Heterotic Supersymmetry: the Legacy of $D = 10$ and $\mathcal{N} = 4$

Hans Peter Nilles

Physikalisches Institut
Universität Bonn
A Zip code for MSSM fields

Localization properties of quarks, leptons and Higgses

- Higgs bosons and top-quark in the “bulk” lead to large top-quark Yukawa coupling
- first 2 families localized (exhibiting family symmetries)
A Zip code for MSSM fields

Localization properties of quarks, leptons and Higgses

- Higgs bosons and top-quark in the “bulk” lead to large top-quark Yukawa coupling
- First 2 families localized (exhibiting family symmetries)

The legacy of higher dimensions

- Mirage Mediation (compressed spectrum for gauginos)
- Natural Susy
- Discrete (nonabelian) family symmetries

Remnants of N=4 SUSY from higher dimensions that might hide Susy at the LHC!
Spinors if SO(10) might be important even in absence of GUT gauge group

gauge-top Yukawa unification in the MSSM

presence of discrete symmetries with many applications

(Kobayashi, HPN, Ploeger, Raby, Ratz, 2006)
Spinors if SO(10) might be important even in absence of GUT gauge group

gauge-top Yukawa unification in the MSSM

presence of discrete symmetries with many applications

(Kobayashi, HPN, Ploeger, Raby, Ratz, 2006)

From the mathematical structure we might prefer exceptional groups

There is a maximal group: \( E_8 \),

but \( E_8 \) and \( E_7 \) do not allow chiral fermions in \( d = 4 \).

How does this fit with our usual picture of unification based on SU(5) or SO(10)?
$E_8$ is the maximal group.

There are, however, no chiral representations in $d = 4$. 
Next smaller is $E_7$.

No chiral representations in $d = 4$ either
$E_6$ allows for chiral representations even in $d = 4$. 

![Graph Diagram](image)
$E_5 = D_5$

$E_5$ is usually not called exceptional.

It coincides with $D_5 = SO(10)$. 
\[ E_4 = A_4 \]

\[ E_4 \text{ coincides with } A_4 = SU(5) \]
$E_3$ coincides with $A_2 \times A_1$ which is $SU(3) \times SU(2)$. 
Exceptional groups in string theory

String theory “favours” $E_8$

- $E_8 \times E_8$ heterotic string
- $E_8$ enhancement as a nonperturbative effect (M- or F-theory)
Exceptional groups in string theory

String theory “favours” $E_8$

- $E_8 \times E_8$ heterotic string
- $E_8$ enhancement as a nonperturbative effect (M- or F-theory)

Strings live in higher dimensions:

- chiral spectrum possible even with $E_8$
- $E_8$ broken in process of compactification
- provides source for (nonabelian) discrete symmetries
- from $E_8/\text{SO}(10)$ and $\text{SO}(6)$ of the higher dimensional Lorentz group
Geography

Many properties of the models depend on the geography of extra dimensions, such as

- the location of quarks and leptons,

- the relative location of Higgs bosons,
Geography

Many properties of the models depend on the geography of extra dimensions, such as

- the location of quarks and leptons,
- the relative location of Higgs bosons,

but there is also a “localization” of gauge fields

- $E_8 \times E_8$ in the bulk
- smaller gauge groups on various branes

Observed 4-dimensional gauge group is common subgroup of the various localized gauge groups!
Calabi Yau Manifold
Orbifold

(Dixon, Harvey, Vafa, Witten, 1985)
We need calculability that goes beyond the effective supergravity field theory approach, e.g. in the form exact conformal field theory. This requires:

- perturbative string theory
- explicit knowledge of metric of the manifold
We need calculability that goes beyond the effective supergravity field theory approach, e.g. in the form exact conformal field theory. This requires:

- perturbative string theory
- explicit knowledge of metric of the manifold

Approximations corresponds to points of enhanced symmetries and enhanced particle spectra

- slightly broken symmetries (Frogatt-Nielsen) provide
- small parameters that appear in particle physics

Hopefully nature is close to points with full calculability.
Enhanced symmetries

This approximate scheme allows model building with geometric intuition. Sectors might exhibit

- enhanced gauge and discrete symmetries
- enhanced supersymmetry
Enhanced symmetries

This approximate scheme allows model building with geometric intuition. Sectors might exhibit

- enhanced gauge and discrete symmetries
- enhanced supersymmetry

If we move away from these points of enhanced calculability, we might still keep “Berechenbarkeit”

- sectors with $N = 4$ or $N = 2$ supersymmetry
- conformal field theory calculations still useful after “blow-up”
- special role of the Green-Schwarz anomaly polynomial.

(Blaszczyk, Cabo, HPN, Ruehle, 2011)
Localization

Quarks, Leptons and Higgs fields can be localized:

- in the Bulk \((d = 10\) untwisted sector\)
- on 3-Branes \((d = 4\) twisted sector fixed points\)
- on 5-Branes \((d = 6\) twisted sector fixed tori\)
Localization

Quarks, Leptons and Higgs fields can be localized:

- in the Bulk ($d = 10$ untwisted sector)
- on 3-Branes ($d = 4$ twisted sector fixed points)
- on 5-Branes ($d = 6$ twisted sector fixed tori)

but there is also a “localization” of gauge fields

- $E_8 \times E_8$ in the bulk
- smaller gauge groups on various branes

Observed 4-dimensional gauge group is common subgroup of the various localized gauge groups!
Localized gauge symmetries

SU(4)$^2$

SO(10)

SU(6) × SU(2) × SU(6) × SU(2)

(Förste, HPN, Vaudrevange, Wingert, 2004)
Standard Model Gauge Group
The MiniLandscape

- many models with the *exact spectrum of the MSSM* (absence of chiral exotics)
  
  (Lebedev, HPN, Raby, Ramos-Sanchez, Ratz, Vaudrevange, Wingerter, 2006-2009)

- family symmetries for the first two families
  
  (Kobayashi, HPN, Ploeger, Raby, Ratz, 2006)

- gauge- and (partial) Yukawa unification
  
  (Raby, Wingerter, 2007)

- large top quark Yukawa coupling

- models with *R-parity* + solution to the \( \mu \)-problem
  
  (Lebedev, HPN, Raby, Ramos-Sanchez, Ratz, Vaudrevange, Wingerter, 2007)

- gaugino condensation and *mirage mediation*
  
  (Löwen, HPN, 2008)
Sectors

The underlying $Z_6II$ orbifold has the following sectors:

- the untwisted sector (bulk $D = 10$, $N = 4$ Susy)
- three twisted sectors corresponding to $\theta$, $\theta^2$ and $\theta^3$

The $\theta$ sector has $4 \times 3 = 12$ fixed points, corresponding to “3-branes” confined to D=4 space-time ($N = 1$ Susy).
The $\theta^2$ sector contains $2 \times 3$ fixed tori corresponding to

- “5-branes” confined to 6 space-time dimensions
  (remnants of $\mathcal{N} = 2$ Susy)
The $\theta^3$ sector contains 2 x 4 fixed tori:

- “5-branes” confined to 6 space-time dimensions
  (sector with $N = 2$ Susy)

Where do we find quarks, leptons and Higgs bosons in the models of the MiniLandscape?
A Benchmark Model

At the orbifold point the gauge group is

\[ SU(3) \times SU(2) \times U(1)^9 \times SU(4) \times SU(2) \]

- one \( U(1) \) is anomalous
- there are singlets and vectorlike exotics
- decoupling of exotics and breakdown of gauge group has been verified
- remaining gauge group

\[ SU(3) \times SU(2) \times U(1)_Y \times SU(4)_{\text{hidden}} \]

- for discussion of neutrinos and R-parity we keep also the \( U(1)_{B-L} \) charges
<table>
<thead>
<tr>
<th>#</th>
<th>irrep</th>
<th>label</th>
<th>#</th>
<th>irrep</th>
<th>label</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>$(3, 2; 1, 1)_{(1/6,1/3)}$</td>
<td>$q_i$</td>
<td>3</td>
<td>$(\bar{3}, 1; 1, 1)_{(-2/3,-1/3)}$</td>
<td>$\bar{u}_i$</td>
</tr>
<tr>
<td>3</td>
<td>$(1, 1; 1, 1)_{(1,1)}$</td>
<td>$\bar{e}_i$</td>
<td>8</td>
<td>$(1, 2; 1, 1)_{(0,*)}$</td>
<td>$m_i$</td>
</tr>
<tr>
<td>3 + 1</td>
<td>$(\bar{3}, 1; 1, 1)_{(1/3,-1/3)}$</td>
<td>$\bar{d}_i$</td>
<td>1</td>
<td>$(3, 1; 1, 1)_{(-1/3,1/3)}$</td>
<td>$d_i$</td>
</tr>
<tr>
<td>3 + 1</td>
<td>$(1, 2; 1, 1)_{(-1/2,-1)}$</td>
<td>$\ell_i$</td>
<td>1</td>
<td>$(1, 2; 1, 1)_{(1/2,1)}$</td>
<td>$\bar{\ell}_i$</td>
</tr>
<tr>
<td>1</td>
<td>$(1, 2; 1, 1)_{(-1/2,0)}$</td>
<td>$h_d$</td>
<td>1</td>
<td>$(1, 2; 1, 1)_{(1/2,0)}$</td>
<td>$h_u$</td>
</tr>
<tr>
<td>6</td>
<td>$(\bar{3}, 1; 1, 1)_{(1/3,2/3)}$</td>
<td>$\bar{\delta}_i$</td>
<td>6</td>
<td>$(3, 1; 1, 1)_{(-1/3,-2/3)}$</td>
<td>$\delta_i$</td>
</tr>
<tr>
<td>14</td>
<td>$(1, 1; 1, 1)_{(1/2,*)}$</td>
<td>$s_i^+$</td>
<td>14</td>
<td>$(1, 1; 1, 1)_{(-1/2,*)}$</td>
<td>$s_i^-$</td>
</tr>
<tr>
<td>16</td>
<td>$(1, 1; 1, 1)_{(0,1)}$</td>
<td>$\bar{n}_i$</td>
<td>13</td>
<td>$(1, 1; 1, 1)_{(0,-1)}$</td>
<td>$n_i$</td>
</tr>
<tr>
<td>5</td>
<td>$(1, 1; 1, 2)_{(0,1)}$</td>
<td>$\bar{\eta}_i$</td>
<td>5</td>
<td>$(1, 1; 1, 2)_{(0,-1)}$</td>
<td>$\eta_i$</td>
</tr>
<tr>
<td>10</td>
<td>$(1, 1; 1, 2)_{(0,0)}$</td>
<td>$h_i$</td>
<td>2</td>
<td>$(1, 2; 1, 2)_{(0,0)}$</td>
<td>$y_i$</td>
</tr>
<tr>
<td>6</td>
<td>$(1, 1; 1, 4, 1)_{(0,*)}$</td>
<td>$f_i$</td>
<td>6</td>
<td>$(1, 1; \bar{4}, 1)_{(0,*)}$</td>
<td>$\bar{f}_i$</td>
</tr>
<tr>
<td>2</td>
<td>$(1, 1; 4, 1)_{(-1/2,-1)}$</td>
<td>$f_i^-$</td>
<td>2</td>
<td>$(1, 1; \bar{4}, 1)_{(1/2,1)}$</td>
<td>$\bar{f}_i^+$</td>
</tr>
<tr>
<td>4</td>
<td>$(1, 1; 1, 1)_{(0,\pm2)}$</td>
<td>$\chi_i$</td>
<td>32</td>
<td>$(1, 1; 1, 1)_{(0,0)}$</td>
<td>$s_i^0$</td>
</tr>
<tr>
<td>2</td>
<td>$(\bar{3}, 1; 1, 1)_{(-1/6,2/3)}$</td>
<td>$\bar{v}_i$</td>
<td>2</td>
<td>$(3, 1; 1, 1)_{(1/6,-2/3)}$</td>
<td>$v_i$</td>
</tr>
</tbody>
</table>
The location of Higgs bosons

Typically there could be a multitude of Higgs doublets (and triplets) in the spectrum

- triplets heavy or projected out
- exactly two Higgs doublet multiplets should remain light
- all other heavy

This is the so-called $\mu$ problem
The location of Higgs bosons

Typically there could be a multitude of Higgs doublets (and triplets) in the spectrum

- triplets heavy or projected out
- exactly two Higgs doublet multiplets should remain light
- all other heavy

This is the so-called $\mu$ problem

The MiniLandscape identifies exactly one Higgs pair protected by a discrete $R$-symmetry and provides a unique solution to the $\mu$ problem, because the Higgs bosons live in the untwisted sector (delocalized Higgs as in torus compactification: remnants of $\mathcal{N} = 4$ susy)
Location of top quark

Given the fact that the Higgs multiplets live in the bulk we now explore how to obtain a large top quark Yukawa coupling

- need maximum “overlap” with the Higgs multiplet
- results of the MiniLandscape teach us that this requires the top quark to live in the untwisted sector as well
Location of top quark

Given the fact that the Higgs multiplets live in the bulk we now explore how to obtain a large top quark Yukawa coupling

- need maximum “overlap” with the Higgs multiplet
- results of the MiniLandscape teach us that this requires the top quark to live in the untwisted sector as well

Top quark in untwisted sector (bulk). The third family is usually distributed over various sectors (it is not in a complete localized SO(10) representation).

Side remark:
3 “complete” families impossible within $Z_6 II$ orbifold
First and second family

The first and second families are in complete localized 16-dimensional representation of SO(10) (at points of “enhanced” gauge symmetry)

They live in the $\theta$ twisted sector and are localized at the fixed points $\alpha = 1, \beta = 1, \gamma = 1, 3$

exhibiting a $D_4$ family symmetry.
Unification

- Higgs doublets live in the bulk
- Heavy top quark live in the bulk as well.
- $\mu-$term protected by a discrete R-symmetry

Minkowski vacuum before Susy breakdown (no AdS)

- Solution to $\mu$-problem (Casas, Munoz, 1993)
- First two families localized (smaller Yukawa couplings) exhibiting a discrete family symmetry
Emergent localization properties

The benchmark model illustrates some of the general properties of the MiniLandscape

- exactly two Higgs multiplets (no triplets)
- the top quark lives in the untwisted sector (as well as the Higgs multiplets)
- only one trilinear Yukawa coupling (all others suppressed)
Emergent localization properties

The benchmark model illustrates some of the general properties of the MiniLandscape

- exactly two Higgs multiplets (no triplets)
- the top quark lives in the untwisted sector (as well as the Higgs multiplets)
- only one trilinear Yukawa coupling (all others suppressed)

The fact that the top-quark has this unique property among all the quarks and leptons has important consequences for the phenomenological predictions including supersymmetry breakdown.

(Krippendorf, HPN, Ratz, Winkler, 2012)
Heterotic string: gaugino condensation

Gravitino mass $m_{3/2} = \frac{\Lambda^3}{M_{\text{Planck}}^2}$ and $\Lambda \sim \exp(-\tau)$

SU(4) in hidden sector predicts gravitino mass in TeV range

(Lebedev, HPN, Raby, Ramos-Sanchez, Ratz, Vaudrevange, Wingerter, 2006)
 Mirage scheme

In string theory we have (from flux and gaugino condensate)

\[ W = \text{flux} - \exp(-X) \]

- modulus mediation suppressed (in the process of adjusting the vacuum energy)
  \[ X \sim \log(M_{\text{Planck}}/m_{3/2}) \sim 4\pi^2 \]

- radiative corrections become relevant (\(\beta\) function)

- Mixed mediation scheme: Mirage Mediation (MMAM) with mirage pattern for gaugino masses:
  \[ m_{1/2} \sim m_{3/2}/4\pi^2 \]
  (Choi, Falkowski, Nilles, Olechowski, 2005)

generic in the framework of Type IIB and heterotic theory.
The overall pattern

This provides a specific pattern for the soft masses with a large gravitino mass in the multi-TeV range

- normal squarks and sleptons in Multi-TeV range
- top squarks \((\tilde{t}_L, \tilde{b}_L)\) and \(\tilde{t}_R\) in TeV-range (suppressed by \(\log(M_{\text{Planck}}/m_{3/2}) \sim 4\pi^2\))
- A-parameters in TeV range
- gaugino masses in TeV range
- mirage pattern for gaugino masses (compressed spectrum)
- heavy moduli (enhanced by \(\log(M_{\text{Planck}}/m_{3/2})\) compared to the gravitino mass)
Parameter scan for a gravitino mass of 15 TeV. The coloured regions are excluded while the hatched region indicates the current reach of the LHC. The contours indicate the mass of the lightest stop.
Spectrum

Mass [GeV]

11000

5000

2865

2000

783

126

H_0, A_0, H^2

\tilde{g}

\tilde{t}_2

\tilde{\chi}^0_2, \tilde{\chi}^+_2

\tilde{\chi}^0_1

\tilde{t}_1 \tilde{b}_1

heavy scalars
Parameter scan for a gravitino mass of 15 TeV. The coloured regions are excluded while the hatched region indicates the current reach of the LHC. The contours indicate the mass of the lightest stop.
Messages

- large gravitino mass (multi TeV-range)
- gaugino masses and stops suppressed by
  \( \log(M_{\text{Planck}}/m_{3/2}) \)
- other sfermion masses are of order \( m_{3/2} \)
- the heterotic string yields “Natural SUSY” as a remnant of the underlying \( N = 4 \) Susy
  - mirage pattern for gauginos,
  - light stop masses
- and this is a severe challenge for LHC searches.
Conclusions

Localization of quarks, leptons and Higgs bosons

- realistic models require Higgs multiplets and top multiplets in bulk (connected to $\mu$ problem)
- this implies Gauge-Yukawa unification
- other fields tend to be localized at fixed points (tori) and exhibit discrete family symmetries
Conclusions

Localization of quarks, leptons and Higgs bosons

- realistic models require Higgs multiplets and top multiplets in bulk (connected to $\mu$ problem)
- this implies Gauge-Yukawa unification
- other fields tend to be localized at fixed points (tori) and exhibit discrete family symmetries

The legacy from extra dimensions ($D = 10$)

- discrete family symmetries
- mirage mediation (a hierarchy from $\log(M_{\text{Planck}}/m_{3/2})$)
- mass spectrum of “Natural Susy” from $N = 4$
Heterotic supersymmetry

is more than just $N = 1 \text{ Susy in } D = 4$.
It provides the Zip code for the MSSM fields,

- Higgs boson are bulk fields with enhanced susy
- R-symmetries for $\mu$-problem and proton stability
- Gauge-Higgs unification (continuous Wilson lines, shift symmetry of the Kaehler potential)
- top quark as bulk field: gauge-top unification
- discrete (nonabelian) family symmetries
Heterotic supersymmetry

is more than just $N = 1$ Susy in $D = 4$. It provides the Zip code for the MSSM fields,

- Higgs boson are bulk fields with enhanced susy
- R-symmetries for $\mu$-problem and proton stability
- Gauge-Higgs unification (continuous Wilson lines, shift symmetry of the Kaehler potential)
- top quark as bulk field: gauge-top unification
- discrete (nonabelian) family symmetries

The fact that we do not see supersymmetric particles at the LHC does not imply that Susy is absent. It is due to the fact that we have more supersymmetry than originally thought.
The Mirage Scale

\[ \log_{10}(\mu/\text{GeV}) \]

\[ \frac{M}{\text{GeV}} \]

\( M_3 \)
\( M_2 \)
\( M_1 \)

(Lebedev, HPN, Ratz, 2005)
Reading the Gaugino Code

Mixed boundary conditions at the GUT scale characterized by the parameter $\alpha$: the ratio of modulus to anomaly mediation.

- $M_1 : M_2 : M_3 \simeq 1 : 2 : 6$ \quad \text{for } \alpha \simeq 0
- $M_1 : M_2 : M_3 \simeq 1 : 1.3 : 2.5$ \quad \text{for } \alpha \simeq 1
- $M_1 : M_2 : M_3 \simeq 1 : 1 : 1$ \quad \text{for } \alpha \simeq 2
- $M_1 : M_2 : M_3 \simeq 3.3 : 1 : 9$ \quad \text{for } \alpha \simeq \infty

The mirage scheme leads to

- LSP $\chi_1^0$ predominantly Bino
- a “compact” (compressed) gaugino mass pattern.

(Choi, HPN, 2007; Löwen, HPN, 2009)
Gaugino Masses

\[ M_a / |M_0| (1 + \alpha^2)^{1/2} \]

\[ M_1, M_2, M_3 \]

\[ \alpha \]

Anomaly \uparrow