i_n}})G^0_{\mu\nu}(\sigma^a, x^i)|_{x^i=0}\ . \end{aligned}$$

In order to have some non trivial interactions, we will need at least three open string states(CAAA,…) and a single closed string state.

$$
\begin{aligned}
S^{(6)} &= i\lambda\mu_p \int \mathrm{STr}\left(FP\left[C^{(p-1)}(\sigma,\phi)\right]\right) \\
&= i\lambda^2\mu_p \int d^{p+1}\sigma \frac{1}{(p-1)!}(\varepsilon^v)^{a_0\cdots a_p}\left[\mathrm{Tr}\left(F_{a_0a_1}\phi^k\right)\partial_k C^{(p-1)}_{a_2\ldots a_p}(\sigma)\right]
\end{aligned}
$$

$$
\boxed{
\begin{aligned}
S^{(5)} &= i\lambda\mu_p \int \mathrm{STr}\left(FP\left[C^{(p-1)}(\sigma,\phi)\right]\right) \\
&= i\lambda^2\mu_p \int d^{p+1}\sigma \frac{1}{(p-1)!}(\varepsilon^v)^{a_0\cdots a_p}\left[\mathrm{Tr}\left(F_{a_0a_1}\partial_{a_2}\phi^k\right)C^{(p-1)}_{ka_3\ldots a_p}(\sigma)\right]
\end{aligned}
}
$$