

*Recent Applications of the Gauge/Gravity Correspondence
to QCD and Condensed Matter Physics.*

Andreas Karch

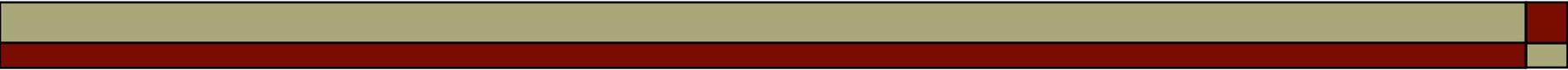
(University of Washington, Seattle)

(talk at STRINGS 2012, Munich, 7/23/12)



Goal:

Why do people outside the string theory community care about the gauge/gravity correspondence = holography?



Holography = Solvable Toy Model

Solvable models of strong coupling dynamics.

- Study Transport, real time
- Study Finite Density

Common Theme: Experimentally relevant, calculations impossible.

Gives us qualitative guidance/intuition.

Challenge for Computers:



We do have methods for strong coupling:

e.g. Lattice QCD



But: typically relies on importance sampling.

e^{-S} weighting in Euclidean path integral.

Monte-Carlo techniques.

Holographic Toy models.

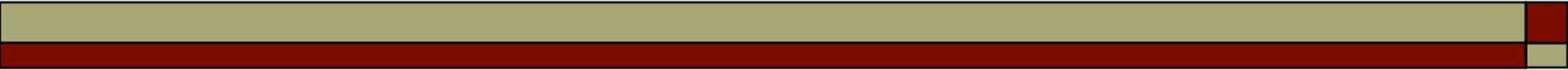


Can we at least
get a qualitative
understanding of
what dynamics look
like at strong coupling?

Holographic Toy models.



Can we at least get a qualitative understanding of what dynamics looks like at strong coupling?



Holographic Theories:

Examples known:

- in $d=1, 2, 3, 4, 5, 6$ space-time dimensions
- with or without super-symmetry
- conformal or confining
- with or without chiral symmetry breaking
- with finite temperature and density

Holographic Theories:

Holographic toy models have two key properties:

“Large N”: theory is essentially classical

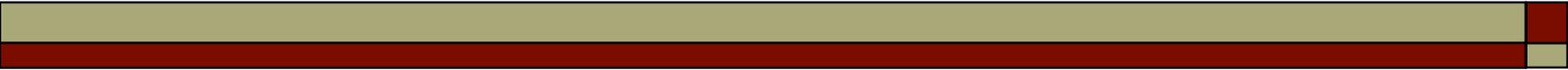
“Large λ ”: large separation of scales
in the spectrum

$$m_{\text{spin-2-meson}} \sim \lambda^{1/4} m_{\text{spin-1-meson}}$$

QCD: **1275 MeV** **775 MeV**

(note: there are some exotic examples where the same parameter N controls both, classicality and separation of scales in spectrum)

Applications to QCD Transport.



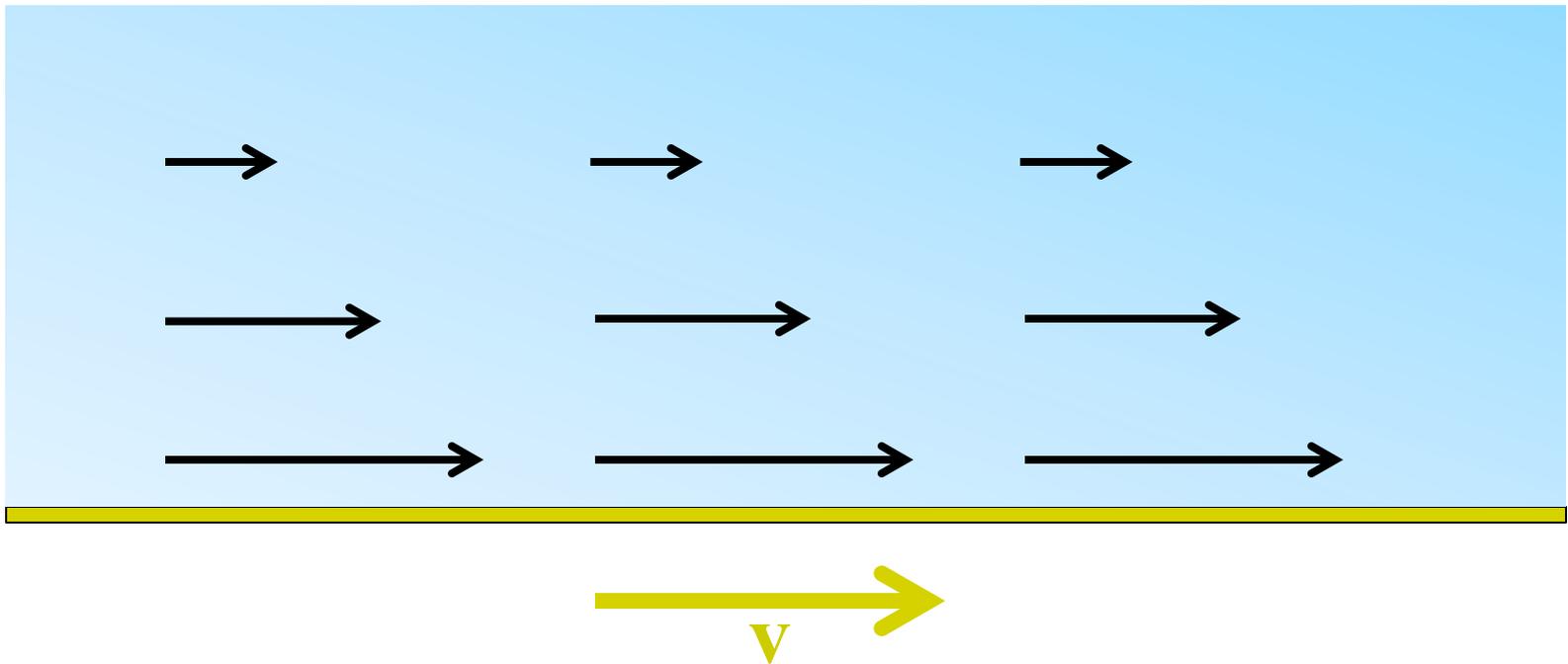
Applications to QCD Transport

(as experimentally probed in Heavy Ion Collisions)

- Viscosity and Hydrodynamics
- Energy Loss
- Thermalization

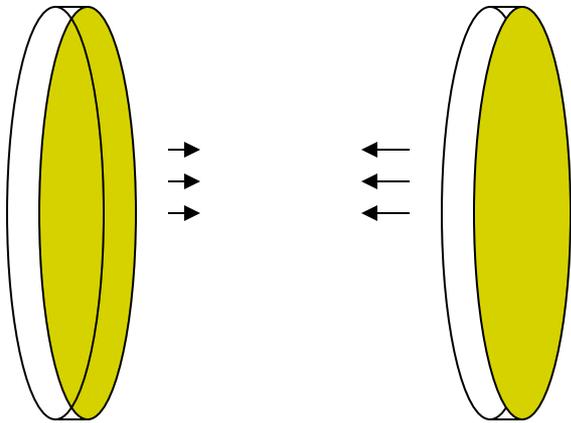
Shear Viscosity

Viscosity = Diffusion constant for momentum



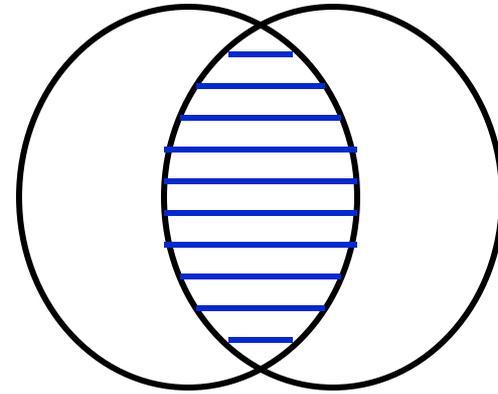
Viscosity = [(force/area)] per unit velocity gradient

Viscosity in Heavy Ions.

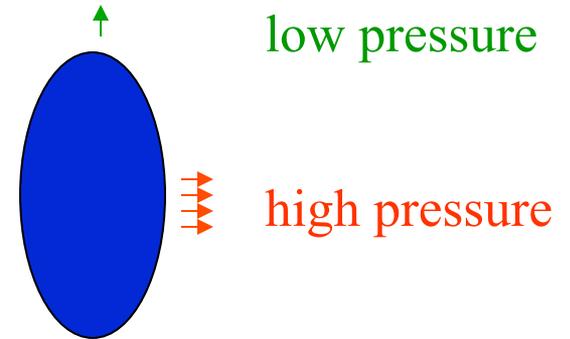


Au

Au



How does the almond shaped fluid expand?



Viscosity

Viscosity can be quantified:

water: 1 centipoise (cp)

air: 0.02 cp

honey: 2000-10000 cp

$$(1 \text{ cp} = 10^{-2} \text{ P} = 10^{-3} \text{ Pa}\cdot\text{s})$$

Measuring Viscosity - an example

Pitch drop experiment



Started in 1930

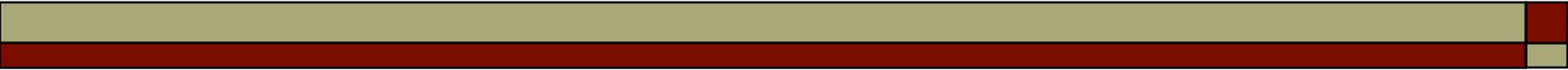
8 drops fell so far

but no one has ever witnessed a drop fall

2005 Ig Nobel Prize in Physics

Viscosity of pitch: 230 billions times that of water

($2.3 \cdot 10^{11}$ cp)



Measuring Viscosity - an example

Recall: Viscosity of pitch: $\sim 2.3 \cdot 10^{11}$ cp

Measuring Viscosity - an example

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RHIC's measurement of QGP (confirmed by LHC):

$$\eta \sim \frac{\hbar}{4\pi} s \sim \frac{10^{-27} \text{ erg} \cdot \text{s}}{(10^{-13} \text{ cm})^3} \sim 10^{14} \text{ cp}$$

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BNL press release 2005:

“The degree of collective interaction, rapid thermalization, and extremely low viscosity of the matter being formed at RHIC makes this the most nearly perfect liquid ever observed.”

Viscosity in Holography:

In a large class of systems:

$$\frac{\eta}{s} = \frac{\hbar}{4\pi}$$

(Kovtun, Son, Starinets)

- pinpoints correct observable
- in contrast to QGP, η/s enormous for pitch
- gives ball-park figure
- large at weak coupling: bound?

Viscosity – Recent Developments

Not a bound!

(Kats, Petrov, 2007)

$$\frac{\eta}{s} = \frac{1}{4\pi} \left(1 - \frac{1}{2N} \right)$$

$\mathcal{N} = 2 \text{ Sp}(N)$
4 fundamental
1 antisymmetric traceless

Higher Curvature corrections violate bound.

(Brigante, Liu, Myers, Shenker, Yaida, Buchel, Sinha,)

Calculations only reliable if violations are small.¹⁹

Hydro – Recent Developments

Viscosity is not the only hydro transport coefficient that can be calculated holographically.

- 2nd order hydro
 - Calculated in 2007 (Romatschke et. al., Batthacharyya et. al.)
 - Needed for stable hydro simulation (causality!)
 - Holographic values/structure routinely used
- anomalous transport

Anomalous Transport in Hydro

(following Kharzeev and Son)

$$\vec{J} = \frac{N_c \mu_5}{2\pi^2} [\text{tr}(VAQ)\vec{B} + \text{tr}(VAB)2\mu\vec{\omega}]$$

Anomalous Transport in Hydro

(following Kharzeev and Son)

$$\vec{J} = \frac{N_c \mu_5}{2\pi^2} [\text{tr}(VAQ)\vec{B} + \text{tr}(VAB)2\mu\vec{\omega}]$$



J: conserved current

- 1) Baryon Number or**
- 2) Electric Charge**

Anomalous Transport in Hydro

(following Kharzeev and Son)

$$\vec{J} = \frac{N_c \mu_5}{2\pi^2} [\text{tr}(VAQ)\vec{B} + \text{tr}(VAB)2\mu\vec{\omega}]$$



B: magnetic field
“Chiral Magnetic Effect”

Anomalous Transport in Hydro

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$$\vec{J} = \frac{N_c \mu_5}{2\pi^2} [\text{tr}(VAQ)\vec{B} + \text{tr}(VAB)2\mu\vec{\omega}]$$

ω : vorticity (= curl of velocity)
“Chiral Vortical Effect”

Anomalous Transport in Hydro

(following Kharzeev and Son)

$$\vec{J} = \frac{N_c \mu_5}{2\pi^2} [\text{tr}(VAQ)\vec{B} + \text{tr}(VAB)2\mu\vec{\omega}]$$

**axial chemical potential
(requires non-zero axial charge)**

$$\langle \mu_5 \rangle = 0$$

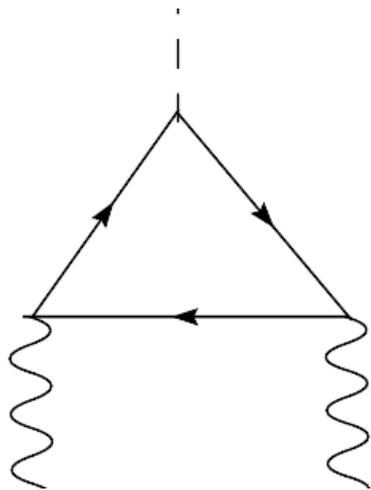
$$\langle \mu_5^2 \rangle \neq 0$$

relies on event
by event fluctuations

Anomalous Transport in Hydro

(following Kharzeev and Son)

$$\vec{J} = \frac{N_c \mu_5}{2\pi^2} [\text{tr}(VAQ)\vec{B} + \text{tr}(VAB)2\mu\vec{\omega}]$$



Coefficients determined by anomaly!

Relative size of baryon versus
charge asymmetry unambiguous.

Anomalous Transport in Hydro

(following Kharzeev and Son)

$$\vec{J} = \frac{N_c \mu_5}{2\pi^2} [\text{tr}(VAQ)\vec{B} + \text{tr}(VAB)2\mu\vec{\omega}]$$

$$J_E^{CME} \sim \frac{2}{3} \quad (N_f = 3) \quad \text{or} \quad \frac{5}{9} \quad (N_f = 2) \quad \quad J_E^{CVE} = 0 \quad (N_f = 3) \quad \text{or} \quad \sim \frac{1}{3} \quad (N_f = 2);$$

$$J_B^{CME} = 0 \quad (N_f = 3) \quad \text{or} \quad \sim \frac{1}{9} \quad (N_f = 2). \quad \quad J_B^{CVE} \sim 1 \quad (N_f = 3) \quad \text{or} \quad \sim \frac{2}{3} \quad (N_f = 2).$$

Predictions

(Kharzeev and Son)

- There should be a baryon number separation of the same sign as the electric charge separation;
- The ratio between the baryon asymmetry and charge asymmetry should increase as the center of mass energy is lowered;
- The magnitude of the ratio of charge and baryon asymmetries allows to discriminate between the CME and CVE mechanisms.

Anomaly and the CVE

connection between CME and anomaly was quantitatively understood before (Kharzeev, ...)

How does the anomaly know about vorticity?

(Erdmenger, Haack, Kaminski, Yarom;
Banerjee, Bhattacharya, Bhattacharyya, Dutta, Loganayagam, Surowka)

In holographic models CVE completely
determined in terms of

Chern-Simons term = anomaly.

Anomaly and the CVE

How does the anomaly know about vorticity?

Son, Surowka: True in general.

**axial anomaly in background
electromagnetic fields**

+

=

CVE

**entropy current with non-negative
divergence**

Beyond the Entropy Current

(Jensen, Kaminski, Kovtun, Meyer, Ritz, Yarom;
Banerjee, Bhattacharya, Bhattacharyya, Minwalla, Sharma)



Idea: “Static Configuration”
should exist in “non-trivial
backgrounds”.

Beyond the Entropy Current

(Jensen, Kaminski, Kovtun, Meyer, Ritz, Yarom;
Banerjee, Bhattacharya, Bhattacharyya, Minwalla, Sharma)



Idea: Static Configuration
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backgrounds.

Metric with timelike Killing Vector

$$ds^2 = -e^{2\sigma(\vec{x})} (dt + a_i(\vec{x})dx^i)^2 + g_{ij}(\vec{x})dx^i dx^j$$

$$\mathcal{A}^\mu = (A^0(\vec{x}), \mathcal{A}^i(\vec{x}))$$

Beyond the Entropy Current

Can be described by Euclidean Generating Functional



Idea: **Static Configuration** should exist in non-trivial **backgrounds.**



Metric with timelike Killing Vector

$$ds^2 = -e^{2\sigma(\vec{x})} (dt + a_i(\vec{x})dx^i)^2 + g_{ij}(\vec{x})dx^i dx^j$$

$$\mathcal{A}^\mu = (A^0(\vec{x}), \mathcal{A}^i(\vec{x}))$$

Beyond the Entropy Current

- Reproduces Son/Surowka results for CVE
- Conjectured to be equivalent to existence of entropy current
- Equivalent to Ward identities on correlators (including “global” ones related to time circle) - **(Jensen)**
- Byproduct: subset of transport coefficient given by **static** correlation functions.

And a puzzle: (Landsteiner, Megias, Melgar, Pena-Benitez)

$$(\sigma^{\mathcal{V}})_A = \frac{1}{8\pi^2} d_{ABC} \mu^B \mu^C + \frac{T^2}{24} b_A.$$

vortical conductivity.
A,B,C: labels axial/vector

**μ_5, μ dependence
of CVE**

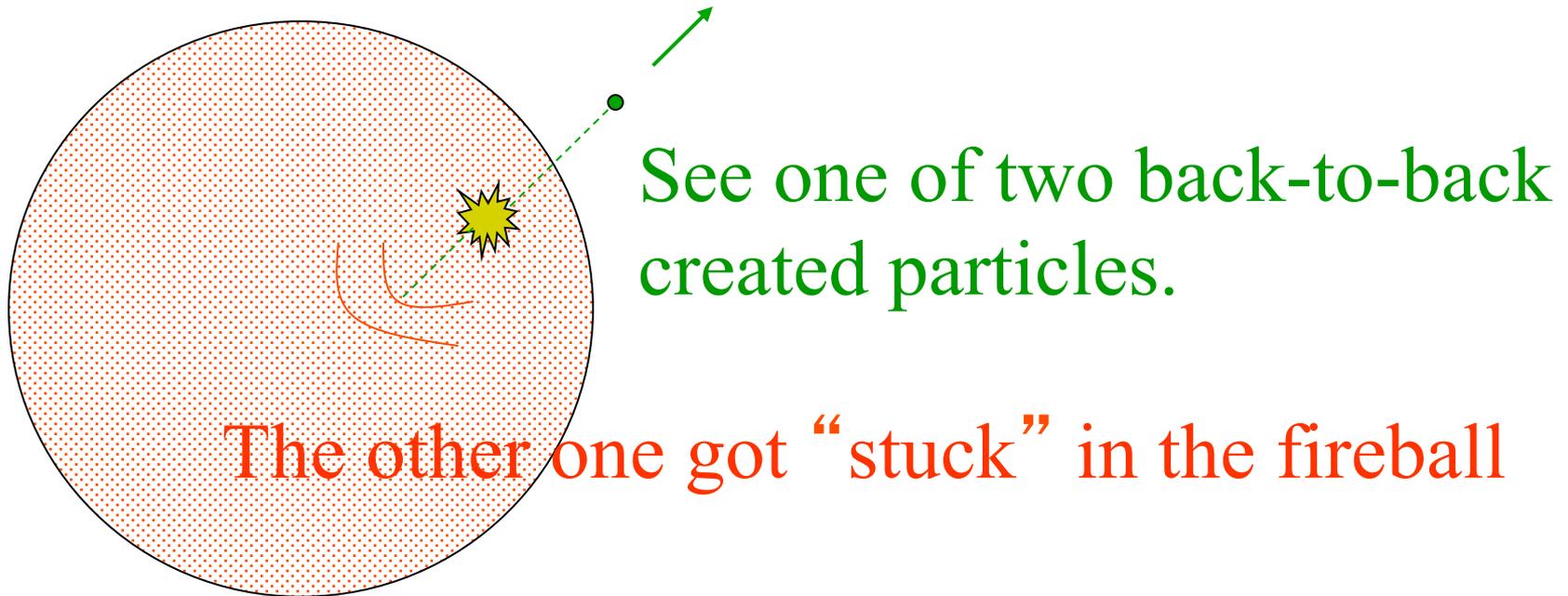
**MIXED GAUGE/
GRAVITATIONAL
ANOMALY**

Coefficient of gravitational anomaly shows up both at weak and strong coupling. WHY?

(would give 10^4 fold enhancement of CVE at RHIC)

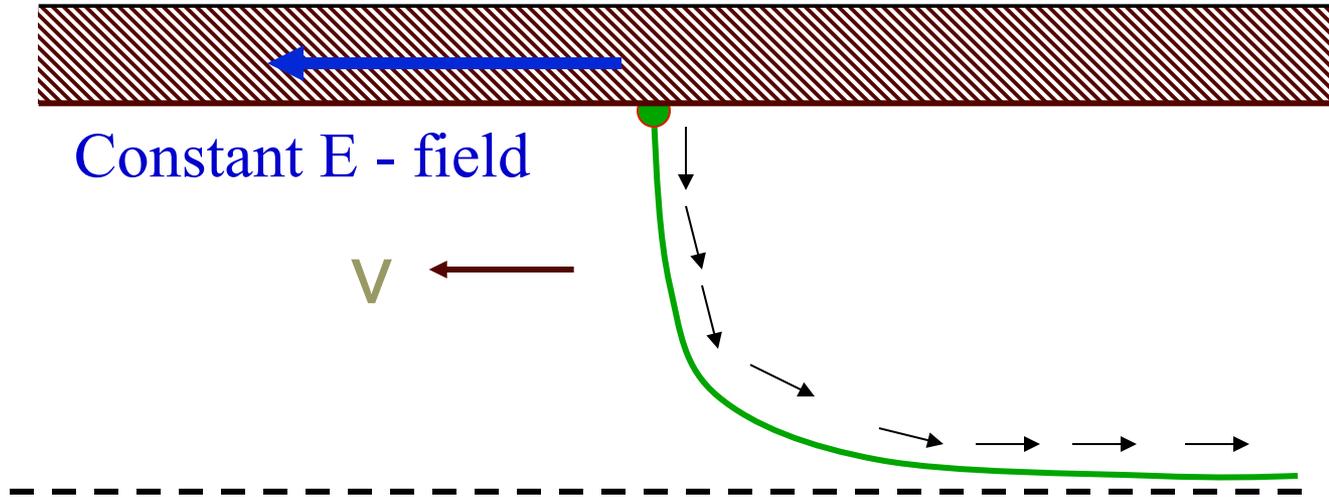
Energy Loss

Energy Loss in Heavy Ions.



Jet quenching is a direct indication of large drag.

Energy Loss (2006): Heavy quarks



$$\frac{dp}{dt} = -\mu p$$

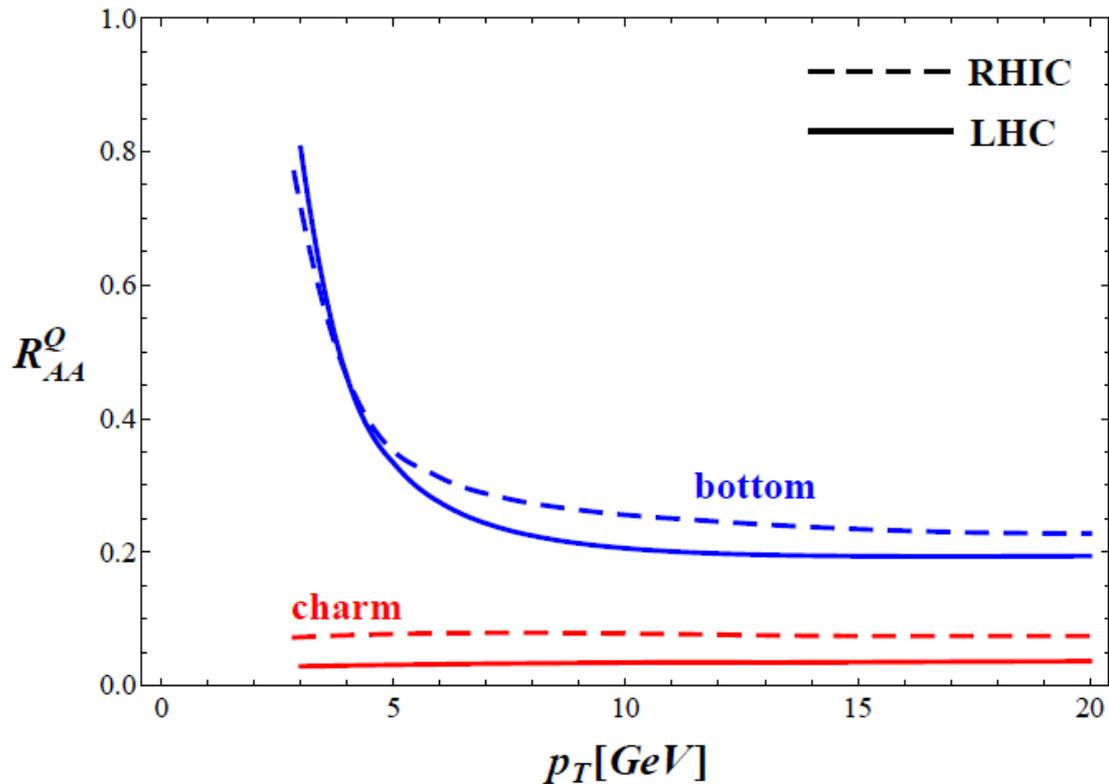
$$\mu = \pi T \frac{\Delta m(T)}{m}$$

$$\Delta m(T) \equiv \frac{1}{2} \sqrt{\lambda} T$$

(Casalderrey-Solana & Teaney; Herzog, AK, Kovtun, Koczkaz, Yaffe; Gubser)

Energy Loss, Recent Developments:

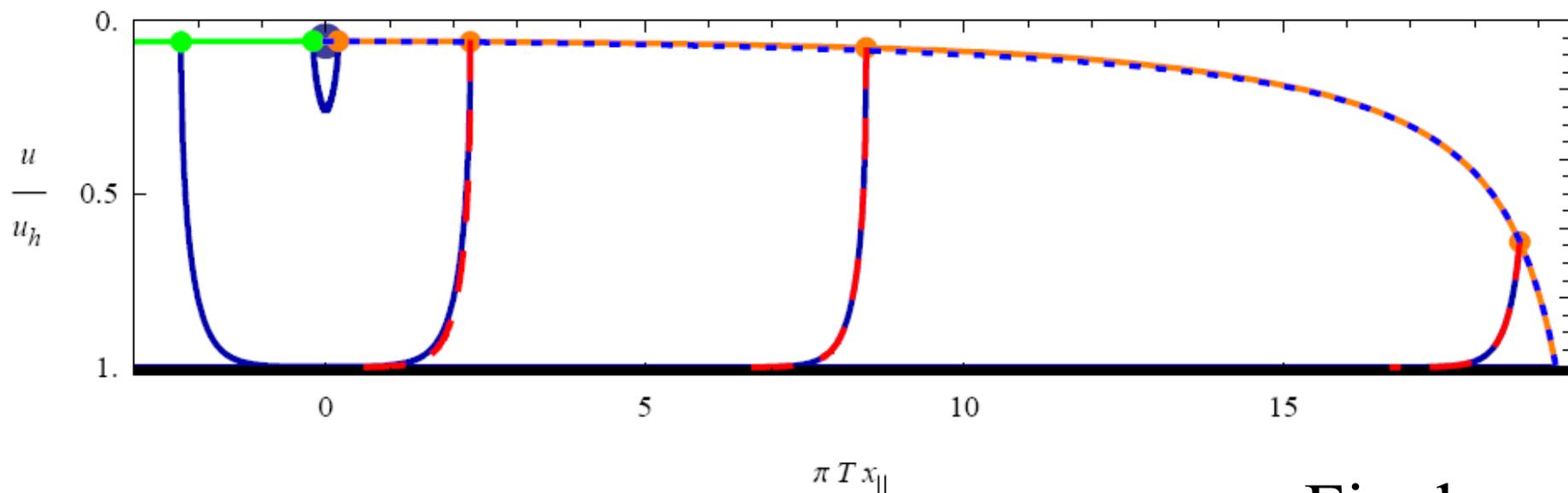
Use holographic models to make LHC “predictions”:



(Ficnar,
Noronha,
Gyulassy)

Energy Loss, Light Quarks (2010)

(Chesler, Jensen, AK, Yaffe; Gubser, Gulotta, Pufu, Rocha)

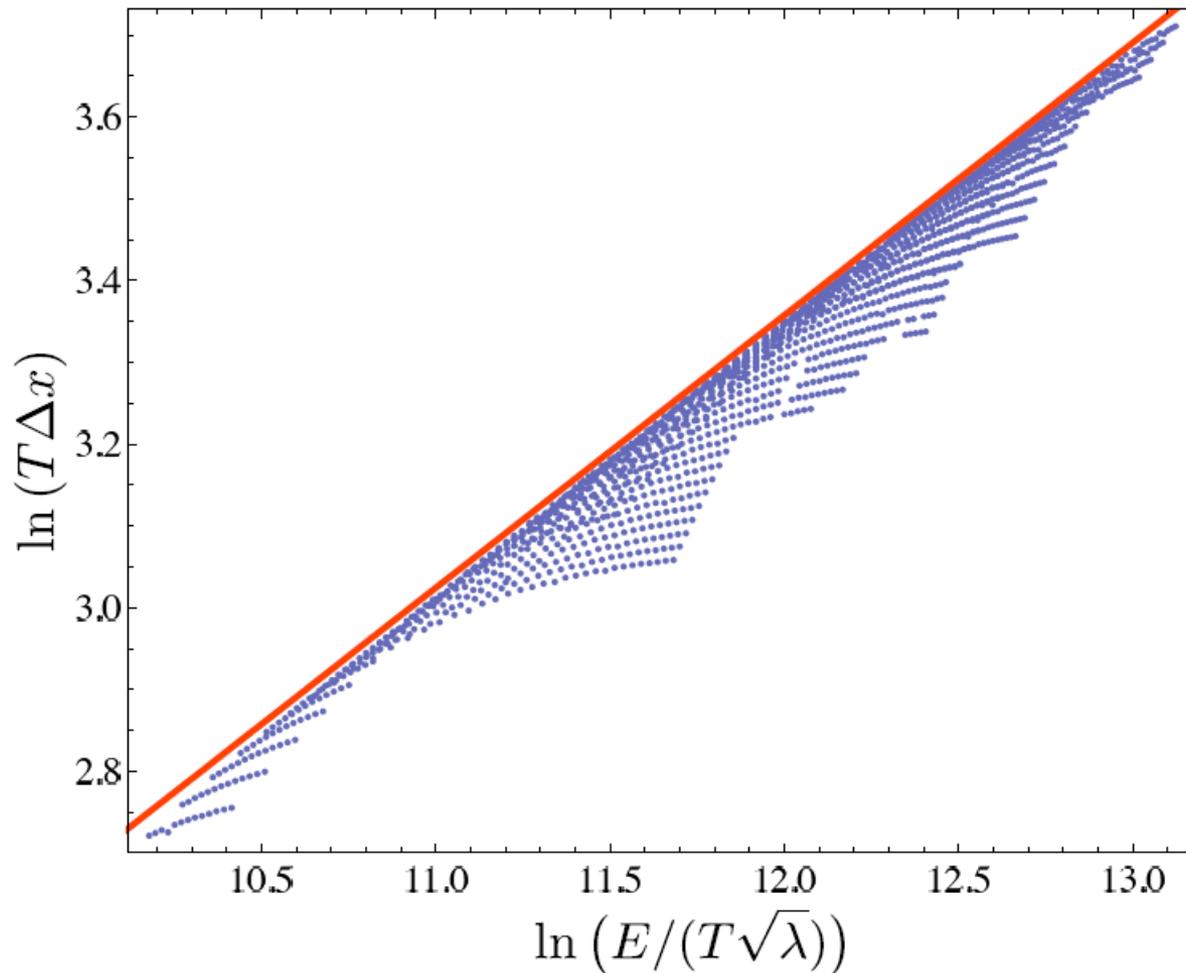


Zero T
Jets

Quasiparticle in Plasma
(for $E \gg T$)

Final
Diffusion

Stopping Distance vs Energy



(Chesler, Jensen,
AK, Yaffe)

Stopping Distance:

Perturbative QCD: $L \sim E^{1/2}$ (BDMPS, ...)

Holography:

Maximal Stopping Distance: $L \sim E^{1/3}$

Typical Stopping Distance: $L \sim E^{1/4}$

(Arnold, Vaman - 2011)

Experiment: ??????

Stopping Distance: Exponents!

Perturbative QCD: $L \sim E^{1/2}$ (BDMPS, ...)

Holography:

Maximal Stopping Distance: $L \sim E^{1/3}$

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Experiment: ??????

Thermalization

Why does the QCD fireball thermalize so rapidly?

Thermalization

Why does the QCD fireball thermalize so rapidly?

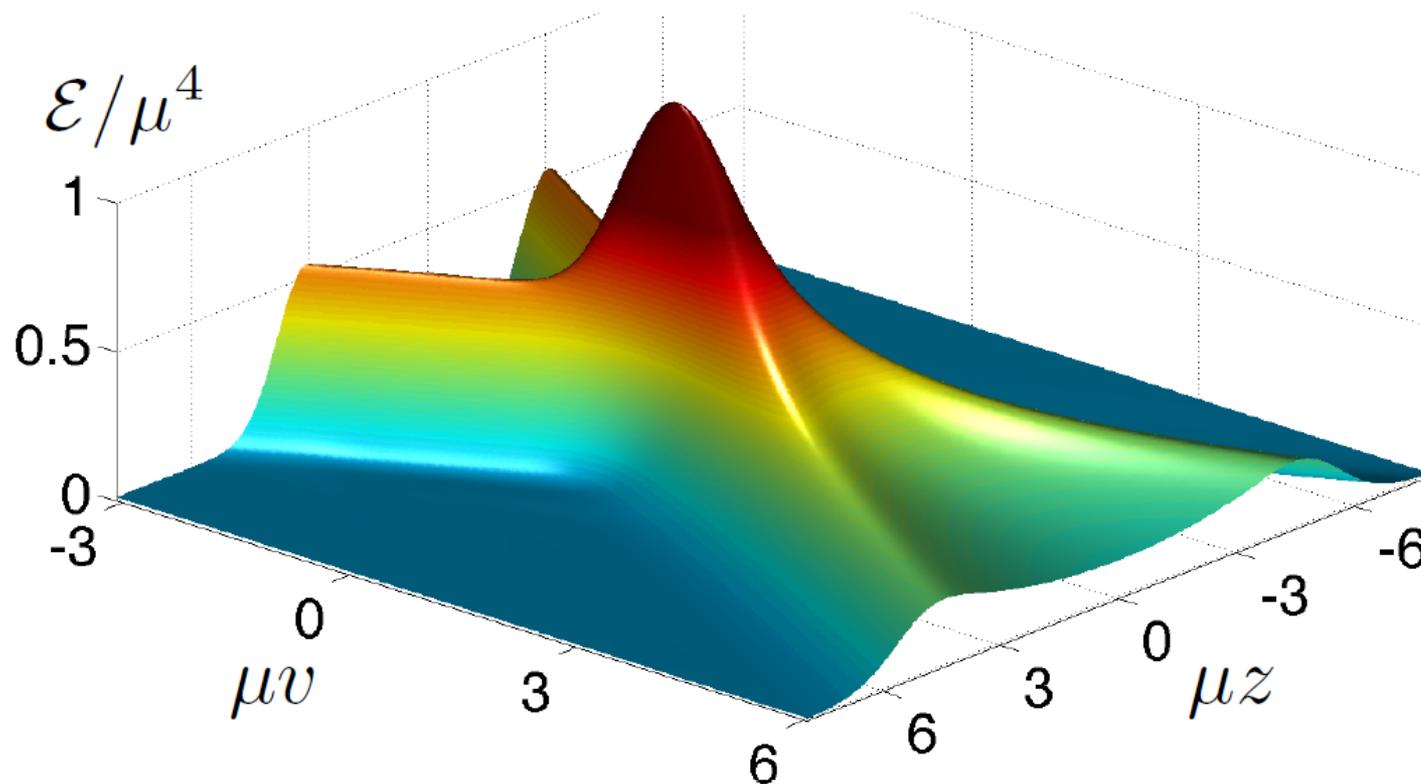
too hard!

Thermalization

How quickly does the holographic fireball thermalize?

Shockwave-collision to black hole

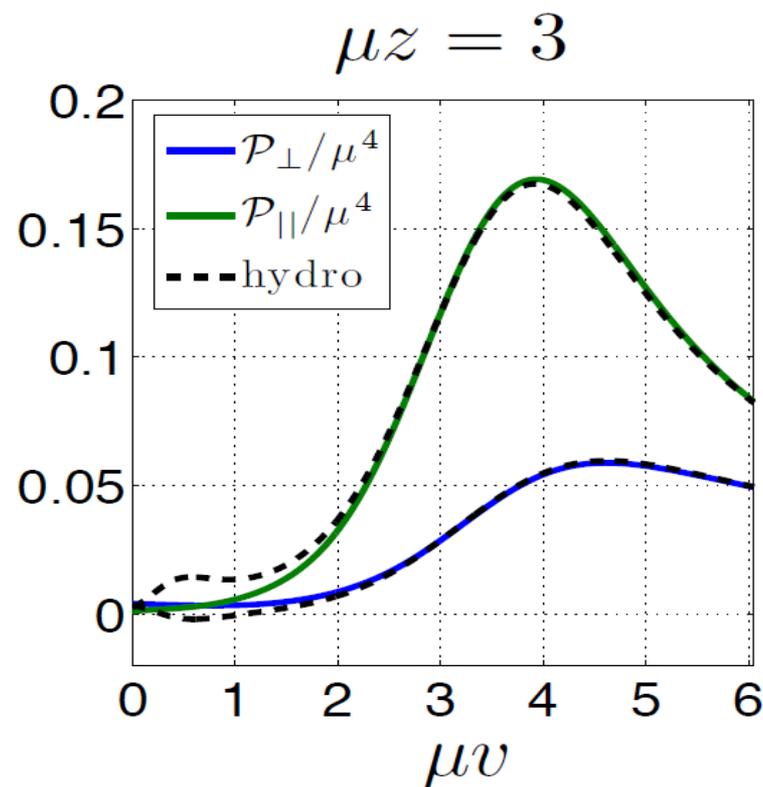
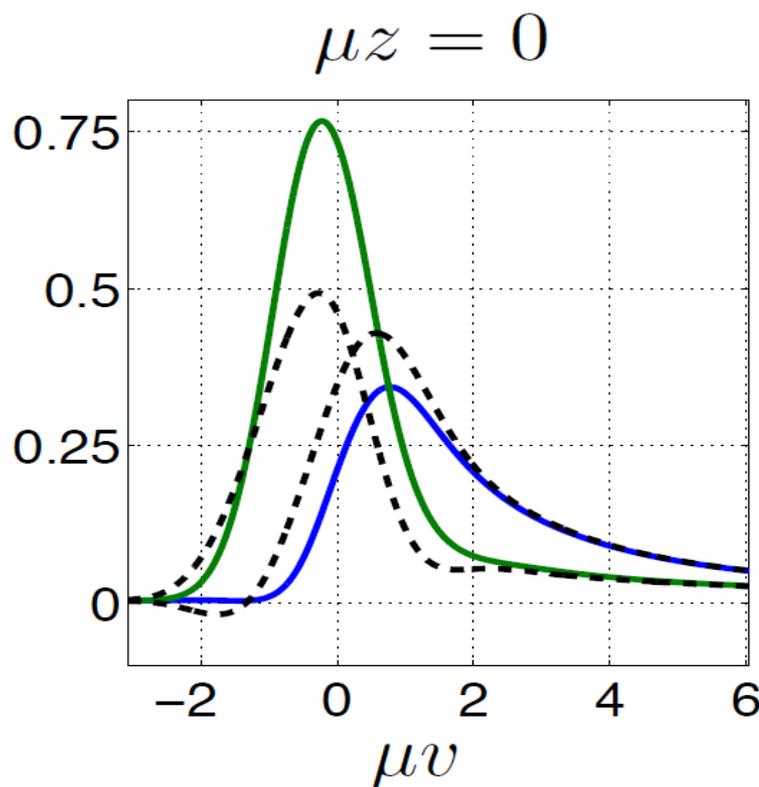
(Chesler, Yaffe)



Energy/area in shock $\sim \mu^3$ ⁴⁷

Shockwave-collision to black hole

(Chesler, Yaffe)



Shockwave-collision to black hole

(Chesler, Yaffe)

“RHIC”:

$$\mu \sim 2.3 \text{ GeV}$$

$$\text{Hydro valid} \sim 0.35 \text{ fm/c} \ll 1 \text{ fm/c}$$

But: there is so much more info in this plot!

What do you want to know?

Hydrolization vs Thermalization

(Chesler, Teaney)

Note: Hydro works when transverse and longitudinal pressure differ by a factor of 2.

Hydrolization before Thermalization!

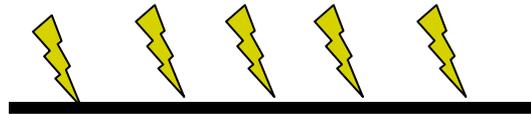


Hydro works. No well defined temperature.

Hydrolization vs Thermalization

(Chesler, Teaney)

UV



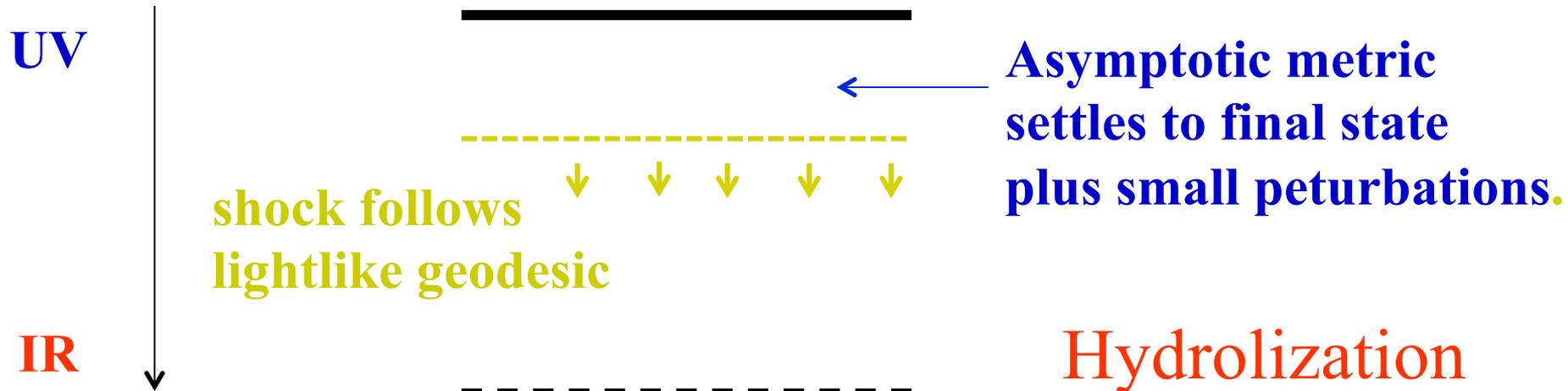
t=0
initial perturbation

IR



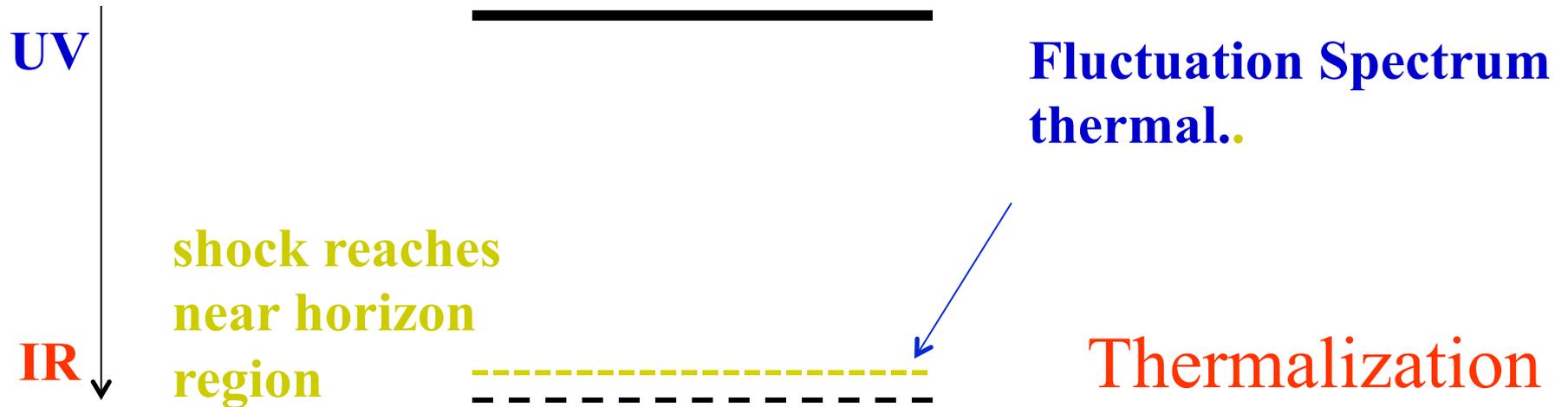
Hydrolization vs Thermalization

(Chesler, Teaney)



Hydrolization vs Thermalization

(Chesler, Teaney)



Hydrolization vs Thermalization

(Chesler, Teaney)

- Generically Hydrolization and Thermalization differ by “infall” time
- For suitable initial condition (lightlike geodesic skimming boundary) thermalization time can be **parametrically large** compared to hydrolization time.

Applications to Condensed Matter Physics.

Strong Coupling in CM.

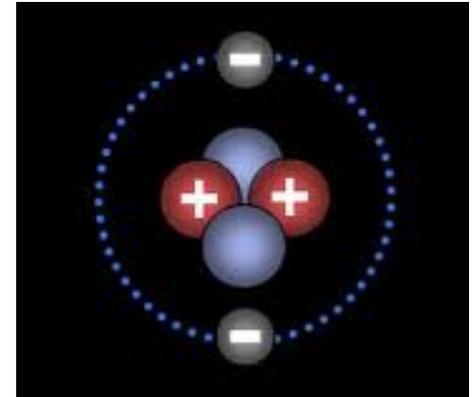
The theory of everything:

$$H = \sum_{\text{Nuclei}, A} \frac{P_A^2}{m_A} + \sum_{\text{electron}, i} \frac{p_i^2}{m_e} - \sum_{A, i} \frac{e^2}{|x_i - x_A|} + \sum_{i \neq j} \frac{e^2}{|x_i - x_j|}$$

How hard can it be?

Strong Coupling in CM

Already Helium too difficult to solve analytically.



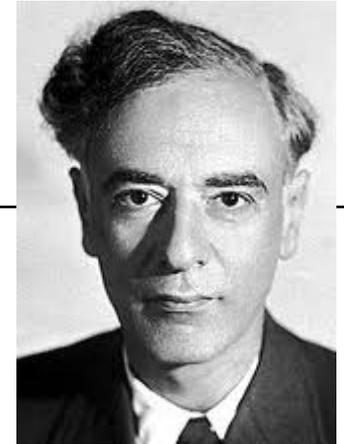
electron/electron Coulomb repulsion not weak!

if it is negligible, we have good theory control:

Band structure! Insulators and conductors.

but what to do when it is not?

Landau's paradigms:



- Identify physical candidates for **low energy** degrees of freedom.

← **dominate transport**

- Write down most general allowed interactions

many interactions “irrelevant” = scale to zero

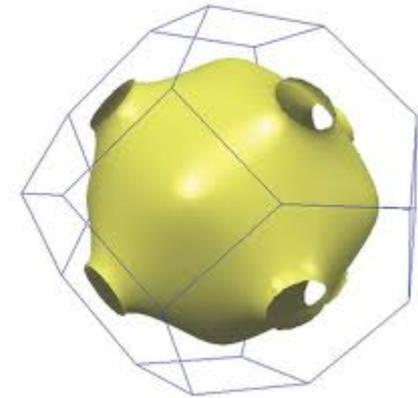


- See how interactions **scale** in low energy limit

What could they be?

1) weakly coupled fermions.

Landau Fermi Liquid



- Fermi Surface
- Low energy excitations near Fermi Surface
- Only Cooper Pair Instability survives at low energies, all other interactions scale to zero

at low temperatures
resistivity grows as T^2

← **universal!**

What could they be?

1) weakly coupled bosons.

Landau's Theory of Phase Transitions

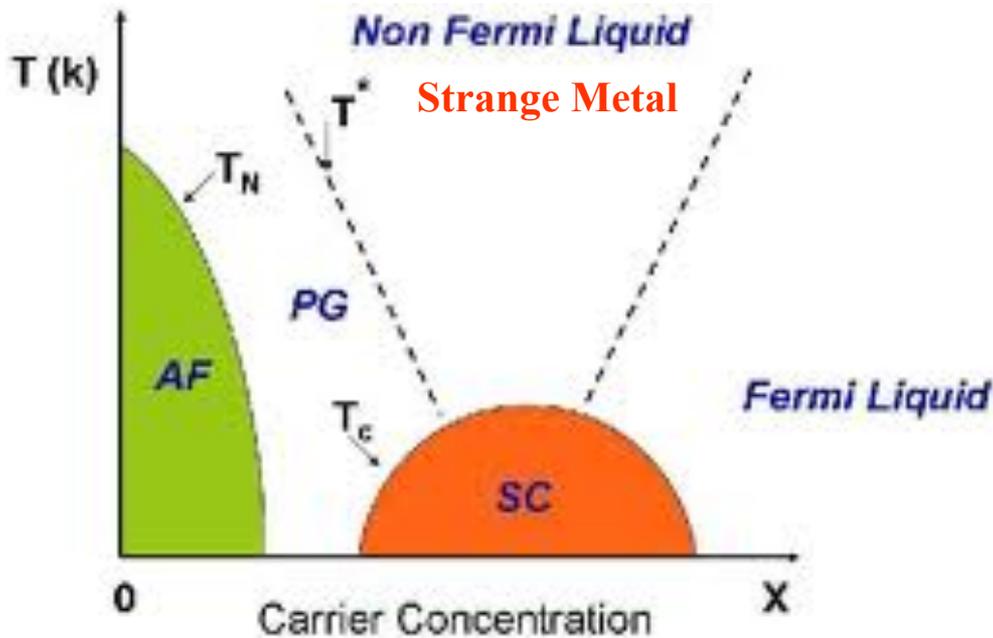
$$G(m, T) = \frac{b_0}{2}(T - T_c)m^2 + \frac{c}{4}m^4 + \frac{d}{6}m^6 + \dots$$

↑
free energy

↑ ↑ ↑
order parameter
= scalar field.

Scalar mass relevant; dominates at low energies.
Can be tuned to zero close to a phase transition.

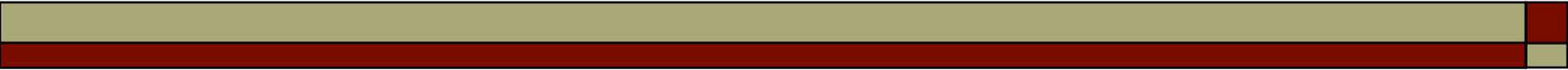
Is this all?



Degrees of freedom
in high T_c
superconductors
are neither!

Non-Fermi Liquid

at low temperatures
resistivity grows as T



What else could it be?

This is the perfect question to ask
a solvable toy model:

Studying matter in holographic
toy models, what are the possible
low energy behaviors?

Matter=finite density of some conserved charge.



MIT/Leiden Fermions.

(Lee)

(Liu, McGreevy, Vegh)

(Cubrovic, Zaanen, Schalm)

Holographic Realization of
a large class of non-Fermi
Liquids.

Fermions in a charged black hole background.

MIT/Leiden Fermions.

Characteristic Features:

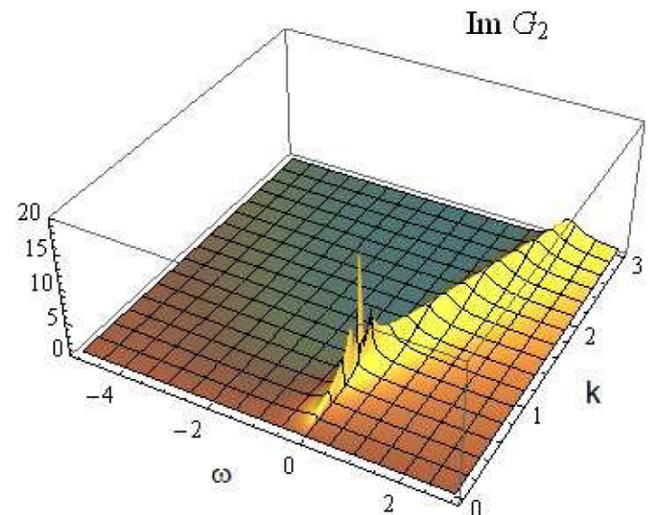
Fermi surface

(singularity in wavevector dependence of correlation functions).

No well defined particle excitation.

(not a Fermi-liquid).

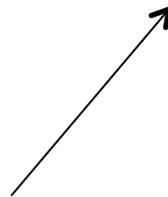
Low temperature resistivity grows as $T^{2\Delta-1}$
(Δ free parameter in model).



Interactions don't scale away?

Fermi-surface, but interactions not irrelevant?

Low energy physics = fermions coupled to other light degrees of freedom!



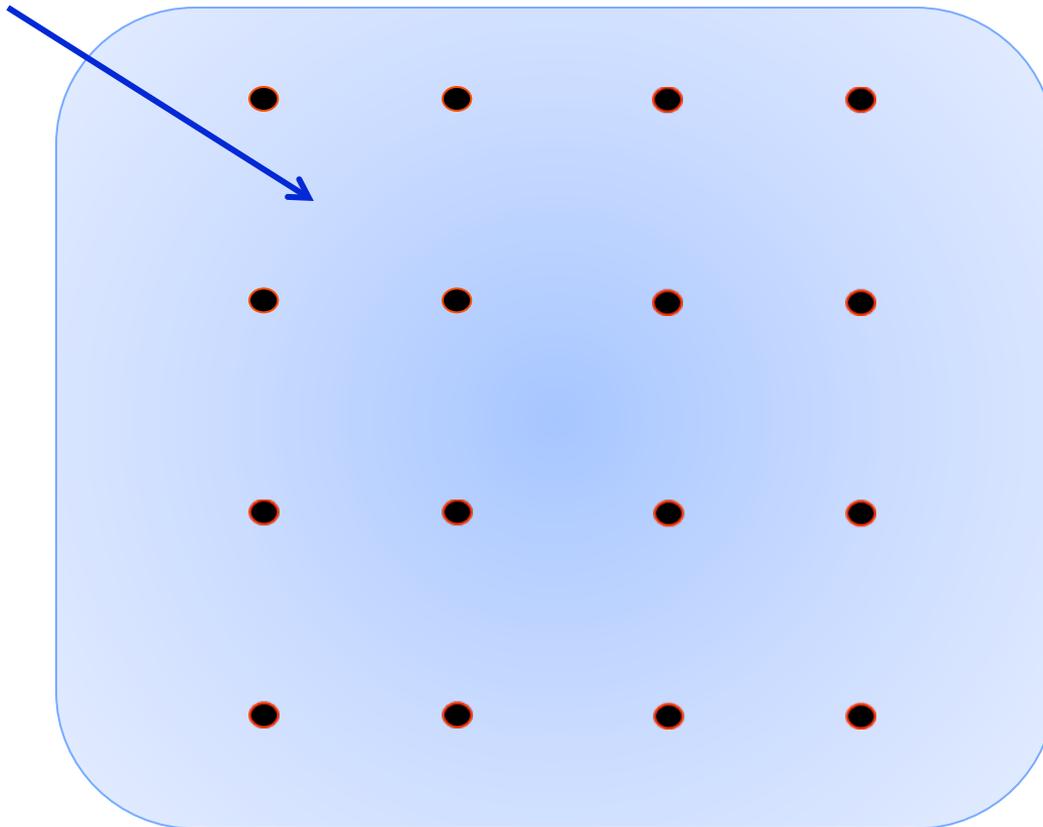
**Local Quantum Criticality.
0+1 dimensional theories close
to a Landau-like phase transition.**

= AdS₂

Local Criticality:

Lattice Kondo model

bulk fermions



**Lattice of
localized defects.**

$$H_J = \sum_{i < j} J_{ij} \hat{S}_i^a \hat{S}_j^a + \dots$$



**Quantum Critical
Point.**

Lattice Kondo model.

CM model for strange metal (heavy fermions)

Supersymmetric Lattice Kondo model gives particularly nice realization of MIT fermions

(Kachru, AK, Yaida)

(Kachru, AK, Polchinski, Silverstein)

- **Explicit Lagrangian of Field Theory is known.**
- **$\Delta=1$ (resistivity as T) arises naturally.**

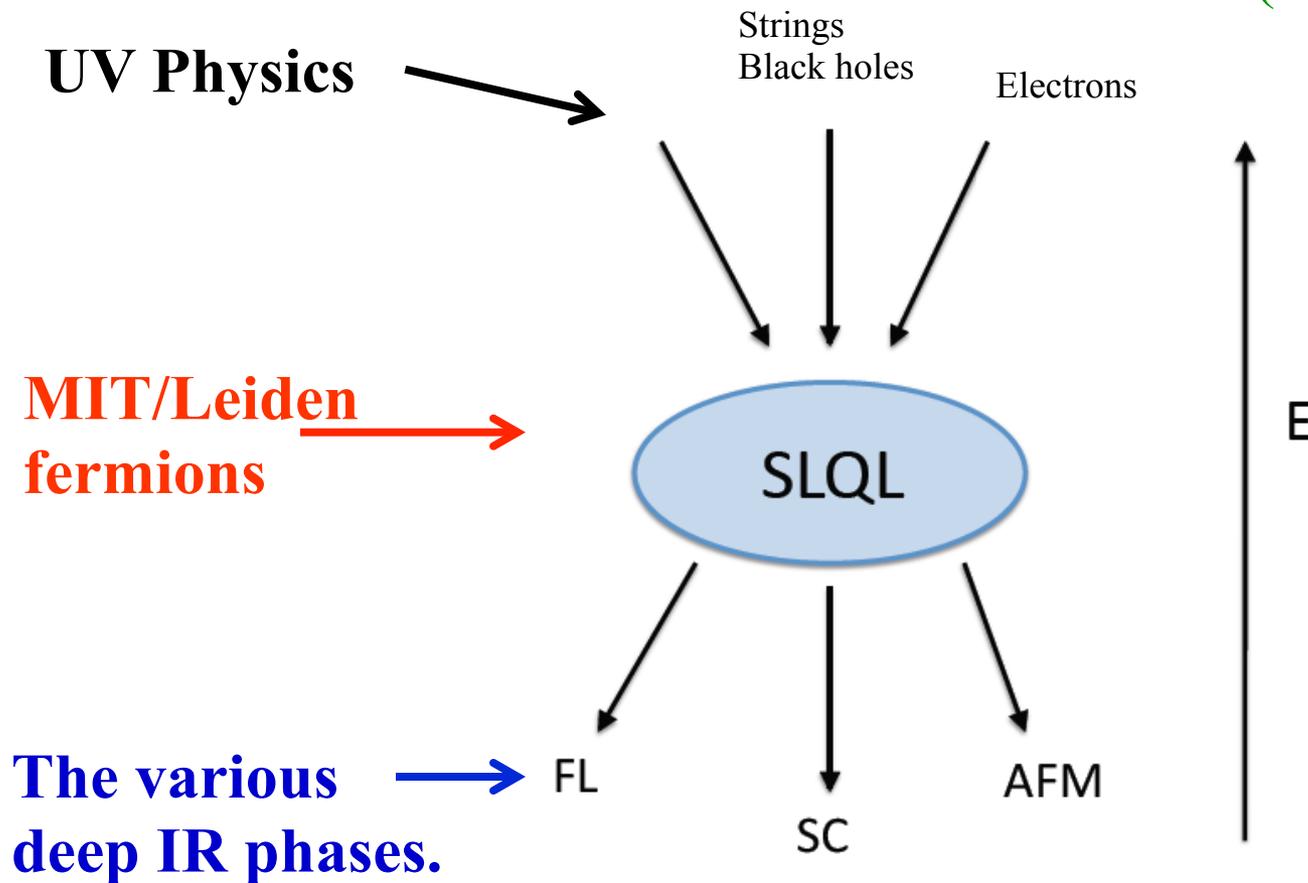
Instabilities

It is perhaps fortunate, therefore, that low temperature charged AdS black holes are found to be unstable towards a range of processes that discharge the black hole and can lead to spacetimes without black hole horizons. The instabilities include condensation of charged scalar fields [26], Cooper pairing of charged fermions [27], emission of D branes [28, 29, 14], backreaction of a bulk fermionic charge density induced by the local chemical potential [14], confinement [30, 31, 32], and perhaps the emergence of underlying lattice degrees of freedom [33]. It is not clear at this stage whether all zero temperature charged AdS black holes with a finite size horizon are unstable [34]. If they are, this fact may be closely tied up with a version of the ‘weak gravity’ conjecture [35]. The instabilities lead to

(from Hartnoll and Tavanfar)

Universal Intermediate Phase

(Iqbal, Liu, Mezei)



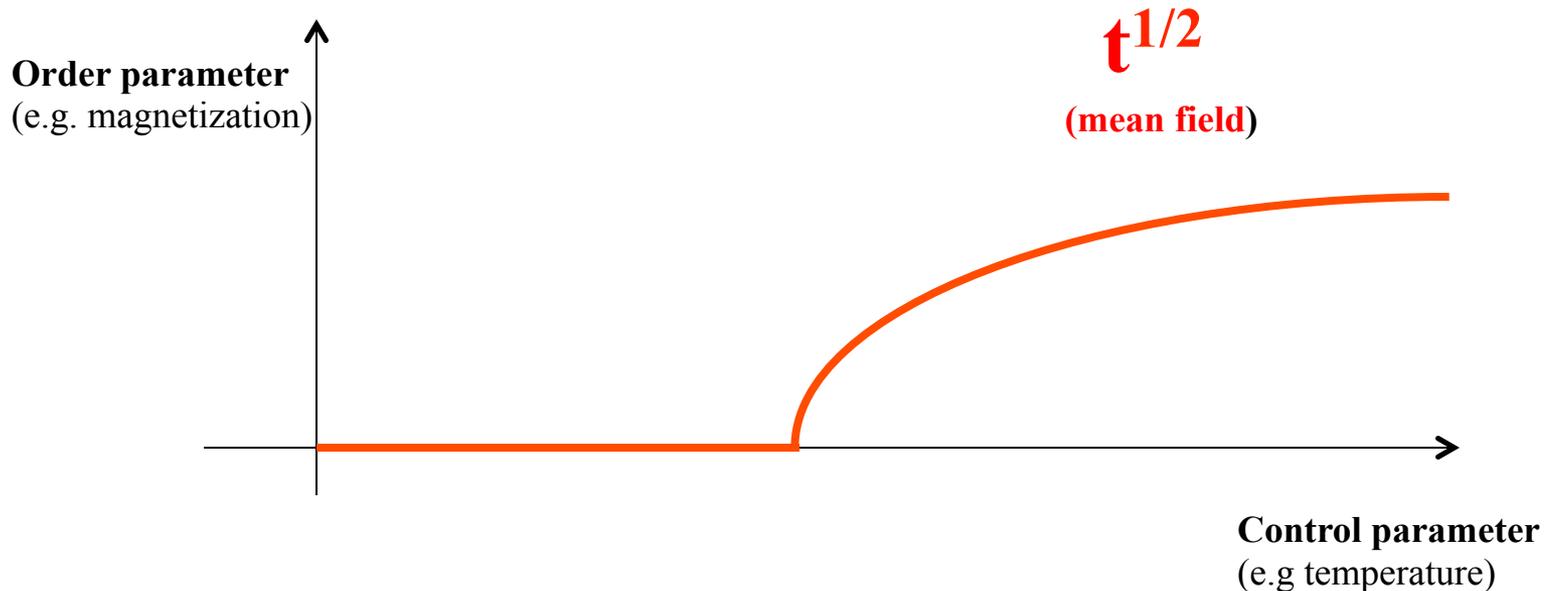


Local Criticality and Bosons

MIT fermions = Local criticality + Landau fermions

????? = Local criticality + Light Scalar

Phase transitions beyond Landau



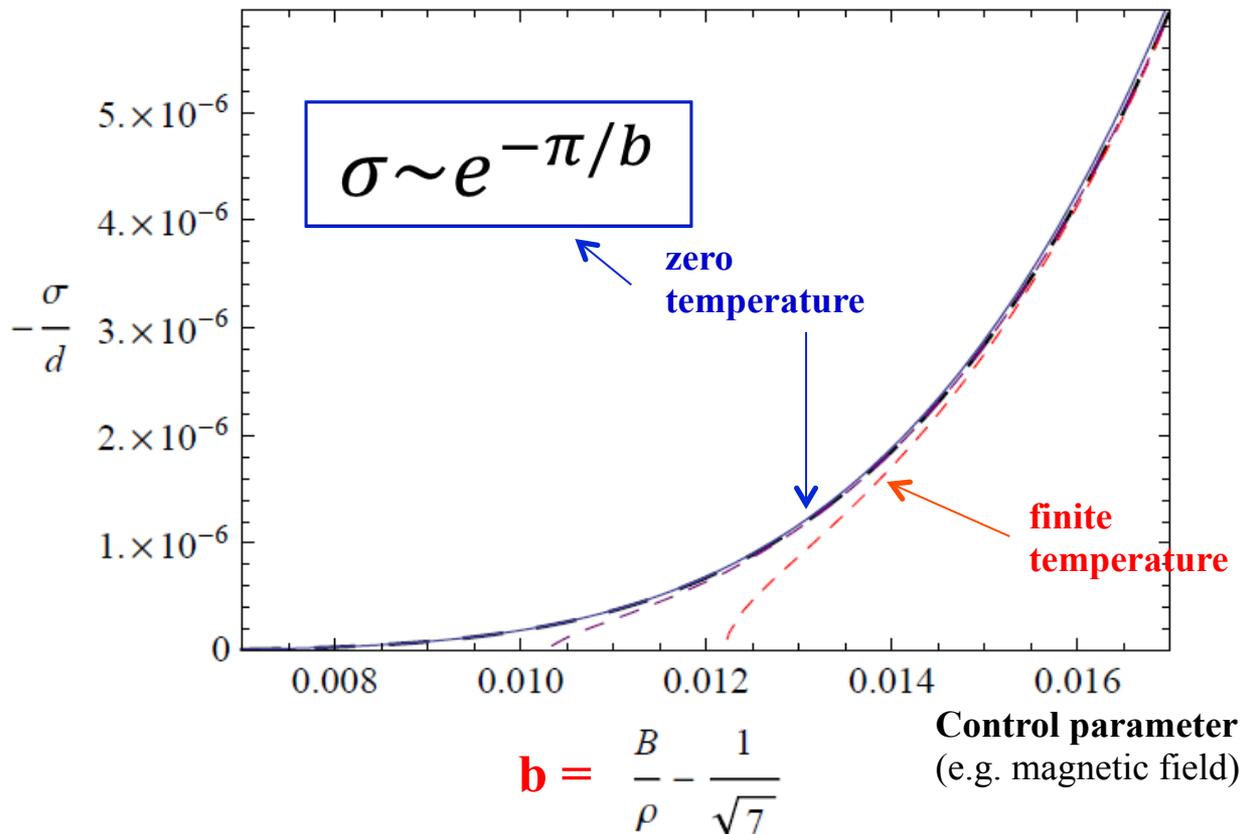
Landau phase transitions:

- **Power Laws**
- **Critical Exponents**

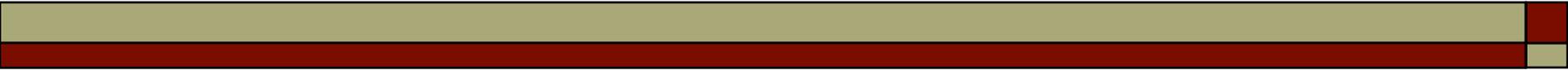
Phase transitions beyond Landau

Order parameter
(e.g. magnetization)

(Jensen, AK, Son, Thompson; Iqbal, Liu, Mezei, Si)



- BKT scaling
(but real 2+1 d quantum phase transition)
- Infinite order



The big question:

Holography provides controlled examples of novel quantum matter.

Is any of this realized (to some approximation) in real systems?

Summary.

Holography

=

Solvable models of strong
coupling dynamics.