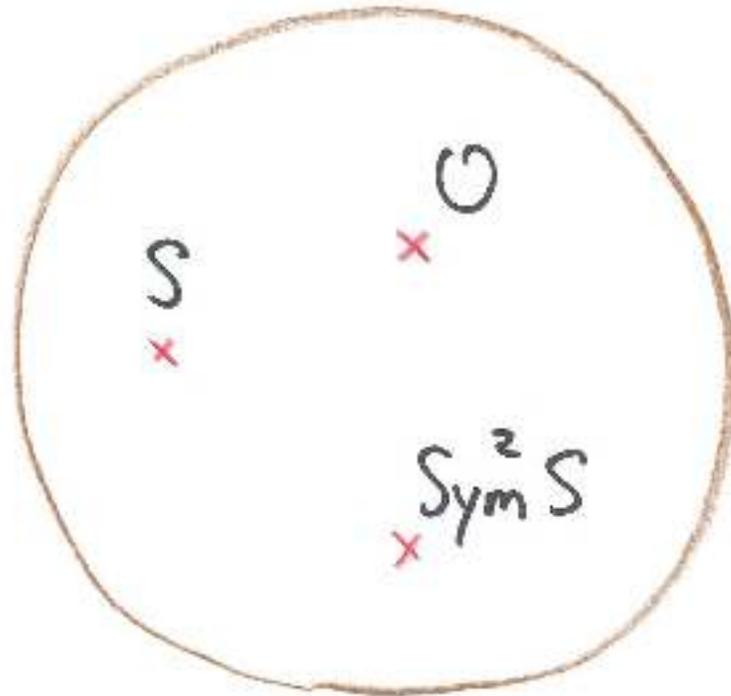


The Grade Restriction Rule



Kentaro Hori (IPMU)

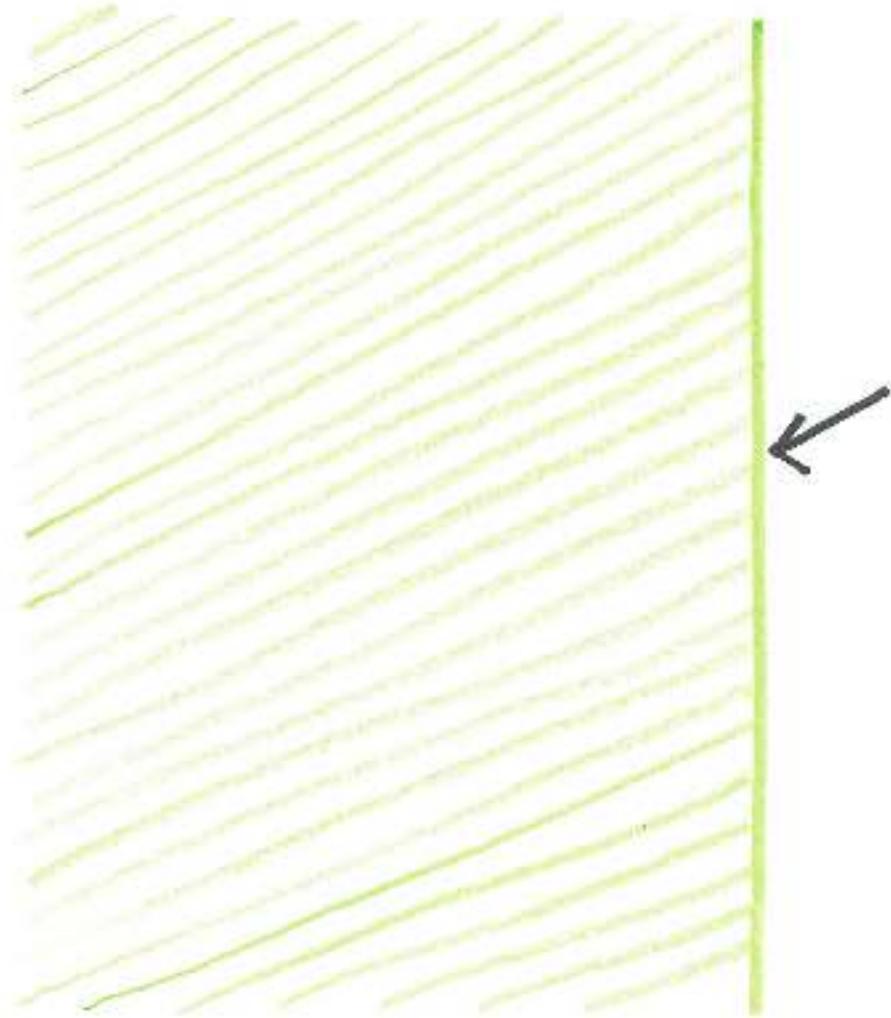
Based on a joint work (to appear) with

Richard Eager, Johanna Knapp, &
Mauricio Romo

Also based on an earlier joint work with

Manfred Herbst & David Page

Boundary conditions in 2d (2,2) gauge theory



Boundary conditions in 2d (2,2) gauge theory

- D-branes in String theory

Boundary conditions in 2d (2,2) gauge theory

- D-branes in String theory

- Duality { Mirror Symmetry
2d Seiberg duality

Boundary conditions in 2d (2,2) gauge theory

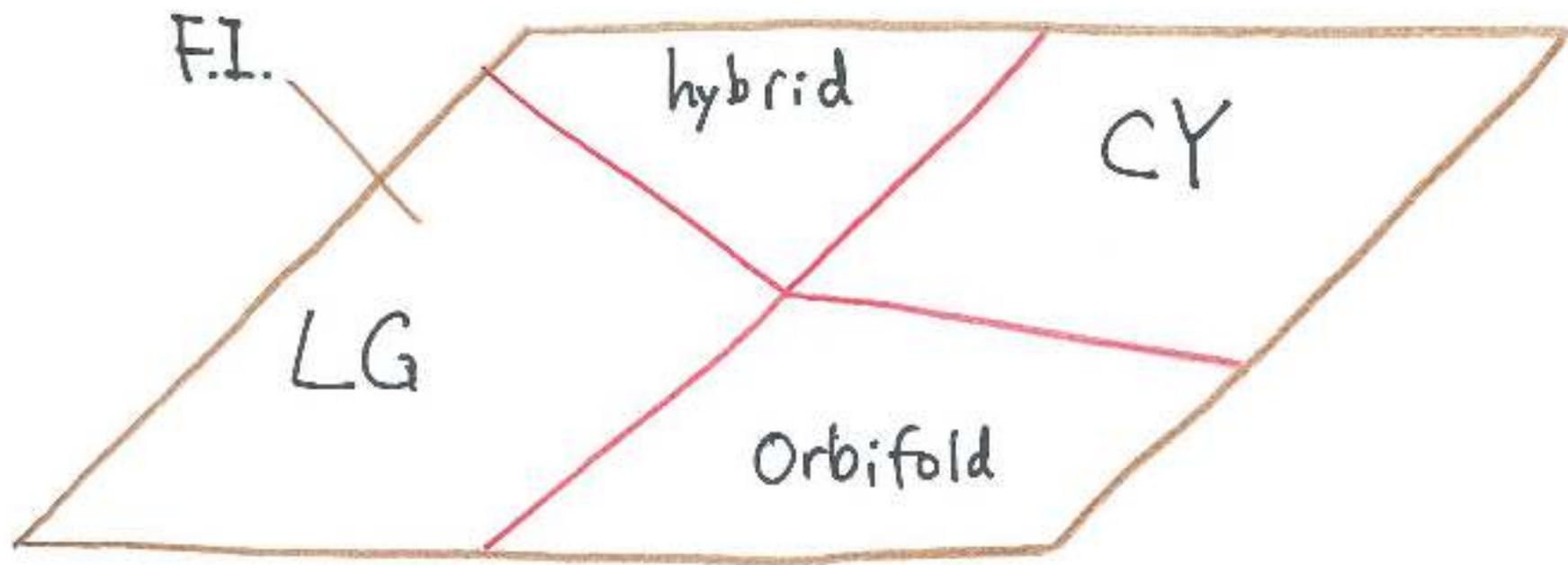
- D-branes in String theory

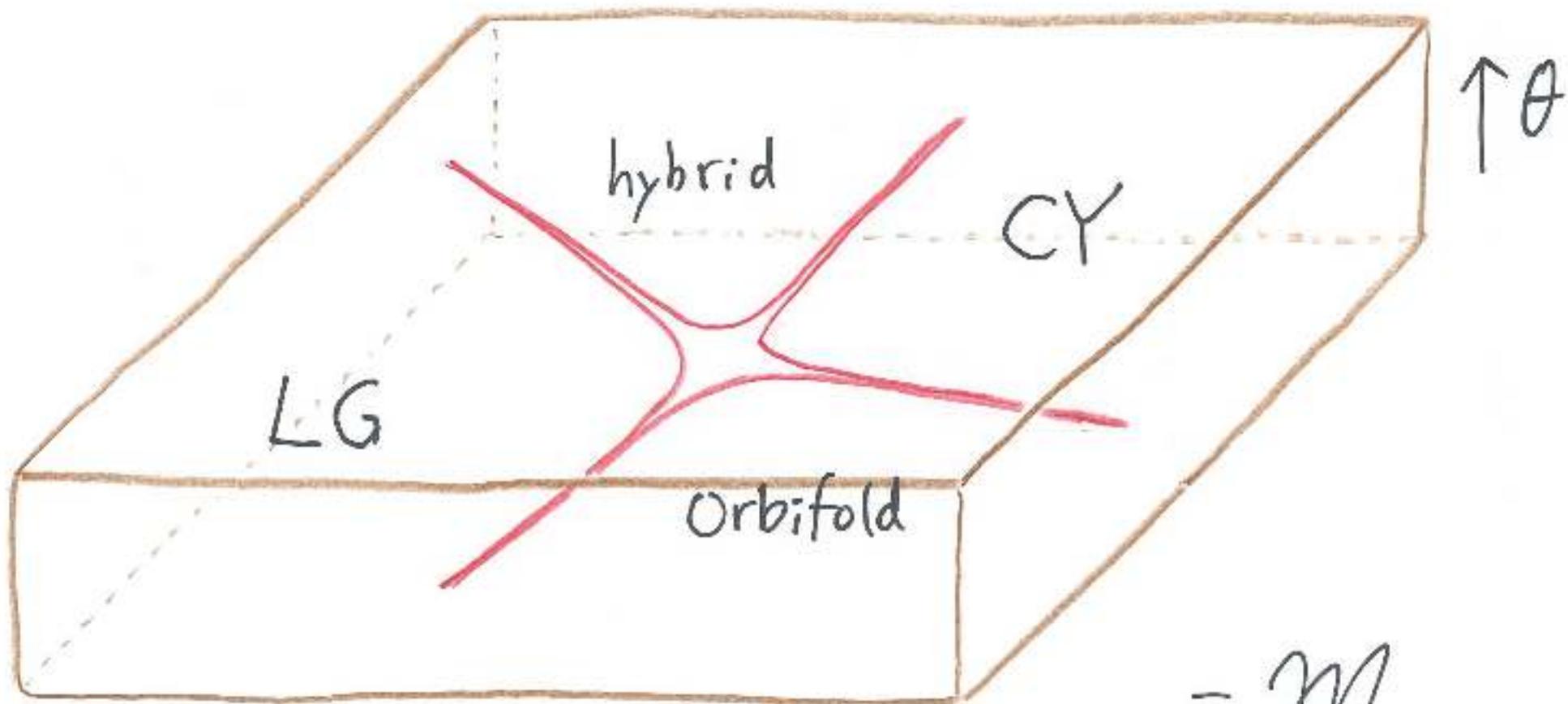
- Duality $\left\{ \begin{array}{l} \text{Mirror Symmetry} \\ \text{2d Seiberg duality} \end{array} \right.$

- Exercise for higher d :

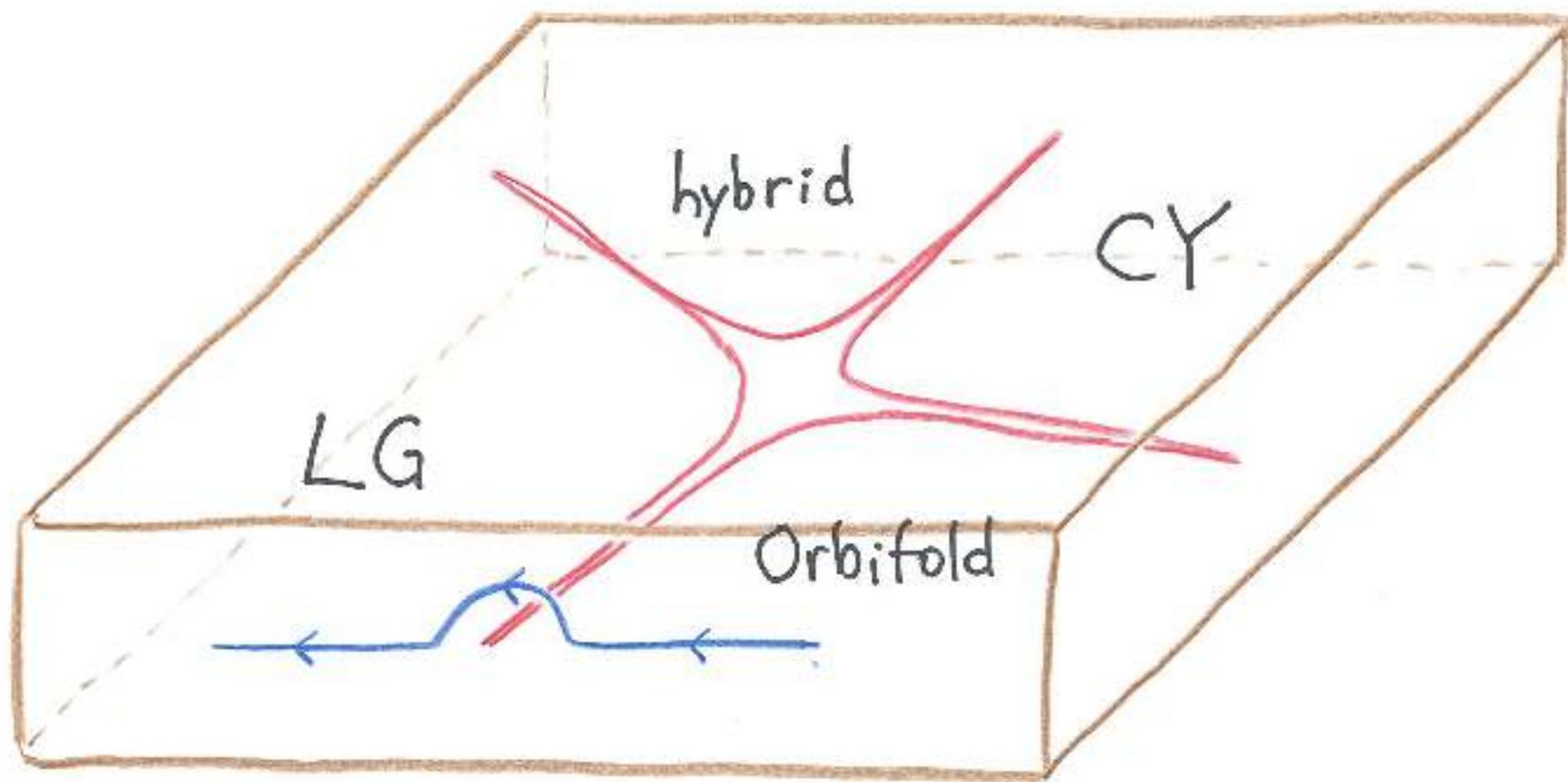
loop / surface / ... / boundary (wall)

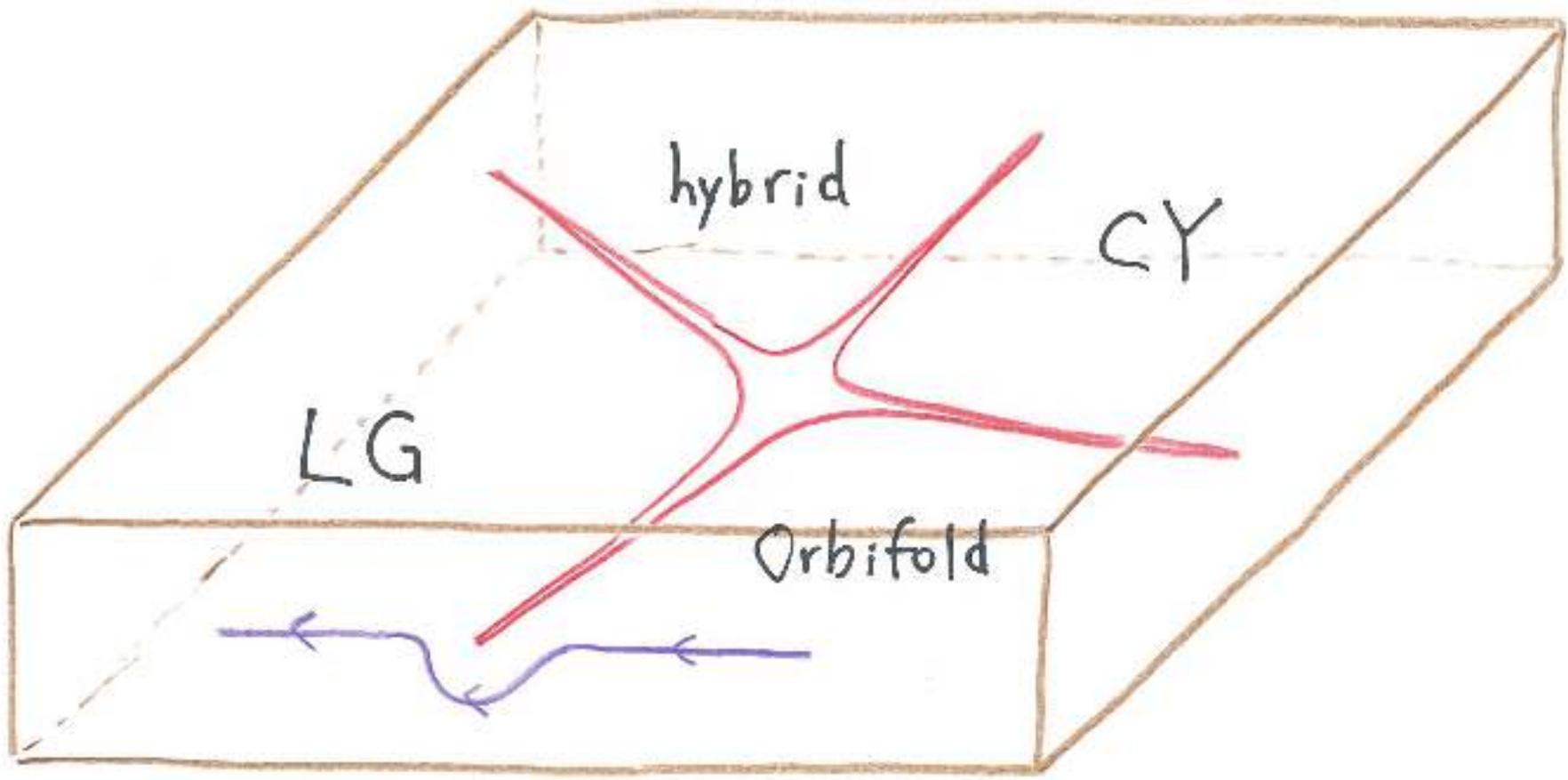
Gauged Linear Sigma Model

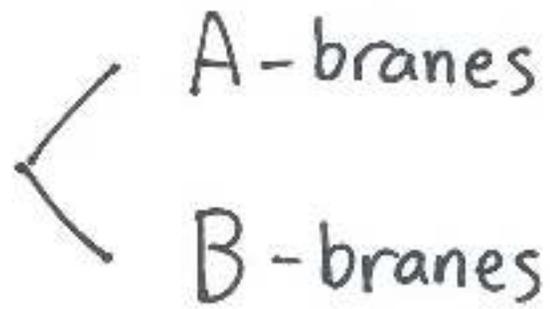




= $\mathcal{M}_{t.c.}$







A-branes
B-branes

A-branes
B-branes

B-branes



e.g.
Complex submanifolds

holomorphic vector bundles

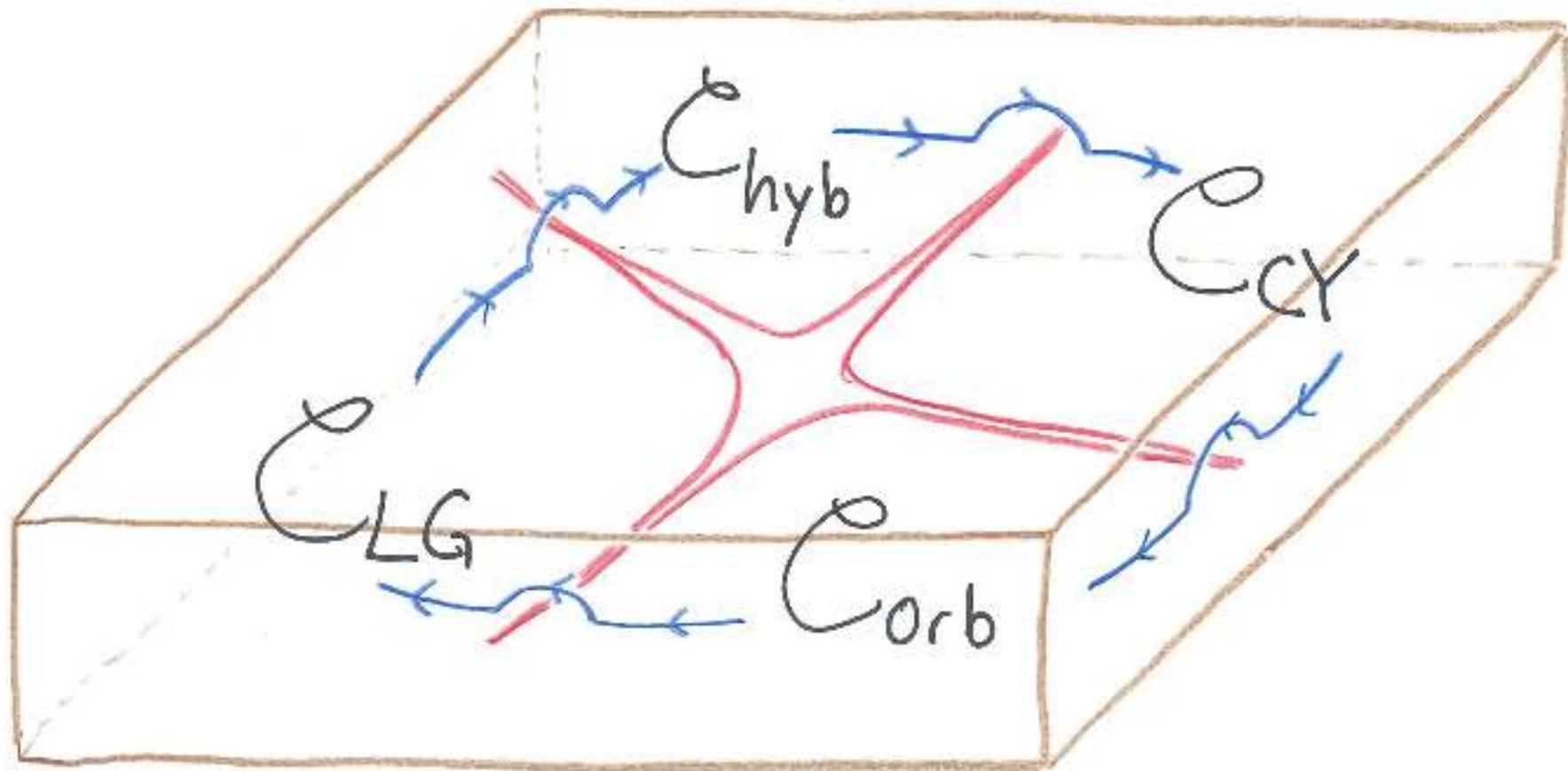
....

in sigma model

B-branes \rightsquigarrow Category \mathcal{C}

↑
invariant under deformation
of twisted chiral parameter

⇒ Parallel transport of \mathcal{C}



Key : Grade Restriction Rule

Abelian gauge group : Herbst-H. Page 2008

Use $\mathbb{Z}(\mathbb{D})$ to find G.R.R.

in non-Abelian LSM

LSM data

gauge group G

matter repr. V

superpotential W

FI-theta $t = \tilde{s} - i\theta$

LSM data

gauge group G

matter repr. V 

$$\underline{\phi \mapsto g\phi}$$

superpotential W

$$\underline{W(g\phi) = W(\phi)}$$

FI-theta $t = \xi - i\theta$

LSM data

gauge group G

matter repr. $V \curvearrowright$ $U(1)_V : \phi \mapsto \lambda^R \phi$

superpotential W

$$\underline{W(\lambda^R \phi) = \lambda^2 W(\phi)}$$

FI-theta $t = \zeta - i\theta$

Charge integrality : $e^{\pi i R} = \mathbb{1} \in G$

$$\underline{U(1)_A : G \subset SL(V)}$$

B-brane data

Chan-Paton vct sp M

Matrix factorization Q of W

B-brane data

Chan-Paton vct sp $M = M^{ev} \oplus M^{od}$ \mathbb{Z}_2 -graded

Matrix factorization $Q(\phi) : M \rightarrow M$ odd

$ev \rightarrow od$
 $od \rightarrow ev$

$$\underline{Q(\phi)^2 = W(\phi) id_M}$$

B-brane data

Chan-Paton vct sp M 

$$\underline{G : m \mapsto \rho(g)m}$$

Matrix factorization Q

$$\underline{\rho(g)^{-1} Q(g\phi) \rho(g) = Q(\phi)}$$

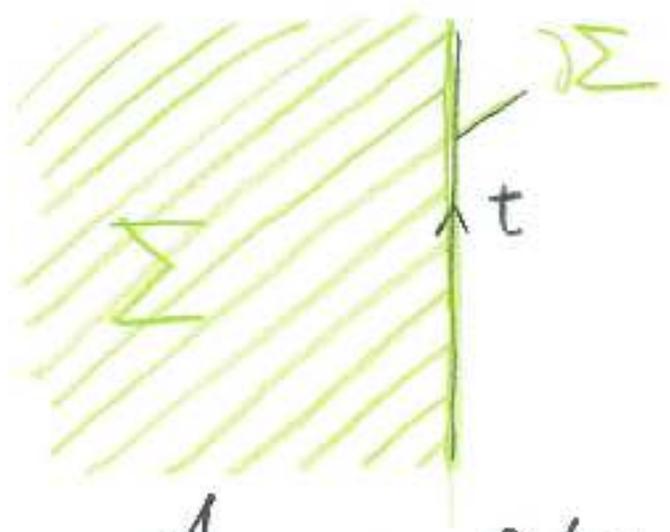
B-brane data

Chan-Paton vct sp M \hookrightarrow $U(1)_v : m \mapsto \lambda^r m$

Matrix factorization Q $\lambda^r Q(\lambda^R \phi) \lambda^{-r} = \lambda Q(\phi)$

Charge integrality

$e^{\pi i r} \rho(\gamma) = \pm 1$ on $M_{\text{odd}}^{\text{ev}}$



$$P \exp \left(-i \int_{\partial \Sigma} A_t dt \right)$$

$$A_t = \rho (v_t - \text{Re } \sigma)$$

$$-\frac{1}{2} \Psi^i \partial_i Q(\phi) + \frac{1}{2} \Psi^{\bar{i}} \partial_{\bar{i}} Q(\phi)^\dagger + \frac{1}{2} \{Q(\phi), Q(\phi)^\dagger\}$$

chiral : Neumann B.C.

vector : $L = \mathfrak{Gr} \subset \mathfrak{g}_0$, $r \subset \mathfrak{t}_0$ W-inv.
Lagrangian

Q : Which $\gamma \subset t_c$
should be chosen ?

Q : Which $\gamma \subset t_c$
should be chosen ?

A Hint : \exists ()



Benini et al

Doroud et al

2012



H-Romo

Sugichita-Terashima

2013

Honda-Okuda

The hemisphere partition function

$$G \supset T \text{ max. for. } V|_{U(1) \times T} = \bigoplus_i \mathbb{C}(R_i, Q_i)$$

$$M|_{U(1) \times T} = \bigoplus_j \mathbb{C}(r_j, q_j)$$

$$Z(\mathbb{D}) = \int_{\gamma} d^4\sigma \prod_{\alpha > 0} d(\sigma) \sinh(\pi d(\sigma)) \prod_i \Gamma(i Q_i(\sigma) + \frac{R_i}{2}) \cdot e^{it(\sigma)} \cdot f_B(\sigma)$$

$$f_B(\sigma) = \text{tr}_M(e^{\pi i r} e^{2\pi \sigma}) = \sum_j e^{\pi i r_j} e^{2\pi q_j(\sigma)}$$

A proposal for $\gamma \subset t_c$

$$\textcircled{1} \quad \zeta(\mathbb{D}) = \int_{\gamma} d^{\rho} \sigma \dots$$

is absolutely convergent.

$$\textcircled{2} \quad \gamma \approx i\mathbb{R} \quad \text{in } t_c \setminus \text{poles}$$

$$\text{poles : } \quad \rho_i(\sigma) = i\left(n_i + \frac{R_i}{2}\right) \quad n_i = 0, 1, 2, \dots$$

Deep inside a phase s.t. $G \rightarrow \text{finite}$,

$\exists \gamma_*$ which is admissible $\forall (M, Q)$

[Take γ_* s.t.
 $e^{it(\sigma)} \searrow \searrow 0$ at $\partial \gamma_*$.]

If $G \rightarrow$ continuous,

- Such γ_* does not exist.
- Existence of admissible γ requires a non-trivial condition on M !

If $G \rightarrow$ Continuous,

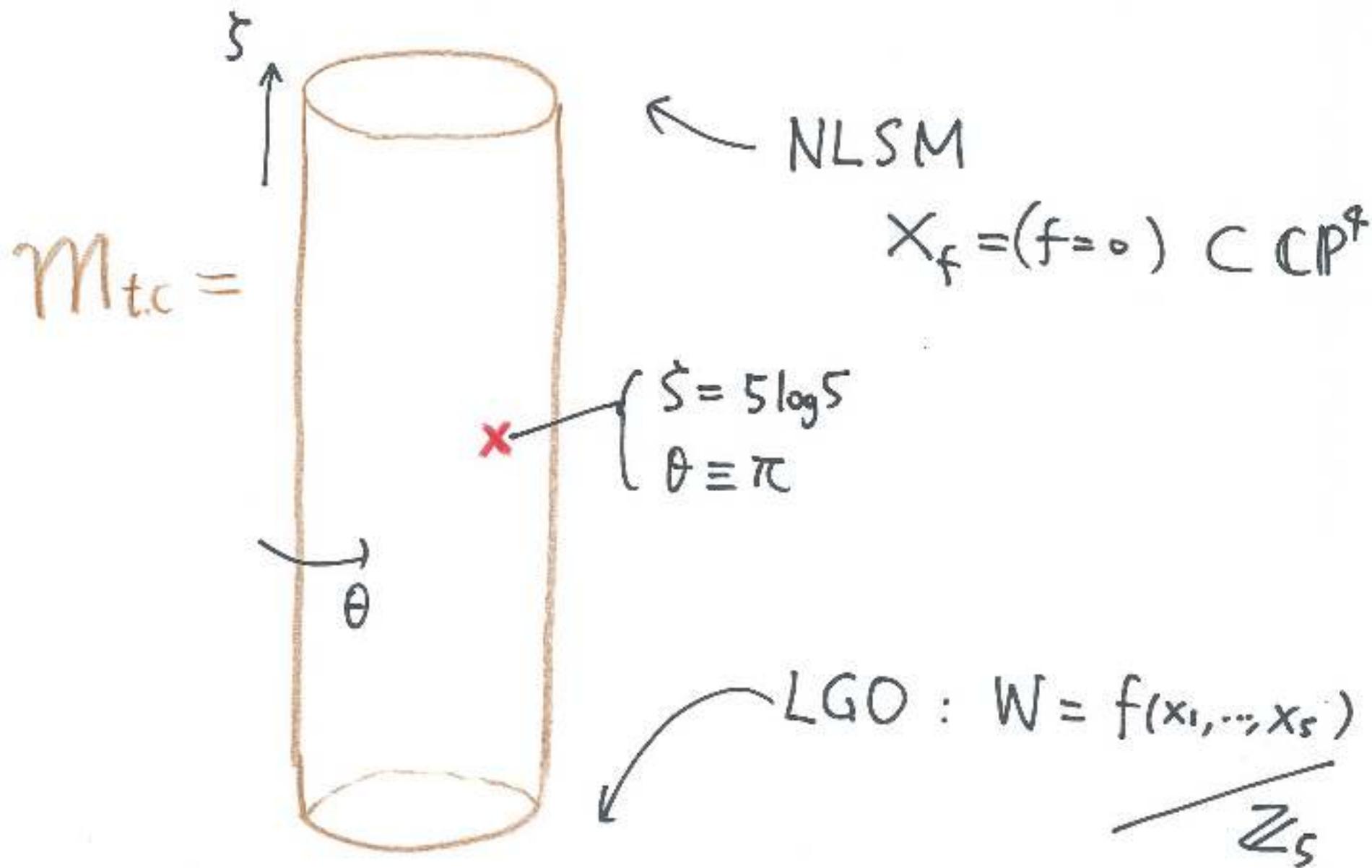
- Such γ_* does not exist.
- Existence of admissible γ requires
a non-trivial condition on M !



Grade Restriction Rule

Warm Up

Quintic $G = U(1)$ P, X_1, \dots, X_5 , $W = p \underline{f(x_1, \dots, x_5)}$
 $-5 \quad 1 \quad \dots \quad 1$ Quintic



$$|\text{Integrand}_q| \sim e^{-\zeta_{\text{eff}} \text{Im} \sigma + (\theta + 2\pi q) \text{Re} \sigma - 5\pi |\text{Re} \sigma|}$$

$$|\text{Integrand}_q| \sim e^{-\xi_{\text{eff}} \text{Im} \sigma + (\theta + 2\pi q) \text{Re} \sigma - \underbrace{5\pi}_{\text{Boundary potential from matters}} |\text{Re} \sigma|}$$

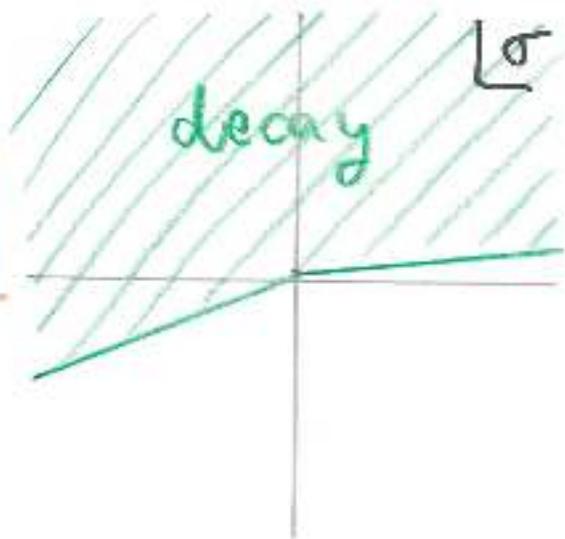
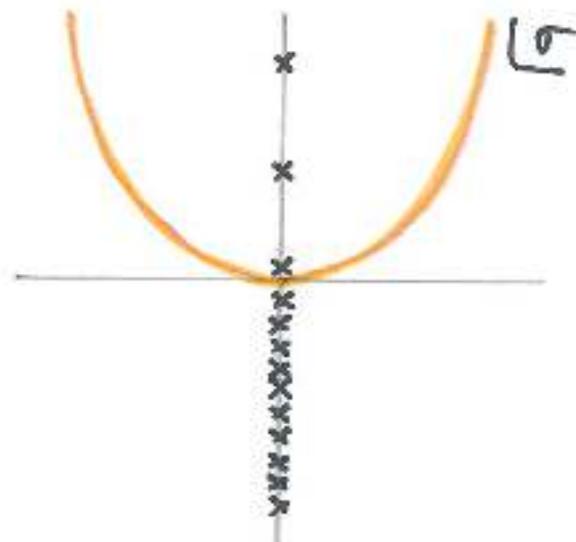
Chan-Paton charge

$\xi_{\text{eff}} = 5 \log 5$

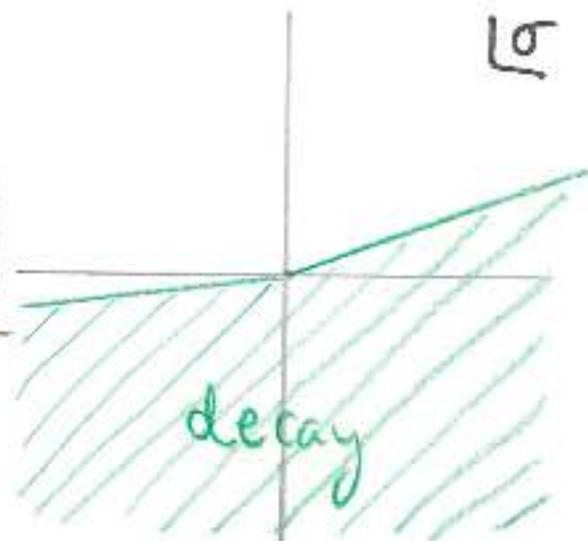
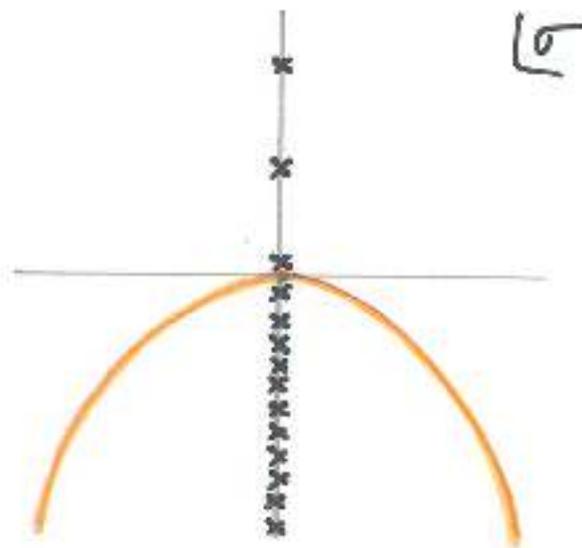
Boundary potential
from matters

$$|\text{Integrand}_q| \sim e^{-\zeta_{\text{eff}} \text{Im} \sigma + (\theta + 2\pi q) \text{Re} \sigma - 5\pi |\text{Re} \sigma|}$$

$$\zeta_{\text{eff}} > 0$$

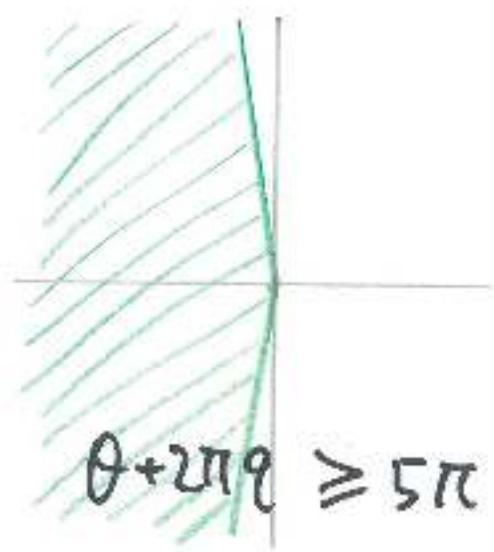
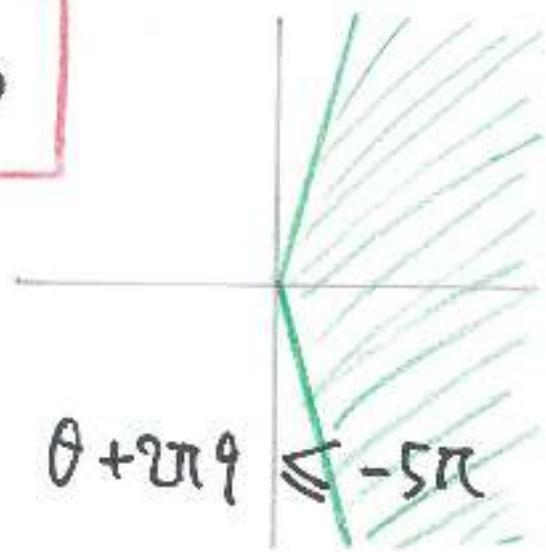

 \rightsquigarrow


$$\zeta_{\text{eff}} < 0$$

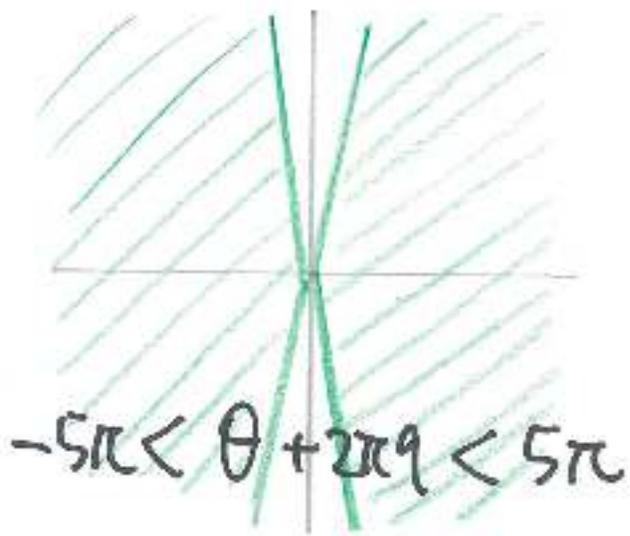

 \rightsquigarrow


$$|\text{Integrand}_q| \sim e^{-\zeta_{\text{eff}} [\text{Im} \sigma + (\theta + 2\pi q) \text{Re} \sigma - 5\pi |\text{Re} \sigma|]}$$

$$\zeta_{\text{eff}} = 0$$

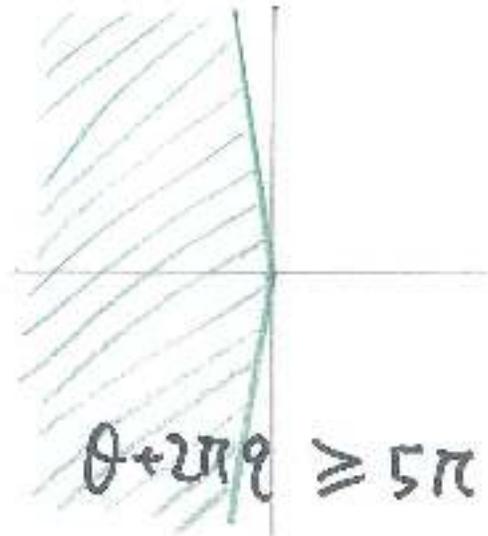
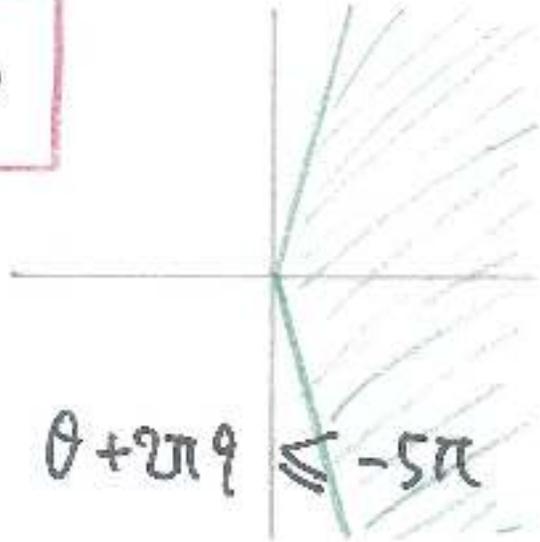


\rightarrow ~~admissible~~ γ

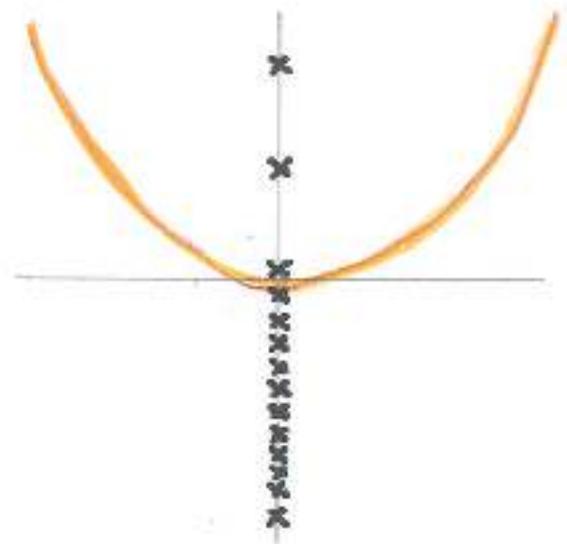
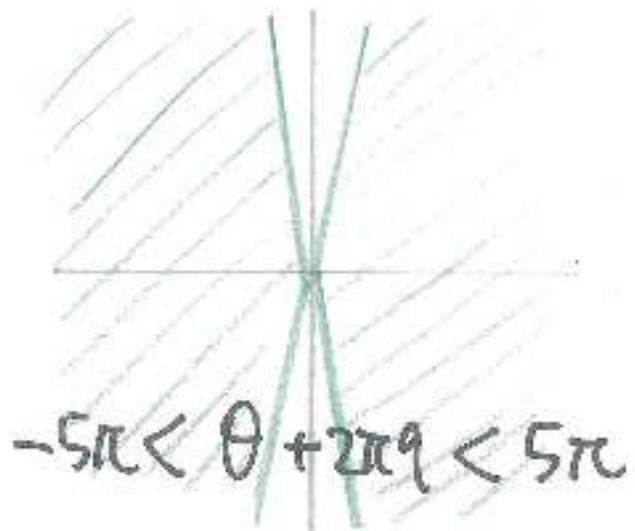


$$|\text{Integrand}_q| \sim e^{-\zeta_{\text{eff}} \text{Im} \sigma + (\theta + 2\pi q) \text{Re} \sigma - 5\pi |\text{Re} \sigma|}$$

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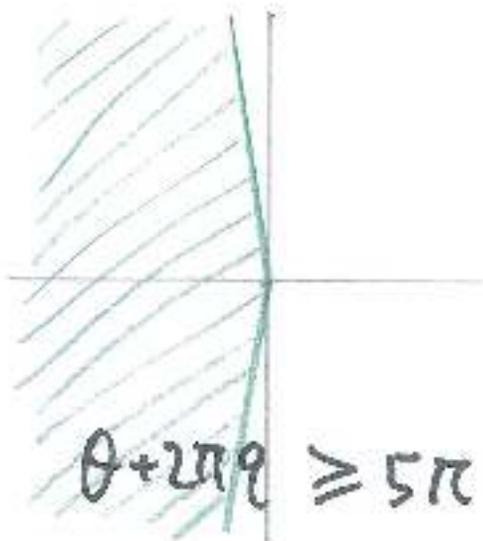
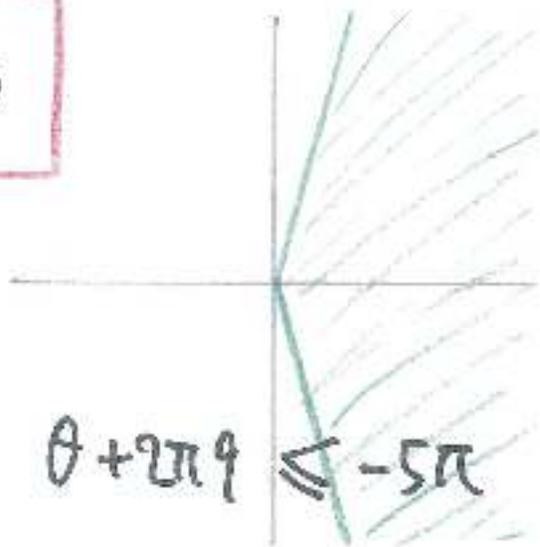


\rightarrow ~~admissible~~ γ

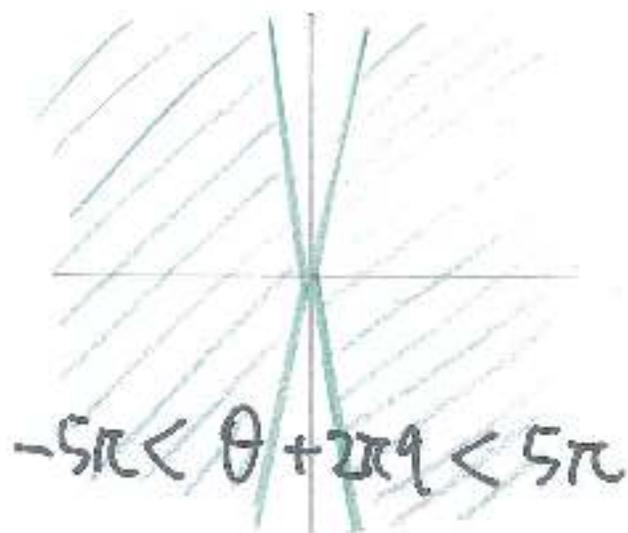


$$|\text{Integrand}_q| \sim e^{-\zeta_{\text{eff}} [\text{Im} \sigma + (\theta + 2\pi q) \text{Re} \sigma - 5\pi |\text{Re} \sigma|]}$$

$$\zeta_{\text{eff}} = 0$$

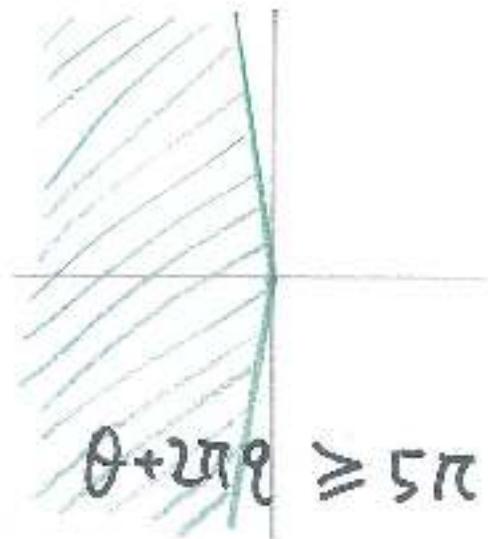
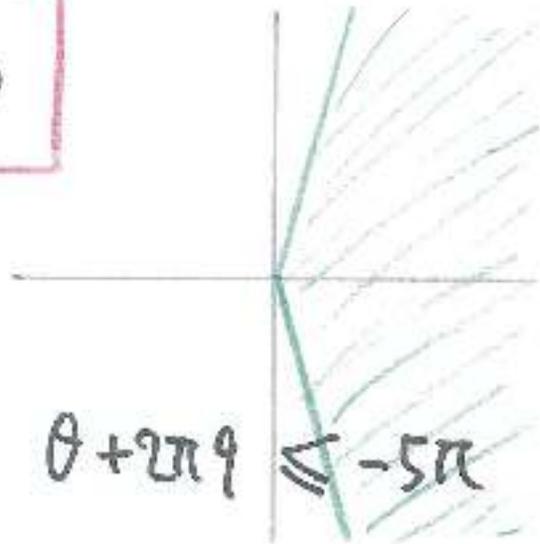


~~admissible γ~~

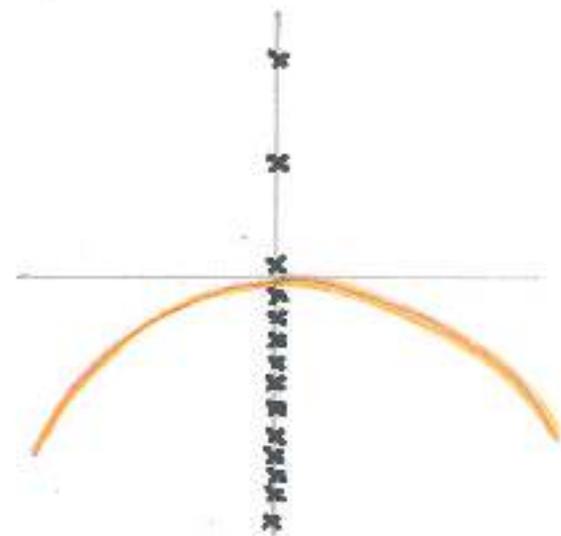
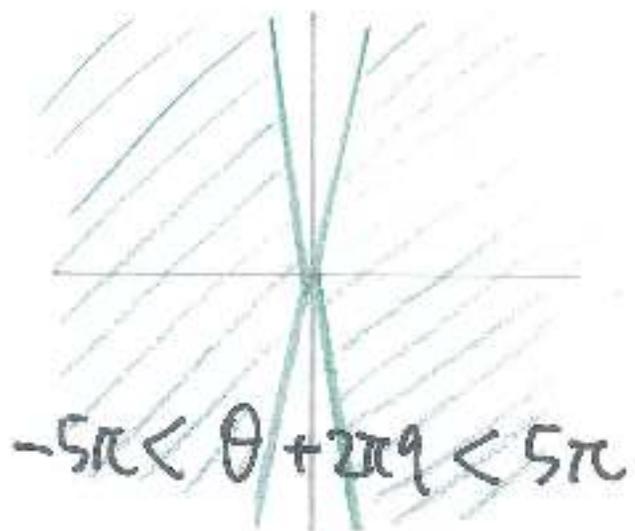


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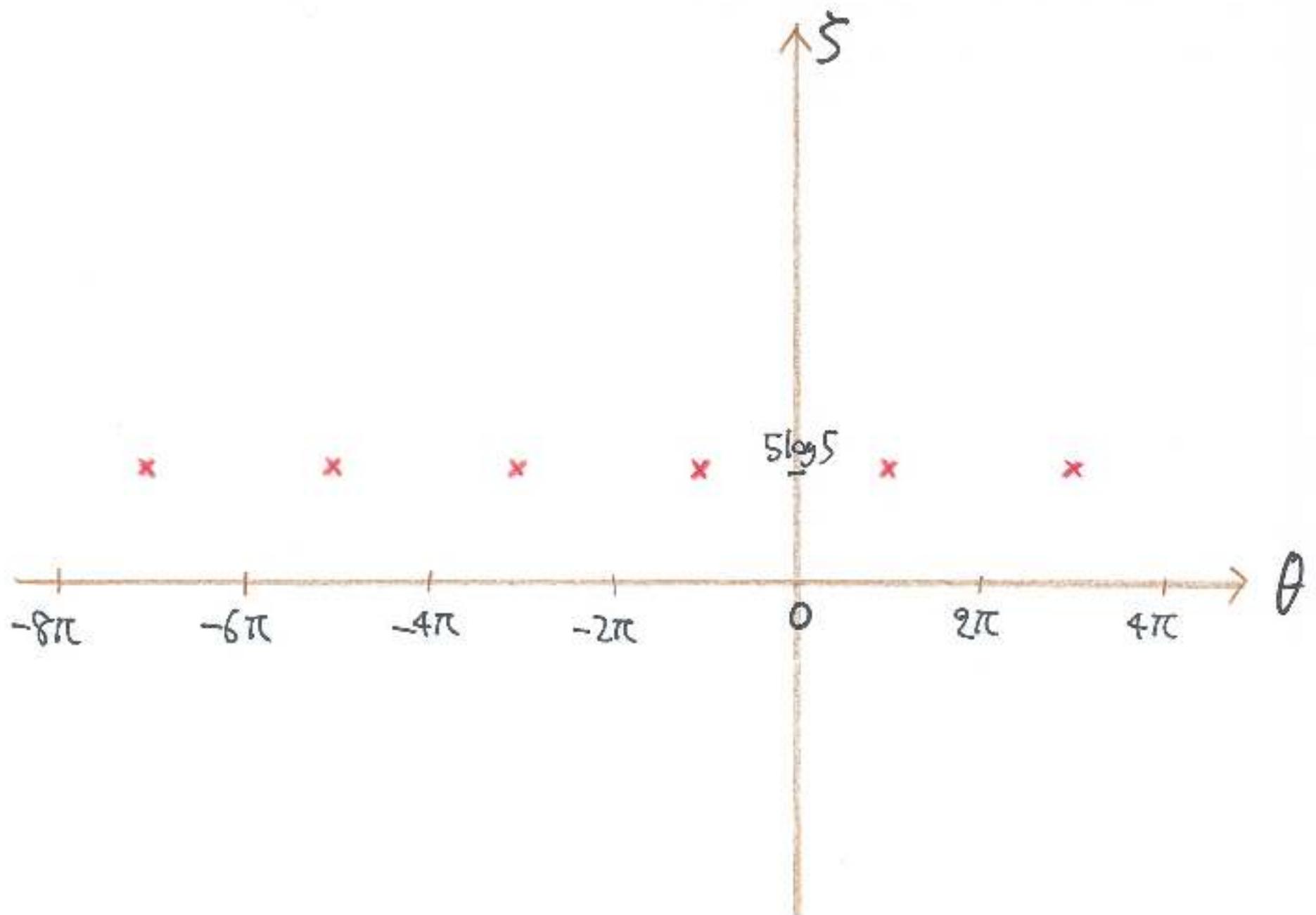
$$\zeta_{\text{eff}} = 0$$



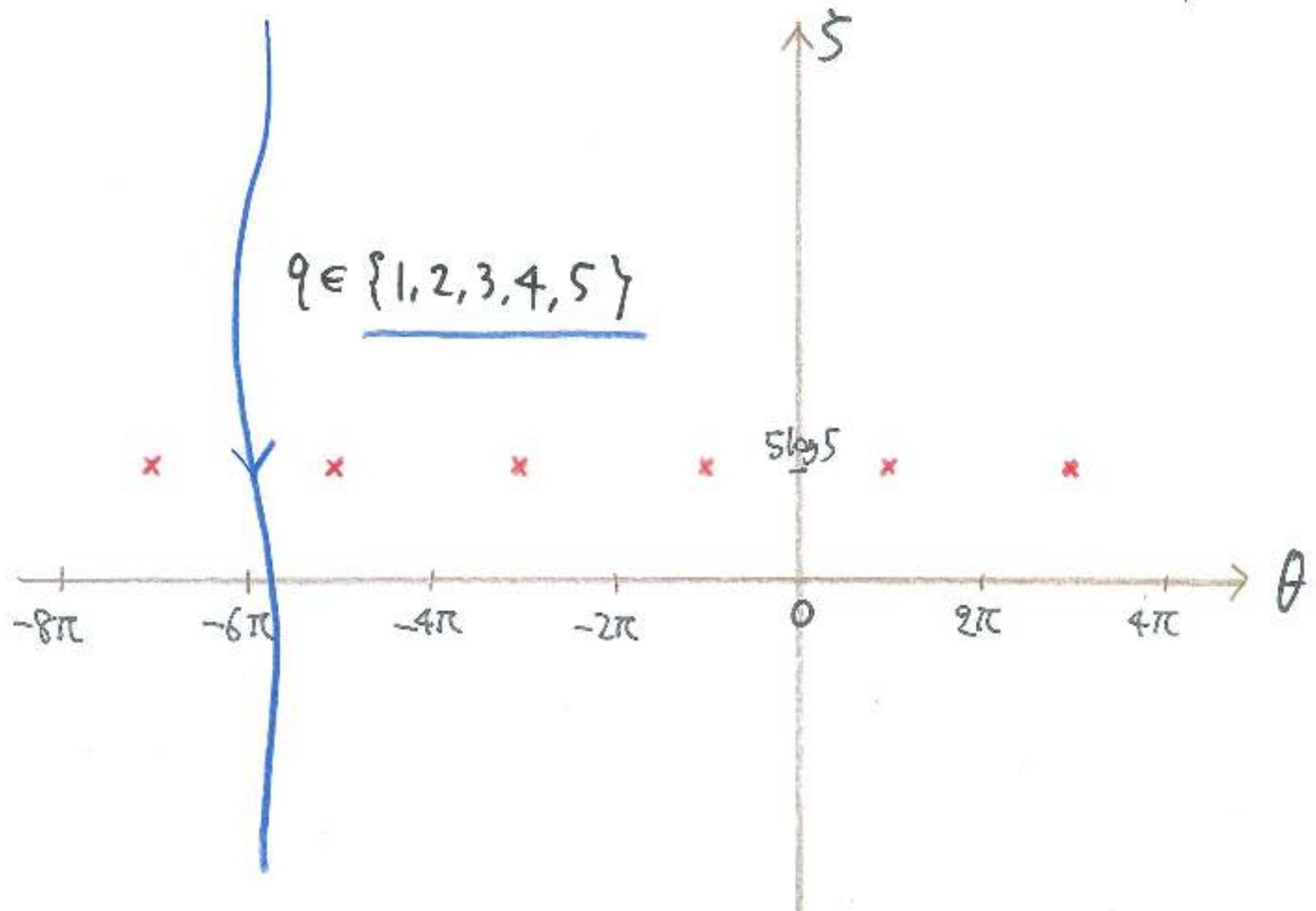
~~admissible γ~~



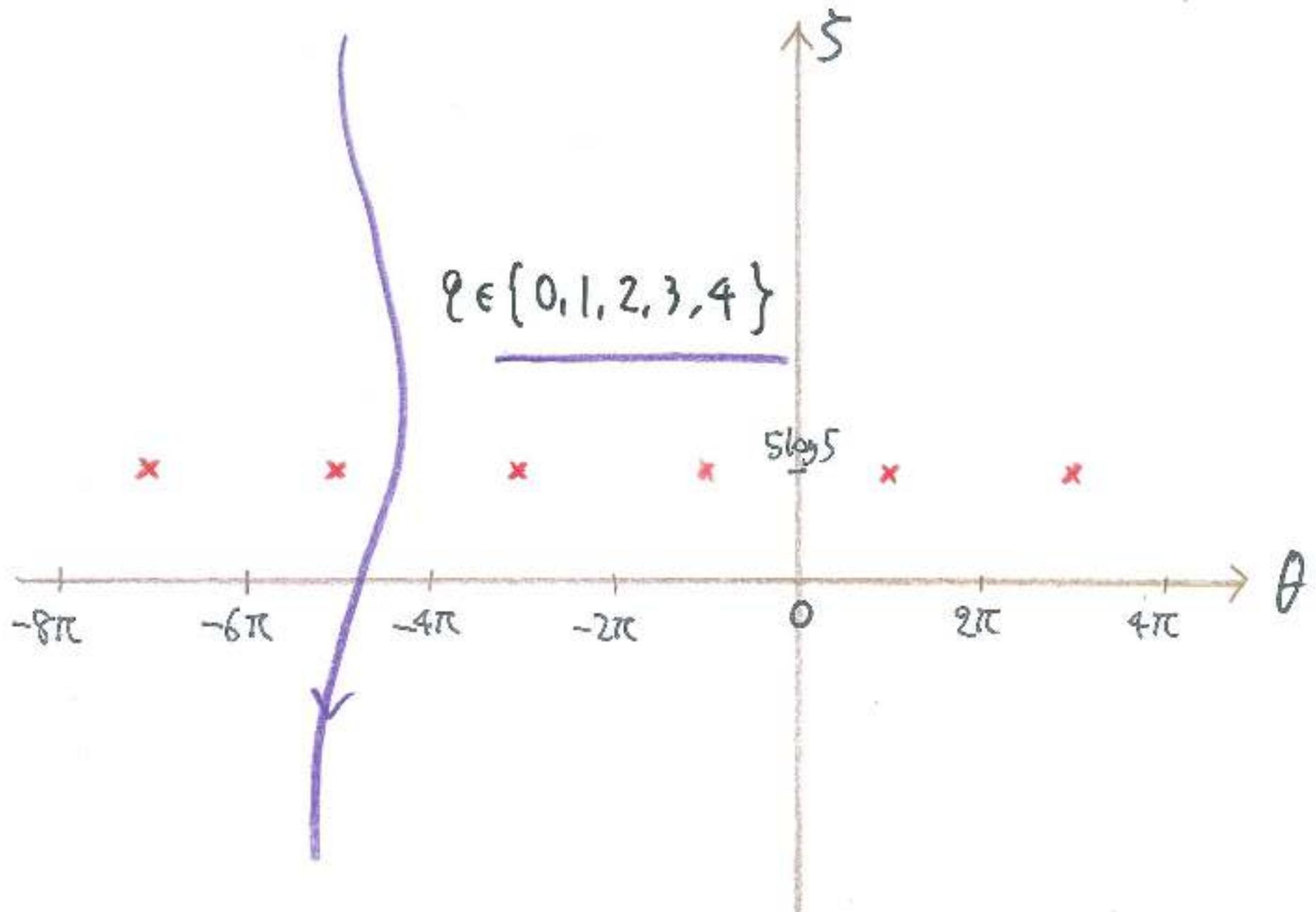
What happens if ζ is moved from $+\infty$ to $-\infty$?



What happens if ζ is moved from $+\infty$ to $-\infty$?



What happens if ζ is moved from $+\infty$ to $-\infty$?



The Main Example

Rødland model

H-Tong 2006

$$G = U(2)$$

$$V : \underbrace{P^1, \dots, P^7}_{\det^{-1}}, \underbrace{X_1, \dots, X_7}_{\text{doublet}}$$

$$W = \sum_{i,j,k} A_{ij}^{ij} P^k [X_i X_j]$$

$$= \sum_{i,j} A^{ij}(p) [X_i X_j]$$

$$[X_i X_j] \\ := X_i^1 X_j^2 - X_i^2 X_j^1$$

SU(2)-invariant

Rødland

$$G = U(2)$$

$$\frac{p^1 \dots p^7}{\det^{-1}} \quad \frac{x_1 \dots x_7}{2}$$

$$W = \sum A_k^{ij} p^k [x_i x_j]$$



NLSM

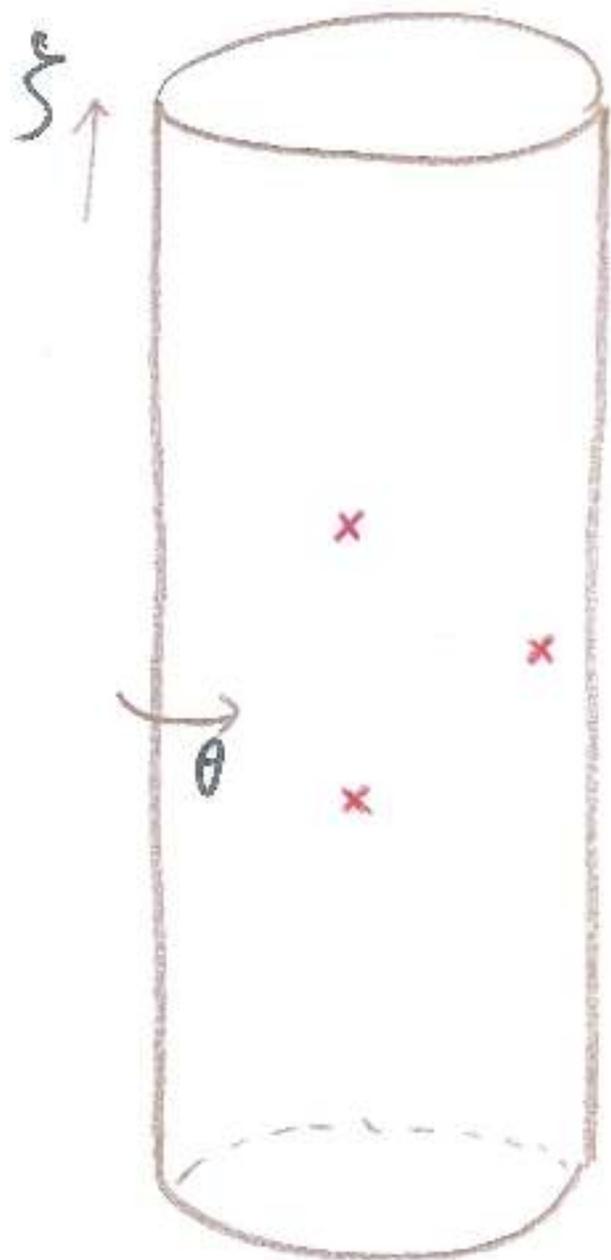
$$X_A = \left\{ \sum_{\substack{ij \\ k=1, \dots, 7}} A_k^{ij} [x_i x_j] = 0 \right\} \subset G(2, 7)$$

Rørdland

$$G = U(2)$$

$$\underbrace{p^1 \dots p^7}_{\det^{-1}} \underbrace{x_1 \dots x_7}_2$$

$$W = \sum A_k^{ij} p^k [x_i x_j]$$



NLSM

$$X_A = \left\{ \sum_{\substack{ij \\ k=1, \dots, 7}} A_k^{ij} [x_i x_j] = 0 \right\} \subset G(2, 7)$$

$$p \neq 0 : G \rightarrow \boxed{SU(2)}$$

$$SU(2) \ x_1 \dots x_7 \quad W = \sum_{ij} A^{ij}(p) [x_i x_j]$$

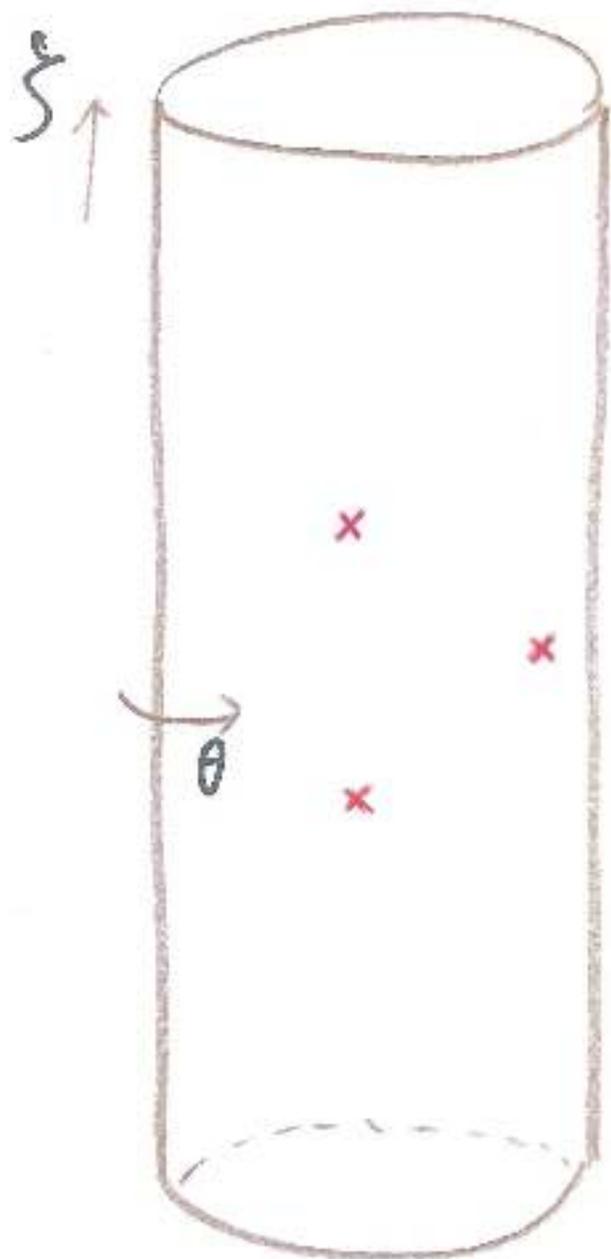
fibered over $\{p\} = \mathbb{C}P^6$

Rørdland

$$G = U(2)$$

$$\frac{P^1 \dots P^7}{\det^{-1}} \quad \frac{X_1 \dots X_7}{2}$$

$$W = \sum A_k^{ij} P^k [X_i X_j]$$



NLSM

$$X_A = \left\{ \sum_{i,j} A_k^{ij} [X_i X_j] = 0 \right\} \subset G(2,7)$$

- $SU(2) N_f = 1$: ~~SUSY~~
- $SU(2) N_f = 3$: Free theory of mesons
[$X_1 X_2$] [$X_2 X_3$], [$X_3 X_1$]

Rødland

$$G = U(2)$$

$$\underbrace{p^1 \dots p^7}_{\det^{-1}} \underbrace{x_1 \dots x_7}_2$$

$$W = \sum A_k^{ij} p^k [x_i x_j]$$



NL SM

$$X_A = \left\{ \sum_{\substack{ij \\ k=1, \dots, 7}} A_k^{ij} [x_i x_j] = 0 \right\} \subset G(2, 7)$$

NL SM

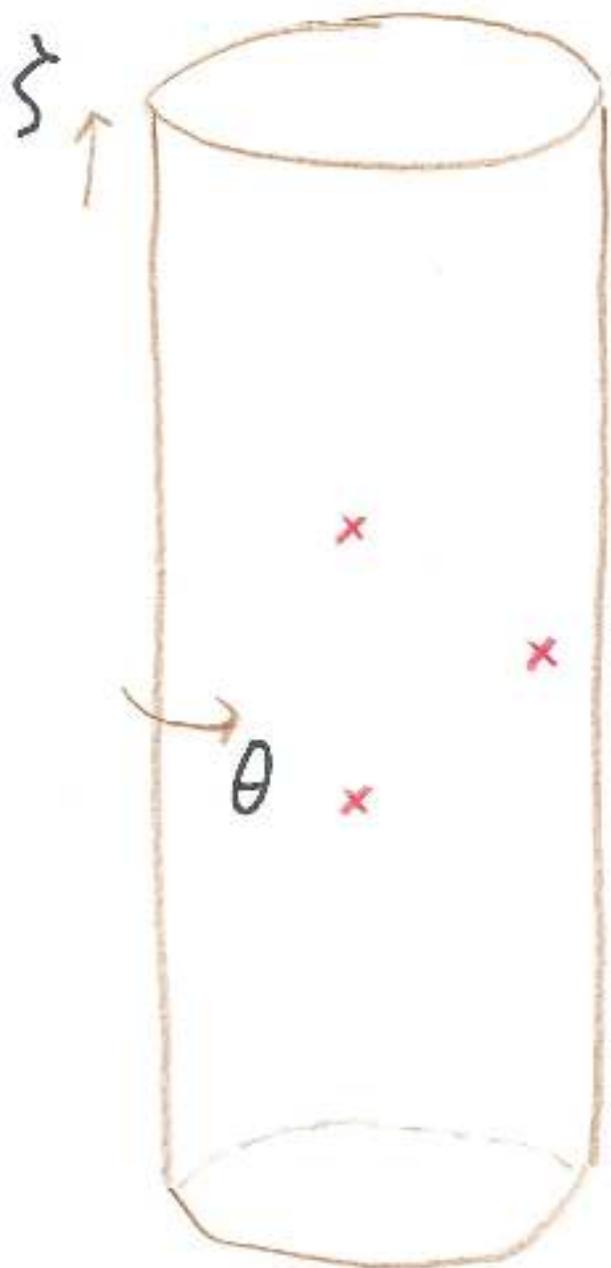
$$Y_A = \{ \text{rk } A(\varphi) = 4 \} \subset \mathbb{C}P^6$$

Rørdland

$$G = U(2)$$

$$\underbrace{p^1 \dots p^7}_{\det^{-1}} \quad \underbrace{x_1 \dots x_7}_2$$

$$W = \sum A_k^{ij} p^k [x_i x_j]$$



Grassmannian Phase

Pfaffian Phase

Representations of $U(2)$

$S := \mathbb{C}^2$ the fundamental doublet

A finite dim irrep of $U(2)$:

$$S^l(i) := \text{Sym}^l S \otimes (\det S)^i \quad \begin{array}{l} l=0,1,2,\dots \\ i \in \mathbb{Z} \end{array}$$

↑
highest weight $(l+i, i)$

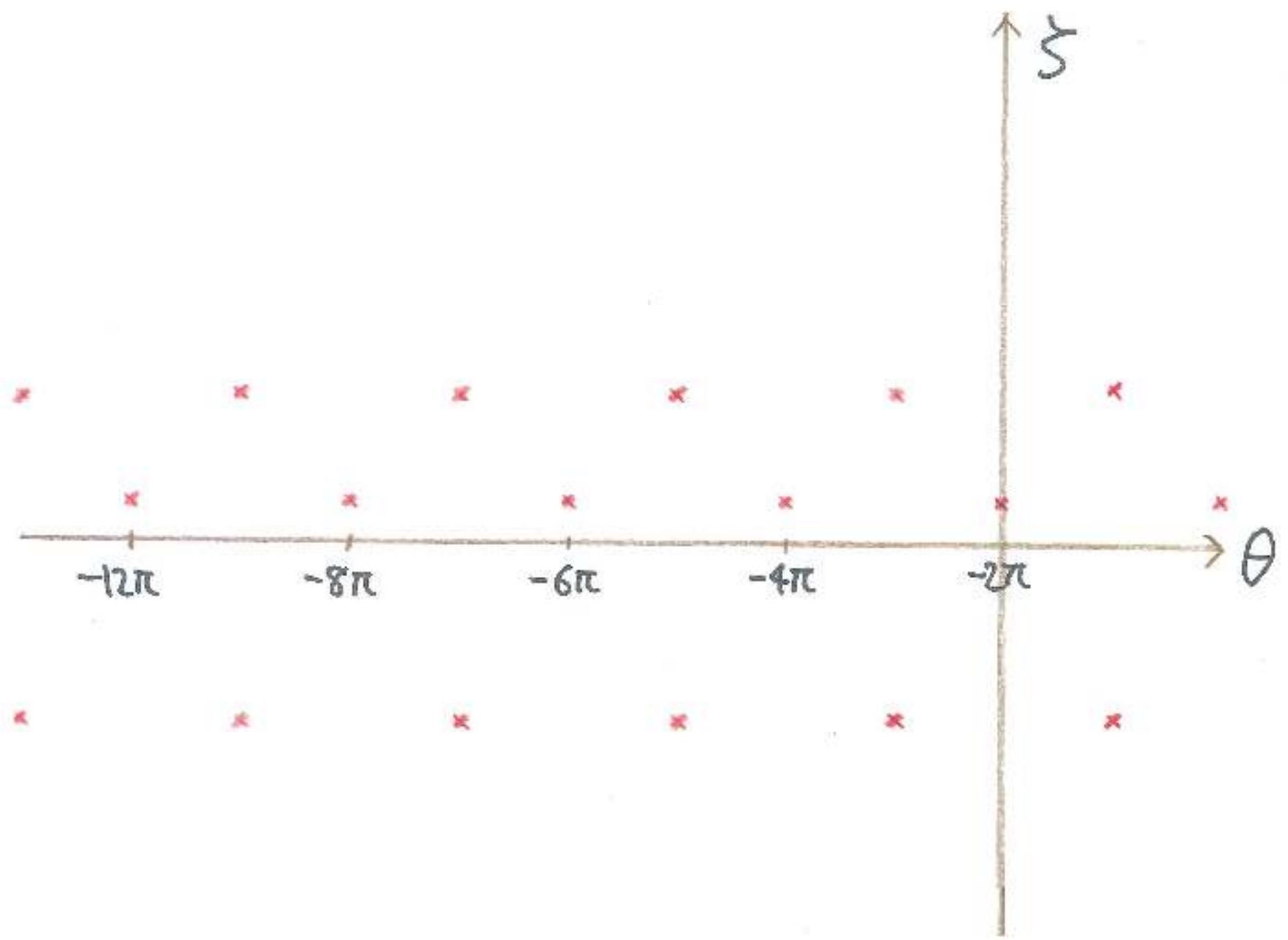
The result

$\xi \gg 0$ $U(2) \rightarrow \{1\}$: Any representation is allowed.

$\xi \ll 0$ $U(2) \rightarrow SU(2)$: Only $l=0, 1, 2$ are allowed.
 $\forall i$ OK

$$\left\{ \mathbb{C}(i), S(i), S^2(i) \right\}_{i \in \mathbb{Z}}$$

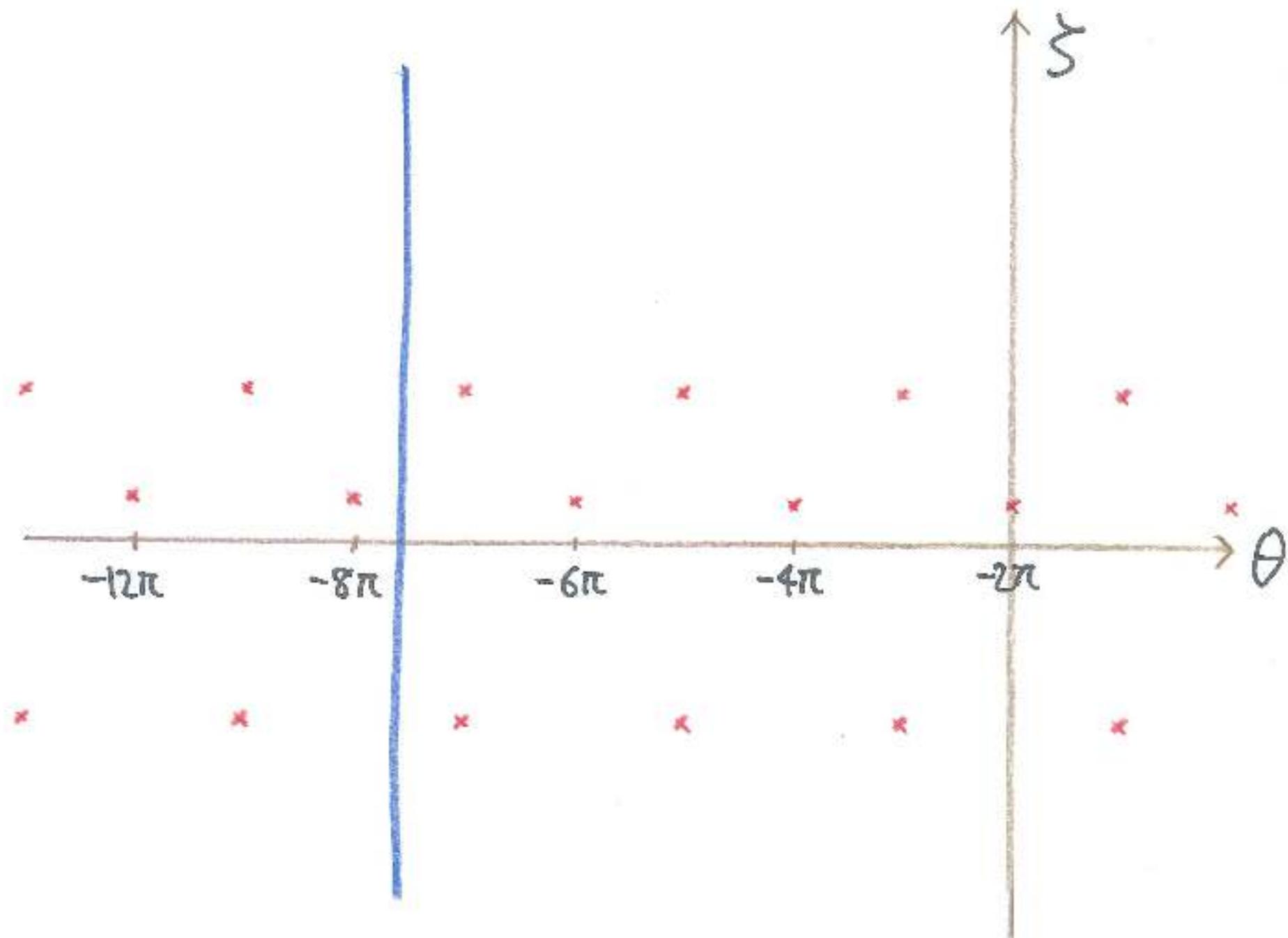
Paths between $\zeta \gg 0$ & $\zeta \ll 0$



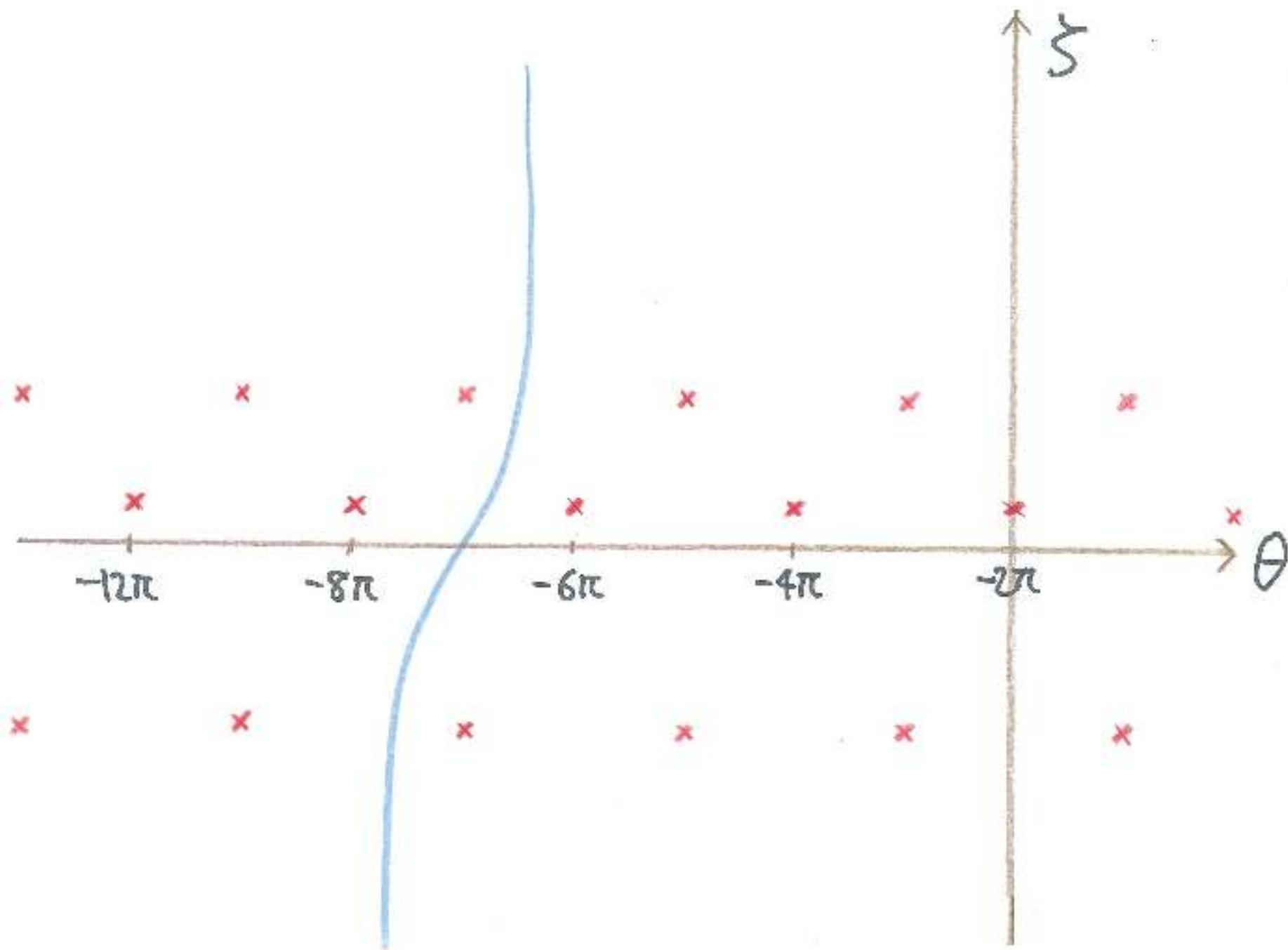
We may only consider $l = 0, 1, 2$.

$$\begin{array}{cccccccccccccc} \dots & C(-2) & C(-1) & C & C(1) & C(2) & C(3) & C(4) & C(5) & C(6) & C(7) & C(8) & C(9) & \dots \\ \dots & S(-2) & S(-1) & S & S(1) & S(2) & S(3) & S(4) & S(5) & S(6) & S(7) & S(8) & S(9) & \dots \\ \dots & S^2(-2) & S^2(-1) & S^2 & S^2(1) & S^2(2) & S^2(3) & S^2(4) & S^2(5) & S^2(6) & S^2(7) & S^2(8) & S^2(9) & \dots \end{array}$$

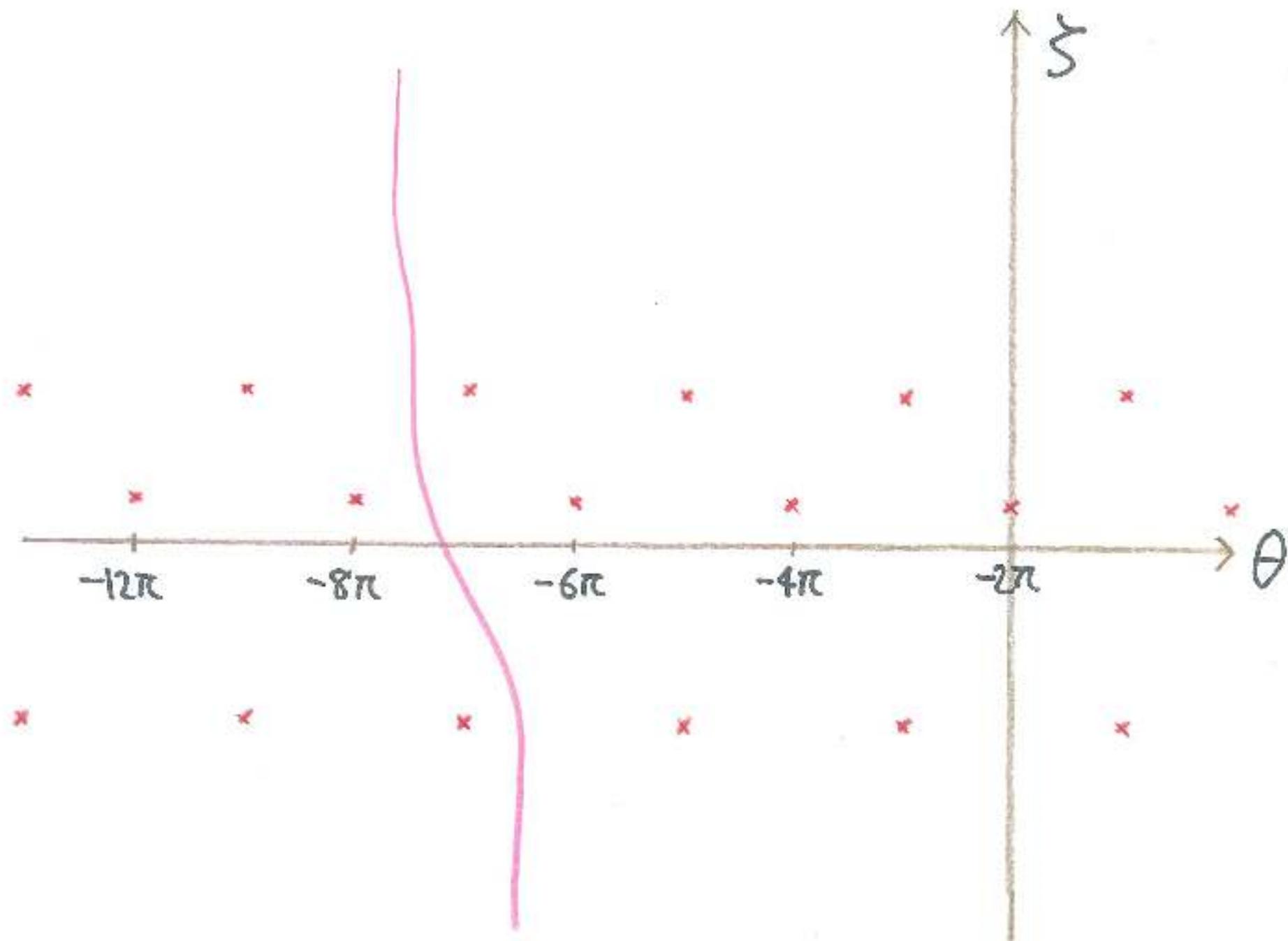
Paths between $\zeta \gg 0$ & $\zeta \ll 0$



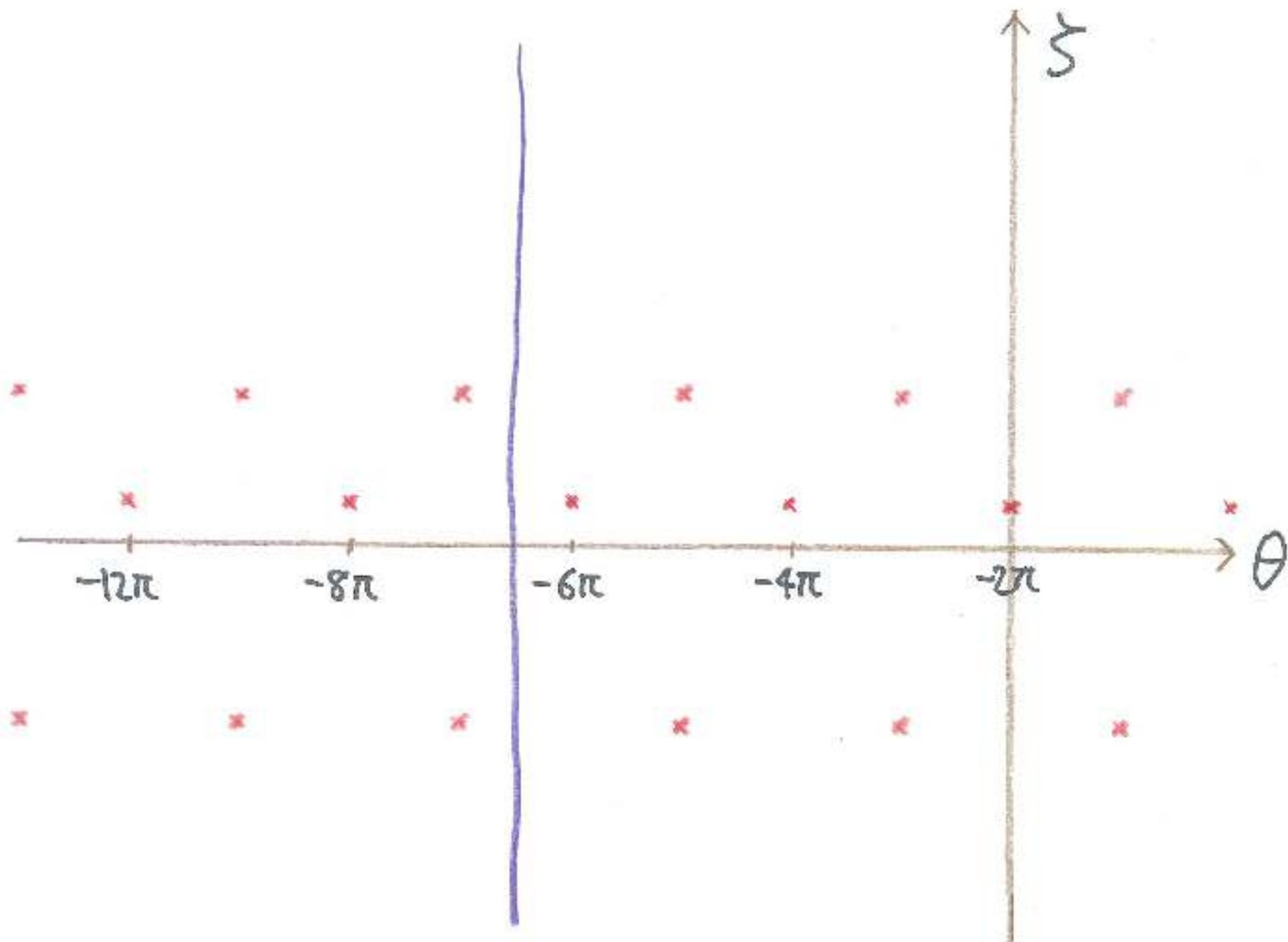
Paths between $\zeta \gg 0$ & $\zeta \ll 0$



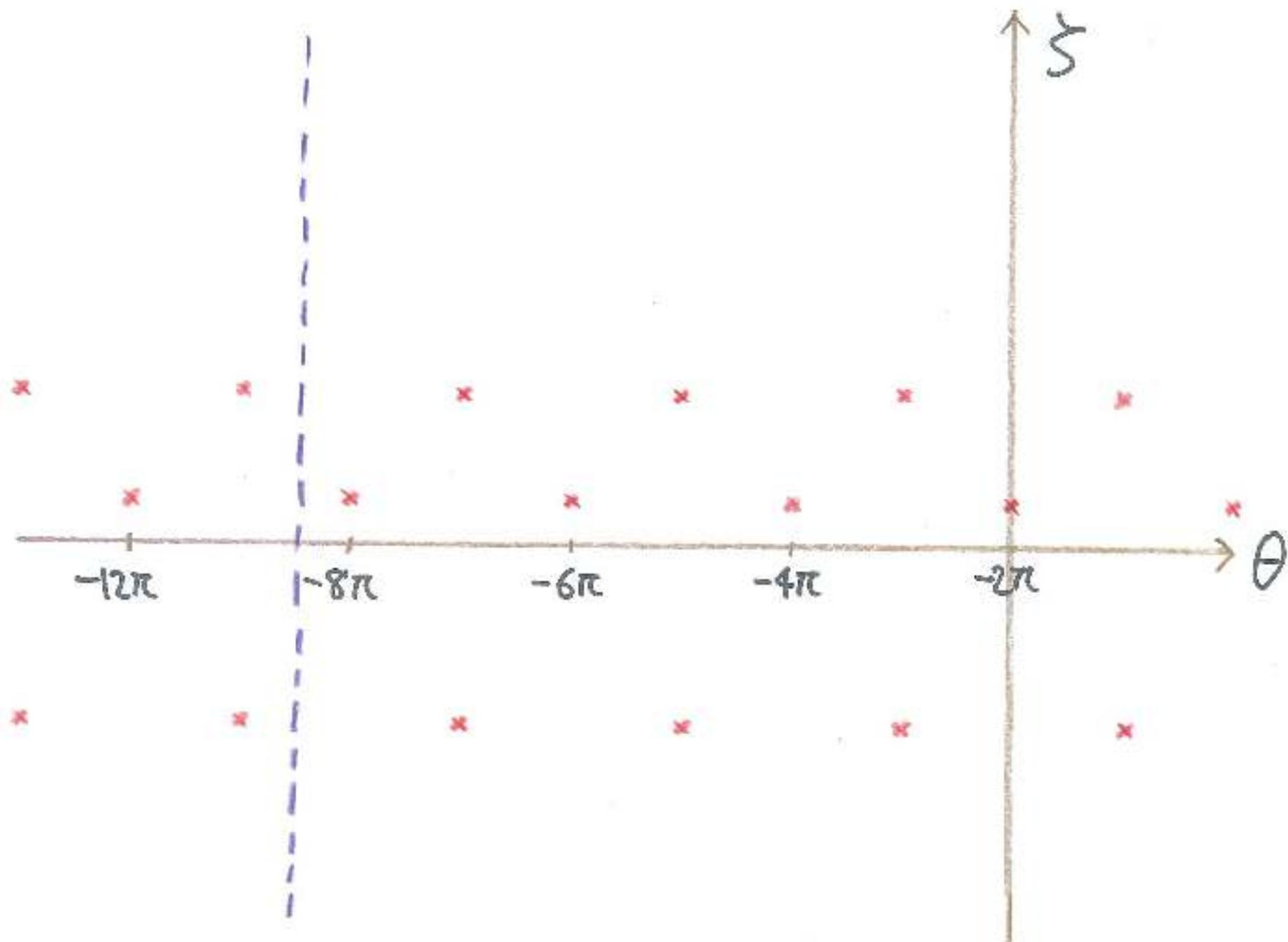
Paths between $\zeta \gg 0$ & $\zeta \ll 0$

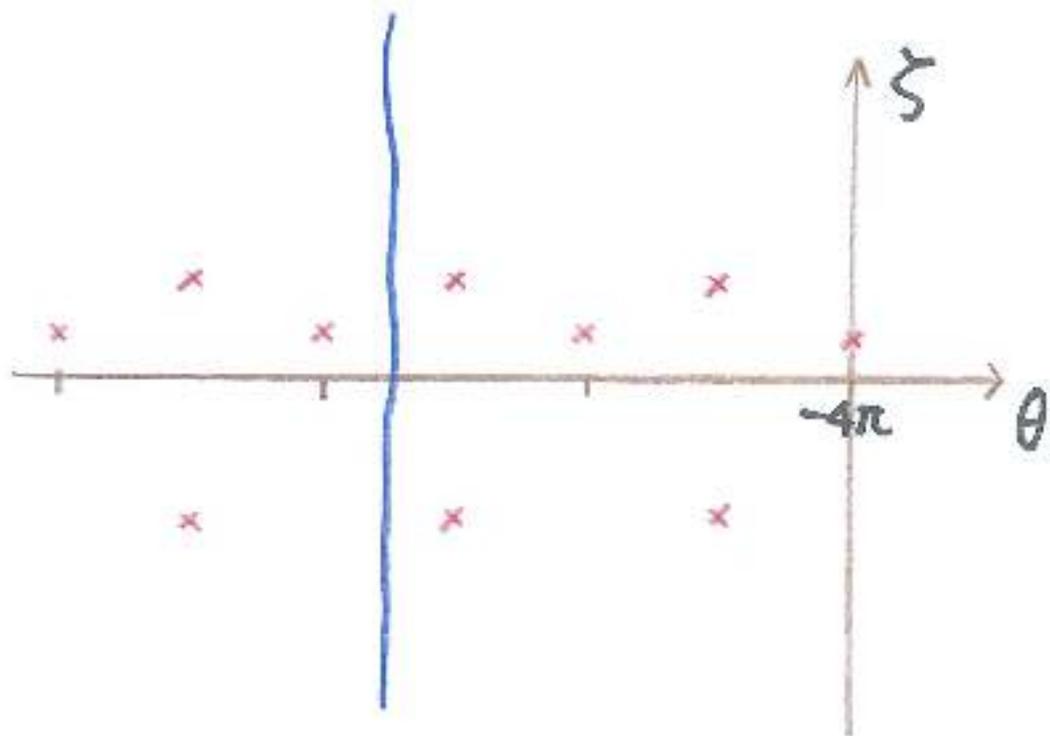


Paths between $\zeta \gg 0$ & $\zeta \ll 0$

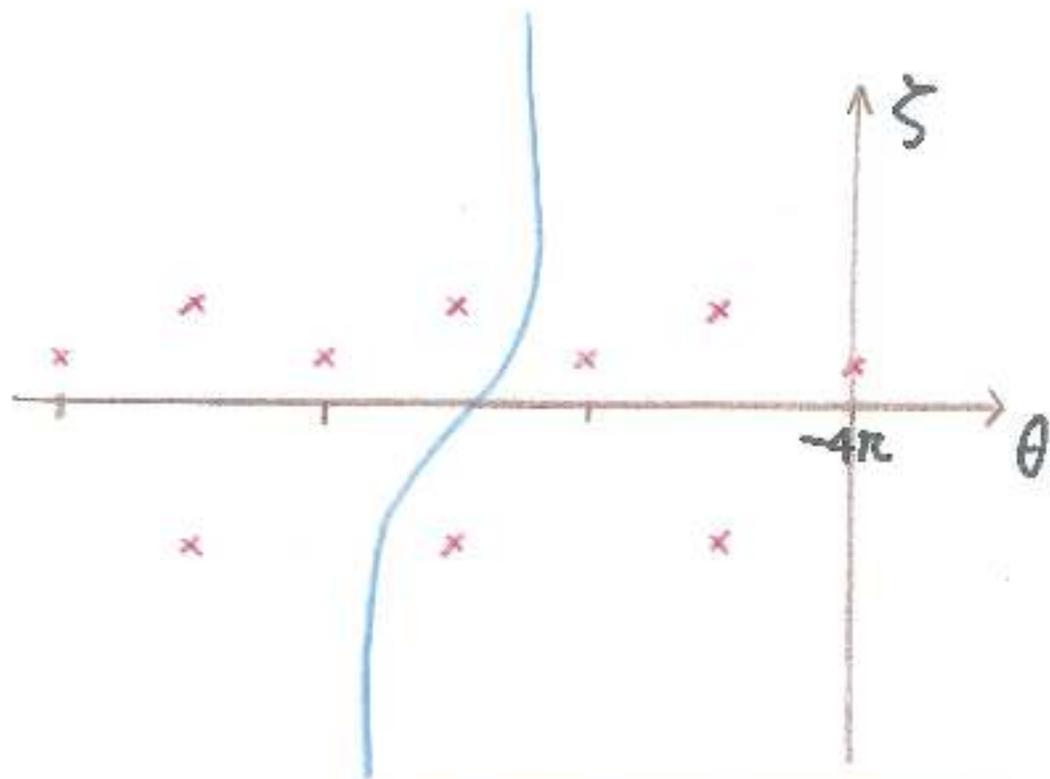


Paths between $\zeta \gg 0$ & $\zeta \ll 0$

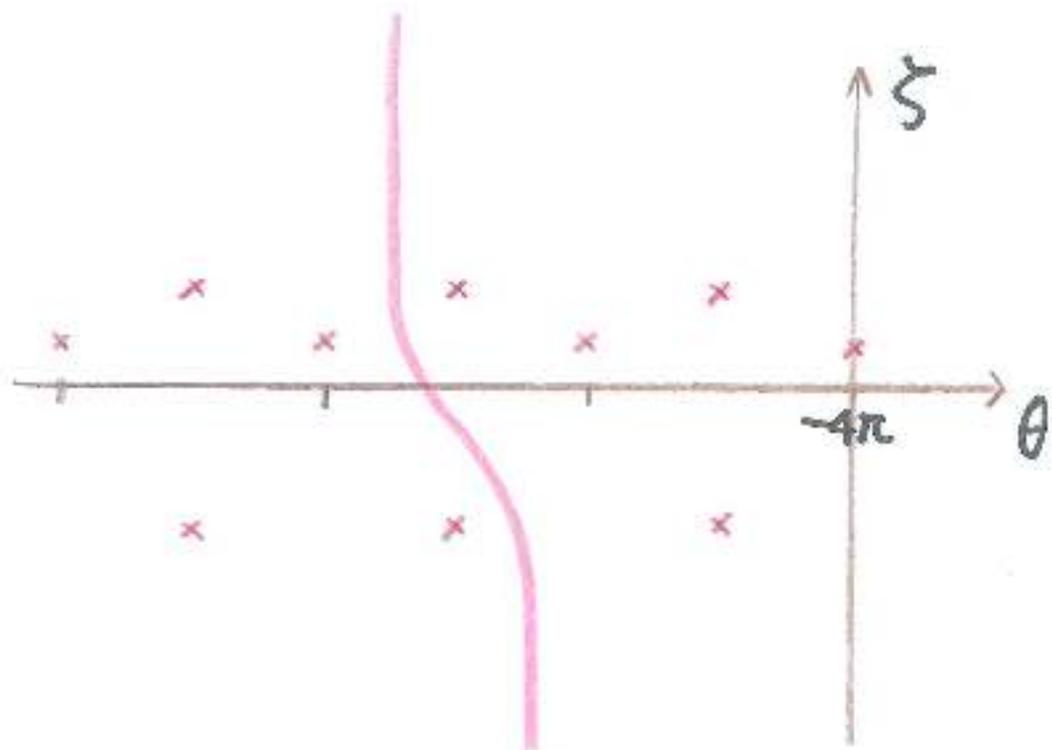




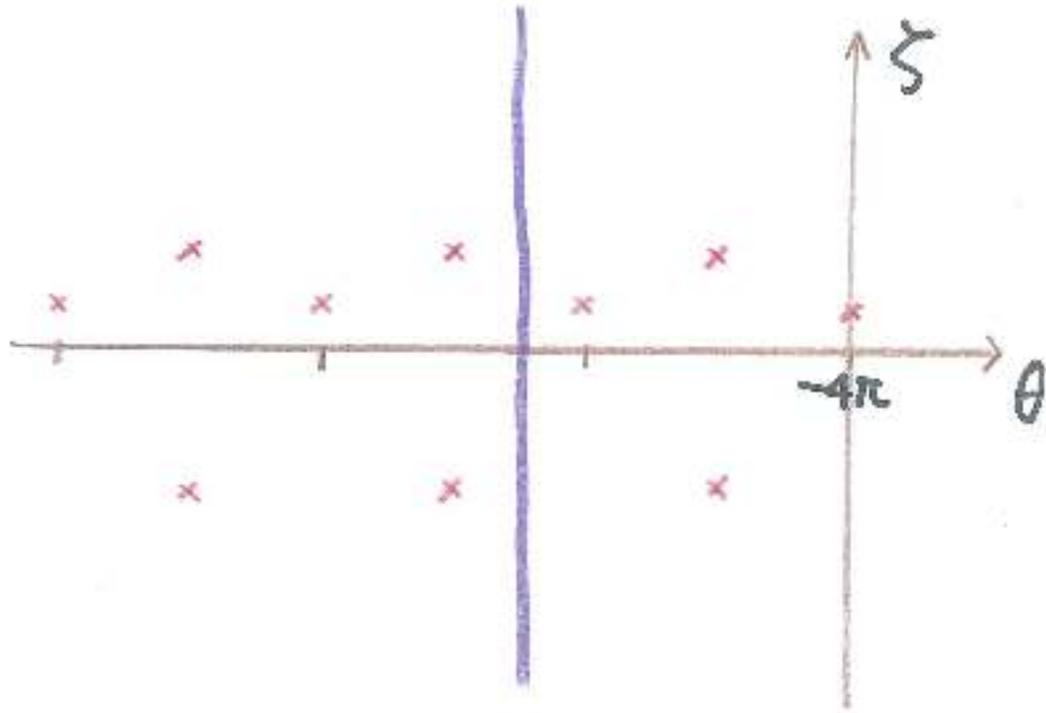
| | | | | | | | | | | | | | |
|-----|-----------|-----------|-------|----------|----------|----------|----------|----------|----------|----------|----------|----------|-----|
| ... | $C(-2)$ | $C(-1)$ | C | $C(1)$ | $C(2)$ | $C(3)$ | $C(4)$ | $C(5)$ | $C(6)$ | $C(7)$ | $C(8)$ | $C(9)$ | ... |
| ... | $S(-2)$ | $S(-1)$ | S | $S(1)$ | $S(2)$ | $S(3)$ | $S(4)$ | $S(5)$ | $S(6)$ | $S(7)$ | $S(8)$ | $S(9)$ | ... |
| ... | $S^2(-2)$ | $S^2(-1)$ | S^2 | $S^2(1)$ | $S^2(2)$ | $S^2(3)$ | $S^2(4)$ | $S^2(5)$ | $S^2(6)$ | $S^2(7)$ | $S^2(8)$ | $S^2(9)$ | ... |



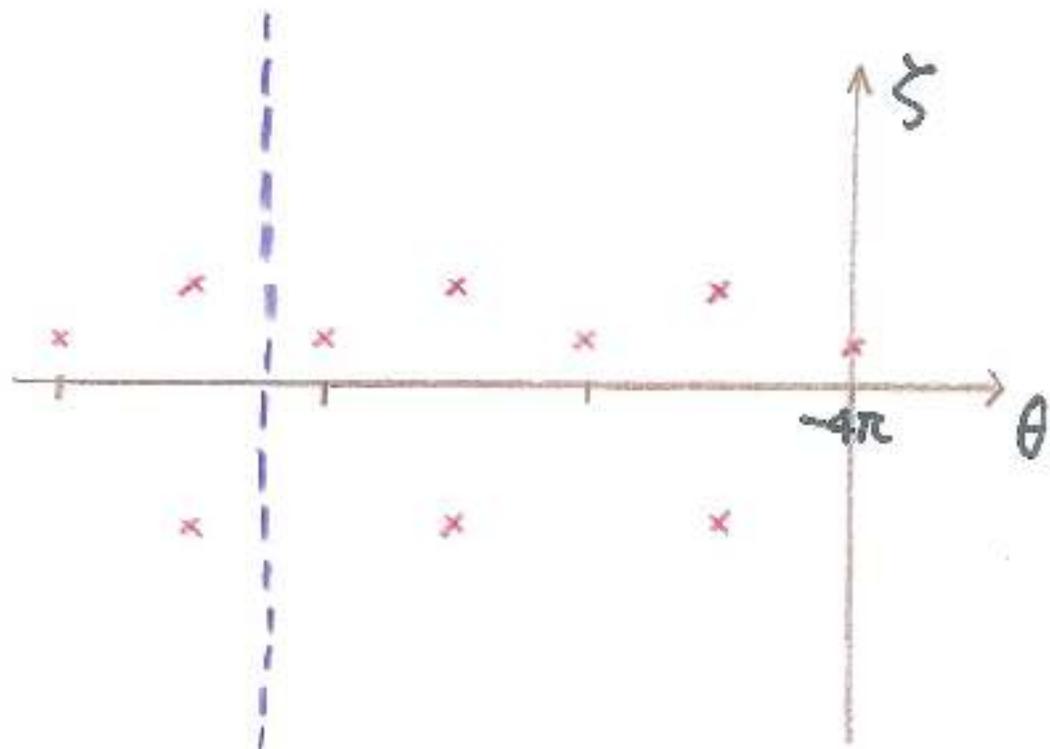
| | | | | | | | | | | | | | |
|-----|-----------|-----------|-------|----------|----------|----------|----------|----------|----------|----------|----------|----------|-----|
| ... | $C(-2)$ | $C(-1)$ | C | $C(1)$ | $C(2)$ | $C(3)$ | $C(4)$ | $C(5)$ | $C(6)$ | $C(7)$ | $C(8)$ | $C(9)$ | ... |
| ... | $S(-2)$ | $S(-1)$ | S | $S(1)$ | $S(2)$ | $S(3)$ | $S(4)$ | $S(5)$ | $S(6)$ | $S(7)$ | $S(8)$ | $S(9)$ | ... |
| ... | $S^2(-2)$ | $S^2(-1)$ | S^2 | $S^2(1)$ | $S^2(2)$ | $S^2(3)$ | $S^2(4)$ | $S^2(5)$ | $S^2(6)$ | $S^2(7)$ | $S^2(8)$ | $S^2(9)$ | ... |



| | | | | | | | | | | | | | |
|-----|-----------|-----------|-------|----------|----------|----------|----------|----------|----------|----------|----------|----------|-----|
| ... | $C(-2)$ | $C(-1)$ | C | $C(1)$ | $C(2)$ | $C(3)$ | $C(4)$ | $C(5)$ | $C(6)$ | $C(7)$ | $C(8)$ | $C(9)$ | ... |
| ... | $S(-2)$ | $S(-1)$ | S | $S(1)$ | $S(2)$ | $S(3)$ | $S(4)$ | $S(5)$ | $S(6)$ | $S(7)$ | $S(8)$ | $S(9)$ | ... |
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| | | | | | | | | | | | | | |
|-----|-----------|-----------|-------|----------|----------|----------|----------|----------|----------|----------|----------|----------|-----|
| ... | $C(-2)$ | $C(-1)$ | C | $C(1)$ | $C(2)$ | $C(3)$ | $C(4)$ | $C(5)$ | $C(6)$ | $C(7)$ | $C(8)$ | $C(9)$ | ... |
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In $\dot{s} \gg 0$, any (M, Q) is OK.

M may have components \notin 

What happens if we transport it to $\dot{s} \ll 0$ along \downarrow ?

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$\exists! (M', Q')$ s.t. ① Components (M') \in 
② $(M, Q) \cong (M', Q')$
at low energies in $\hbar \gg 0$.

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What happens if we transport it to $\zeta \ll 0$ along \downarrow ?

$\exists!$ (M', Q') s.t. ① Components (M') \in 

② $(M, Q) \cong (M', Q')$

at low energies in $\zeta \gg 0$.

(M', Q') can be safely transported along \downarrow .

(M', Q') at $\zeta \ll 0$ is the result of the transport.

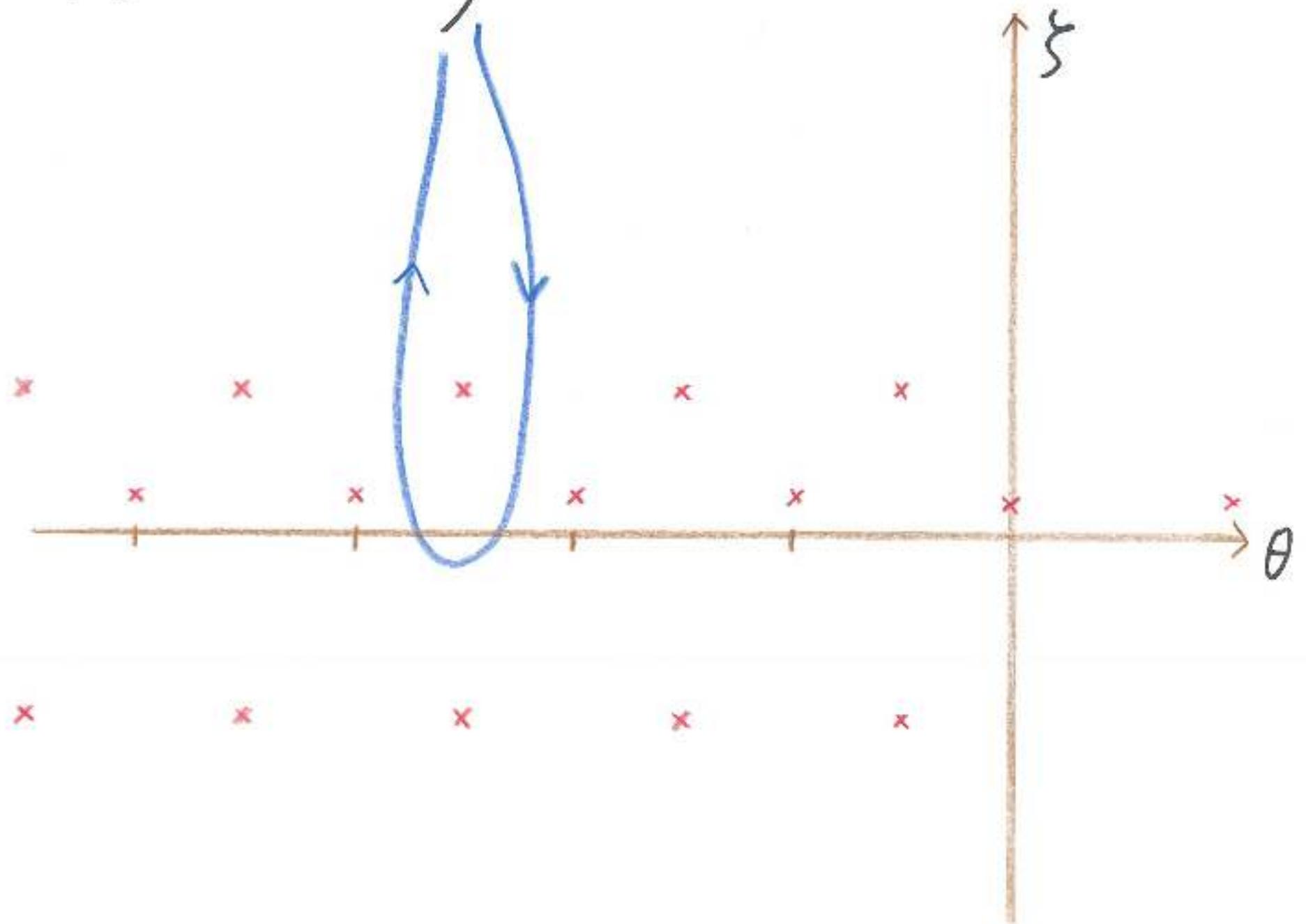
- Replacement $(M, Q) \rightarrow (M', Q')$ works also for the other sets , , ,
- This holds also in the $\zeta \ll 0$ phase.

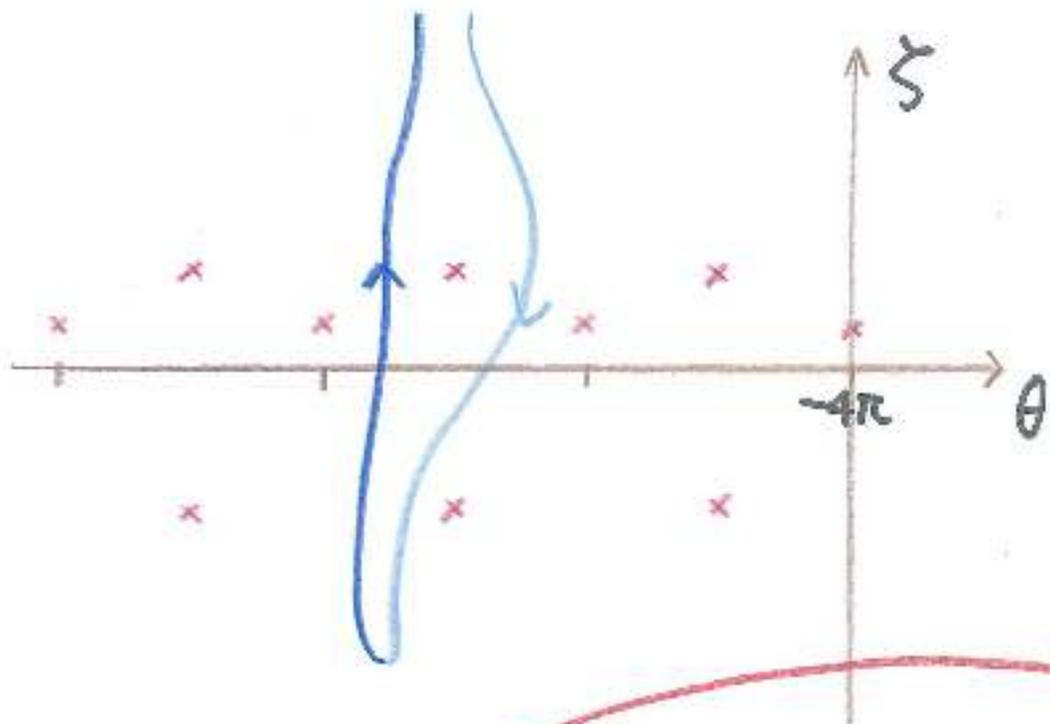
The Key : Empty branes

$\zeta \gg 0$ twisted Lascoux cplx $(X) \rightsquigarrow$ exact on $G(2,7)$

$\zeta \ll 0$ Koszul complex $(P) \rightsquigarrow$ exact on $\mathbb{C}P^6$

Monodromy





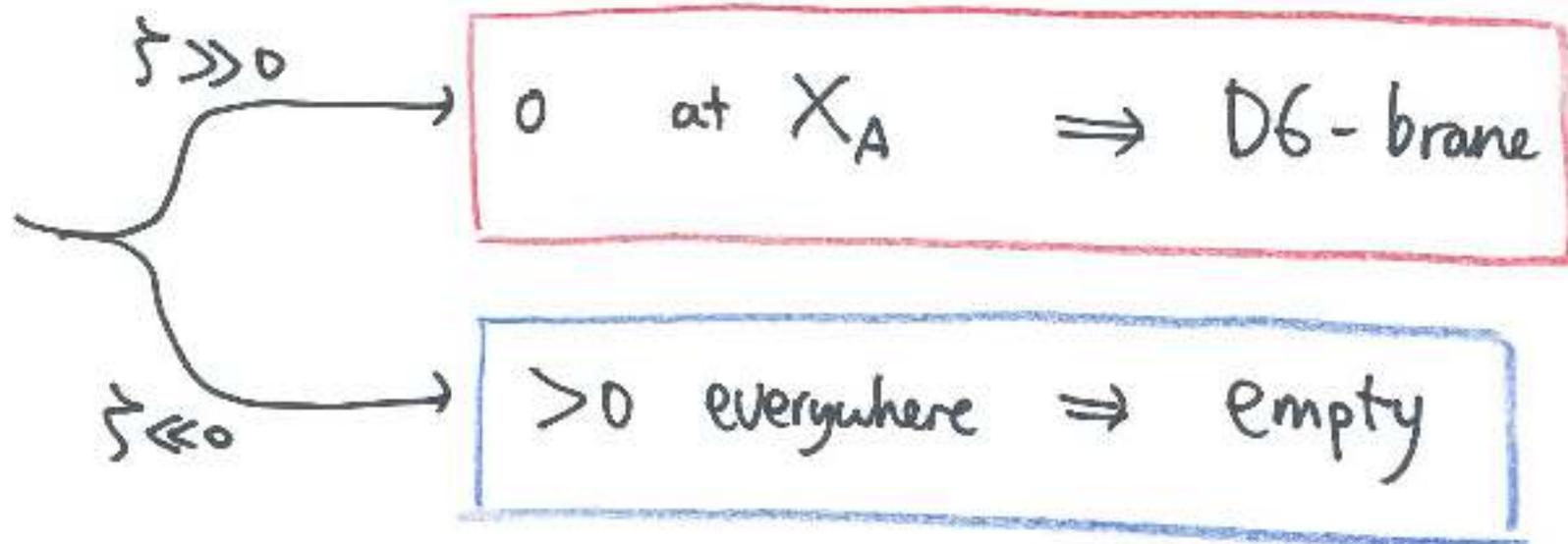
| | | | | | | | | | | | | | |
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| ... | $S(-2)$ | $S(-1)$ | S | $S(1)$ | $S(2)$ | $S(3)$ | $S(4)$ | $S(5)$ | $S(6)$ | $S(7)$ | $S(8)$ | $S(9)$ | ... |
| ... | $S^2(-2)$ | $S^2(-1)$ | S^2 | $S^2(1)$ | $S^2(2)$ | $S^2(3)$ | $S^2(4)$ | $S^2(5)$ | $S^2(6)$ | $S^2(7)$ | $S^2(8)$ | $S^2(9)$ | ... |

The empty brane at $\zeta \ll 0$ that does $\mathbb{C} \rightsquigarrow \mathbb{C}(7)$:

p-Koszul

$$Q = \sum_k \left(p^k \eta_k + \left(\sum_{i,j} A_k^{ij} [X_i X_j] \right) \bar{\eta}^k \right)$$

$$\{Q, Q^+\} = \sum_k |p^k|^2 + \sum_k \left| \sum_{i,j} A_k^{ij} [X_i X_j] \right|^2$$

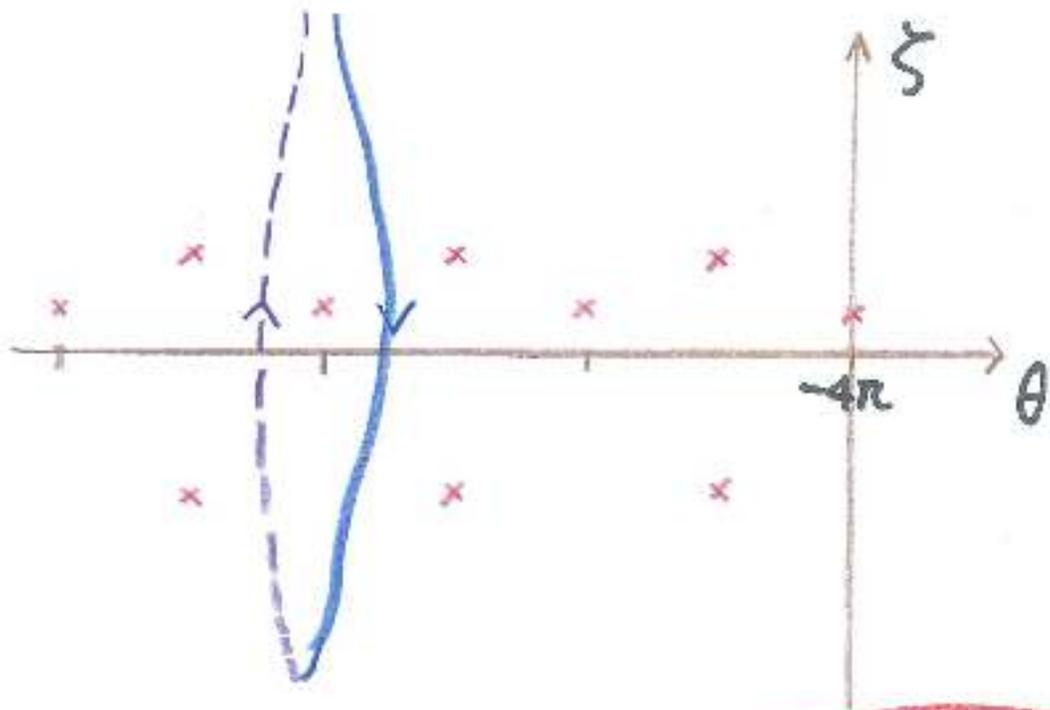


The result :

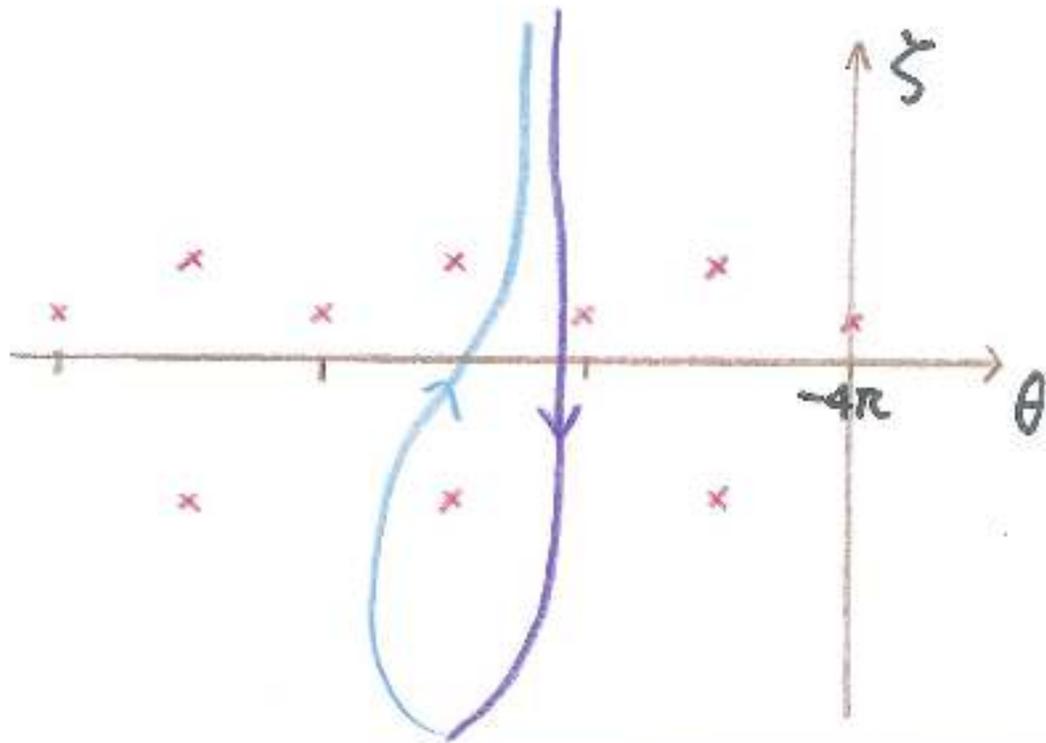


Binding a # of copies of
the D6-brane \mathcal{O}_{XA}

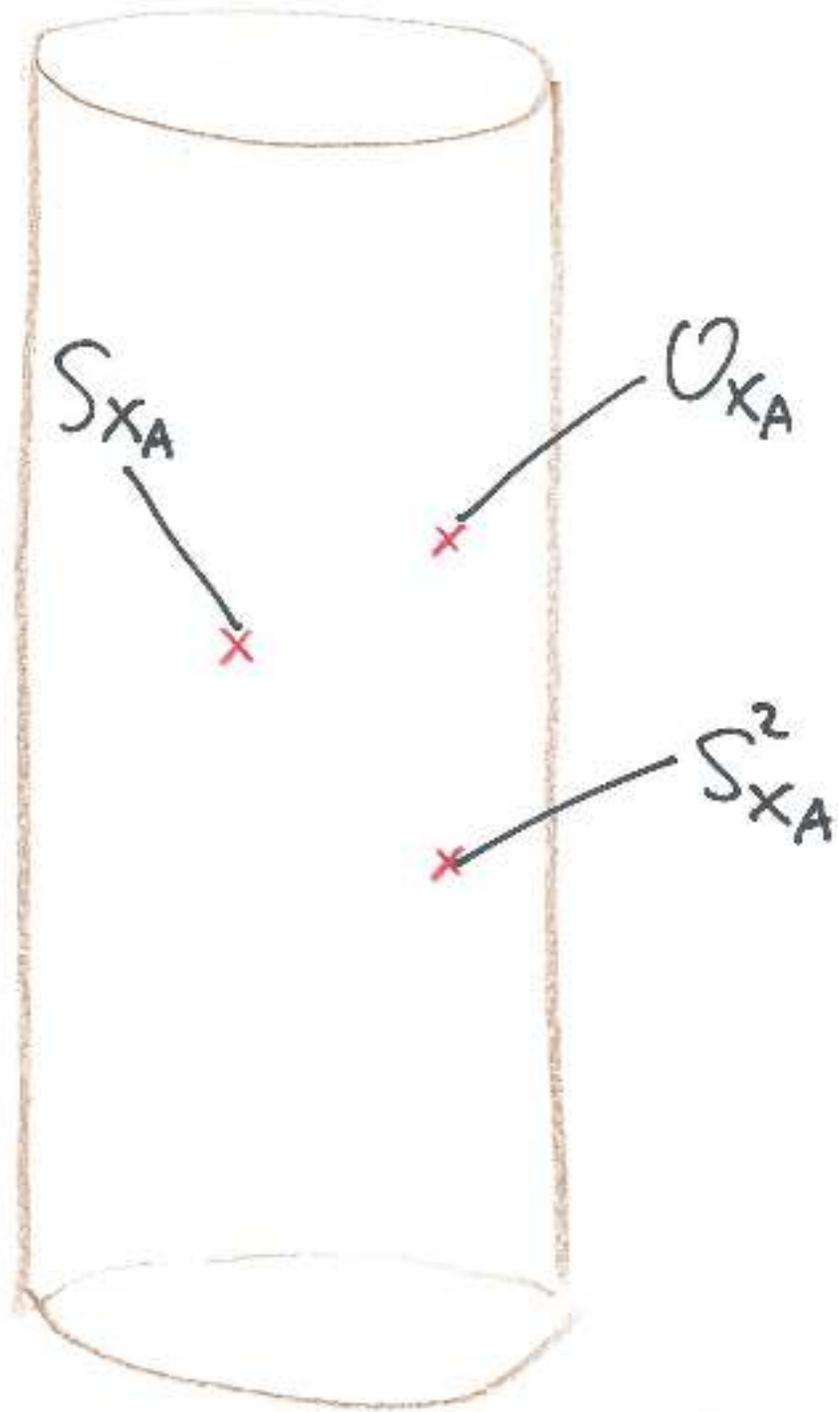
The brane that is massless
at this \otimes



| | | | | | | | | | | | | | |
|-----|-----------|-----------|-------|----------|----------|----------|----------|----------|----------|----------|----------|----------|-----|
| ... | $C(-2)$ | $C(-1)$ | C | $C(1)$ | $C(2)$ | $C(3)$ | $C(4)$ | $C(5)$ | $C(6)$ | $C(7)$ | $C(8)$ | $C(9)$ | ... |
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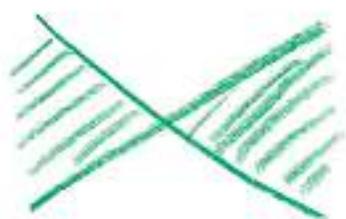


A Key point of derivation of GRR

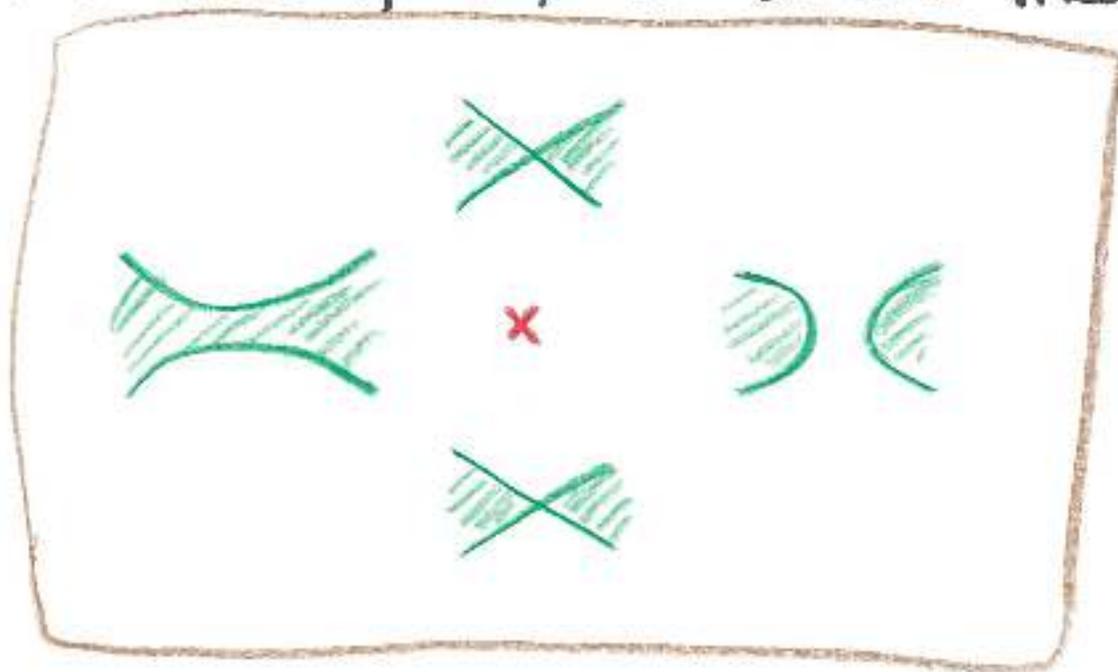
$$\text{Integrand} \sim e^{-i \tilde{W}_{\text{eff}}(\sigma)}$$

- The branch cut of \tilde{W}_{eff} is determined by the Chan-Paton repr.
- Exactly at x , \tilde{W}_{eff} is critical on Coulomb branch.
- But $\tilde{W}_{\text{eff}} \neq 0$ there for generic CP repr.

- $\widetilde{W}_{\text{eff}} \equiv 0$ on a domain of the Coulomb branch
for the distinguished pair of CP repr.

- $\text{Re}(i\widetilde{W}_{\text{eff}}) > 0$ is like 

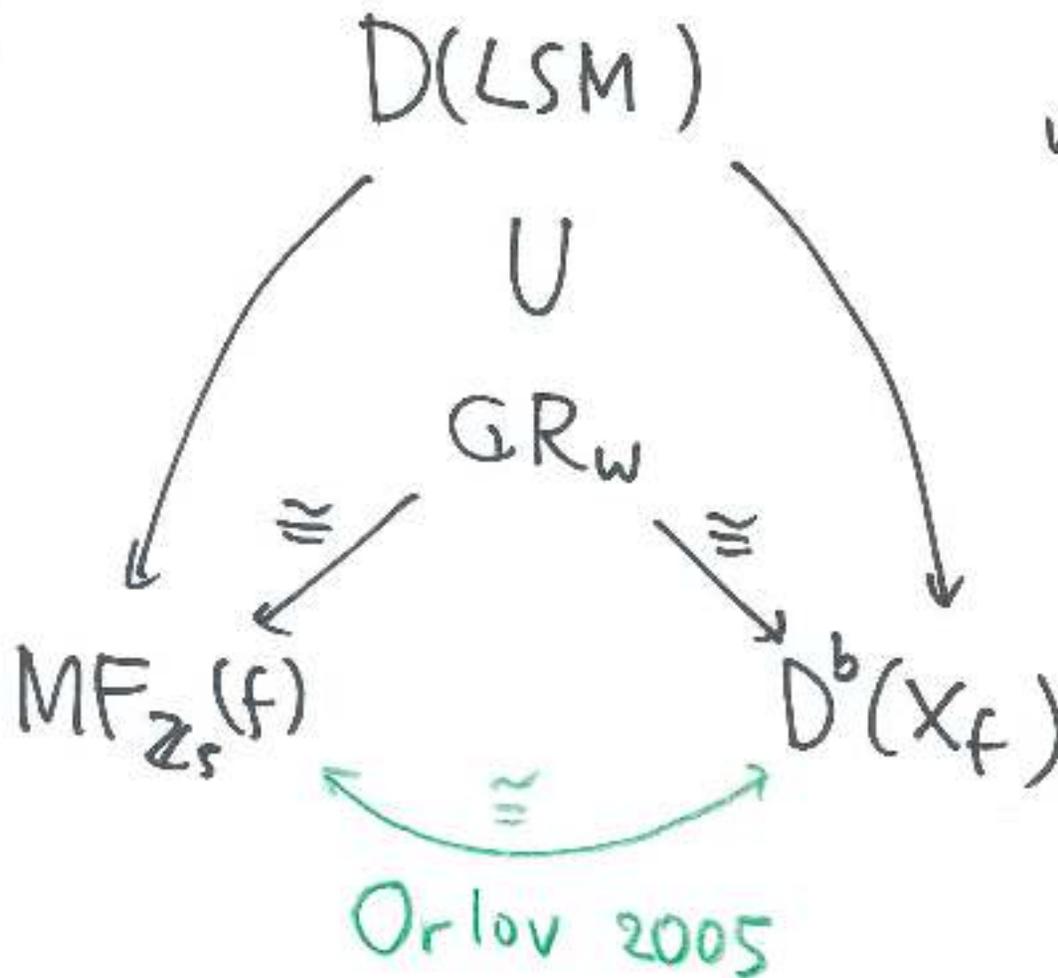
- In a nhd of \times , it behaves like



Math

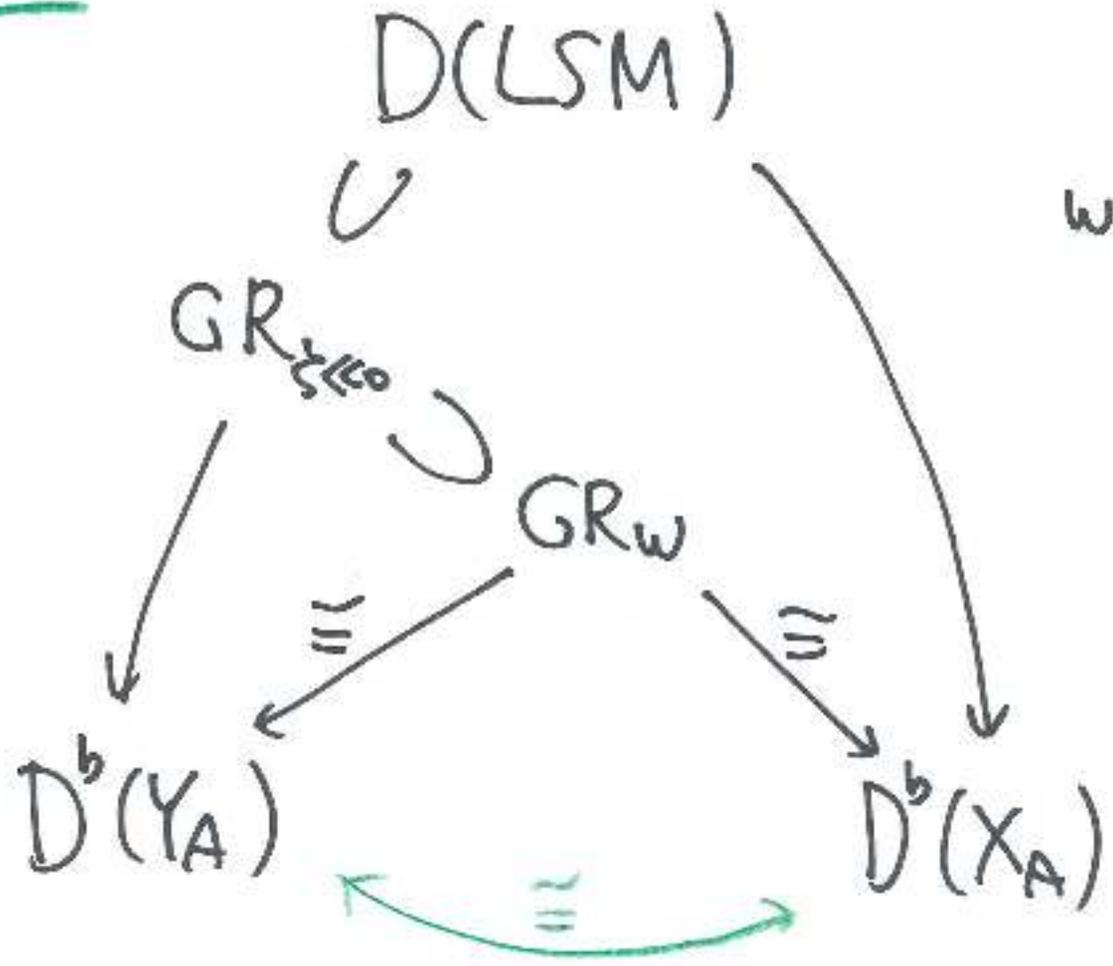
Mathematical Consequence

Quintic



W --- homotopy class
of paths

Rødland



$$w = |, |, |, |, \dots$$

Borisov-Caldararu

Kuznetsov 2006

H.P.D.

- The set  was first discovered by

Addington-Donovan-Segal 2014

and the present work is strongly encouraged
by that.

- The mathematical formulation / generalization
of GRR (in the classical phases)

is given by Segal 2009

Halpern-Leistner 2012

Ballard-Favera-Katzarkov 2012

Outlook

Generalizations of Rørdland

Math

Phys

2006

Kuznetsov

2007

Caldararu et al

2008

2009

2010

Kanazawa

2011

Hosono-Takagi

KH

2012

2013

Miura

H-Knapp

2014

Galkin

2015

Segal-Thomas

Gerhardus - Jockers

(Homological)

Projective Duality

Kuznetsov

(2d)

Seiberg Duality

KH 2011

