Quantum Instability of the de Sitter Space

The Red and The Black

Cosmological const expected:

$$\Lambda \sim (\text{Planck length})^{-2}$$

real

$$\Lambda \sim (\text{size of universe})^{-2}$$

Why it is defined By the IR-cut-off and not UV-cutoff?

„Commutative diagram"

$$
\begin{tikzcd}
\text{super } Z \arrow[rr, leftrightarrow] \arrow[d] & & \text{B.H} \\
\text{Const. } E \arrow[r, leftrightarrow] \arrow[d] & dS \arrow[r] \arrow[d] & \text{NCFT} \\
\text{const } B \arrow[r] & AdS \arrow[r] & \text{UCFT}
\end{tikzcd}
$$

Electric IR/UV mixing

$$\varphi = \sum_k a_k f_k^* + a_k^\dagger f_k$$

$$\ddot{f}_k + \omega_k^2(t) f_k$$

$$\omega_k = \sqrt{(k - A(t))^2 + m^2}$$

$$f_k \to \frac{1}{\sqrt{2\omega_k}} e^{i S_k(t)} \qquad k \gg A(t)$$

$$f_k \to \frac{1}{\sqrt{2\omega_k}} \left(\alpha e^{i S_k} + \beta e^{-i S_k}\right) \qquad k \ll A$$

$$|\alpha|^2 - |\beta|^2 = 1.$$

If $A(-\infty) << k_z << A(\infty)$

β – doesn't depend on k_z

$$\beta = \beta(k_\perp^2 + m^2)$$

The mode k_z is produced at the moment t_k:

$$A(t_k) = k_z$$

The current

$$\langle J_z(t) \rangle$$

$$= \int dk_z \, d\vec{k}_\perp \, (k_z - A)$$

$$\cdot \, |f_k(t)|^2 =$$

$$= \int\limits_{k_z > A(t)} dk_z \int dk_\perp \frac{(k_z - A)}{2\omega_k}$$

$$+ \int dk_\perp \, (1 + 2|\beta|^2) \int \frac{dk_z \, (k_z - A)}{2\omega_k}$$

$$= \int 2|\beta|^2 \, d^2 k_\perp \int\limits_{A(t) - A(-\infty) < p < 0} \frac{dp \, p}{2\omega_p}$$

The result

$$\langle J_z(t)\rangle \sim \left(\int |\beta|^2 d^2k_\perp\right)$$

$$\times\left(A_z(t) - A_z(-\infty)\right)$$

$$\sim E\, t$$

Huge Backreaction

$$F = \langle \varphi^*(z\,t\,\vec{x}_{1\perp})\ \varphi(z\,t\,\vec{x}_{2\perp})\rangle$$

$$\sim F^{(0)} + \int dk_\perp\, e^{ik_\perp(x_{1\perp}-x_{2\perp})}$$

$$\cdot \int_{A(\infty)-A(t)}^{0} \frac{dp}{2\omega_p}$$

T – total time passed

$$F = \Phi(x_{1\perp}-x_{2\perp}) \log \frac{ET}{m}$$

For $\frac{1}{m} \gg |z_1 - z_2| \gg \frac{1}{ET}$

$$F \propto \delta(x_1^\perp - x_2^\perp) \log \frac{1}{m(z_1 - z_2)}$$

singularity at non-coinciding points!

If $ET \sim \Lambda_{UV}$

~~renorm~~ renormaliz. lost

De Sitter space

$$n_+ n_- + \vec{n}_\perp^2 = 1$$

$$n_\pm = n_1 \pm n_0 .$$

dS symmetry:

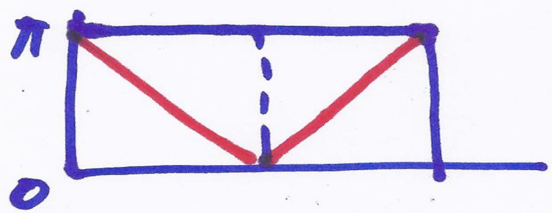
$$\langle \varphi^2(n) \rangle = \text{const}$$

No large Backreaction
unless symmetry
is Broken

But it is!

Poincaré patch

$n_+ \geq 0$



The B.-D. vacuum:
analytic contin. from
a sphere:

$z = n n'$

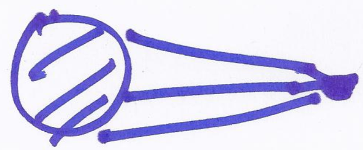
$G_{++}(n, n') = g(z - i0)$

$G_{+-}(n, n') = g(z - i\epsilon(n'_+ - n_+))$

etc

Let us check that
$n_+ \geq 0$ doesn't break
symm. (non-trivial)

$$\delta n_+ = \omega_{+\perp} n_\perp$$



n $\quad \int (dn)\, \theta(n_+) \ldots$

$$\delta F \propto \sum_{\pm} \int dn_\perp\, n_\perp\, dn_-$$

$$* \prod_A G(n_A \cdot n)$$

since $\delta \vartheta(n_+) = \omega_{+\perp} \cdot$
$\cdot\, n_\perp\, \delta(n_+)$

$$\delta F \sim \Im m \int dn_\perp \, n_\perp \cdot$$

$$\cdot \int_{-\infty}^{+\infty} dn_- \; g(n_{A+} n_- + n_A^\perp n^\perp - i0)$$

$$\doteq 0$$

In any NON-BD vacuum symmetry broken. Example

$$G_{++}^{(N)} = (1+N)\, g(z - i0) + N\, g(z + i0)$$

Moreover

The symmetry may be unstable

$$\Delta n(q) \propto \frac{1}{q^{d-2}} \int \frac{d^d k}{k^2}\, n(k)$$

(in linear approxim.)

analogous to negative spec. heat of B.H.

Hyperbolic motion – another analogy

Golden rule

$$w \sim \int_k |J_\mu(k)|^2 \,\theta(k_0)\,\delta(k^2)$$

$$J_\mu = \int e^{ikx}\, \dot{x}_\mu(s)\, ds$$

$$w \sim \int \frac{ds_1 ds_2 \,(\dot{x}(s_1)\dot{x}(s_2))}{(x(s_1) - x(s_2) + i\epsilon)^2}$$

ϵ – time-like vector

$$\Delta P_\mu \sim \int \frac{ds_1 ds_2 \,(\dot{x}(s_1)\,\dot{x}(s_2))}{(x(s_1)-x(s_2)+i\epsilon)^4}$$

$$\cdot (x_\mu(s_1) - x_\mu(s_2))$$

$$\sim \int ds \,(\ddot{v}_\mu - (v\ddot{v})\,v_\mu)$$

$$v_\mu = \dot{x}_\mu \quad \boxed{x_\mu(s+t) = \Lambda_{\mu\nu}\, x_\nu(t)}$$

Lorentz – Dirac :

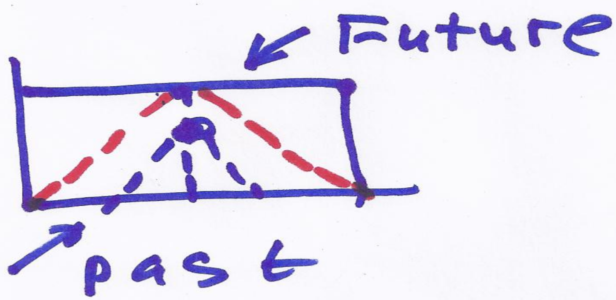
$$\dot{v}_\mu = \epsilon_{\mu\nu} v_\nu + \frac{2\alpha}{3}(\ddot{v}_\mu -$$

$$- (v\ddot{v})\, v_\mu)$$

Runaway

Anti-Poincaré

/Global dS/



The one-point funct. can be calculated in the anti-Poincare patch:

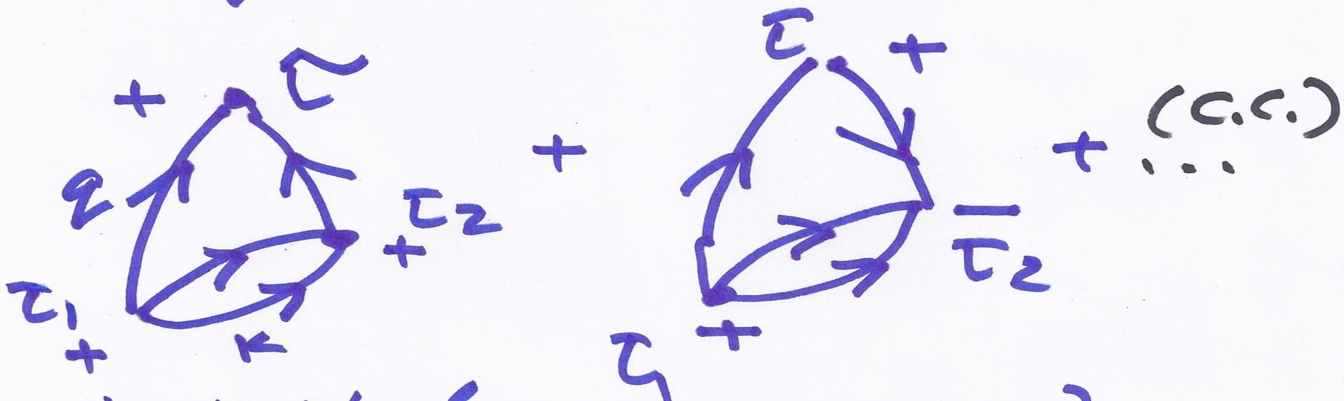
$$n_- = 1/\tau \geq 0$$

$$n_+ = 1/\tau\,(\tau^2 - x^2)$$

$\tau = 0$ $\downarrow$ past

Feynman rules

$$\langle \varphi_q^2 \rangle = ?$$



(individual arrows of time)

$$G = (\tau_1 \tau_2)^{d/2}\, h^*(q\tau_>)\, h(q\tau_<)$$

$$\left(h(x) \propto H^{(1)}_{i\mu}(x) \right).$$

Outgoing arrow $\Longrightarrow h$

Incoming arrow $\Longrightarrow h^*$

The diagrams

$$I = \left(h^*(q\tau)\right)^2 \int A(q,k)\, B^*(q,k) \cdot d^d k \quad (++)$$

$$II = |h(q\tau)|^2 \cdot \int |A(q,k)|^2\, d^d k \quad (+-)$$

$$A = \int_\epsilon^\tau d\tau_1\, \tau_1^{d/2-1} \cdot h(q\tau_1)\, h(k\tau_1)\, h(|q-k|\tau_1)$$

$$B = \int_\epsilon^\tau d\tau_1\, \tau_1^{d/2-1}\, h^*(q\tau_1) \cdot h(k\tau_1)\, h(|q-k|\tau_1)$$

$$h(x) \to a\, x^{i\mu} + \hat{a}\, x^{-i\mu} \qquad x \to 0$$

$$(\hat{a} = a(-\mu))$$

$$h \to \frac{1}{\sqrt{2x}}\, e^{ix} \qquad x \to \infty$$

$$A(q,k) \to k^{-d/2} \cdot$$

$$\cdot \left[a(\mu)\, g(\mu) \left(\frac{q}{k}\right)^{i\mu} + \mu \to -\mu \right]$$

$$g(\mu) = \int_0^\infty dx\, x^{d/2 - 1 + i\mu}\, h^2(x)$$

$$|A(q,k)|^2 \propto k^{-d}$$

for $k \gg q$.

The structure

$$G^{(1)} = 2N(q,\tau)\,|h(q\tau)|^2 + \mathrm{Re}\left(\alpha^*(q\tau)\, h^2(q\tau)\right)$$

$$N(q,\tau) \sim \int |A(q,k)|^2 d^2k$$

$$\alpha(q,\tau) \sim \int A B^* d^2k$$

The dominant region

$$\boxed{1/\varepsilon \gg k \gg \max(q, 1/\tau)}$$

First order:

$$N(q,\tau) \propto \{ |a g|^2 + |\tilde{a} \tilde{g}|^2 \} \times \underline{\log \tau/\epsilon}$$

$$\alpha(q,\tau) \propto a\tilde{a} [|g|^2 + |\tilde{g}|^2] \wedge \underline{\log \tau/\epsilon}$$

ϵ = IR cut-off

We get slowly changing α-vacuum and the Fock space over it

The one point function

$$\langle \varphi^2(x) \rangle \sim \tau^d \int d^d q \, |h(q\tau)|^2 \cdot \int d^2 k \, |A(q,k)|^2$$

UV cut-off $p = q\tau < \Lambda_{UV}$

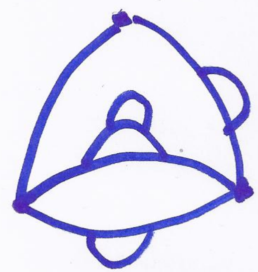
Dominant contribution from

$$m \ll p = q\tau \ll \min\left(m\frac{\tau}{\varepsilon}, \Lambda_{UV}\right)$$

If $\Lambda_{UV} > m\tau/\varepsilon$

$$\langle \varphi^2(x) \rangle \sim m^{d-1}\left(\left(\frac{\tau}{\epsilon}\right)^{d-1} + O\left(\log\frac{\tau}{\epsilon}\right)\right)$$

Leading logs and direct cascade



$$q \ll k \ll k' \ll \dots$$

The ansatz

$$G(q,\tau) = (1+N(\ell))\, f^*(q,\tau_>)\, f(q,\tau_<) + N(\ell)\, f^*(q,\tau_<)\, f(q,\tau_>)$$

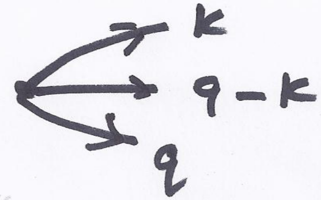
$$f = \alpha(\ell)\, h + \beta(\ell)\, h^*$$

$$\ell = \log \tau/G$$

Kinetic equation

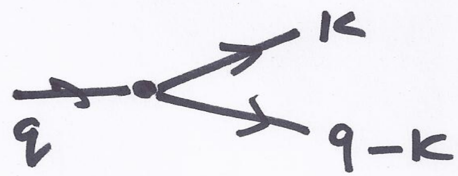
Processes

(A)



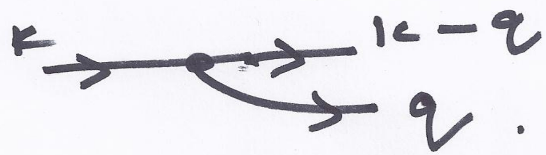
creation

(B)



decay

(C)



Bremsstral.

$$N^{(A)}(\tau) = \int_{\epsilon}^{\tau} \frac{d\tau_1}{\tau_1} \, |A(\tau_1)|^2 \cdot$$

$$\left((1+N(\tau_1))^2 (1+N(\tau)) - \right.$$
$$\left. - N^2(\tau_1)\, N(\tau) \right)$$

Gibbons–Hawking temperature and subjective idealism

Consider

$$a(t) = e^{T \tanh t/T}$$

$$\left(\partial_t^2 + m^2 + k^2/a^2(t)\right) f = 0$$

Jost function:

$$f \to \frac{1}{\sqrt{2\omega_k^-}} e^{i\omega_k^- t}$$

$$f \to \frac{1}{\sqrt{2\omega_k^+}} \left\{ \alpha e^{+i\omega_k^+ t} + \right.$$
$$\left. + \beta e^{-i\omega_k^+ t} \right\}$$

For $ma(-\infty) \ll k \ll ma(\infty)$

α, β – k-independent

(dS symmetry)

As $t, t' \to +\infty$

Thermal

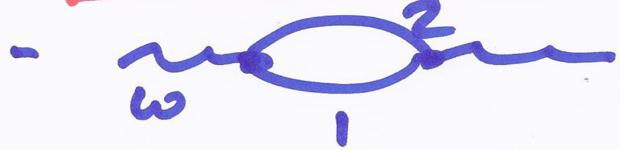
$$\langle T\varphi(t)\varphi(t')\rangle$$

$$\to \frac{1}{2\omega_k}\Big[(1+|\beta|^2)e^{-i\omega_k|t-t'|} + |\beta|^2 e^{i\omega_k|t-t'|}\Big]$$

$$+ \frac{1}{2\omega_k}\Big[\alpha^*\beta\, e^{-i\omega_k(t+t')} + c.c.\Big]$$

removed by coarse graining

Jeans instability and plasma waves



$= \Pi_{\mu\nu}(q)$

$q_\mu \Pi_{\mu\nu} = 0$

But $\omega = E_1 \pm E_2$

and $q_\mu = 0$ is singular point

$\Rightarrow$ real mass of photon, imaginary mass of graviton

But the temperature not G./H with interaction

Inflation

Standard view

$$-\int R + \int (\nabla\phi)^2 + V(\phi)$$

① zero cosm. const

② Pumped up intidl state.
Why not the vacuum?

· **Present picture**

Global dS can not be stable. It slowly inflates and the curvature depletes.
„Radiation damping" for cosmic a.

dS/CFT

$AdS_5 \times S_5 = (N{=}4)\,CFT(\lambda)$

$\lambda = g_{YM}^2 N.$

$R_{ADS} \propto \lambda^{1/4}$ (large λ)

Continue $\lambda \Rightarrow e^{2\pi i}\lambda$

$AdS \Rightarrow dS$

$S^5 \Rightarrow L_5$

(Lobachevsky space

with euclid. signature

$dS_5 \times L_5 = NCFT$

Singularities in λ

Planar diagrams
$F \sim \sum c_n \lambda^n$, radius of convergence.
Must be a scaling lim $\lambda \to \lambda_*$ (dense graphs)
describing non-unitary CFT through the σ-model

(dS) sigma model

Positive curvature

— Negative β-funct.

$$m^2 \propto \Lambda^2 e^{-\frac{const}{\alpha_0}}$$

$$\mathcal{L} = \frac{1}{\alpha_0}(\partial n)^2$$

On a lattice

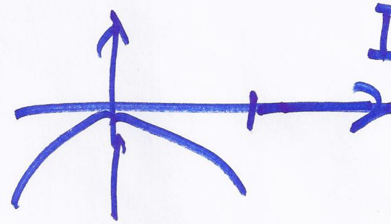
$$\mathcal{H} = -\frac{1}{\alpha}\sum (n_x n_{x+1})$$

For a sphere

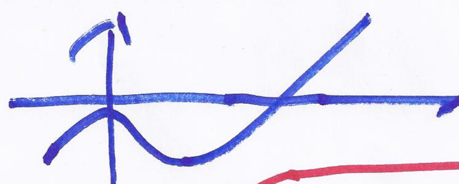
$\alpha \to \infty$
$$E = \frac{1}{\alpha}\sum l_x(l_x+1)$$

We have a mass gap Both for small and large α_0.

Sphere:



For (dS)
No mass gap for strong coupling
$l_x = -1/2 + i\rho_x$



Fixed point