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Higher Spin Theories in AdS and Their Role for Holography

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Intro

The spectrum of string theory includes an infinite tower of massive particles of arbitrarily high spins.

In the high energy/tensionless limit, these higher spin particles look massless.

Are there emergent higher spin gauge symmetries?

The search for a theory of interacting higher spin fields goes back a long way.

Ginzburg and Tamm '47, Fradkin '50, Fronsdal '78, ...

While an effective theory of massive higher spin fields clearly exists, it is difficult to construct theories of interacting massless higher spin fields.

No-Go Theorems

Weinberg: amplitudes of soft massless HS particles forbidden by Lorentz invariance - assuming minimal coupling (also ruled out by **Aragone-Deser '79**).

Weinberg-Witten: no conserved Lorentz invariant and gauge invariant stress-energy tensor carried by massless particles of spin >1 .

Coleman-Mandula: no conserved higher spin charge - assuming nontrivial S-matrix and mass gap.

These no-go theorems could be evaded with non-minimal coupling and non-local interactions of massless higher spin fields.

Gauge invariant cubic vertex of massless higher spin fields in flat space does exist. For spins $s_1 \geq s_2 \geq s_3$, n -derivative interaction, $s_1 + s_2 - s_3 \leq n \leq s_1 + s_2 + s_3$. [Metsaev '05, Manvelyan-Mkrtchyan-Rühl '10, Sagnotti-Taronna '10]

We know from AdS/CFT that higher spin gauge theory in AdS must exist, because there are dual large N free CFTs with conserved higher spin currents.

e.g. $d=4$, $N=4$ SYM in the zero 't Hooft coupling limit

\Rightarrow tensionless limit of type IIB string theory in $AdS_5 \times S^5$ must reduce to a higher spin gauge theory (coupled to infinite towers of massive fields.)

[Ferrara-Fronsdal '98, Haggi-Mani-Sundborg '00, Konstein-Vasiliev-Zaikin '00, Witten '01 talk, Beisert-Bianchi-Morales-Samtleben '04]

Long before AdS/CFT...

Fradkin-Vasiliev '87: cubic vertex for higher spin gauge fields in AdS_4 .

Vasiliev '90-92: constructed full nonlinear equation of motion for higher spin gauge fields in AdS_4 .

Vasiliev's system: classical nonlinear gauge invariant equations describing an infinite tower of higher spin fields in AdS_4 .

Spins $s=0,1,2,3,\dots$ (or only the even ones.)

Frame-like formalism - Lagrangian not known explicitly (not a problem at classical/tree level, but obstacle to quantization.)

The role of Vasiliev's theory in AdS/CFT was clarified by **Sezgin-Sundell** and **Klebanov-Polyakov '02**.

K-P observed that the dual of a pure higher spin gauge theory should be a (large N) vector model,

and that by changing AdS boundary condition the dual CFT can be either a free theory or an interacting critical theory.

The AdS_4 / CFT_3 Story

Vasiliev's system

Spacetime coordinates x^μ , twister variables: $Y=(y^A, \bar{y}^{\dot{A}})$, $Z=(z^A, \bar{z}^{\dot{A}})$
 $(A, \dot{A}=1,2)$, noncommutative $*$ product

$$f(y,z)*g(y,z) = f(y,z) \exp(\epsilon^{AB}(\overleftarrow{\partial}_{y_A} + \overleftarrow{\partial}_{z_A})(\overrightarrow{\partial}_{y_B} - \overrightarrow{\partial}_{z_B})) g(y,z)$$

Master fields: $\Lambda(x|Y,Z) = W_\mu dx^\mu + S_A dz^A + S_{\dot{A}} d\bar{z}^{\dot{A}}$, $B(x|Y,Z)$,
 both contain symmetric traceless tensor fields of all spins along
 with infinitely many auxiliary fields.

Equation of motion: $d\Lambda + \Lambda * \Lambda = f_*(B * K) dz^2 + f_*(B * \bar{K}) d\bar{z}^2$. Implies
 a Bianchi identity of the form $d(B + \Lambda * B - B * \Lambda) = 0$.

K, \bar{K} "Kleinian operators".

Parity and ~~Parity~~

The function $f(X)$ can be put to the form $X \exp(i \theta(X))$ by field redefinition.

$\theta(X) = \theta_0 + \theta_2 X^2 + \theta_4 X^4 + \dots$ e.g. θ_0 controls cubic and higher interaction vertices, θ_2 controls 5-point and higher, etc.

Parity invariance \Rightarrow only two possibilities: $f(X) = X$ (A-type theory) or $f(X) = iX$ (B-type theory)

Parity violating theories much richer.

A higher spin gauge theory in AdS_4

AdS_4 vacuum: $W=W_0(x|Y)$, $S=0$, $B=0$.

W_0 constructed out of the spin connection and vierbein of AdS_4 .

Linearizing Vasiliev's equation around W_0 , one finds an infinite tower of higher spin gauge fields propagating in AdS_4 .

Roughly speaking, \mathbb{B} contains HS Weyl tensors, whereas \mathbb{A} contains the HS symmetric traceless tensor fields themselves.

Nonlinear terms can be taken into account by solving auxiliary fields order by order, yielding equation of motion for physical higher spin gauge fields of the standard form

$$(\square - m^2)\varphi_{\mu_1\mu_2\cdots\mu_s} + \cdots = \mathcal{O}(\varphi^2)$$

Vasiliev's system describes classical interacting higher spin gauge fields in AdS_4 . The interactions are highly constrained (and almost uniquely fixed) by higher spin symmetry, which makes it plausible that the quantum theory is renormalizable and possibly finite, despite the higher derivative (and seemingly nonlocal) interactions. If Vasiliev's theory is a consistent quantum theory of gravity, then it should have a three-dimensional CFT dual.

HS/VM Duality

Conjecture (Klebanov-Polyakov, Sezgin-Sundell '02):
Vasiliev's minimal bosonic theory in AdS_4 is holographically dual to the free or critical $O(N)$ vector model.

AdS boundary condition on bulk scalar field

	A-type	B-type
$\Delta=1$	free $O(N)$ boson	critical $O(N)$ fermion (Gross-Neveu)
$\Delta=2$	critical $O(N)$ boson (Wilson-Fisher)	free $O(N)$ fermion

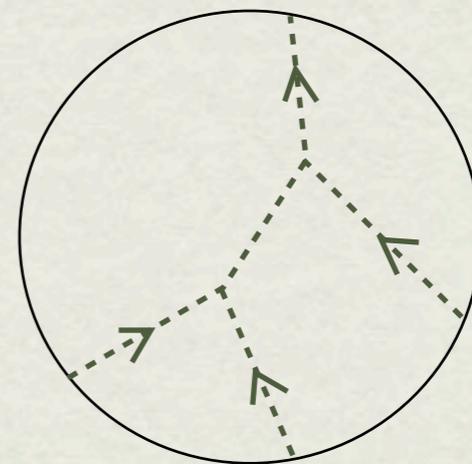
$O(N)$ Vector Model

Free $O(N)$ vector model: N free massless scalar fields ϕ_i ($i=1,\dots,N$) in 3d, restrict to the $O(N)$ singlet sector. This CFT has conserved currents of the form $J_{\mu_1\dots\mu_s} = \phi_i \partial_{(\mu_1\dots\mu_s)} \phi_i + \dots$ for each even integer s .

The critical $O(N)$ model can be described as the IR (Wilson-Fisher) fixed point of a double trace deformation of the free CFT. Restriction to $O(N)$ singlet sector important for large N factorization. Higher spin symmetry broken by $1/N$ effects.

Nontrivial test: tree level 3-point function of HS currents in Vasiliev theory matched to free/critical $O(N)$ vector model [Giombi-XY '09,'10]

Tree level n -point correlators: treat $n-1$ boundary currents as sources, solve bulk equation of the schematic form $D\varphi = \varphi * \varphi$ to $(n-1)$ -th order and extract n -point function from boundary value of $\varphi^{(n-1)}$.



What does it take to prove HS/VM duality?

While at least tree level correlators can in principle be computed from Vasiliev's equations, in practice this appears to be quite hard. 3-point function done in parity invariant case. Parity violating case already subtle [work in progress]. Explicit computation of 4-point function and loop corrections (should be absent for $\Delta=1$ b.c.) in Vasiliev theory yet to be performed (gauge fixing, ghosts?).

Maldacena-Zhiboedov '11,'12:

exactly conserved higher spin current \Rightarrow CFT is free.

"Approximate" HS symmetry \Rightarrow three-point functions constrained.

Girardello-Porrati-Zaffaroni '02, Hartman-Rastelli '06, Giombi-XY '10: In A-type Vasiliev theory, higher spin symmetry broken by $\Delta=2$ boundary condition. Duality with free $O(N)$ theory for $\Delta=1$ boundary condition implies the duality with critical $O(N)$ theory for $\Delta=2$ boundary condition, to all order in perturbation theory ($1/N$).

What remains to be shown: Vasiliev's system can be quantized in a manner in which higher spin symmetry is not anomalous nor broken by boundary conditions.

Alternative ideas of deriving the bulk theory
from boundary:

collective field theory [de Mello Koch-Jevicki-Jin-Rodrigues '10]

holographic RG [Douglas-Mazzucato-Razamat '10]

Generalizations

Gaiotto-XY '07: Chern-Simons-matter theories provide a large class of 3d CFTs \Leftrightarrow various supersymmetric or non-supersymmetric string theories in AdS_4 .

In some examples, there is a semi-classical gravity limit (**Aharony-Bergman-Jafferis-Maldacena '08** + many more). In many other examples, the dual must always involve higher spin fields (**Minwalla-Narayan-Sharma-Umesh-XY '11**).

Giombi-Minwalla-Prakash-Trivedi-Wadia-XY, Aharony-Gur-Ari-Yacoby '11: Chern-Simons vector models have approximately conserved higher spin currents at large N .

Conjecture: Chern-Simons vector models are dual to ~~parity~~ Vasiliev theories in AdS_4 .

Chern-Simons vector model

U(N) or SU(N) Chern-Simons theories coupled to massless scalars (AGY '11) or massless fermions (GMPTWY '11) in the fundamental representation are exactly conformal vector models.

e.g. CS-fermion vector model

$$S = \frac{k}{4\pi} \int (A \wedge dA + \frac{2}{3} A \wedge A \wedge A) + \int \bar{\psi} \gamma^\mu D_\mu \psi$$

't Hooft limit: N large, $\lambda = N/k$ finite.

~~Higher spin symmetry~~

Spin- s operator $J^{(s)}_{\mu_1 \dots \mu_s}$ made out of bilinears of fundamental matter fields, conserved at infinite N . Current conservation broken by $1/N$ effects, through mixing with double-trace and triple-trace operators:

$$\partial^\mu J^{(s)}_{\mu \dots} = f(\lambda) \sum \partial^{n_1} J^{(s_1)} \partial^{n_2} J^{(s_2)} + g(\lambda) \sum \partial^{n_1} J^{(s_1)} \partial^{n_2} J^{(s_2)} \partial^{n_3} J^{(s_3)}$$

The higher spin dual to $\mathcal{N}=6$ ABJ vector model, and more

n -extended supersymmetric ~~parity~~ Vasiliev theory: introduce Grassmannian auxiliary variables ψ_1, \dots, ψ_n in the master fields. They obey Clifford algebra $\{\psi_i, \psi_j\} = \delta_{ij}$. Equation of motion modified to

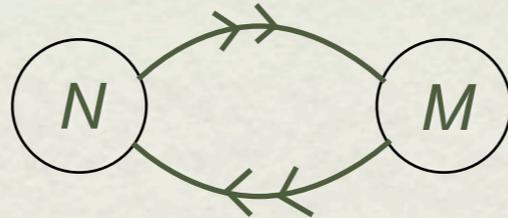
$$dA + A * A = f_*(B * K) dz^2 + f_*(B * \bar{K} \Gamma) d\bar{z}^2.$$

The $\mathcal{N}=0,1,2,3,4,6$ CS vector models differ merely by double trace and triple trace deformations, and gauging a flavor symmetry with Chern-Simons coupling. These correspond to, in the holographic dual, simply changes of boundary condition on the bulk fields.

[Chang-Minwalla-Sharma-XY '12]

Not a tautology: the duality makes nontrivial predictions on two and three point function coefficients that do not follow from known symmetries.

A Triality



$U(N)_k \times U(M)_{-k}$ ABJ theory

$$M \ll N, \theta_0 = \pi\lambda/2$$

$$R_{AdS}/\ell_{string} = \lambda^{1/4}$$

$$\int_{CP^1} B = (N-M)/k$$

$n=6$ supersymmetric ~~parity~~
 Vasiliev theory with $U(M)$
 Chan-Paton factor and $\mathcal{N}=6$
 boundary condition

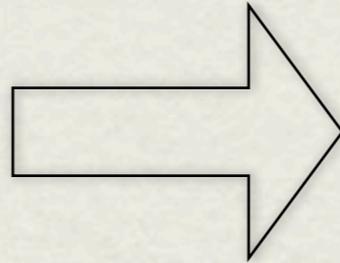
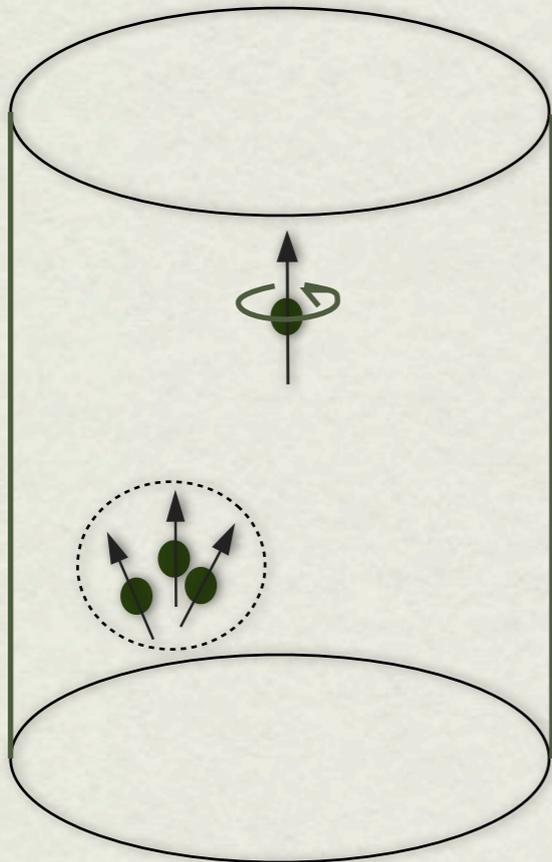
strong **bulk** 't Hooft coupling
 $\lambda_{BULK} = M/N$

IIA string theory
 in $AdS_4 \times CP^3$

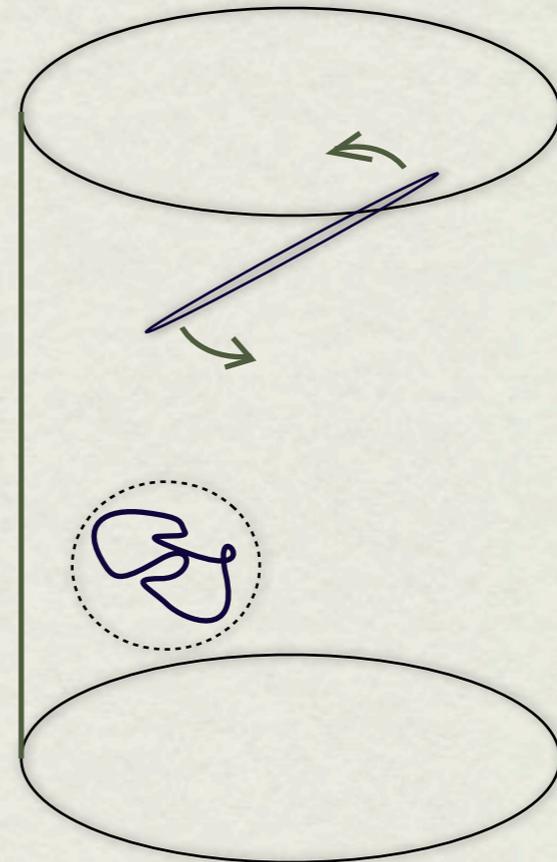
bound states of
 higher spin particles

strings

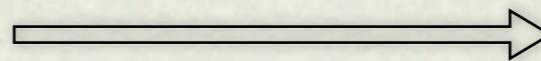
Vasiliev theory as a string theory



strong **bulk** 't Hooft coupling
 $\lambda_{\text{BULK}} = M/N$



$n=6$ supersymmetric ~~parity~~
 Vasiliev theory with $U(M)$
 Chan-Paton factor and $\mathcal{N} = 6$
 boundary condition



IIA string theory
 in $AdS_4 \times CP^3$

The AdS_3 / CFT_2 Story

AdS_3 is simple

Gravity in 3d has no propagating degrees of freedom. Almost a trivial theory. (Or is it?)

Have much better handle on dual 2d CFT.
(Or so we thought.)

AdS₃ is not that simple

AdS₃ gravity admits BTZ black holes. Microstates from quantizing phase space? [Maloney, unpublished]

Pure gravity: is there a CFT dual? Don't know.

- lacking knowledge of irrational CFTs, non-perturbative effects in gravity [Witten, XY, Maloney-Witten, Gaiotto, Gaberdiel '07]

String theory on AdS₃, sometimes exactly solvable (purely NS background), but dual CFT₂ hard (little is known about non-BPS physics away from orbifold point).

Gravity dual to rational CFTs?

We know a lot about rational CFTs in 2d. Do they have sensible gravity duals?

Black holes, thermalization, ...? Possible if there is a large N limit - need not just large central charge, but large N factorization of correlators.

Higher spin AdS_3/CFT_2 duality

Gaberdiel-Gopakumar '11: The 2d version of vector model is the W_N minimal model.

They further conjectured its holographic dual to be the AdS_3 Vasiliev system (an ∞ tower of HS fields coupled to massive scalars).

Higher spin gauge theories in AdS_3

Pure higher spin gauge theory in AdS_3 can be constructed as $SL(N,R) \times SL(N,R)$ Chern-Simons theory, with appropriate boundary condition [Henneaux-Rey, Campoleoni-Fredenhagen-Pfenninger-Theisen '10, Gaberdiel-Hartman '11].
Spin $s=2, \dots, N$.

Unclear whether the pure HSGT in AdS_3 is by itself well defined non-perturbatively. (Don't know answer even in the $N=2$ case, i.e. pure Einstein gravity in AdS_3).

Vasiliev's system in AdS_3 , on the other hand, is not a pure higher spin gauge theory (unlike the AdS_4 theory). It contains an infinite tower of HS fields $s=2, 3, \dots, \infty$, and two massive scalar fields.

HS interaction controlled by $hs[\lambda]$ algebra. The parameter λ also determines the scalar mass.

W_N minimal model

W_N algebra generated by conserved (holomorphic and anti-holomorphic) currents of conformal weights $(s,0)$ and $(0,s)$, $s=2,3,\dots,N$.

W_N minimal model can be realized as the coset model $(SU(N)_k \times SU(N)_1) / SU(N)_{k+1}$.

Primaries w.r.t. W_N algebra are labeled by a pair of representations of (affine) $SU(N)$, denoted $(R;R')$.

“Basic” primaries $\phi_1 = (\square; 0)$, $\tilde{\phi}_1 = (0; \square)$ generate all primaries via OPE.

't Hooft limit: $N \rightarrow \infty$, $\lambda \equiv N/(k+N)$ finite ($0 < \lambda < 1$).

For representations R and R' that do not grow with N , the primary $(R;R')$ has finite dimension in the 't Hooft limit.

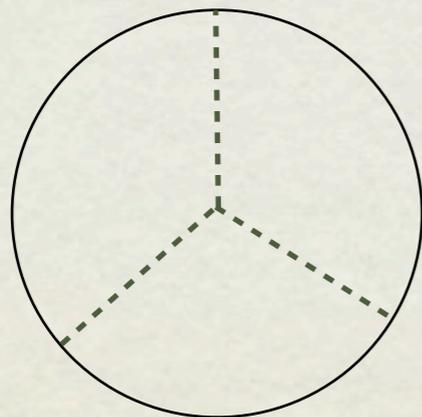
Large N factorization of W_N minimal model

[Chang-XY '11] In the large N limit, the primaries ($R; R'$) are organized into “single-trace” and “multi-trace” operators.

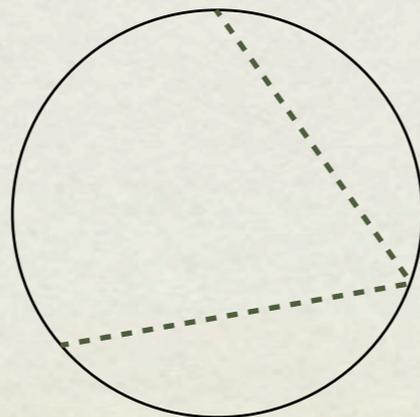
For instance, correlators of a single-trace operator \mathcal{O}_1 and a double-trace operator \mathcal{O}_2 have different scaling with N :

If we normalize the operators by $\langle \mathcal{O} \mathcal{O} \rangle \sim 1$, then in a large N vector model,

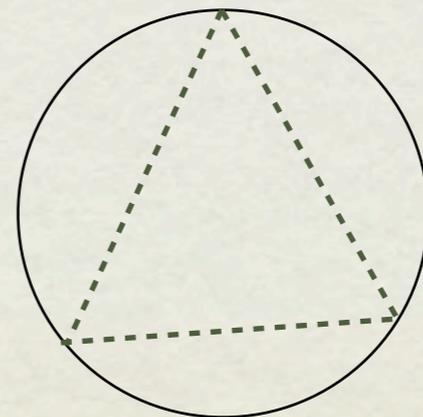
$$\langle \mathcal{O}_1 \mathcal{O}_1 \mathcal{O}_1 \rangle \sim N^{-1/2}$$



$$\langle \mathcal{O}_1 \mathcal{O}_1 \mathcal{O}_2 \rangle \sim 1$$



$$\langle \mathcal{O}_2 \mathcal{O}_2 \mathcal{O}_2 \rangle \sim 1$$



“Single-trace” vs “multi-trace” in W_N minimal model

Single-trace operators

$$\phi_1 = (\square ; 0)$$

$$\phi_2 = (\square\square ; \square) - (\square ; \square)$$

$$\phi_3 = \sqrt{2} (\square\square\square ; \square\square) - (\square\square ; \square\square) - (\square\square ; \square) + \sqrt{2} (\square ; \square)$$

.....

$$\tilde{\phi}_1 = (0 ; \square), \dots$$

$$\omega_1 = (\square ; \square)$$

$$\omega_2 = (\square\square ; \square\square) - (\square ; \square)$$

$$\omega_3 = (\square\square\square ; \square\square\square) - (\square\square ; \square\square) + (\square ; \square)$$

.....

Multi-trace operators

$$(\square ; 0) \sim \phi_1^2$$

$$(\square\square ; 0) \sim \phi_1 \partial \bar{\partial} \phi_1 - \partial \phi_1 \bar{\partial} \phi_1$$

$$(\text{adj} ; 0) \sim \phi_1 \bar{\phi}_1$$

$$(\square\square ; \square\square) + (\square ; \square) \sim \omega_1^2$$

$$(\square\square ; \square) + (\square ; \square) \sim \phi_1 \omega_1$$

$$(\square\square ; \square) + (\square ; \square\square) \sim \omega_1 \partial \bar{\partial} \omega_1 - \partial \omega_1 \bar{\partial} \omega_1$$

$$\partial \bar{\partial} \omega_1 \sim \frac{\lambda^2}{N} \phi_1 \tilde{\phi}_1$$

$$\partial \bar{\partial} \omega_2 \sim \sqrt{2} \frac{\lambda^2}{N} (\phi_1 \tilde{\phi}_2 + \phi_2 \tilde{\phi}_1)$$

$$\partial \bar{\partial} \omega_3 \sim \sqrt{3} \frac{\lambda^2}{N} (\phi_1 \tilde{\phi}_3 + \phi_2 \tilde{\phi}_2 + \phi_3 \tilde{\phi}_1)$$

.....

Light states

A primary operator of the form $(R; R)$ has scaling dimension $\Delta = \lambda^2 B(R)/N$ at large N , where $B(R) = \#$ boxes in the Young tableaux of R .

\Rightarrow near continuum of low lying states at large N

- does large N factorization really hold?

- YES! correlators do obey large N factorization, provided that one makes the identifications [Raju-Papadodimas, Chang-XY, '11]

$$N \partial \bar{\partial} \omega_1 \sim \lambda^2 \phi_1 \tilde{\phi}_1$$

$$N \partial \bar{\partial} \omega_2 \sim \sqrt{2} \lambda^2 (\phi_1 \tilde{\phi}_2 + \phi_2 \tilde{\phi}_1)$$

$$N \partial \bar{\partial} \omega_3 \sim \sqrt{3} \lambda^2 (\phi_1 \tilde{\phi}_3 + \phi_2 \tilde{\phi}_2 + \phi_3 \tilde{\phi}_1)$$

.....

$\partial \omega$ becomes null in the large N limit, and $j = \sqrt{N} \partial \omega$ effectively becomes a new primary operator.

Light states \Rightarrow a lot of hidden symmetries at large N !

It appears that the holographic dual to the large N W_N minimal model should be Vasiliev theory in AdS_3 extended by an ∞ tower of massive matter fields and an ∞ dimensional Chern-Simons gauge group broken by boundary conditions. [Chang-XY, work in progress]

A perturbative duality

Vasiliev theory in AdS_3 contains gauge fields of spin $s=2,3,\dots,\infty$, and a single complex massive scalar field φ . φ should be dual to one of the basic primaries of W_N minimal model, namely $(\square; 0)$ or $(0; \square)$ depending on the boundary condition.

A “modest” version of Gaberdiel–Gopakumar conjecture [Chang-XY '11]: Vasiliev theory in AdS_3 is dual perturbatively to the subsector of W_N minimal model generated by primaries of the form $(R; 0)$ (or the subsector generated by $(0; R)$, with the alternative boundary condition on φ .)

This sector closes perturbatively in $1/N$ on the sphere/plane, but is not modular invariant and non-perturbatively incomplete.

The non-perturbative dual of W_N minimal model ?

...remains illusive.

How to couple the infinite tower of elementary matter fields (dual to what we identified as single trace primaries) to the higher spin gauge fields?

Does the BTZ black hole dominate the thermodynamics at high temperature? On the CFT side, exact torus two-point function known [Chang-XY '11] but large N behavior not yet understood. Note: despite rational CFT, thermalization behavior possible at large N (though Poincaré recurrence time $\sim N^\#$ rather than e^N).

An alternative, non-perturbative but non-unitary duality?

Unitary W_N minimal model requires $0 < \lambda < 1$. But, analytic continue to $\lambda = N$, $hs[M] = SL(N)$. **Now keep N finite but c large.** [Gaberdiel-Gopakumar '12]

No more “light” states. ϕ_1 remains the single complex massive scalar of Vasiliev’s system (highly non-minimal coupling to gravity!), whereas $\tilde{\phi}_1$ becomes a non-perturbative state - identified with “conical surplus” [Castro-Gopakumar-Gutperle-Raeymaekers '11]. Both negative energy, however.

It could be that Vasiliev’s system by itself at $\lambda = N$ is a non-perturbatively complete theory, in the sense that it is dual to a modular-invariant but non-unitary CFT_2 .

Black Holes

in higher spin gravity

AdS₃ black holes with higher spin hair

In higher spin gravity based on $SL(N) \times SL(N)$ Chern-Simons: alternative embeddings of $SL(2) \subset SL(N)$ give black hole solutions that carry higher spin charge. [Gutperle-Kraus, Castro-Hijano-Lepage-Jutier-Maloney]

But “metric horizon” not a gauge invariant notion in higher spin gravity. horizon \Leftrightarrow wormhole? [Ammon-Gutperle-Kraus-Perlmutter]

What is a HS black hole? Horizon characterized by $SL(N)$ -holonomy (in Euclidean continuation).

Black hole entropy consistent with HS generalization of Cardy formula. [Kraus-Perlmutter, Gaberdiel-Hartman-Jin]

Black holes in AdS_4 higher spin gravity

[Didenko-Vasiliev](#) solution: a one-parameter family of extremal black holes in A-type Vasiliev theory with $\Delta=1$ boundary condition.

Generalization by [Iazeolla-Sundell](#) that (presumably) allows for more general higher spin charges.

But are these really black holes? Metric looks like AdS-Schwarzschild in some gauge. Need gauge invariant characterization.

From the dual CFT on S^2 , singlet constraint \Rightarrow no black hole at $\mathcal{O}(N^0)$ temperature. [[Shenker-XY](#)]

More intriguing features

Chern-Simons vector model on $g \geq 1$ Riemann surface
 \Rightarrow large density of states [Banerjee-Hellerman-Maltz-Shenker]
dual to wrapped topological strings?

Reminiscent of light states in W_N minimal model
(twisted states from non-contractible cycles) [Gaberdiel-Suchanek].

Higher spin black holes with nontrivial horizon topology?

higher spin

$dS / CFT?$

A concrete realization of dS/CFT

Vasiliev's higher spin gravity in dS_4 is conjectured to be dual to ghost-like $Sp(N)$ model [Anninos-Hartman-Strominger, Ng-Strominger]. See also [Ouyang, Das-Das-Jevicki-Ye].

Can compute exact wave function of the universe in this model as partition function of dual CFT [Anninos-Denef-Harlow].

Can this toy model help resolving puzzles with de Sitter holography and quantum cosmology?

Many open problems

I highlight two of them

Derive (supersymmetric) Vasiliev system from open string field theory on D6-branes wrapping $\text{AdS}_4 \times \text{RP}^3 \subset \text{AdS}_4 \times \text{CP}^3$ (Jafferis-Gaiotto setup) in small radius/tensionless limit. Lessons for (a useful formulation of) closed string field theory?

Thermalization behavior of higher spin black holes? Can they be used to understand black holes in semi-classical gravity regime via the gauge/HS/string triality?

A general lesson

It is useful to explore limits/families of theories that exhibit hidden symmetries.

e.g. supersymmetry, conformal symmetry.

And higher spin symmetry.