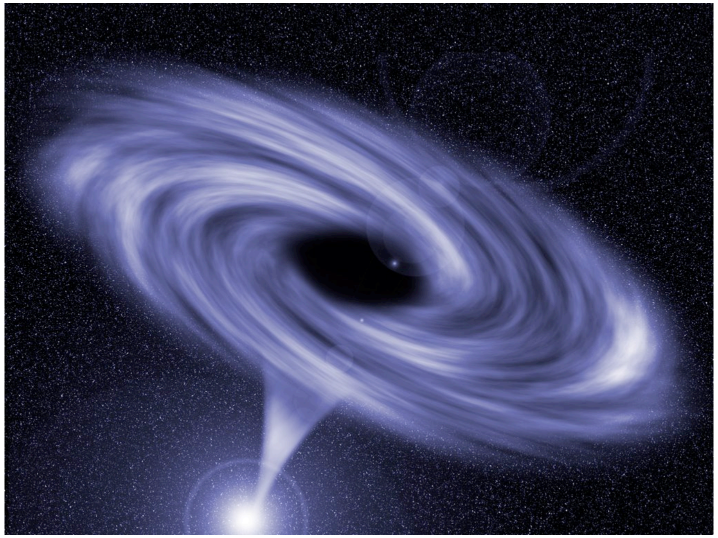
New Horizons in Finite Density Field Theory and String Theory



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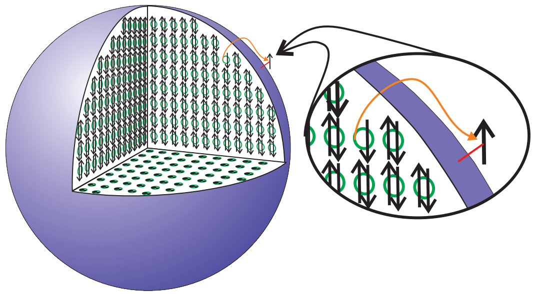
Based in part on:
arXiv:1201.4861 (with Iizuka, Narayan, Prakash, Sircar, Trivedi)
arXiv:1202.6635 (with Harrison, Wang)
arXiv:1201.1905 (with Dong, Harrison, Torroba, Wang)

1. Some interesting physics problems

Typical metals (including those which become superconducting at low temperatures) are governed by Fermi liquid theory:



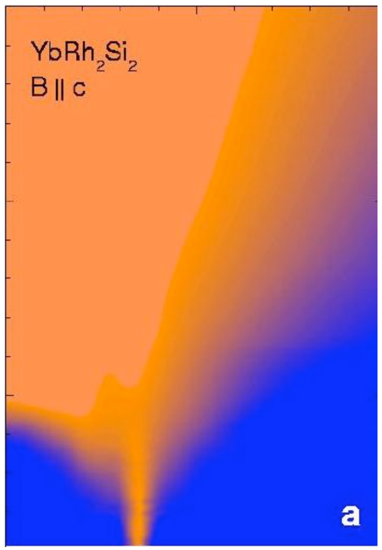
Basic quantum mechanics together with the kinematics of the Fermi surface determines robustly:

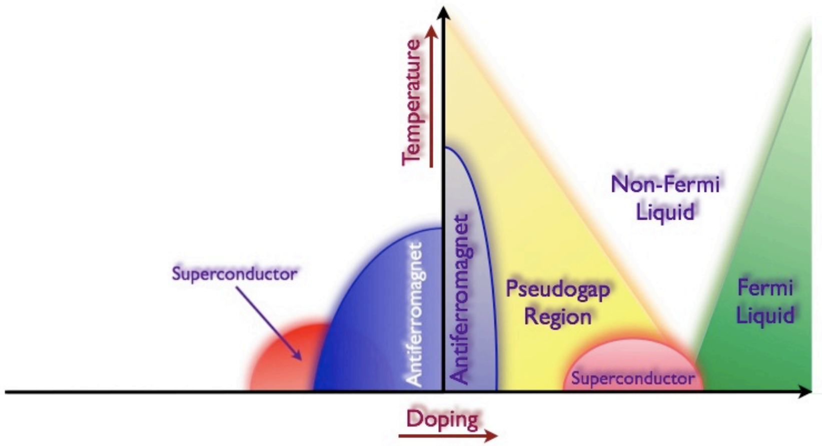
$$C_V(T) \sim T, \;\; \rho(T) \sim T^2$$

The Fermi liquid fixed point is infrared stable (except for the Cooper channel instability). This explains its ubiquity in metals.

Landau; ..... ; Shankar; Polchinski

But a variety of experiments yield phase diagrams with decidedly "non-Fermi liquid" behavior, even in the regime where Fermi liquid theory would be expected to apply:





These materials suggest a challenge: can we classify IR stable (or almost stable) fixed points of finite density quantum matter?

We have few tools to do this. Perhaps the newest one is holography. Here, I'll describe new results from studies of holography at finite density. Reductively, this can just be viewed as an attempt to better understand the zoo of black brane solutions in string theory (and their dual physics).

II. Holography at finite charge density

A. Simplest case

Doped finite T holographic matter is mapped, in AdS/CFT, to a charged black brane geometry.

The simplest action that supports a ("bottom-up") holographic dictionary for such doped matter is:

$$S = \frac{1}{2\kappa^2} \int d^{d+1}x \sqrt{-g} \left[\mathcal{R} + \frac{d(d-1)}{R^2} - \frac{R^2}{g_F^2} F_{MN} F^{MN} \right]$$

The AdS/RN solution has a metric:

$$ds^2 \equiv g_{MN} dx^M dx^N = \frac{r^2}{R^2}(-f dt^2 + d\vec{x}^2) + \frac{R^2}{r^2} \frac{dr^2}{f}$$

Chamblin, Emparan, Johnson, Myers

with gauge field and "emblackening factor":

$$f = 1 + \frac{Q^2}{r^{2d-2}} - \frac{M}{r^d}, \qquad A_t = \mu \left(1 - \frac{r_0^{d-2}}{r^{d-2}} \right) .$$

In the case of a 4d bulk, the extremal limit (the ground state of the doped theory) has a near-horizon geometry:

$$ds^2 = -r^2 dt^2 + \frac{dr^2}{r^2} + dx^2 + dy^2$$

The failure of the R^2 to shrink, means that there is an extensive ground-state entropy.

While e.g. free spins or phonons above the Debye temperature could do this, it seems very exotic for a state of strongly coupled quantum matter.

Fascinating "1/N" non-Fermi liquids have been found in this context, in a way that relies strongly on the AdS2 quantum critical region.

S.S. Lee; MIT group; Leiden group; ... ; DeWolfe, Gubser, Rosen

B. Other homogeneous, isotropic scaling geometries

In order to investigate whether the features of the Einstein-Maxwell extremal black brane are generic, it is reasonable to write down simple toy actions with more than the two fields, and see what the extremal solutions are like.

One simple extension is to add a scalar "dilaton":

$$\mathcal{L} = \sqrt{-g}\left(R - 2(\nabla\phi)^2 - e^{2\alpha\phi}F^2 - 2\Lambda - V(\phi)\right)$$

Various simple choices with such an action yield qualitatively new near-horizon geometries for charged black branes.

Case i) No dilaton potential

In this case, one can find asymptotically AdS solutions with near-horizon geometry (at extremality) governed by the solution:

$$ds^2 = -r^{2z}dt^2 + r^2(dx^2 + dy^2) + \frac{dr^2}{r^2}$$

$$\phi = - K \log(w)$$

M. Taylor;
Goldstein, S.K,
Prakash, Trivedi

The "dynamical critical exponent" z, and the constant K, are determined by the dilaton coupling parameter in the action.

The solution runs to weak/strong coupling at the horizon in the electrically/magnetically charged solution.

Physically, there are several interesting differences from the AdS/RN geometry:

* Vanishing entropy density at T=0.

* The metric exhibits scaling with finite z; the near-horizon geometry of AdS/RN is recovered in the $z \to \infty$ limit.

* The dilaton breaks the scaling symmetry. However, similar exactly "Lifshitz-invariant" solutions with various z arise in slightly different low-energy theories. S.K., Liu, Mulligan; ....

* One can still obtain non-Fermi liquids in these geometries, by considering transport of fermions on probe D-branes. z=2 yields linear resistivity. Hartnoll, Polchinski, Silverstein, Tong

Case ii) Including a dilaton potential

Charmousis, Gouteraux, Kim, Kiritsis, Meyer

If one arranges for a global solution where in the near-horizon geometry a dilaton potential of the schematic form

$$V(\phi) \sim e^{\beta\phi}$$

dominates, then one finds a more general class of homogeneous, isotropic near-horizon geometries. They are characterized by both a "dynamical critical exponent" z and a "hyperscaling violation exponent" θ :

$$ds^2_{d+2} = r^{-2(d-\theta)/d}\left(-r^{-2(z-1)}dt^2 + dr^2 + dx_i^2\right)$$

With hyperscaling violation, such metrics are not scale invariant; they are however conformal to Lifshitz metrics.

Again, they are supported by a running scalar field whose trajectory typically hits extreme values (very weak or very strong coupling) at the extremal horizon, signaling a breakdown of the low-energy action used.

Before expanding on this point, let us describe the most interesting features of such geometries:

a) The free energy density scales in a way different from naive dimensional analysis, given the dimension. In a scale invariant field theory in d space dimensions:

$$\frac{F}{V} \sim T^{d+1}$$

These metrics instead have (finite T deformed) horizons which yield:

$$\frac{F}{V} \sim T^{d+1-\theta} \ .$$

In many respects, then, the thermodynamics of the dual theories is such that there are $d-\theta$ "effective" space dimensions. ⟵ dim'l analysis made up by powers of e.g. Fermi momentum in a Fermi liquid

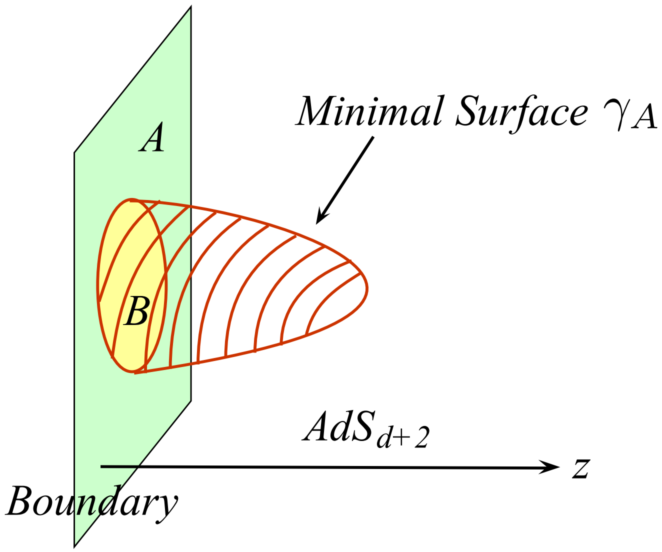
Notably:

$$\theta = d-1 \rightarrow C \sim T^{\frac{1}{z}}$$

Ogawa, Takayanagi, Ugajin; Huijse, Sachdev, Swingle

This matches (at leading large N) the scaling expected for a theory with a Fermi surface.

b) The entanglement entropy shows interesting behavior. Recall that there is a holographic entanglement entropy formula:



Ryu, Takayanagi

$$S_A = \frac{\text{Area of minimal surface } \gamma_A}{4G_N}$$

“Typical” phases of matter are expected to obey a (UV-cutoff dependent) “area law” for the entanglement entropy. But these holographic geometries yield:

$$S_A \sim L\ log(L)$$

in the case when

$$\theta = d - 1.$$

Ogawa, Takayanagi, Ugajin; Huijse, Sachdev, Swingle

These are hence doubly promising as a step towards a bulk representation of a large N theory which has a Fermi surface, for the leading large N degrees of freedom.

(However, no spectral weight at low energy and finite k in the supergravity approximation....)

Hartnoll, Shaghoulian

So, while there's progress, much remains to be done to find a convincing putative large N dual for a theory with a Fermi surface (at leading order).

C. On "IR incompleteness"

Many of the most interesting solutions we just discussed in B are "IR incomplete" due to the running dilaton. It runs either to very strong or very weak coupling at the horizon.

Either extreme is potentially problematic.

Strong coupling for obvious reasons. Weak coupling because of what the "dilaton" means in full constructions:

* In the case it is the literal string dilaton, because of the fact that

$$M_{\rm string} = g_s M_{\rm Planck}$$

extreme weak coupling means that the string tower is becoming light. This shows up in the action in unsuppressed higher curvature terms.

* Often in "consistent truncations," the dilaton is a radion. Then weak coupling is large volume, and a Kaluza-Klein tower comes down in mass.

In fact, black hole solutions in string theory where such corrections are crucial have been understood. Dabholkar, Kallosh, Maloney

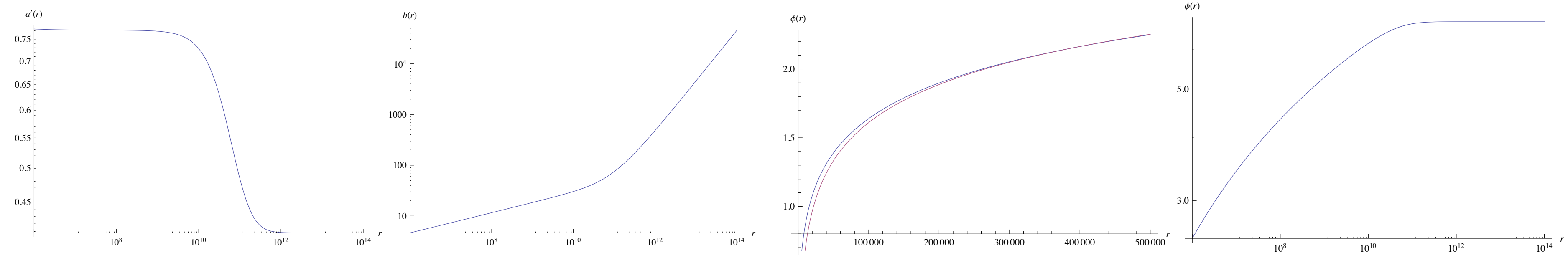
For magnetically charged black branes, where one is in the strong-coupling situation, we were able to show in a toy model that the "IR incompleteness" is cured in the running-dilaton solutions. They terminate in a constant-coupling AdS2 geometry (with parametrically smaller entropy than its Maxwell-Einstein cousin).

Harrison, S.K., Wang

$$S = \int d^4x \, \sqrt{-g} \left(R - 2(\nabla\phi)^2 - e^{2\alpha\phi} F^2 - 2\Lambda \right) .$$

$$e^{2\alpha\phi} F^2 \to f(\phi) F^2$$

$$f(\phi) = e^{2\alpha\phi} + \xi_1 + \xi_2 \; e^{-2\alpha\phi} + \xi_3 \; e^{-4\alpha\phi} + \cdots = \frac{1}{g^2} + \xi_1 + \xi_2 \; g^2 + \xi_3 \; g^4 + \cdots$$



Such metrics also do arise in microscopic D-brane constructions.

Perlmutter; Dong, Harrison, S.K., Torroba, Wang; Narayan; Ammon, Kaminski, Karch; ...

A prototypical simple example is the D2 solution, which has $(z = 1, \theta = -\frac{1}{3})$:

$$ds^2 = \alpha' \left(\frac{U^{5/2}}{g_{YM}\sqrt{6\pi^2 N}} dx_{||}^2 + \frac{g_{YM}\sqrt{6\pi^2 N}}{U^{5/2}} dU^2 + g_{YM}\sqrt{6\pi^2 N/U}\, d\Omega_6^2 \right)$$

$$e^{\phi} = \left(\frac{g_{YM}^{10} 6\pi^2 N}{U^5} \right)^{1/4} .$$

Itzhaki, Maldacena, Sonnenschein, Yankielowicz

It is trustworthy when:

$$g_{YM}^2 N^{1/5} \ll U \ll g_{YM}^2 N.$$

In this case, the "IR incompleteness" is saved by the flow to a strong coupling limit which is the M2-brane CFT. (The UV incomplete weak coupling region is 2+1 SYM).

In fact, even without the running dilaton, the e.g. Lifshitz horizons have a mystery; while curvature invariants are constant, there are strong tidal forces at the extremal horizon. The first mechanism just discussed can "resolve" this issue in the dilatonic solutions.

Other resolutions in other contexts also exist. Bao, Dong, Harrison, Silverstein; Way

III. Less symmetric horizons

Low-T phases with translation/rotation breaking are ubiquitous in condensed matter systems -- charge/spin density waves, nematic phases, ...

Can we classify, at least coarsely, dual gravity horizons?

As a starting point for a classification, we would like to classify the most general homogeneous, anisotropic extremal black brane geometries.

Iizuka, SK, Kundu, Narayan, Sircar, Trivedi

Here, by homogeneous, we do not mean that there is normal translation symmetry. Rather, we mean that for a theory in d spatial dimensions, there should be a d-dimensional group action which relates each point to its neighbors.

That, is there should be d Killing vectors whose commutators give rise to a Lie algebra. Only the trivial algebra gives "normal" translations.

Example:

Imagine in our three-dimensional space, we enjoy usual translation symmetries along two of the directions. But along the third, one must translate as well as rotating in the transverse plane, to get a symmetry.

$$\xi_1 = \partial_2, \quad \xi_2 = \partial_3,$$

$$\xi_3 = \partial_1 + x^2 \partial_3 - x^3 \partial_2.$$

← vector fields which generate our generalised translations

These generate a homogeneous space, in the sense that each point in an infinitesimal neighborhood can be transported to each other point.

However, the commutation relations define a Lie algebra which is invariantly distinct from the algebra of translations:

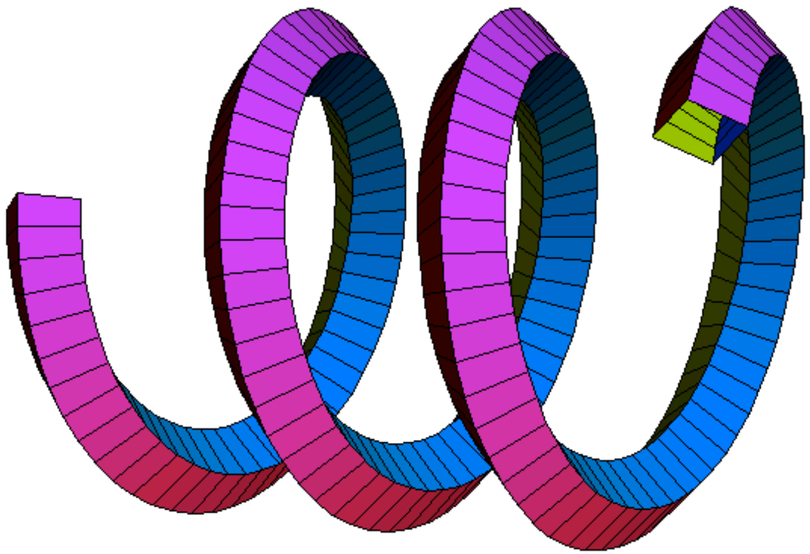
$$[\xi_1, \xi_2] = 0; \quad [\xi_1, \xi_3] = \xi_2; \quad [\xi_2, \xi_3] = -\xi_1,$$

In fact, these generalised translations could leave invariant a vector order parameter (whose expectation value "breaks" normal translations):

$$\delta V = \epsilon[\xi_i, V].$$

$$V^1 = \text{constant}, \quad V^2 = V_0 \cos(x^1 + \delta), V^3 = V_0 \sin(x^1 + \delta)$$

Or in a picture:



Happily, for the application to phases in 3 spatial dimensions, all possible algebras of this sort have been classified. This is the Bianchi classification, also of use in theoretical cosmology.

The basic mathematical structure is as follows:

* For each of the 9 inequivalent algebras, there are three "invariant one-forms" left invariant under all three isometries.

* A metric expressed in terms of these forms with constant coefficients will automatically be invariant, then.

* The one-forms actually satisfy the relations:

$$d\omega^i = \frac{1}{2} C^i_{jk} \omega^j \wedge \omega^k$$

with C the structure constants of the algebra.

Now, in holography we really have an extra spatial dimension. We also have time. Natural assumptions: we maintain also the (near-horizon) symmetries:

$$r \to r + \epsilon,\ \ t \to e^{-\beta_t \epsilon} t\ ;$$
$$t \to t + \text{const}\ .$$

Then the general "allowed" near-horizon metric takes the form:

$$ds^2 = R^2[dr^2 - e^{2\beta_t r} dt^2 + \eta_{ij} e^{(\beta_i + \beta_j) r} \omega^i \otimes \omega^j]$$

I.e. given the Bianchi type, there is a finite set of constants one must solve for to get the scaling metric.

* One can find solutions in 8 of the 9 Bianchi types just by considering Einstein gravity coupled to a massive vector field. The equations reduce to algebraic equations, not differential equations.

* 7 of the types are, as far as we can tell, entirely new classes of black brane horizons not considered heretofore.

* Close analogues of our Bianchi VII solutions were found previously, arising from instabilities in other holographic phases.

Domokos, Harvey;
Nakamura, Ooguri, Park;
Donos, Gauntlett

* Gluing these solutions into AdS seems possible; we did it explicitly for type VII. Though, we did it numerically....

Work in Progress:

* We are refining the classification of homogeneous horizons, and demonstrating connections to AdS. Stanford + TIFR

* We are trying to import some of the lessons from AdS constructions directly back to field theory, studying quantum critical bosonic systems coupled to a Fermi surface using field theory techniques. SK, Mahajan, Raghu, Ramirez

* We are trying to find general classes of black brane solutions in AdS flux vacua of string theory (e.g. type IIA flux vacua arising on Calabi-Yau threefolds). These may shed light on issues of microscopic interpretation of some singular configurations, for instance. SK, Torroba, Wang