Physical Mathematics and the Future

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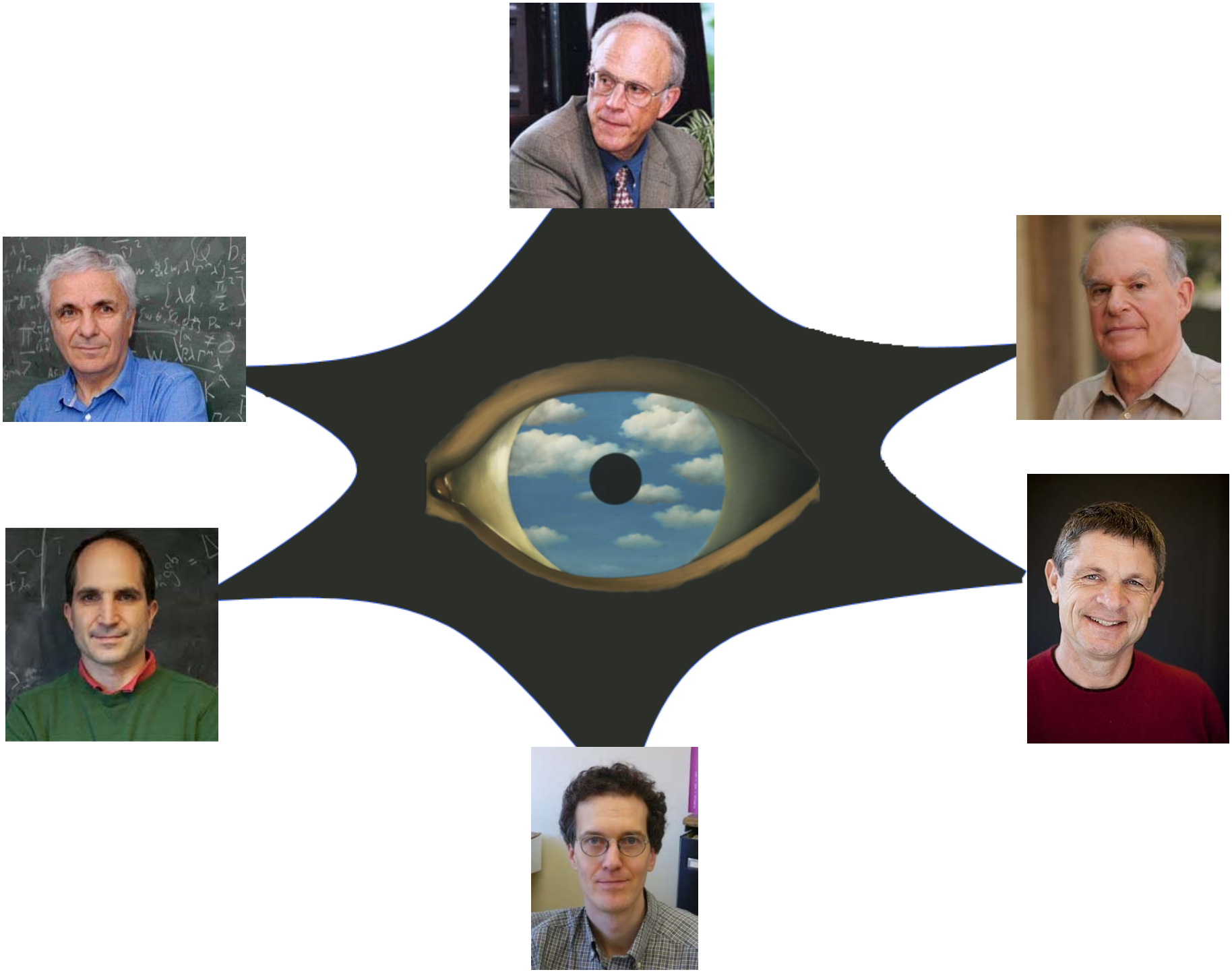
written version available at
http://www.physics.rutgers.edu/~gmoore/



Hilbert's Problem Set

Mathematische Probleme.

Vortrag, gehalten auf dem internationalen Mathematiker-Kongreß zu Paris 1900.

Von

D. Hilbert.

Wer von uns würde nicht gern den Schleier lüften, unter dem die Zukunft verborgen liegt, um einen Blick zu werfen auf die

Who of us would not be glad to lift the veil behind which the future lies hidden; to cast a glance at the next advances of our science and at the secrets of its development during future centuries? What particular goals will there be toward which the leading mathematical spirits of coming generations will strive? What new methods and new facts in the wide and rich field of mathematical thought will the new centuries disclose?

1900: Hilbert's 6th Problem

He stated his famous 23 problems for the 20th century, on August 8, 1900

To treat […] by means of axioms, those physical sciences in which mathematics plays an important part […]

October 7, 1900: Planck's formula, introducing h.

Prerequisite: 750:502 Quantum Mechanics, or equivalent. Lorentz group; relativistic wave-equations; second quantization; global and local symmetries; QED and gauge invariance; spontaneous symmetry breaking; nonabelian gauge theories; Standard Model; Feynman diagrams; cross sections, decay rates; renormalization group.

If then, Herr Doktor Hilbert, who possessed such deep mathematical insight, could not read the simplest aspects of the quantum relativistic future in its profounder and more subtle meanings, how may unlettered Ishmael hope to read the awful Chaldee of String Theory's future?

I will do what little I can.

One thing I can do is look to the past. Let us do so briefly so that I can explain the term ``Physical Mathematics''

Kepler

Snapshots from the

Great Debate

over

the relation between

Galileo

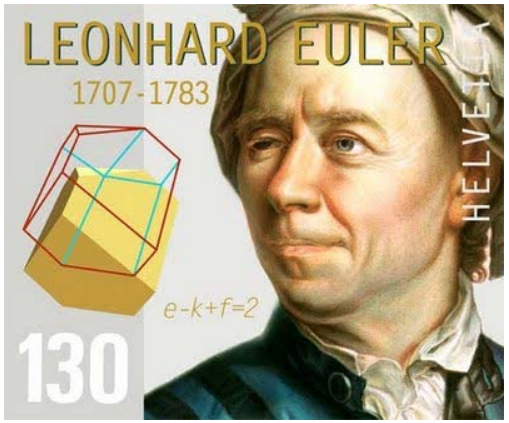
Mathematics and Physics

Newton

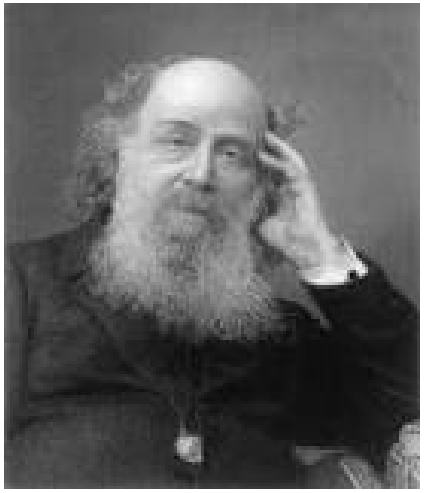
Leibniz

When did Natural Philosophers become either physicists or mathematicians?

Even around the turn of the 19th century …



But, we can read in volume 2 of the journal Nature ….



1869: Sylvester's Challenge

A pure mathematician speaks:

of physical philosophy ; the one here in print," says Professor Sylvester, "is an attempted faint adumbration of the nature of mathematical science in the abstract. What is wanting (like a fourth sphere resting on three others in contact) to build up the ideal pyramid is a discourse on the relation of the two branches (mathematics and physics) to, and their action and reaction upon, one another—a magnificent theme, with which it is to be hoped that some future president of Section A will crown the edifice, and make the tetralogy (symbolisable by A + A′, A, A′, AA′) complete."

1870: Maxwell's Answer

An undoubted physicist responds,

SECTIONAL PROCEEDINGS

SECTION A.—*Mathematical and Physical Science.*—President, Prof. J. Clerk Maxwell, F.R.S.

The president delivered the following address :—

Maxwell recommends his somewhat-neglected dynamical theory of the electromagnetic field to the mathematical community:

phenomena must be studied in order to be appreciated. Another theory of electricity which I prefer denies action at a distance and attributes electric action to tensions and pressures in an all-pervading medium, these stresses being the same in kind with those familiar to engineers, and the medium being identical with that in which light is supposed to be propagated."

1897: The First ICM

A. Vorträge der ersten Hauptversammlung.

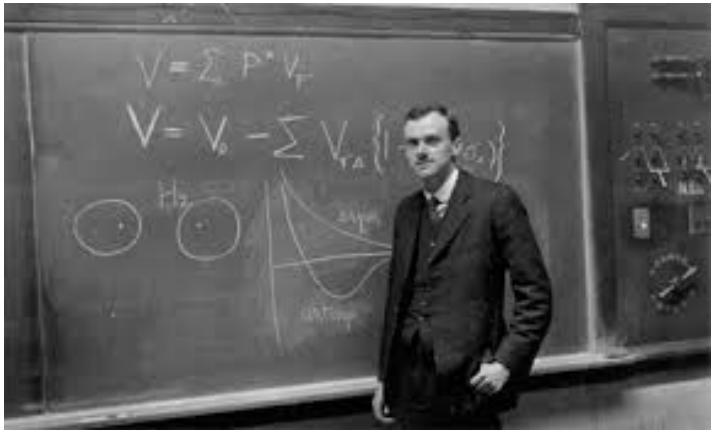
Sur les rapports de l'analyse pure et de la physique mathématique.

Par

H. Poincaré à Paris.

C'est que Maxwell était profondément imprégné du sentiment de la symétrie mathématique; en aurait-il été de même, si d'autres n'avaient

Maxwell en un mot n'était peut-être pas un habile analyste, mais cette habileté n'aurait été pour lui qu'un bagage inutile et gênant. Au contraire il avait au plus haut degré le sens intime des analogies mathématiques. C'est pour cela qu'il a fait de bonne physique mathématique.



1931: Dirac's Paper on Monopoles

Quantised Singularities in the Electromagnetic Field

P.A.M. Dirac

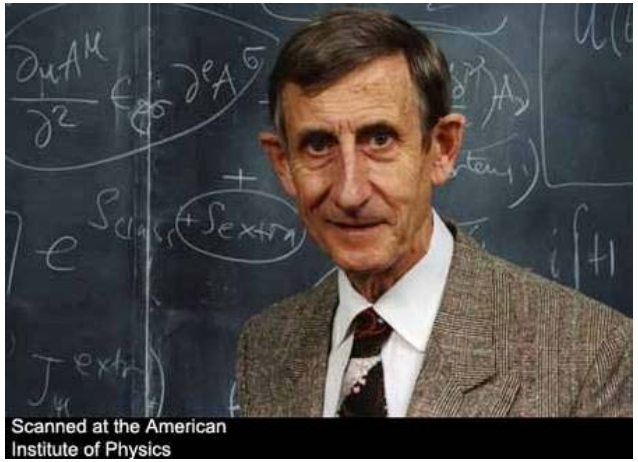
Received May 29, 1931

§ 1. *Introduction*

The steady progress of physics requires for its theoretical formulation a mathematics that gets continually more advanced. This is only natural and to be expected. What, however, was not expected by the scientific workers

for the description of general facts of the physical world. It seems likely that this process of increasing abstraction will continue in the future and that advance in physics is to be associated with a continual modification and generalisation of the axioms at the base of the mathematics rather than with a

1972: Dyson's Announcement



MISSED OPPORTUNITIES[1]

BY FREEMAN J. DYSON

It is important for him who wants to discover not to confine himself to one chapter of science, but to keep in touch with various others.
JACQUES HADAMARD

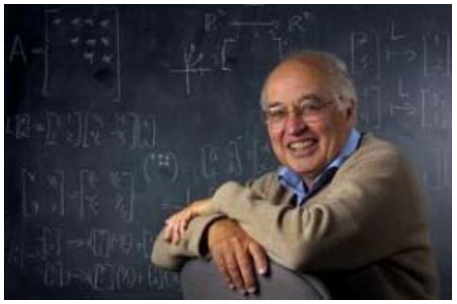
1. **Introduction.** The purpose of the Gibbs lectures is officially defined as "to enable the public and the academic community to become aware of the contribution that mathematics is making to present-day thinking and to modern civilization." This puts me in a difficult position. I happen to be a physicist who started life as a mathematician. As a working physicist, I am acutely aware of the fact that the marriage between mathematics and physics, which was so enormously fruitful in past centuries, has recently ended in divorce. Discussing this divorce, the

Well, I am happy to report that Mathematics and Physics have remarried!

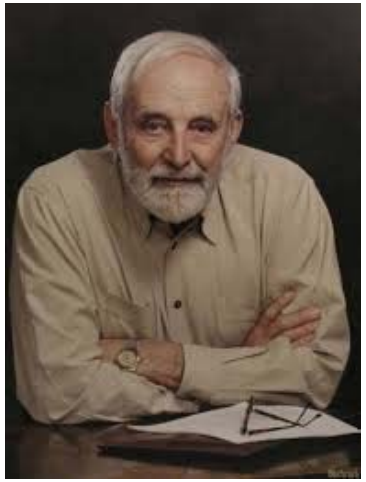
But, the relationship has altered somewhat…

A sea change began in the 1970's …..

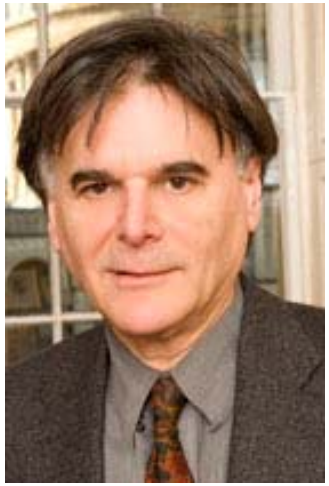
A number of great mathematicians got interested in the physics of gauge theory and string theory, among them,



M. Atiyah



R. Bott



G. Segal

I. Singer

+ . . .

And at the same time a number of great physicists started producing results requiring increasing mathematical sophistication, among them

S. Coleman

B. Zumino

+ . . .

many of you in the audience…

We know who you are!

Physical Mathematics

With a great boost from string theory, after 40 years of intellectual ferment a new field has emerged with its own distinctive character, its own aims and values, its own standards of proof.

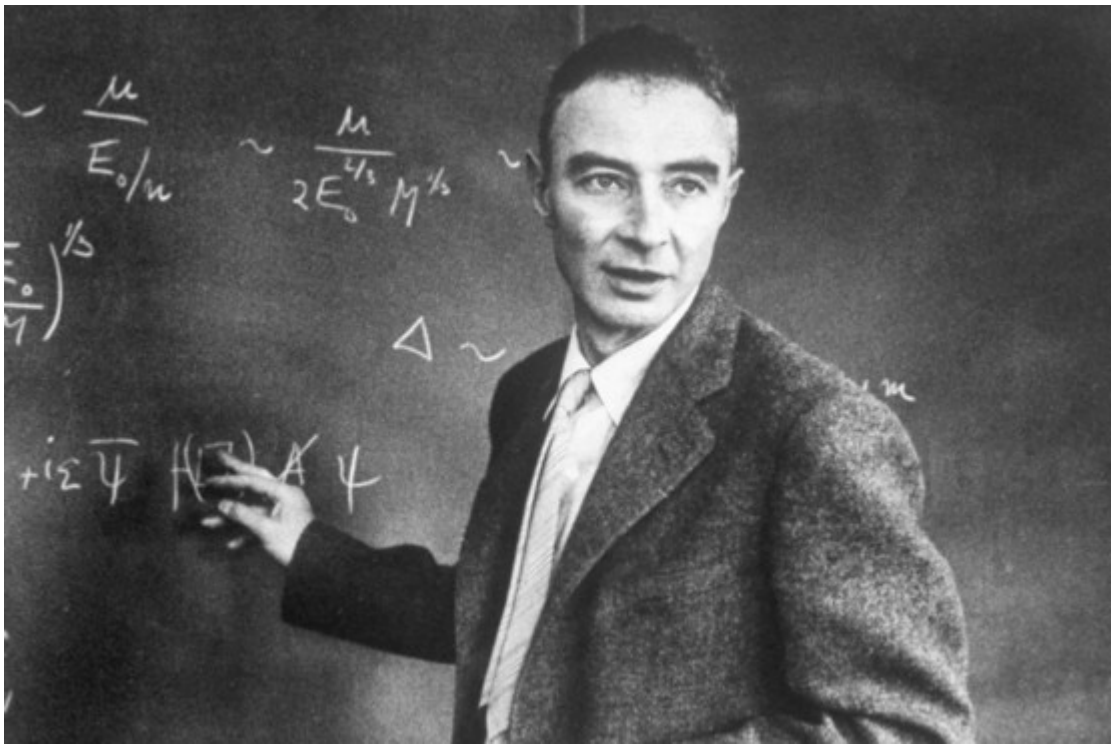
One of the guiding principles is certainly Hilbert's 6th Problem (generously interpreted): Discover the ultimate foundations of physics.

As predicted by Dirac, this quest has led to ever more sophisticated mathematics…

But getting there is more than half the fun: If a physical insight leads to an important new result in mathematics – that is considered a great success.

It is a success just as profound and notable as an experimental confirmation of a theoretical prediction.

Strings vs. String-Math



This is a world in which each of us, knowing his limitations, knowing the evils of superficiality and the terrors of fatigue, will have to cling to what is close to him, to what he knows, to what he can do, ...


Future Directions

List of concrete problems at:

http://www.physics.rutgers.edu/~gmoore/PhysicalMathematicsAndFuture.pdf


NOVA
BELGICA
ET
ANGLIA
NOVA
Grand Lac
NOVÆ FRANCIÆ PARS
Norembegua
CADIE
N. SCOTIA
NIEUW ENGELAND
NIEUW NEDERLAND
MAR DEL NORT
VIRGI
NIA
Weapemeoc
Secotan
La Bermude
Amstelodami
Johannes Janssonius
excudit


M5

Some QFT's have no action principle and no obvious``fundamental'' degrees of freedom.

Some QFT's have many action principles with totally different ``fundamental'' degrees of freedom.

Even with an action principle, mathematical construction of interacting QFT with running couplings is hard.

S-matrix amplitudes display much magic, encoding locality in unusual ways.

There are ``theories'' which are neither local QFT's nor fully string theories:
Little string theory, brane probe theories, noncommutative field theories,…

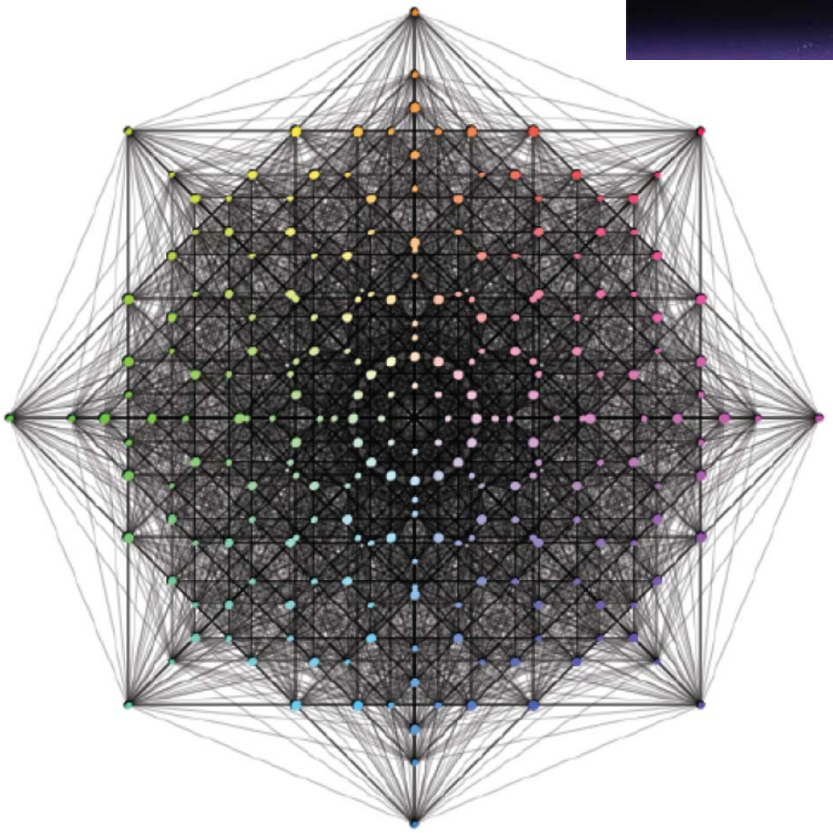
Many nontrivial field theoretic phenomena have simple geometrical reformulations (using compactification and branes)

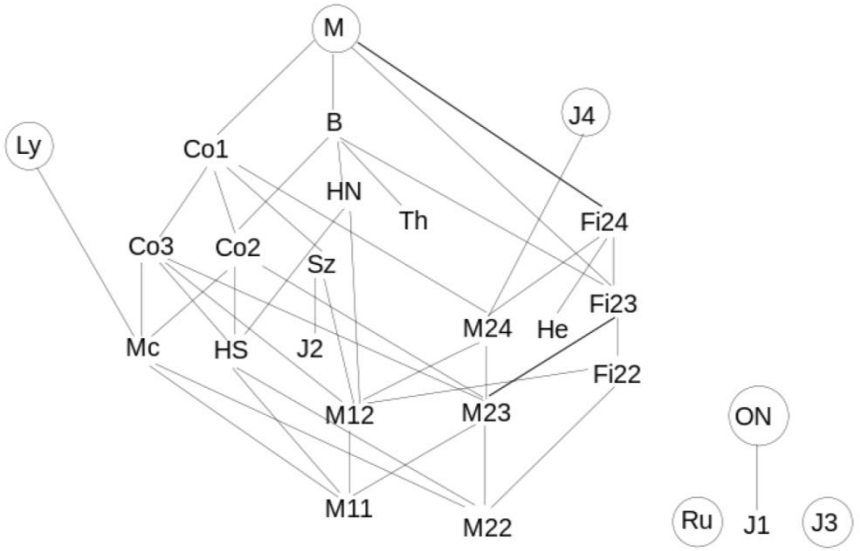
Fully local theories should have gluing rules for all codimensions.

Field theories are not fully defined without specifying their categories of defects.

All this evidence suggests that there are new, possibly more geometric, ways of formulating Quantum Field Theory

Possibly one way forward would be to find a general conceptual mathematical framework for the AdS/CFT correspondence.





I cannot forecast what stormy weather our field is destined to endure, but I can forecast with confidence that there will be abundant moonshine ahead.

Enduring Question: M-Theory

D-branes teach us to replace spacetime with D-brane categories.

Space-***time*** becomes noncommutative…