

# FLUXES & WARPING IN F-THEORY

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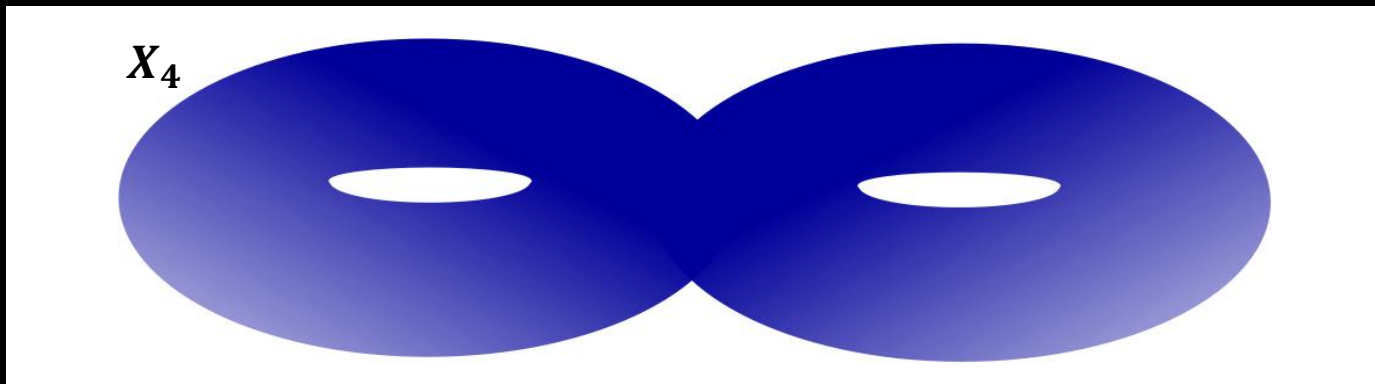
“Strings 2012”, Munich

26<sup>th</sup> of July, 2012

Based on: T.W. Grimm, M. Poretschkin, DK.: [arXiv:1202.0285 \[hep-th\]](#).  
T.W. Grimm, M. Cvetič, DK: work in progress; T.W. Grimm, DK: work in progress.

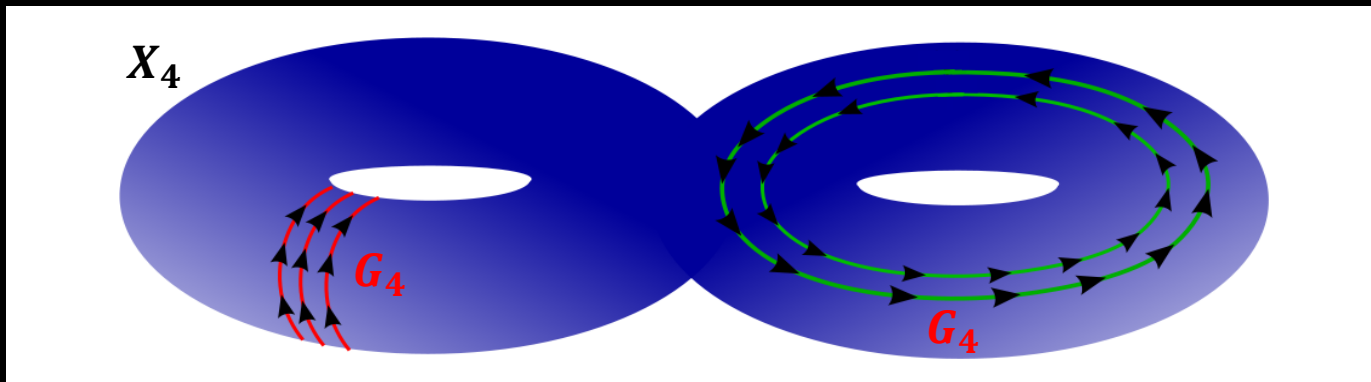
# F-THEORY WITH FLUXES

Elliptically fibered Calabi-Yau fourfold  $X_4$



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Elliptically fibered Calabi-Yau fourfold  $X_4$  + **G-flux**  $G_4$



# FLUXES & THEIR EFFECTIVE 4D PHYSICS

## Chirality generating fluxes

- Fluxes  $G_4$  are naturally constructed on resolved fourfold  $\hat{X}_4$ .
- Fluxes of the form  $G_4 = m^{IJ} \omega_I \wedge \omega_J$  (for  $\omega_I$  (1,1)-forms on  $\hat{X}_4$ ) determine 4D chirality  $\chi(\mathbf{R})$  in dual 3D M-theory compactification.

## Backreaction of fluxes: warping in F-theory

Grimm,DK,Poretschkin '12

- Analytic calculation of flux-induced warp-factor in local models.
- Warping induces essential corrections to 4D effective couplings.

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# CHIRALITY FROM 3D

- Derivation of 4D chirality  $\chi(\mathbf{R})$  on Coulomb branch of 3D N=2 effective theory :

4D F-theory on  $X_4$   
 $N = 1$  gauge theory  
 Chiral matter in rep  $\mathbf{R}$



F-theory on  $X_4 \times S^1 =$  3D M-theory on  $\hat{X}_4$   
 $N = 2$  gauge theory on Coulomb branch  
**Massive matter** **No matter**

Comparison of two dual 3D effective actions:

- 1) M-theory side: Chern-Simons terms  $\theta_{IJ} A^I \wedge F^J$  are classical

$$\Theta_{IJ}^{flux} = \int_{\hat{X}_4} G_4 \wedge \omega_I \wedge \omega_J, \quad \omega_I \in H^{1,1}(\hat{X}_4)$$

- 2) F-theory side: CS-terms classically absent, from integrating out massive matter  $\mathbf{R}$

$$\Theta_{IJ}^{loop} \sim \chi(\mathbf{R}) \quad \text{4D chiral index}$$

General form of loop-corr.:  
 Aharony, Hanany, Intriligator,  
 Seiberg, Strassler '97





- Matching of **CS-terms** at 1-loop

$$\Theta_{IJ}^{flux} \equiv \Theta_{IJ}^{loop}: \quad \chi(\mathbf{R}) = C_R^{IJ} \int_{\hat{X}_4} G_4 \wedge \omega_I \wedge \omega_J$$

Grimm, Hayashi '11; Grimm, DK: in preparation

'11: Braun, Collinucci, Valandro;  
 Esole, Yau;  
 Marsano, Schäfer-Nameki;  
 Krause, Mayrhofer, Weigand

# CS-TERMS & ANOMALIES

- **Algorithmic computation** of chiralities  $\chi(R)$  from **CS-terms in toric examples**.  
Grimm,DK:in preparation
  - 1) flux integrals  $\Theta_{IJ}^{flux}$   exceptional divisors on  $\hat{X}_4$
  - 2) loop-formula  $\Theta_{IJ}^{loop}$   effective curves on  $\hat{X}_4$
- **Relations between different 3D CS-terms** required by **4D anomaly cancellation**  
Cvetic,Grimm,DK:in preparation
  - 4D Green-Schwarz mechanism **in F-theory**:
    - CS-terms encoding 4D chiralities  $\chi(R)$ .  
 
    - CS-terms encoding gaugings of 4D axions.
- **Interpretation** of these relations in 3D?

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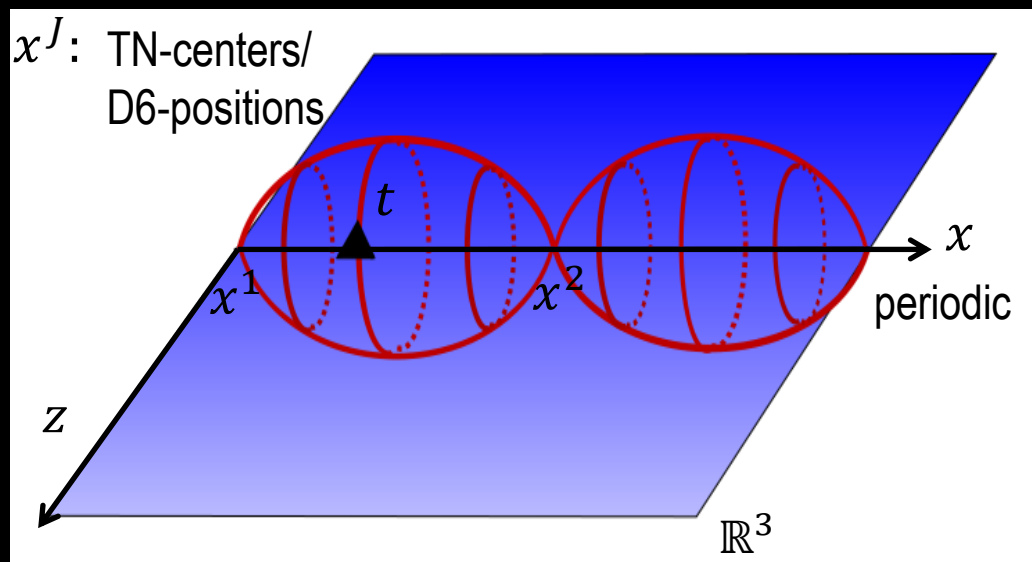
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# LOCAL GEOMETRY OF 7-BRANES

- Construct a local model of  $X_4$  for a **stack of  $k$  7-branes** as follows

$k$  7-brane stack on divisor  $S$ 
 $\xrightarrow[\text{T-duality}]{S_x^1}$ 
 $k$  6-brane stack on divisor  $S$ 
 $\xrightarrow[\text{M-theory}]{S_t^1}$ 
 Periodic multi-center **Taub-NUT** over  $S$



- Metric on periodic Taub-NUT** specified by harmonic functions on  $\mathbb{R}^2 \times S^1$

$$V_I = \log(|z|) - \sum_{n>0} K_0(2\pi|z|n) \cos(2\pi n(x - x_I))$$

# THE WARP-FACTOR ON TAUB-NUT

- **Explicit solution** of warp-factor eq.

$$\Delta_{X_4} e^{3A/2} = *_{X_4} (G_4 \wedge G_4)$$

Becker,Becker '96

Warp-factor on periodic Taub-NUT **analytically** determined

$$e^{3A/2} = 1 - \frac{n^I}{\text{vol}(S)} \left( \frac{V_I^2}{V} - V_I \right) \quad \text{\textcolor{red}{$n^I$ brane-flux number on $S$}}$$

- **M-theory reduction** on this warped geometry **to 3D/4D**:
  - flux  $\alpha'$ - corrections to **D7-brane gauge coupling** in F-theory
  - Warp-factor **dependence on auxiliary torus** direction  $x$  is **essential**.
- Extension to **other 4D couplings** + inclusion of **more sources** to warp-factor eq?

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