Why General Relativity is like a High Temperature Superconductor

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1204.0519, and to appear

Many properties of condensed matter systems can be described using general relativity:

1) Fermi surfaces
2) Non-Fermi liquids
3) Superconducting phase transitions
4) …

This is a consequence of gauge/gravity duality, in the limit where the bulk can be described by classical general relativity.

It is remarkable that general relativity can reproduce qualitative features of condensed matter systems, but can it do even more?

Can general relativity provide a quantitative explanation of some mysterious property of real materials?

We will see evidence that the answer is yes!

Most previous applications have assumed translational symmetry.

Unfortunate consequence: Any state with nonzero charge density has infinite DC conductivity. (A simple boost produces a nonzero current with no applied electric field.)

This can be avoided in a probe approximation (Karch, O'Bannon, 2007;….).

Plan: Add a lattice to the simplest holographic model of a conductor and calculate $\sigma(\omega)$.

A perfect lattice still has infinite conductivity due to Bloch waves. So we work at nonzero T and include dissipation. (Cf: Kachru et al; Maeda et al; Hartnoll and Hoffman; Liu et al.)

Result: We will find a surprising similarity to the optical conductivity in the normal phase of the cuprates.

Simple model of a conductor

Suppose electrons in a metal satisfy

$$m\frac{dv}{dt} = eE - m\frac{v}{\tau}$$

If there are n electrons per unit volume, the current density is J = nev. Letting $E(t) = Ee^{-i\omega t}$, find J = σ E, with

$$\sigma(\omega) = \frac{K\tau}{1 - i\omega\tau}$$

where $K = ne^2/m$. This is the Drude model.

$$\mathrm{Re}(\sigma) = \frac{K\tau}{1+(\omega\tau)^2}, \quad \mathrm{Im}(\sigma) = \frac{K\omega\tau^2}{1+(\omega\tau)^2}$$

Note:

(1) For $\omega\tau \gg 1, \quad |\sigma| \approx K/\omega$

(2) In the limit $\tau \to \infty$:

$$\mathrm{Re}(\sigma) \propto \delta(\omega), \quad \mathrm{Im}(\sigma) = K/\omega$$

This can be derived more generally from Kramers-Kronig relation.

Our gravity model

$$S = \int d^4x \sqrt{-g} \left[R + \frac{6}{L^2} - \frac{1}{2} F_{ab} F^{ab} - 2 \nabla_a \Phi \nabla^a \Phi + \frac{4\Phi^2}{L^2} \right]$$

We work in Poincare coordinates with boundary at z=0. Then

$$\Phi \to z\phi_1 + z^2 \phi_2 + \mathcal{O}(z^3)$$

We introduce the lattice by requiring:

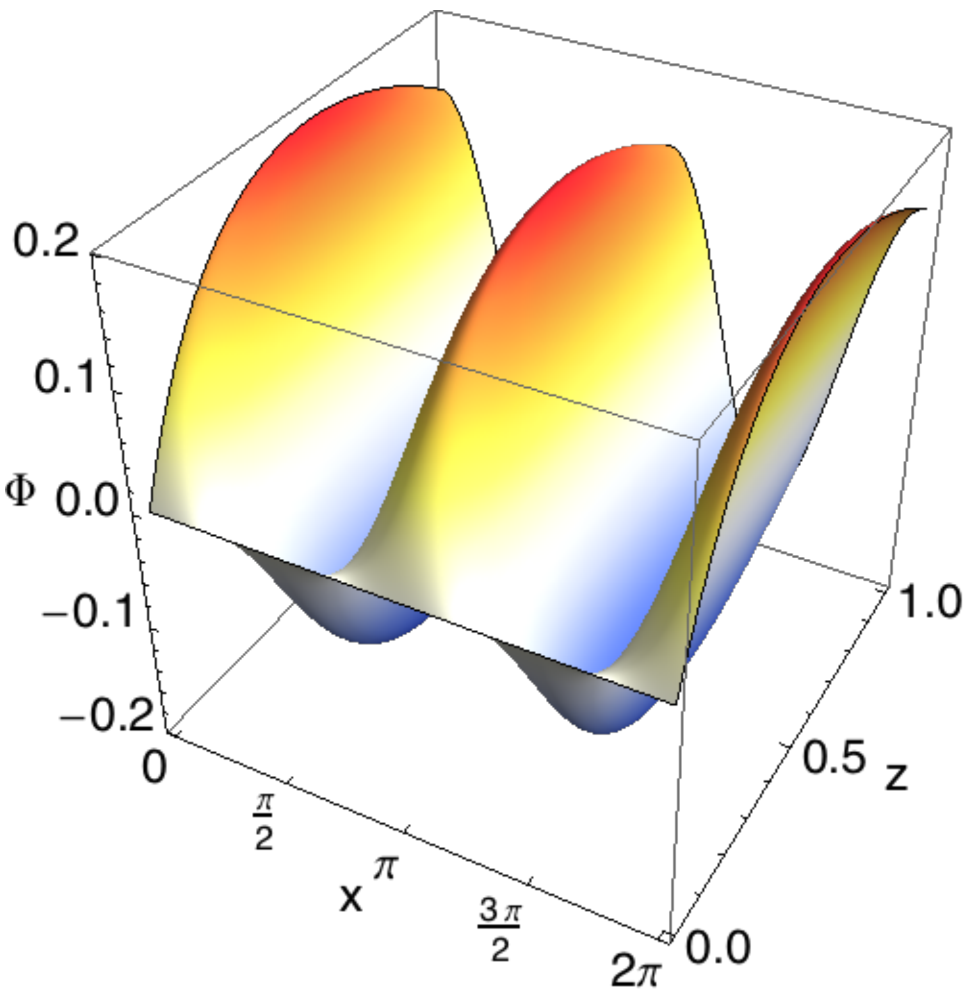
$$\phi_1(x) = \mathcal{A}_0 \cos(k_0 x)$$

Want finite temperature: Add black hole

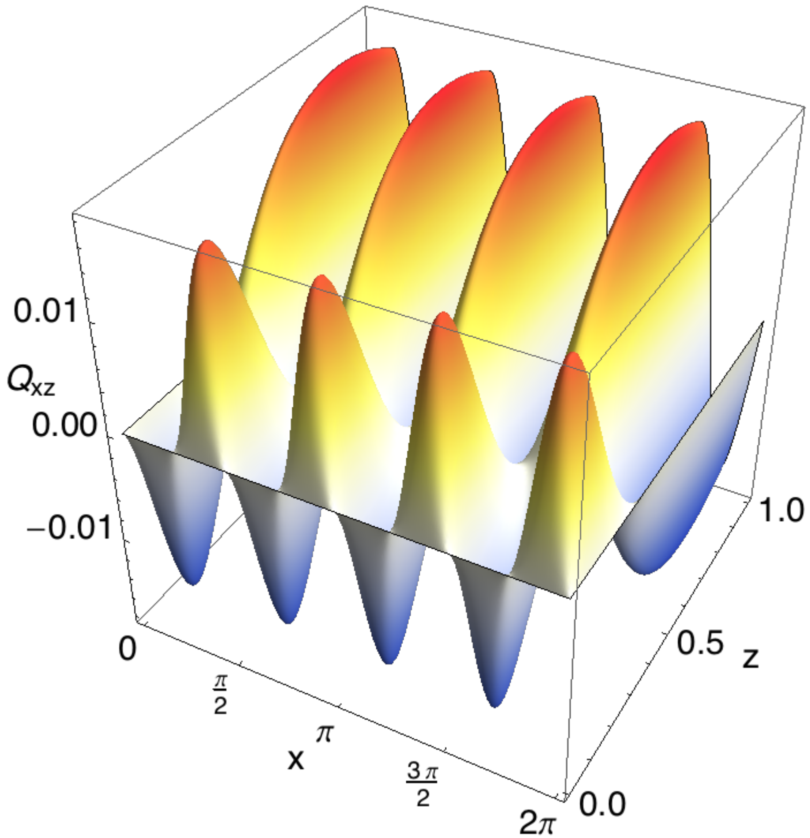
Want finite density: Add chemical potential

$$A_t\,(z=0) = \mu$$

We numerically find solutions with smooth horizons that are static and translationally invariant in one direction. (Have to solve 7 nonlinear coupled PDE's in 2D.)

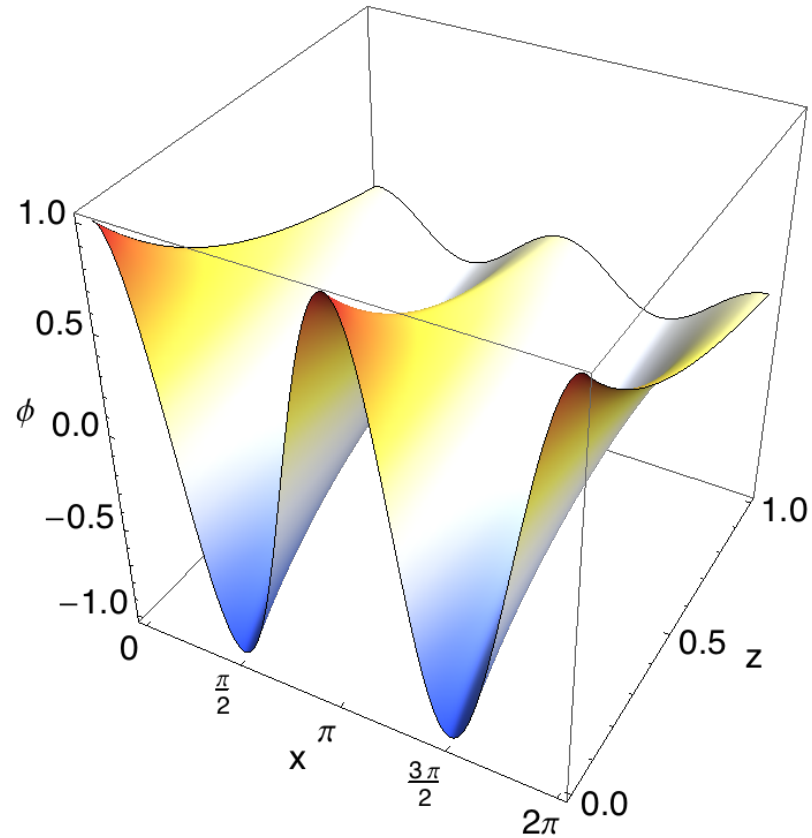


Solution with
$T/\mu = .1$,
$k_0 = 2$ and unit
amplitude



metric component

Lattice induced on metric has $k_0 = 4$.



scalar field

Lattice with $k_0 = 2$.

Conductivity

To compute the optical conductivity using linear response, we perturb the solution

$$g_{ab} = \widehat{g}_{ab} + h_{ab}, \quad A_a = \widehat{A}_a + b_a, \quad \Phi = \widehat{\Phi} + \eta$$

Boundary conditions:

ingoing waves at the horizon

h_{ab} and η normalizable at infinity

$b_t \sim O(z), \quad b_x = b_0 e^{-i\omega t} + O(z)$

↑ induced current

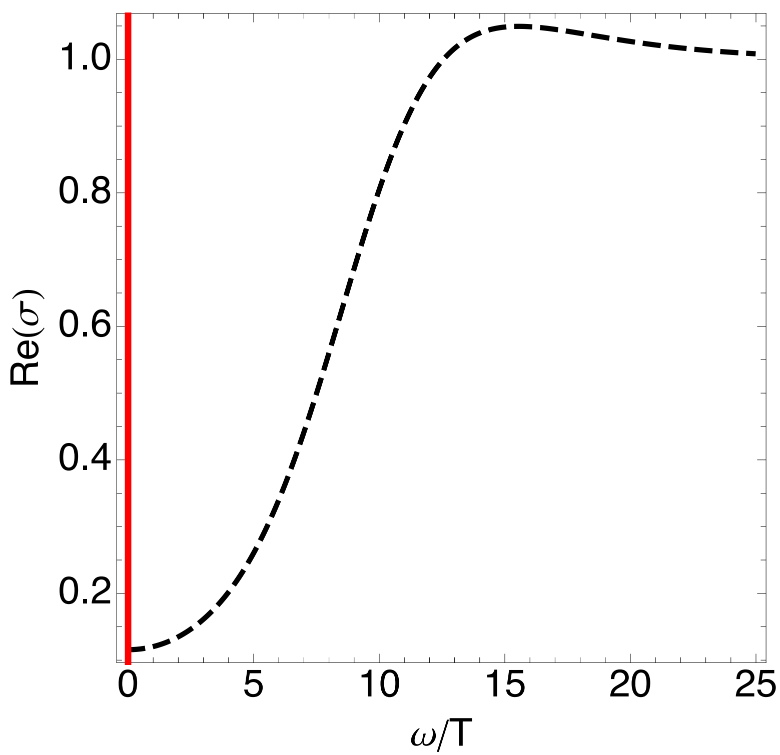
Using Ohm's law, J = σE, the optical conductivity is given by

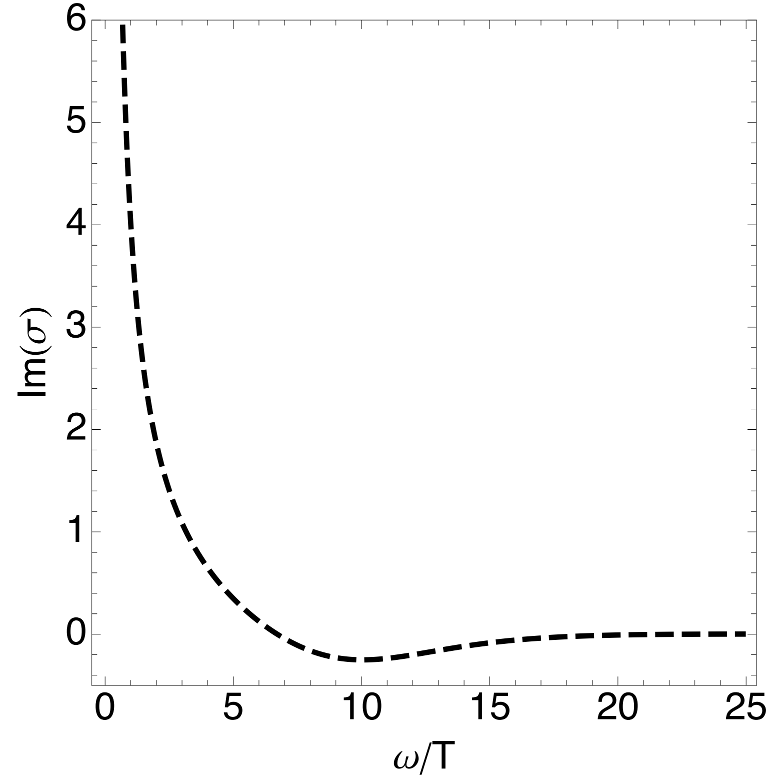
$$\tilde{\sigma}(\omega, x) \equiv \lim_{z \to 0} \frac{f_{zx}(x,z)}{f_{xt}(x,z)}$$

where f = db.

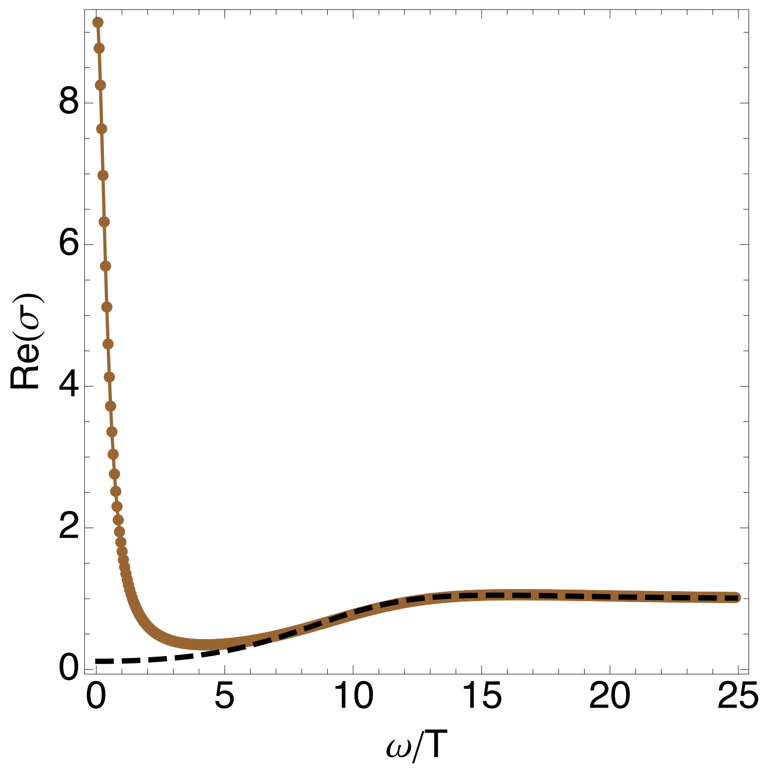
Since we impose a homogeneous electric field, we are interested in the homogeneous part of the conductivity σ(ω).

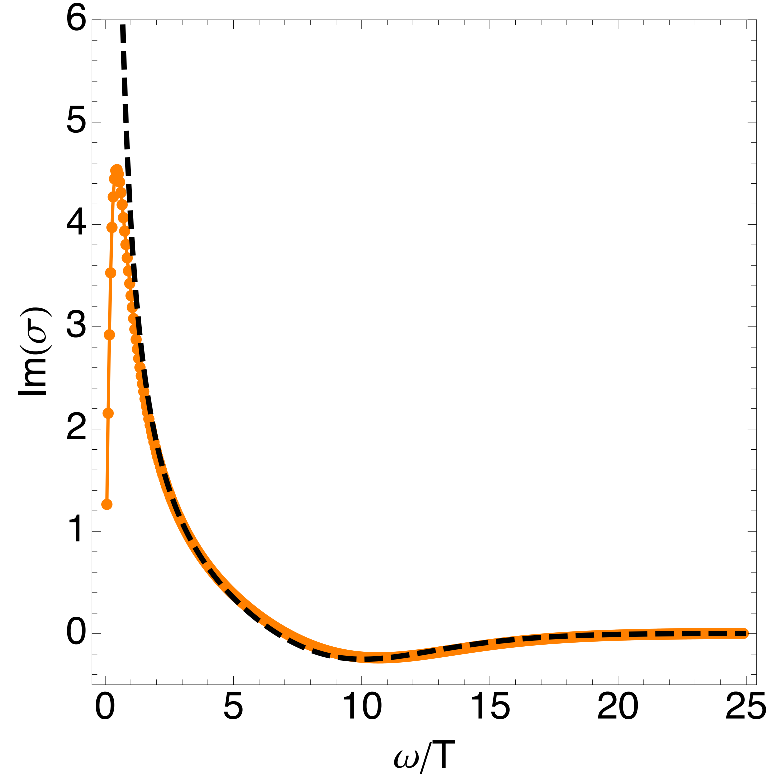
Review: optical conductivity with no lattice
(T/μ = .115)





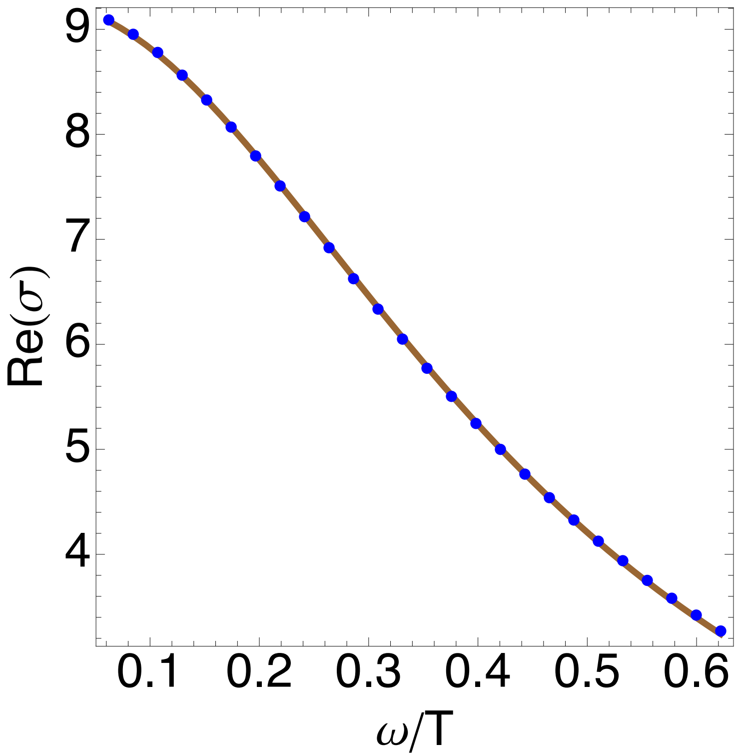
With the lattice, the delta function is smeared out

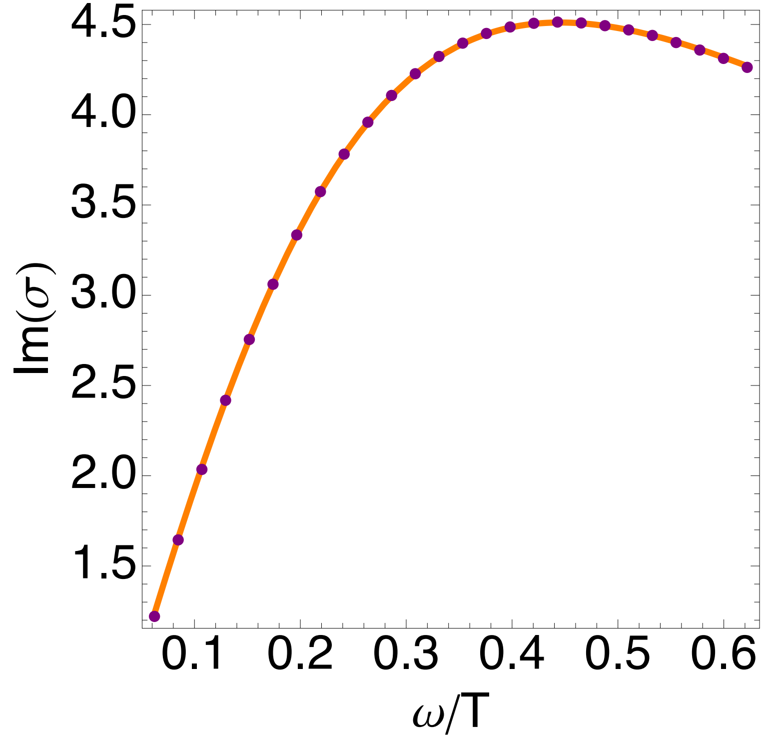




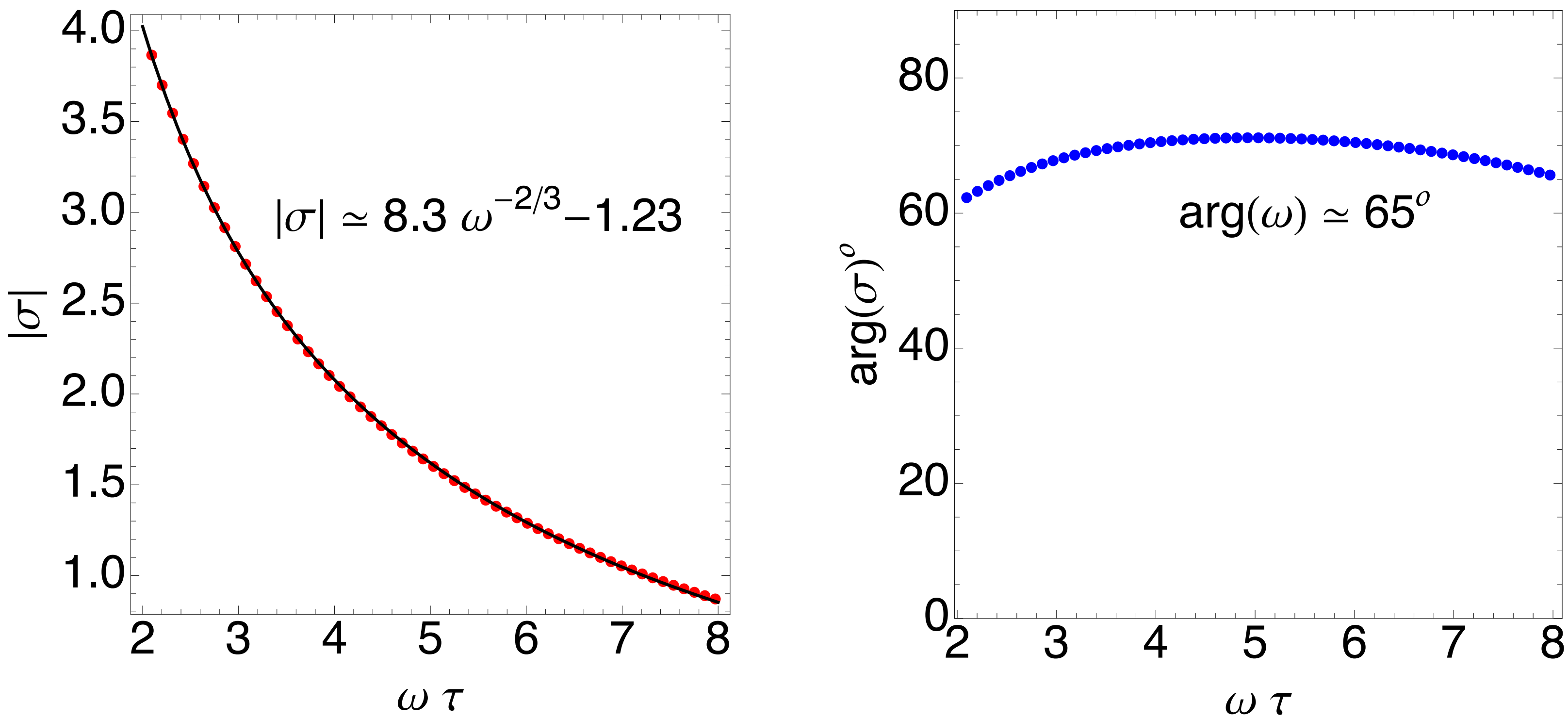
The low frequency conductivity takes the simple Drude form:

$$\sigma(\omega) = \frac{K\tau}{1 - i\omega\tau}$$



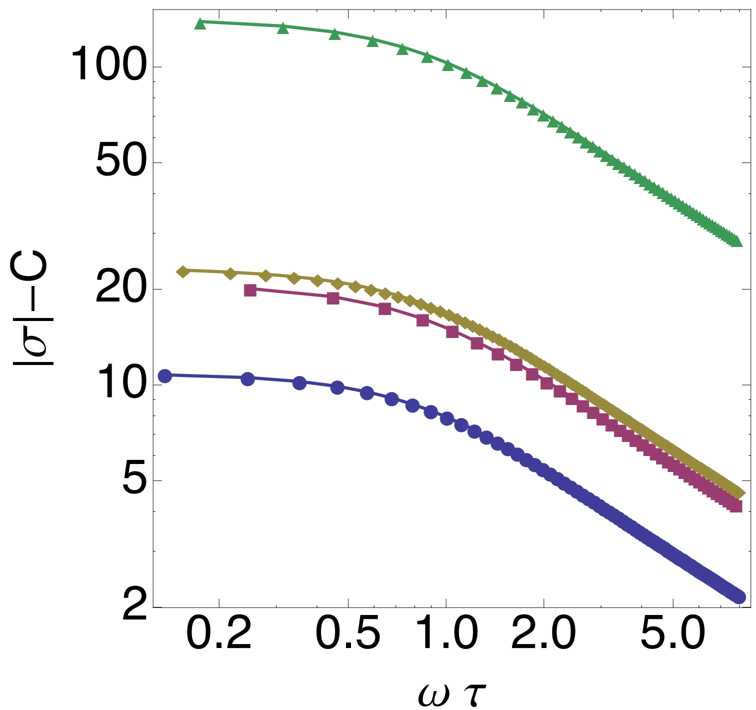


Intermediate frequency shows scaling regime

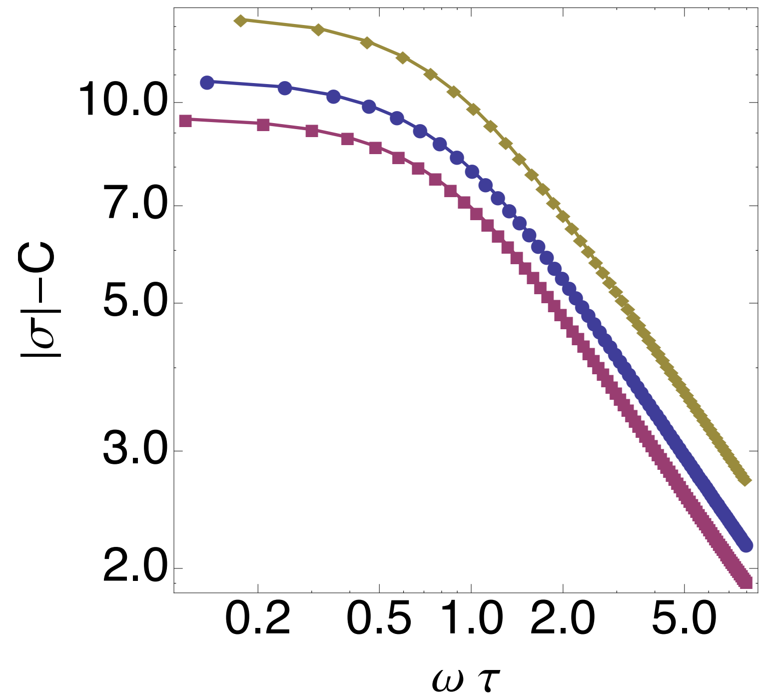


The data is very well fit by $$|\sigma| = \frac{B}{\omega^{2/3}} + C$$

The exponent 2/3 is robust

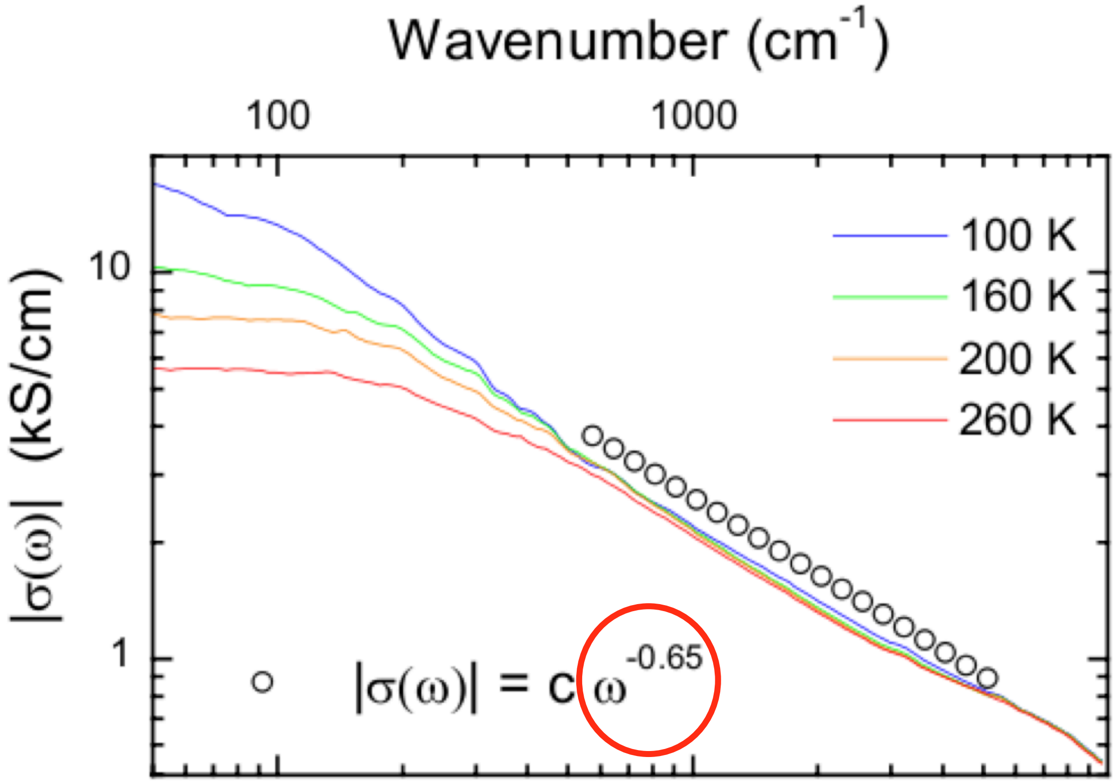


different wavenumbers
k_0 = .5, 1, 2, 3



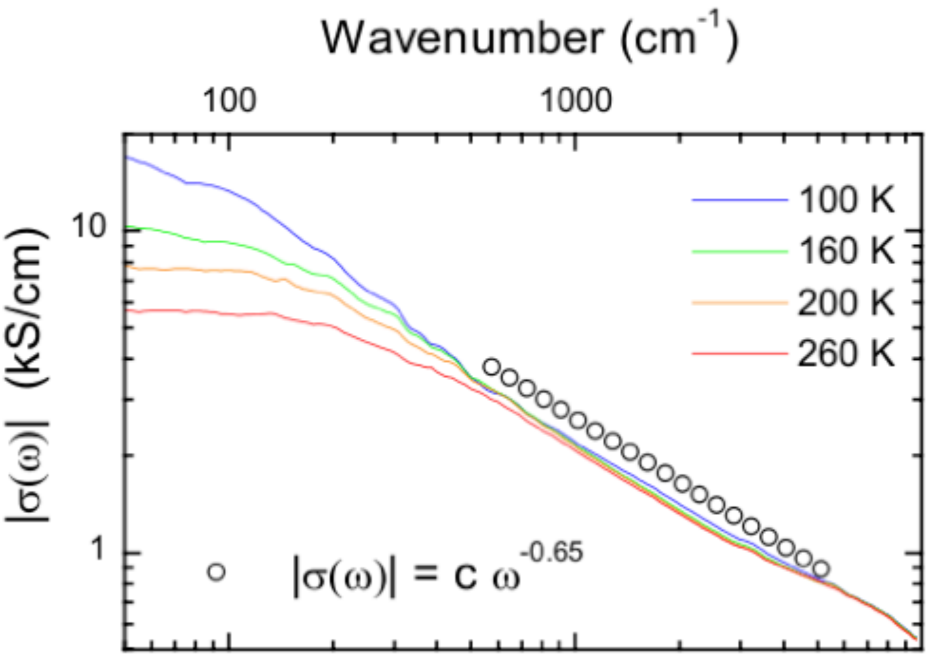
different temperatures
T/μ = .098, .115, .13

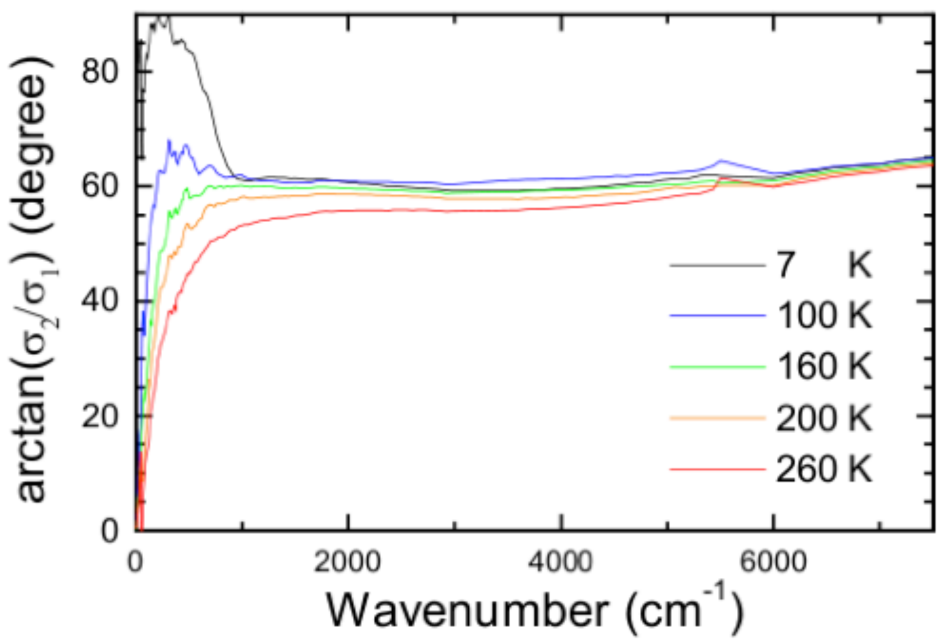
Comparison with the cuprates
(van der Marel, et al 2003)



$$Bi_2Sr_2Ca_{0.92}Y_{0.08}Cu_2O_{8+\delta}$$

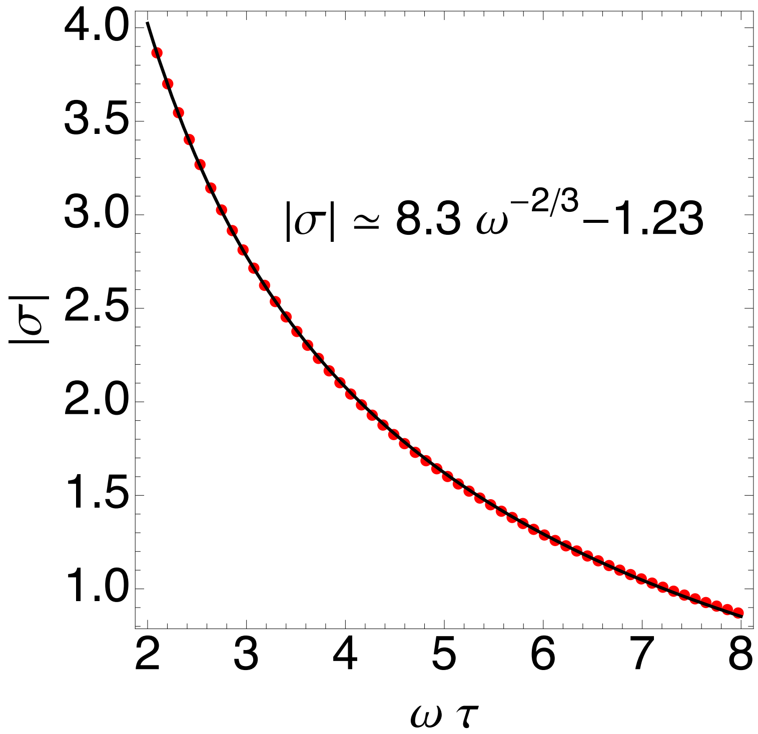
The phase remains approximately constant

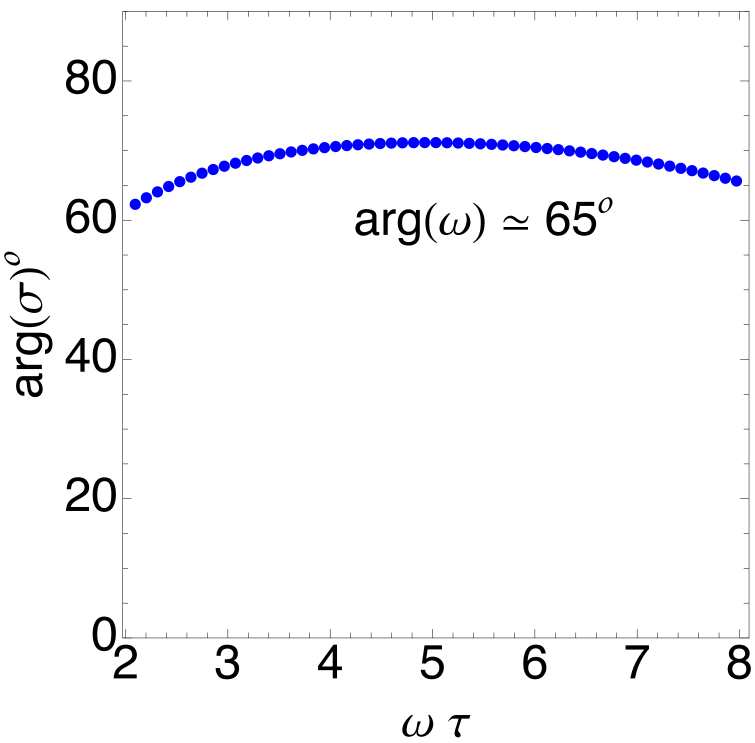




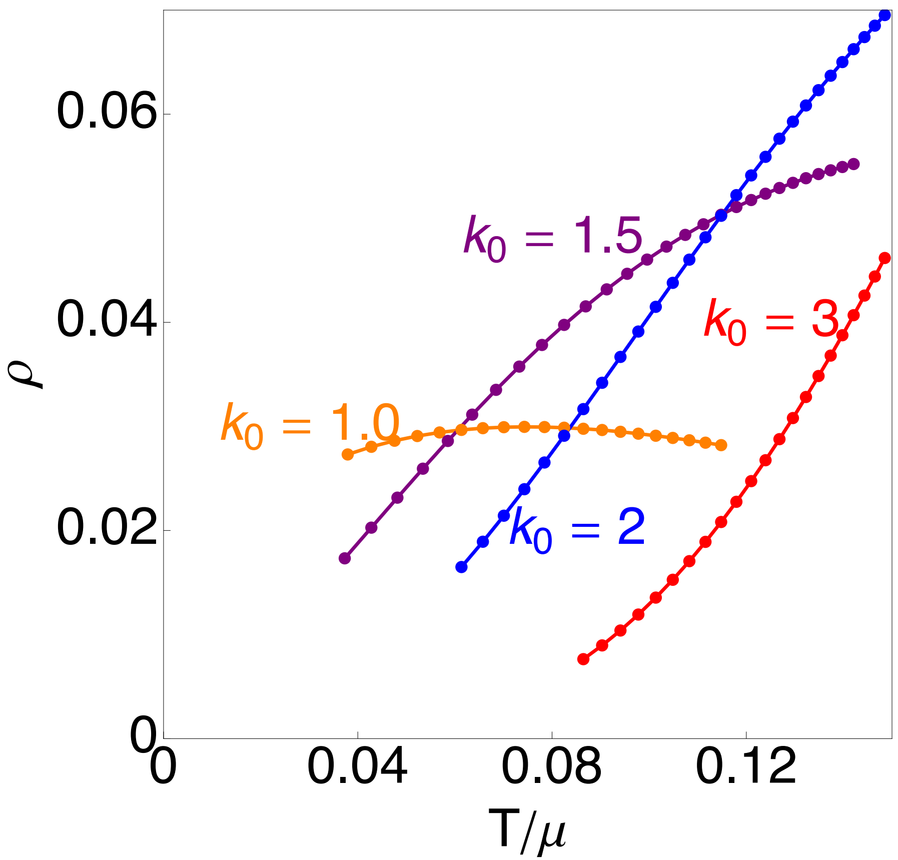
$$Bi_2Sr_2Ca_{0.92}Y_{0.08}Cu_2O_{8+\delta}$$

Just like our data (but our phase varies slightly with k_0)





DC resistivity



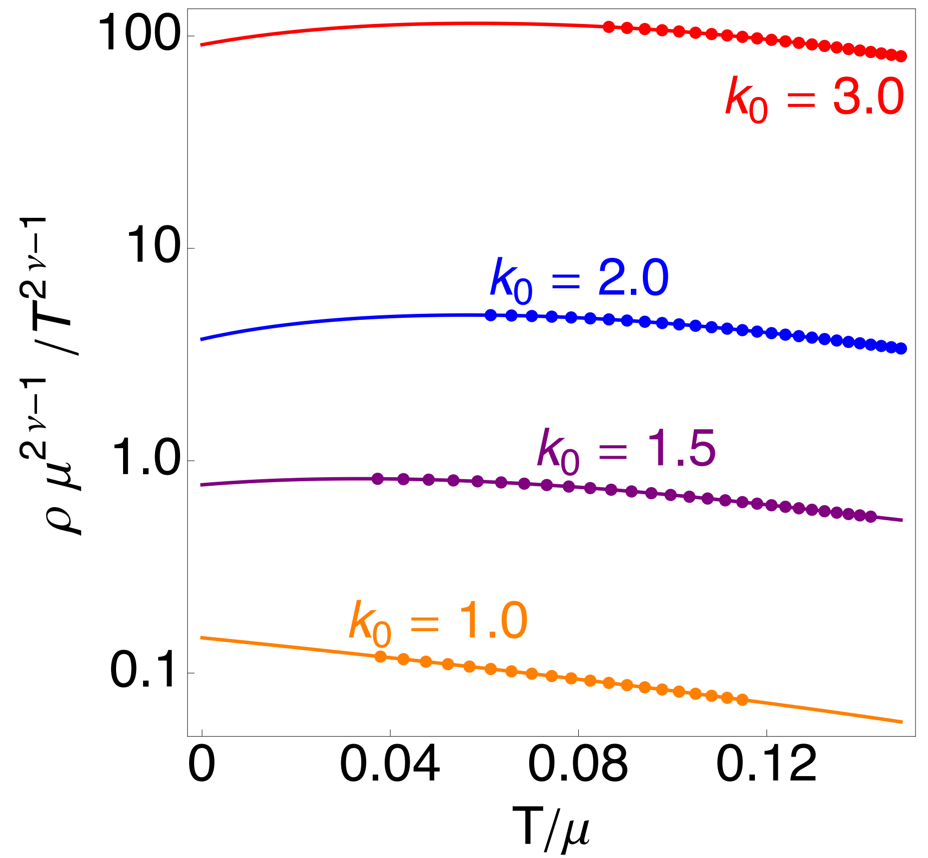
The DC resistivity $\rho = (K\,\tau\,)^{-1}$ depends on the lattice wavenumber k_0 as well as T.

Near horizon geometry of T = 0 black hole is AdS_2 x R^2. Hartnoll and Hoffman (2012) showed that at low T, ρ can be extracted from the two point function of the charge density evaluated at the lattice wavenumber:

$$\rho \propto T^{2\nu-1}$$

$$\nu = \frac{1}{2}\sqrt{5 + 2(k/\mu)^2 - 4\sqrt{1 + (k/\mu)^2}}$$

Our data is in good agreement with the Hartnoll-Hoffman result (with k = $2k_0$)



A tentative prediction

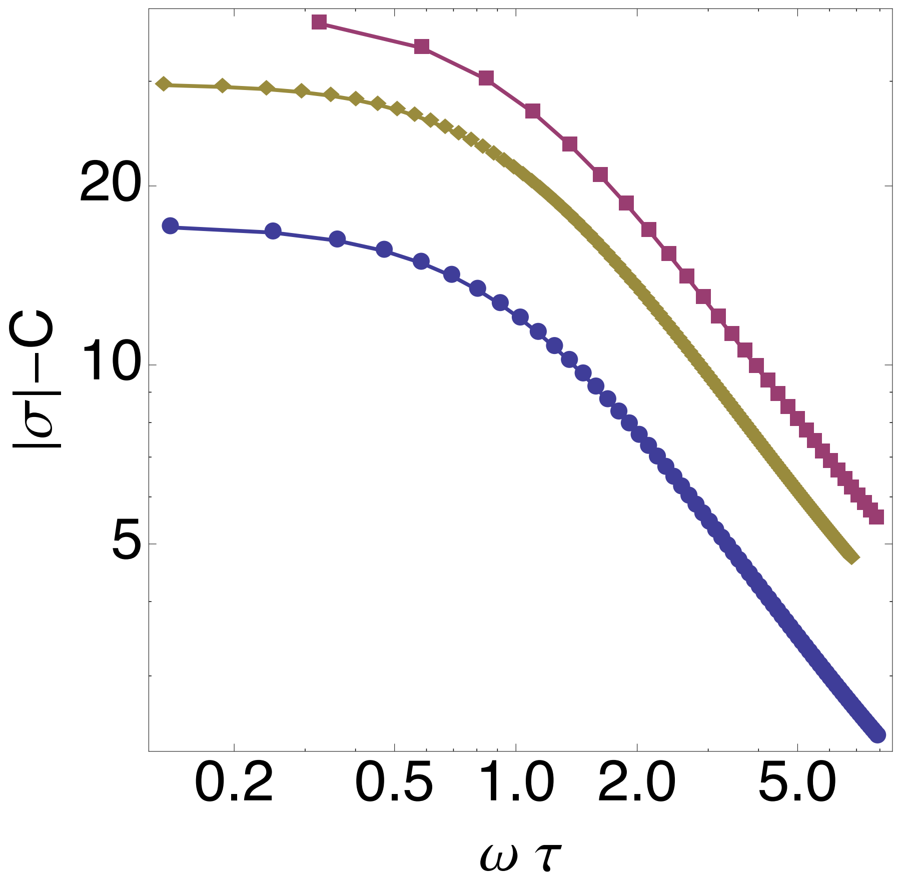
One can do the same thing in D = 5, to describe a more isotropic 3+1 material.

One again finds Drude behavior at low frequency and a power law fall-off, but the exponent is different:

$$|\sigma| = \frac{B}{\omega^{\sqrt{3}/2}} + C$$

It would be great to find a 3+1 analog of the cuprates to compare this to!

The exponent is again robust against changing the parameters in our model:



3+1 conductivity for k_0 = 1, 2, 3

Further evidence for a lattice induced scaling regime

1) We have repeated our calculation for an ionic lattice: $\mu = \mu_0[1 + \mu_1 \cos(k_0 x)]$ and we again find a power law fall-off with the same exponent.

2) We have computed the thermoelectric coefficient and it also has a power-law fall at intermediate frequencies.

Summary

- We have constructed the gravitational dual of a lattice
- We perturbed the solution and computed the optical conductivity
- Simple Drude behavior at low frequencies
- Intermediate power law with exponent that agrees with the normal phase of the cuprates
- DC resistivity scales like a power of T which depends on the lattice spacing