Chern-Simons Contact Terms

Nathan Seiberg

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Outline

Throughout the talk we will be in $3d$.

- Chern-Simons contact terms
- Currents in $\mathcal{N} = 2$ supersymmetry
- Various SUSY Chern-Simons contact terms
- Anomaly: superconformal vs. (compact) $U(1)_R$ symmetry
- The partition function of $\mathcal{N} = 2$ on a three sphere
- F-maximization
- Tests of duality
- Conclusions

Contact Terms

Contact terms are correlation functions at coincident points.

- Some of them are determined; e.g.
  - The seagull term (needed for gauge invariance)
  - The $2d$ conformal anomaly (in a CFT $T^\mu_\mu = 0$, but it must have nonzero contact terms)
- Most of them are arbitrary (not universal).
  - They reflect short distance physics.
  - They depend on the regularization scheme.
  - They are associated with local counterterms constructed out of the dynamical fields and background fields.
  - They change under dynamical and background fields redefinitions (coupling constants redefinitions).

An Important Exception

Consider a three-dimensional field theory with a global (compact) $U(1)$ symmetry.
The conserved current j_μ can be coupled to a classical background $U(1)$ gauge field a_μ.
A contact term in the two-point function

$$\langle j_\mu(x) j_\nu(0) \rangle = \cdots + \frac{i\kappa}{2\pi} \epsilon_{\mu\nu\rho} \partial^\rho \delta^{(3)}(x)$$

can be interpreted as due to a Chern-Simons (CS) counterterm in the background fields

$$\frac{i\kappa}{4\pi} \int \varepsilon^{\mu\nu\rho} a_\mu \partial_\nu a_\rho \ .$$

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- However, since $U(1)$ is compact, and we would like the theory to make sense on arbitrary manifolds with arbitrary $U(1)$ bundles, the freedom in κ is quantized.
- Sometime, a consistent definition of the theory forces us to add such a bare counterterm with fixed value of $\kappa \bmod(1)$; still, the integer part of κ is arbitrary.
- A physical observable: $\kappa \bmod(1)$.

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- The same story can be repeated for the energy-momentum tensor and the Lorentz CS term.

Currents in $\mathcal{N}=2$ Supersymmetry in $3d$

Supersymmetrizing the previous discussion, we distinguish between ordinary global symmetries and an R-symmetry.

A global non-R $U(1)$ symmetry can be coupled to a classical background $U(1)$ gauge superfield: $(a_\mu, \sigma, D, \lambda_\alpha)$.

The supersymmetric CS term is

$$\frac{\kappa}{4\pi}\left(i\epsilon^{\mu\nu\rho}a_\mu\partial_\nu a_\rho - 2\sigma D + \text{fermions}\right) \ .$$

Supersymmetry relates the contact term in two currents to a contact term between two scalar operators.

Again, $\kappa \bmod(1)$ is physical.

R-Symmetry

The $U(1)_R$ current is in the same supermultiplet as the energy-momentum tensor and the supersymmetry current. The classical background fields include the metric $g_{\mu\nu}$, a $U(1)_R$ gauge field A_μ and others.

The relevant CS contact terms are

- Flavor-R: $Ada + \cdots$. It is not superconformal.
- R-R: $AdA + \cdots$. It is not superconformal.
- Lorentz: $\omega d\omega + \cdots$. It is superconformal.

As above, their fractional parts are physical.

The two non-superconformal terms have interesting consequences.

A New Anomaly

Given a superconformal field theory we would like to impose:

- Supersymmetry
- Conformal symmetry
- All flavor and R-symmetries are compact

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The most conservative approach is to cancel the contact terms by adding CS terms with fractional coefficients.
(This is similar to the known framing anomaly [Witten].)

Then, the functional integral is not fully gauge invariant – it can change by a phase.

More precisely, defining the CS terms by extending the fields to $4d$, the results depend on the extension.

Example: $\mathcal{N} = 2$ SQED

This is a $U(1)$ gauge theory with N_f flavors and a CS level k.

Explicit computations uncover nonzero CS contact terms (for background fields) in the IR CFT.

- Flavor-flavor 2-pt function leads to $\kappa^{ff} = \frac{\pi^2 N_f}{4k} + \mathcal{O}(\frac{1}{k^3})$.
- Flavor-R 2-pt function leads to $\kappa^{fR} = -\frac{N_f}{2k} + \mathcal{O}(\frac{1}{k^3})$.
- We expect the R-R and the Lorentz terms to be nonzero.

The flavor-R and the R-R contact terms violate the superconformal symmetry of the IR theory.

We can cancel them by adding appropriate counterterms, violating invariance under large gauge transformations of the background fields.

Placing an $\mathcal{N} = 2$ Theory on S^3

- An $\mathcal{N} = 2$ theory with a $U(1)_R$ symmetry can be placed on S^3, while preserving supersymmetry [D. Sen; Romelsberger; Kapustin, Willett, Yaakov; ...].
- This seems straightforward for superconformal theories, but is nontrivial for nonconformal theories. Here we need to turn on imaginary (supergravity) background fields, which violate unitarity [Festuccia, NS].
- The sphere partition function $Z = e^{-F}$ of a unitary field theory should be real (even if it is not parity invariant).
- The imaginary (non-unitary) background fields are expected to be harmless, if the theory is conformal.

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- The sphere partition function $Z = e^{-F}$ of a unitary field theory should be real (even if it is not parity invariant).
- The imaginary (non-unitary) background fields are expected to be harmless, if the theory is conformal.
- Nevertheless, explicit calculations (localization) lead to complex answers for SCFTs on S^3 [Kapustin, Willett, Yaakov; Jafferis; Hama, Hosomichi, Lee; Jafferis, Yin;...].

The Phase of the S^3 Partition Function

Starting in flat space, we find the four supersymmetric CS contact terms. Two of them are not superconformal.

If we do not add bare counterterms to remove them, the superconformal field theory has nonconformal contact terms.

This conformal violation leads to dependence on the unitarity violating background fields, and through them to complex answers.

The phase of the partition function is thus computable using the flat-space values of the contact terms.

The anomaly discussed above is seen now as a clash between unitarity and full background gauge invariance.

F-maximization

Consider an SCFT with some flavor symmetries.
Explore the sphere partition function $Z = e^{-F}$ as a function of background gauge superfields for these symmetries $(\sigma, a_\mu, \dots)$.
The parameter $t = \mathrm{Im}(\sigma)$ shifts the choice of R-symmetry in the superconformal algebra.

We add local counterterms to restore the superconformal symmetry and unitarity.

- They are incompatible with the full background gauge invariance.
- They set $\mathrm{Im}(F)$ to zero and shift $\mathrm{Re}(F)$.

Then (in terms of the original F),

- Vanishing of the one-point function leads to

$$\partial_t \mathrm{Re} F = 0$$

  (conjectured by [Jafferis]).
- Positivity of the flat-space (separated) two-point functions lead to

$$\partial_t^2 \mathrm{Re} F < 0 \quad .$$

  Hence, F is at a maximum (conjectured by [Jafferis, Klebanov, Pufu, Safdi]).

F-maximization is closely related to the "F-theorem."

Tests of Dualities

There are several conjectured dualities between different $3d$ theories; e.g. $3d$ mirror symmetry, Aharony duality, Giveon-Kutasov duality, etc.

They can be tested by comparing the S^3 or the $S^2 \times S^1$ partition functions of the two dual theories [Kapustin, Willett, Yaakov; Benini, Closset, Cremonesi; ...].

The results almost agree.

- They can be made to agree by adding to one of the sides of the duality CS counterterms for the background fields. (Equivalently, these are corrections to the definition of the global symmetry currents.)
- Following our discussion, these counterterms should have quantized coefficients.
  (Generalization of matching the parity anomaly in dual $3d$ theories [Aharony, Hanany, Intriligator, NS, Strassler].)
- Furthermore, their quantized coefficients can be determined independently.
  They can be calculated at one loop on R^3 by comparing different dual pairs, which are related by renormalization group flows.

Example

One-loop flat-space computations in Giveon-Kutasov duality ($U(N_c)_k$ with N_f flavors) predict the correction term

$$\delta F = \pi\Big[iN_f(N_f-k)m^2 + i\xi^2 + N_f(N_f+k-2N_c)m\Big] + \cdots \ ,$$

where m is a real mass and ξ is an FI-parameter.
Note, the correction is not merely a phase.

With this correction term the partition functions of the two dual theories match!

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The agreement is a nontrivial test of:

- The dualities
- Our entire understanding of these contact terms

Conclusions

- Contact terms are usually arbitrary.
- Chern-Simons contact terms lead to new computable observables.
- The natural way to describe them is in terms of counterterms of background gauge and (super)gravity (super)fields.
- Some Chern-Simons contact terms are not superconformal – like an anomaly.
- In order to preserve supersymmetry on curved space, we should turn on various supergravity background fields.

- The non-conformal Chern-Simons terms lead to a phase of the S^3 partition function (violation of unitarity) even for conformal theories.
- Removing these terms (by tentatively sacrificing full background gauge invariance) we proved the conjectured F-maximization principle.
- This understanding leads to new non-trivial tests of conjectured dualities.