Deconfinement Transition
As Black Hole Formation
By The Condensation Of
QCD Strings

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In gauge/gravity duality the deconfinement transition of a gauge theory is dual to the formation of a Black Hole in the gravity bulk [Witten -1998]

We want to describe an intuitive way of understanding this Duality without referring to a sophisticated duality dictionary. Our initial motivation was to study a simple Matrix Model for a Black Hole by looking at the deconfinement transition of 4d $\mathcal{N} = 4$ SYM on an $S^3$ and the Hawking-Page Transition of the Black hole in the corresponding AdS bulk [Hawking,Page -1983]

Such a black can be modeled as a long and winding string [Susskind,Teitelboim - 1993; Horowitz,Polchinski - 1997]. Since we do not assume the dual gravity description, our argument is applicable to a generic Gauge theories. We do this by paying attention to the behavior of the stringy degrees of freedom of a gauge theory (the Wilson Lines) as the gauge theory undergoes a deconfinement transition. This was achieved through a Monte-Carlo Lattice gauge theory simulation of the transition.
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As concrete example consider \((D + 1)\) pure \(U(N)\) YM Theory on a discrete lattice

\[
H = K + V \quad K = \frac{\lambda N}{2} \sum_{\vec{x}} \sum_{\mu} \sum_{\alpha = 1}^{N^2} \left( E_{\mu,\vec{x}}^{\alpha} \right)^2
\]

\[
V = \frac{N}{\lambda} \sum_{\vec{x}} \sum_{\mu < \nu} \left( N - \text{Tr}(U_{\mu,\vec{x}} U_{\nu,\vec{x}+\hat{\mu}} U_{\nu,\vec{x}+\hat{\mu}}^\dagger U_{\mu,\vec{x}+\hat{\nu}}^\dagger) \right). \]

\[
[E_{\mu,\vec{x}}, U_{\nu,\vec{y}}] = \delta_{\mu \nu} \delta_{\vec{x} \vec{y}} \cdot \tau^\alpha U_{\nu,\vec{y}},
\]

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[E_{\mu,\vec{x}}, E_{\nu,\vec{y}}] = [U_{\mu,\vec{x}}, U_{\nu,\vec{y}}] = [U_{\mu,\vec{x}}, U_{\nu,\vec{y}}^\dagger] = 0. \quad E_{\mu,\vec{x}}^{\alpha} |0\rangle
\]

\[
W_{C_1} W_{C_2} \cdots W_{C_k} |0\rangle \quad W_C = \text{Tr} \left( U_{\mu,\vec{x}} U_{\nu,\vec{x}+\hat{\mu}} \cdots U_{\rho,\vec{x}+\hat{\rho}} \right)
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E = K = \frac{\lambda}{2} L_{\text{total}}(T), \quad S = L_{\text{total}} \log(2D - 1),
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F = L_{\text{total}}(T) \left( \frac{\lambda}{2} - T \log(2D - 1) \right), \quad T_c = \frac{\lambda}{(2 \log(2D - 1))}.
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Strictly speaking D+1 YM is dual to a D-dimensional black brane rather then a black hole as the string condensation fills the whole D-dimensional space.

In order to describe a black hole 0-brane let us consider two lattice models.

First the dimensionally reduced D-matrix model. This is the Eguchi-Kawai model with continuous time direction. At strong coupling the $U(1)^D$ center symmetry is not broken, then this theory is then known to be equivalent to the D+1 dim. YM at large N. In the sense that translationally invariant observables are reproduced from the former at leading order.

At weak coupling this model is equivalent to the bosonic part of the BFSS matrix model of M-theory, which is dual to black 0-branes in type IIA supergravity. In the 't Hooft large N limit.

For $D \leq 2$ this theory exhibits a deconfinement transition, characterized by the non-vanishing expectation value of the absolute value of the Polyakov loop. The energy and entropy are of order $N^2$ and a typical state contains a long winding string such as $\text{Tr}(U_1 U_2 U_1^\dagger U_1^\dagger U_2^\dagger \ldots)$.
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The second Model is the tetrahedron Lattice, here the entropy and temperature scale as

\[ S = L_{total} \log 2 \] and \[ T_c = \lambda/(2 \log 2) \] This system also possesses a deconfinement transition with a long string described as

\[ Tr(U_{12} U_{23} U_{31} U_{14} U_{42} \ldots) \]
\[ S_{\text{lattice}} = -\frac{N}{2a\lambda} \sum_{\mu, t} \text{Tr} \left(V_t U_{\mu, t} + a V_{\mu, t}^{\dagger} U_{\mu, t} + c.c.\right) + \frac{aN}{\lambda} \sum_{\mu \neq \nu, t} \left(N - \text{Tr}(U_{\mu, t} U_{\nu, t} U_{\mu, t}^{\dagger} U_{\nu, t}^{\dagger})\right) \]

\[ S_{\text{tet}} = -\frac{N}{2a\lambda} \sum_{t} \sum_{m<n} \left(\text{Tr}(V_{m, t} U_{mn, t} + a V_{n, t}^{\dagger} U_{nm, t}) + c.c.\right) - \frac{aN}{\lambda} \sum_{t} \sum_{l<m<n} \left(\left(N - \text{Tr}(U_{lm, t} U_{mn, t} U_{nl, t})\right) + c.c.\right). \]

\[ P_{\text{tet}} = \frac{1}{4N} \sum_{m=1}^{4} \text{Tr}(V_{m, t=a} V_{m, t=2a} \cdots V_{m, t=n_t a}) \]

\[ P = \frac{1}{N} \text{Tr}(V_{t=a} V_{t=2a} \cdots V_{t=n_t a}). \]

We use the absolute value of \( P \) in order to eliminate the \( U(1) \)
\[
S_{\text{lattice}} = -\frac{N}{2\alpha \lambda} \sum_{\mu, t} \text{Tr} \left( V_{t} U_{\mu, t} a V_{\mu, t}^{\dagger} U_{\mu, t} + \text{c.c.} \right) + aN \lambda \sum_{\mu \neq \nu, t} \left( N - \text{Tr}(U_{\mu, t} U_{\nu, t} U_{\mu, t}^{\dagger} U_{\nu, t}^{\dagger}) \right)
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The expectation value of the tetrahedron with $N = 64 \ n = 12$.

There is strong hysteresis about the theoretically predicted critical temperature $\langle T_c/\lambda \rangle = 1/(2 \log 2) \approx 0.721$.

The range for hysteresis of the EK model with $N = 64 \ n = 12$ for various dimensions.

The dashed curve is the critical temperature $\langle T_c/\lambda \rangle = 1/(2 \log(2D - 1))$. 
The End


