Brownian branes, emergent symmetries, and hydrodynamics

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[1502.00636], work in progress

[1412.1090], [1312.0610]

JUNE 23, 2015

Loganayagam's Talk
Thursday 02:30 (Parallel session I)

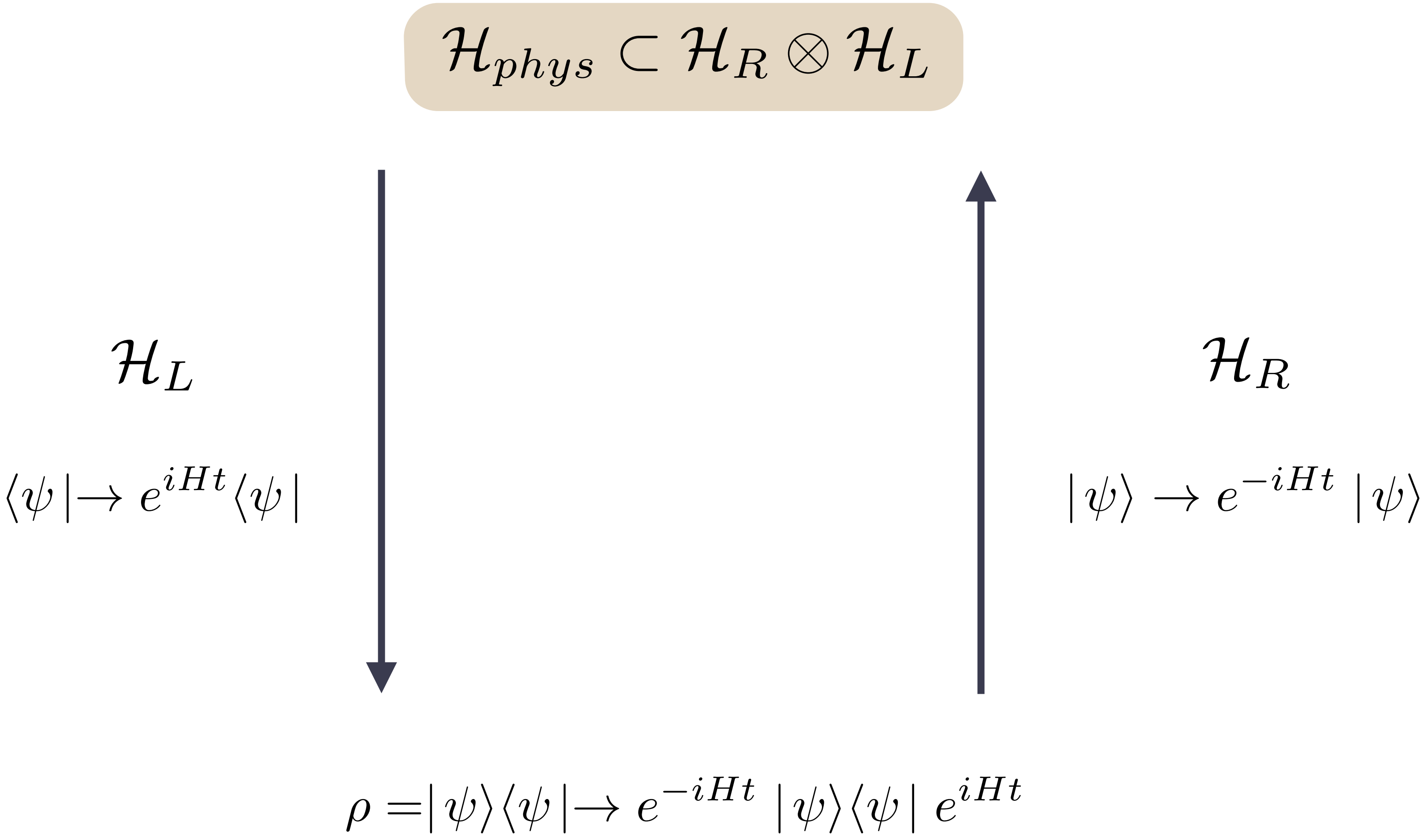
Motivation: non-equilibrium QFT dynamics

⁂ What is the correct Wilsonian treatment of low energy dynamics in mixed states of a QFT?

- There is a reasonably good phenomenological understanding, but the theoretical underpinnings are not yet fully understood.
- The entanglement of the system with an external reservoir is central to the discussion.
- There are many reasons to be interested in this question:
  - intrinsic interest from QFT and many-body physics standpoint.
  - dynamics of black holes via AdS/CFT.
  - cosmology.

A microscopic perspective

- **Doubling**: Mixed states of a QFT can be purified by introducing an ancillary system. Focus on pure states in tensor product Hilbert space.
- Central to the Schwinger-Keldysh formalism developed to compute real time correlation functions in QFTs.



$$\mathcal{H}_{phys} \subset \mathcal{H}_R \otimes \mathcal{H}_L$$

$\mathcal{H}_L$

$\langle\psi| \to e^{iHt}\langle\psi|$

$\mathcal{H}_R$

$|\psi\rangle \to e^{-iHt}\,|\psi\rangle$

$$\rho = |\psi\rangle\langle\psi| \to e^{-iHt}\,|\psi\rangle\langle\psi|\,e^{iHt}$$

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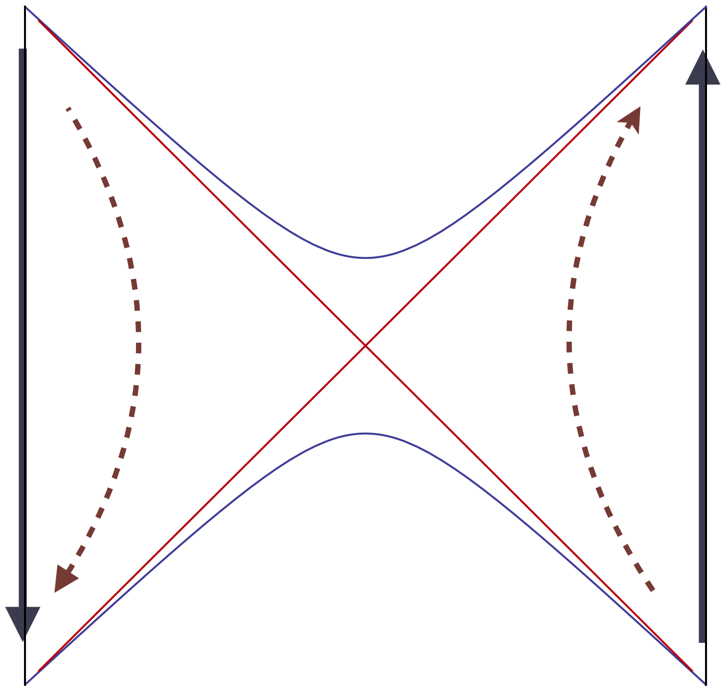

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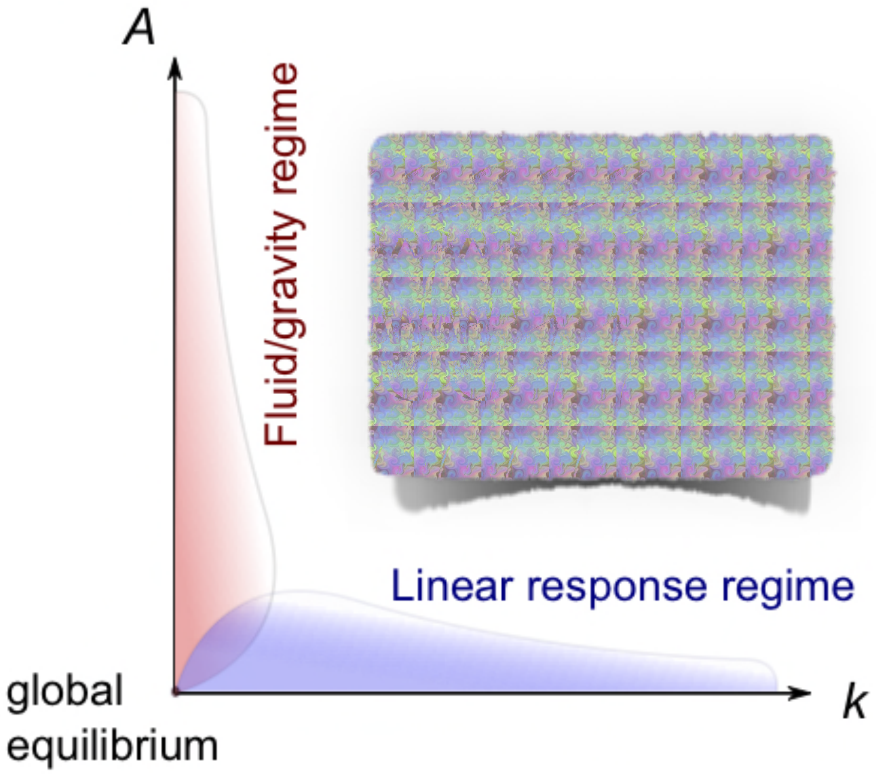
Macroscopic phenomenology

- Equilibrium dynamics can be understood by working with Euclidean generating functions, etc..
- Linear fluctuations are captured by Schwinger-Keldysh, while long-wavelength fluctuations are described by hydrodynamic effective field theory.
- General non-equilibrium dynamics is theoretical terra incognita.

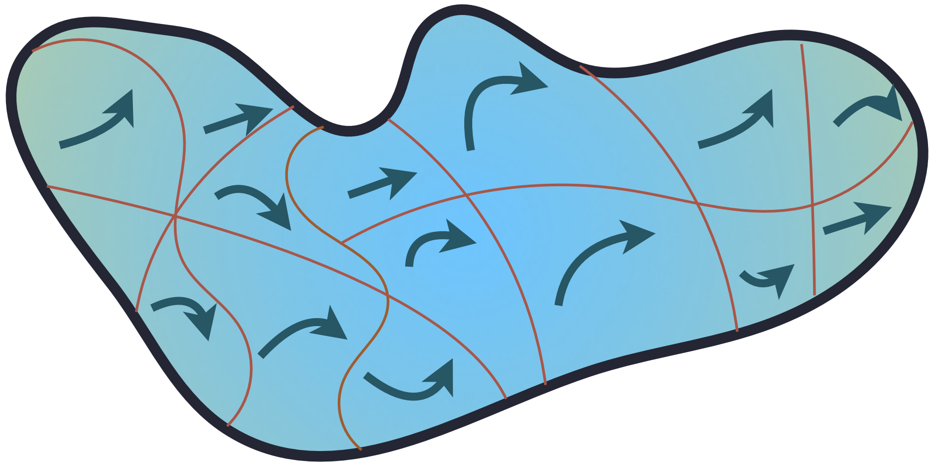


- Integrating out high energy modes starting from microscopic Schwinger-Keldysh leads to coupling between L and R encoded in *influence functionals.*

Feynman, Vernon '63

- What influence functionals are consistent with microscopic unitarity?

Hydrodynamics I: macroscopic fields



- Hydrodynamics describes near-equilibrium dynamics, capturing long-wavelength fluctuations about a Gibbs density matrix.

$$\ell_{\text{mfp}} \ll L\,, \qquad t_{\text{mfp}} \ll t$$

- The doubled microscopic variables are replaced by collective coordinates Ψ:
  - temperature and chemical potential and a flux vector (fluid velocity) $\quad T, \mu, u^\mu, \qquad u^\mu\, u_\mu = -1$
  - background metric and electromagnetic potential $\quad g_{\mu\nu}, A_\mu$

$$\boldsymbol{\beta}^\mu \equiv \frac{u^\mu}{T}\,, \qquad \Lambda_{\boldsymbol{\beta}} \equiv \frac{\mu}{T} - \frac{u^\sigma}{T} A_\sigma$$

thermal vector

thermal twist

Hydrodynamics II: Constrained dynamics

- *Constitutive relations*: monitor conserved currents, energy momentum, charge, etc.. as functionals of the hydrodynamic fields.

- Dynamics is conservation modulo work and anomaly terms, subject to a constraint: local form of the second law of thermodynamics is upheld.

$$\mathcal{E}^{\mu}_{T} = \nabla_\nu T^{\mu\nu} - J_\nu \cdot F^{\mu\nu} - T^{\mu\perp}_{H} = 0 \qquad\qquad \mathcal{E}_J = D_\mu J^\mu - J^{\perp}_{H} = 0$$

work term

covariant anomalies

$$\exists \;\; J^\mu_S[\mathbf{\Psi}] : \;\; \forall \;\; \mathbf{\Psi}_{\text{on-shell}} \, , \qquad \nabla_\mu J^\mu_S[\mathbf{\Psi}] \geq 0$$

- Ample evidence from kinetic theory, fluid/gravity correspondence etc., that this is the correct macroscopic picture.

Entropy from an emergent symmetry

- A-priori the entropy current is curious; a current not associated with any underlying symmetry principle, but emergent at low energies.
- Clue from gravity: black hole entropy is a Noether charge. Iyer, Wald '94
- Posit existence of a macroscopic Abelian symmetry, *KMS gauge symmetry*, which couples to the entropy current.
- The symmetry is dynamical and Higgsed at the thermal scale, leading to physical effects such as entropy production etc..
- KMS gauge symmetry controls low energy influence functionals ensuring that they respect the second law.

Wherefrom KMS gauge symmetry?

- Q: What are the acceptable solutions to the axioms of hydrodynamics, i.e., what constitutive relations are consistent with the second law?

- **Theorem:** Hydrodynamic transport can be classified in an ***eightfold way.*** There are seven adiabatic classes and a class of dissipative transport. In addition we have a class of forbidden constitutive relations which can be determined by studying hydrostatic equilibrium.

- This theorem was proved by studying an off-shell reformulation of the second law using the *adiabaticity equation:*

$$\nabla_\mu J_S^\mu + \boldsymbol{\beta}_\mu \, \mathcal{E}_{_T}^\mu + (\Lambda_{\boldsymbol{\beta}} + \boldsymbol{\beta}^\alpha A_\alpha) \, \mathcal{E}_{_J} = \Delta \geq 0$$

Aside: Free energy current

- The structures are clearer if we introduce the *Gibbs free energy current*, switching from a microcanonical to grand-canonical language:

$$-\frac{\mathcal{G}^\sigma}{T} = J^\sigma_S + \boldsymbol{\beta}_\nu T^{\nu\sigma} + (\Lambda_{\boldsymbol{\beta}} + \boldsymbol{\beta}^\alpha A_\alpha) \cdot J^\sigma$$

- The off-shell second law encoded in the adiabaticity equation then reads

$$\nabla_\sigma \left(\frac{\mathcal{G}^\sigma}{T}\right) - \frac{\mathcal{G}^\perp_H}{T} = -\frac{1}{2}\, T^{\mu\nu} \delta_{\mathcal{B}} g_{\mu\nu} - J^\mu \cdot \delta_{\mathcal{B}} A_\mu + \Delta$$

$$\delta_{\mathcal{B}} g_{\mu\nu} \equiv \pounds_{\boldsymbol{\beta}} g_{\mu\nu} = \nabla_\mu \boldsymbol{\beta}_\nu + \nabla_\nu \boldsymbol{\beta}_\mu \,,$$

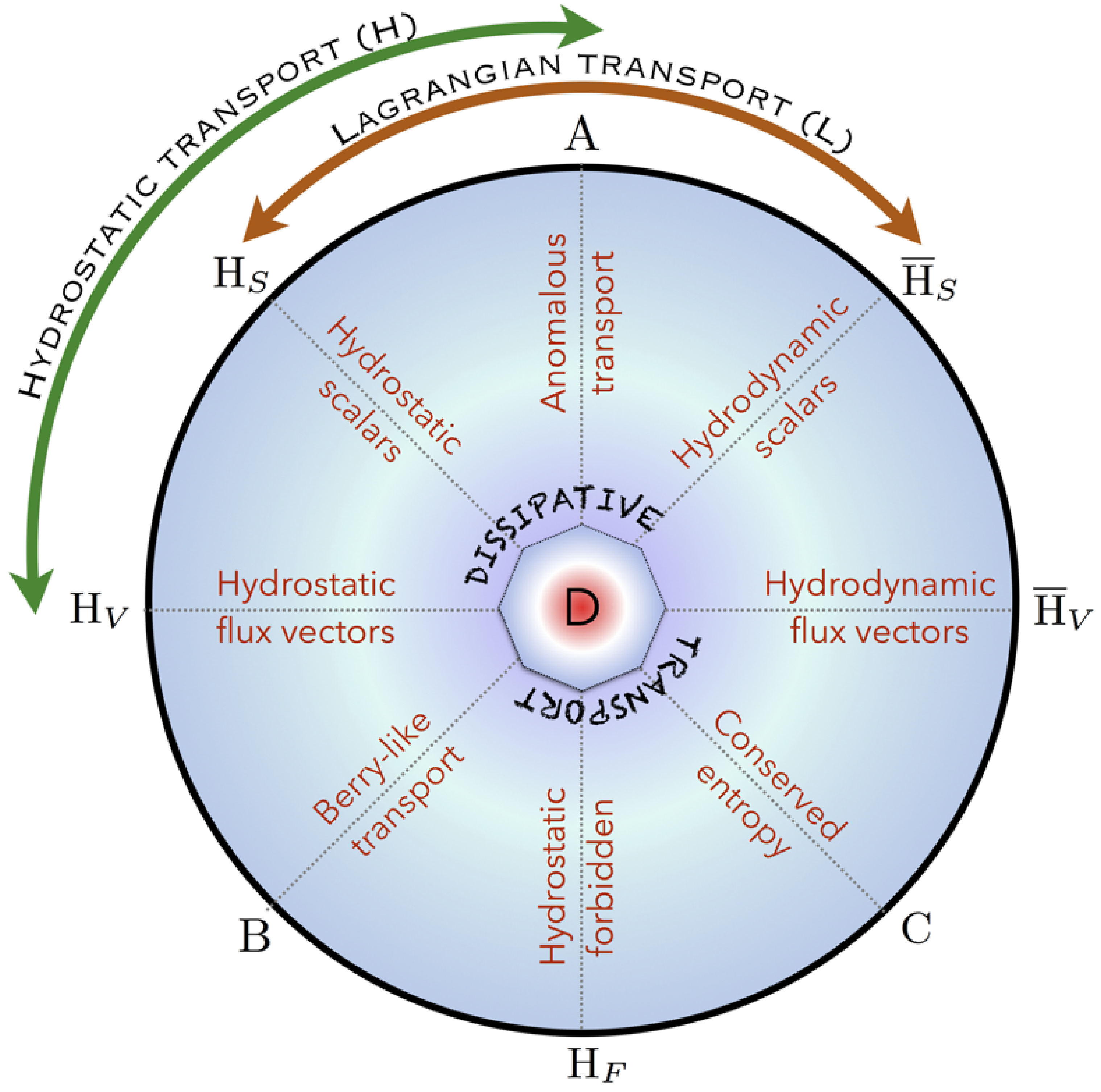
diffeomorphism

$$\delta_{\mathcal{B}} A_\mu \equiv \pounds_{\boldsymbol{\beta}} A_\mu + \partial_\mu \Lambda_{\boldsymbol{\beta}} + [A_\mu, \Lambda_{\boldsymbol{\beta}}]$$

flavour gauge transformation

along the thermal vector & twist.

Eightfold classification of hydrodynamic transport



✦ Second law:

✳ forbids HF.

✳ D terms sign-definite only at leading order.

S. Bhattacharyya ['13-'14]

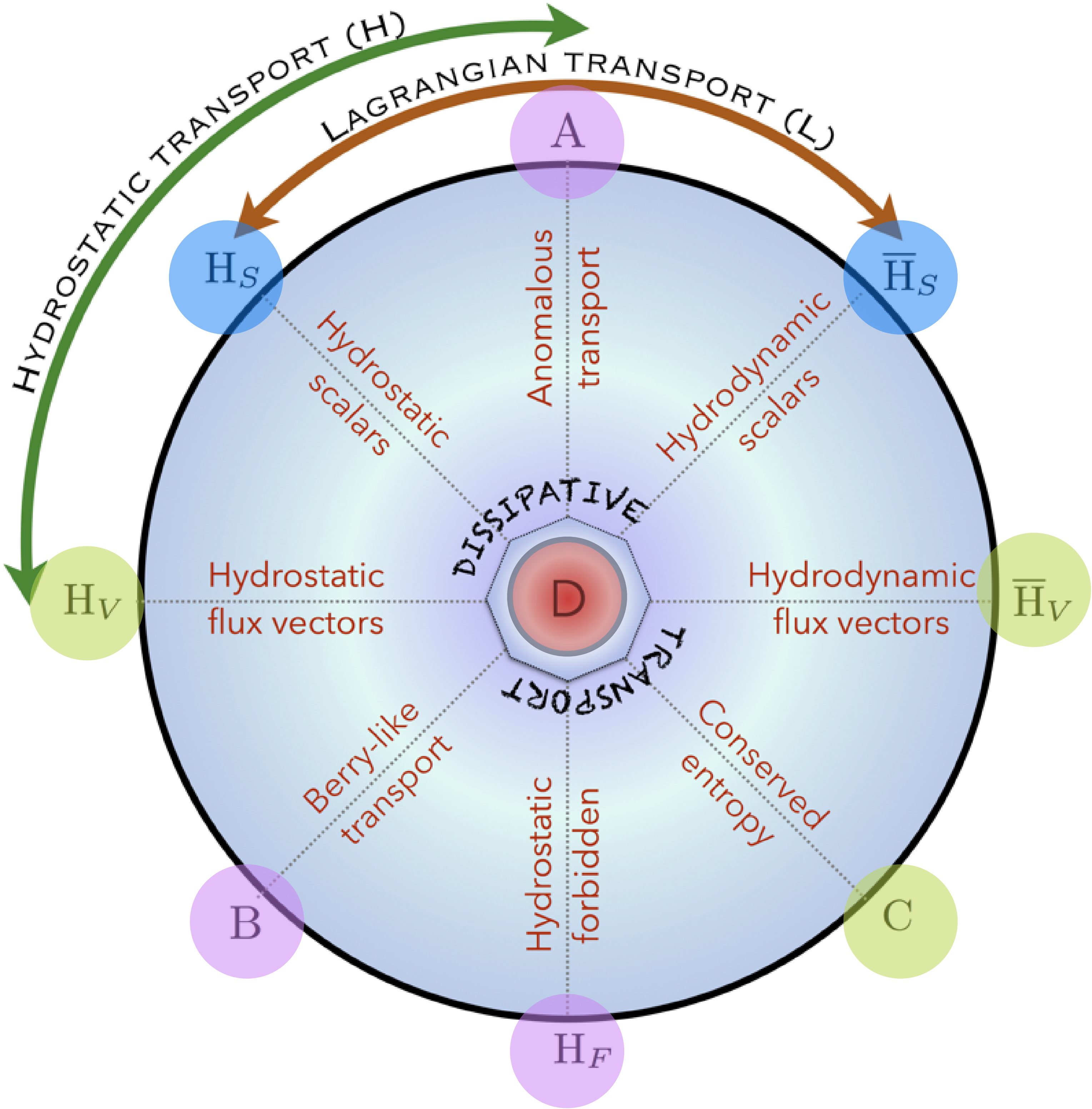
longitudinal vector

$$\mathcal{G}^\mu = \mathfrak{S}\,\beta^\mu + \mathfrak{V}^\mu$$

transverse vector

HLR ['14-'15]

Eightfold effective action?



✓ ?

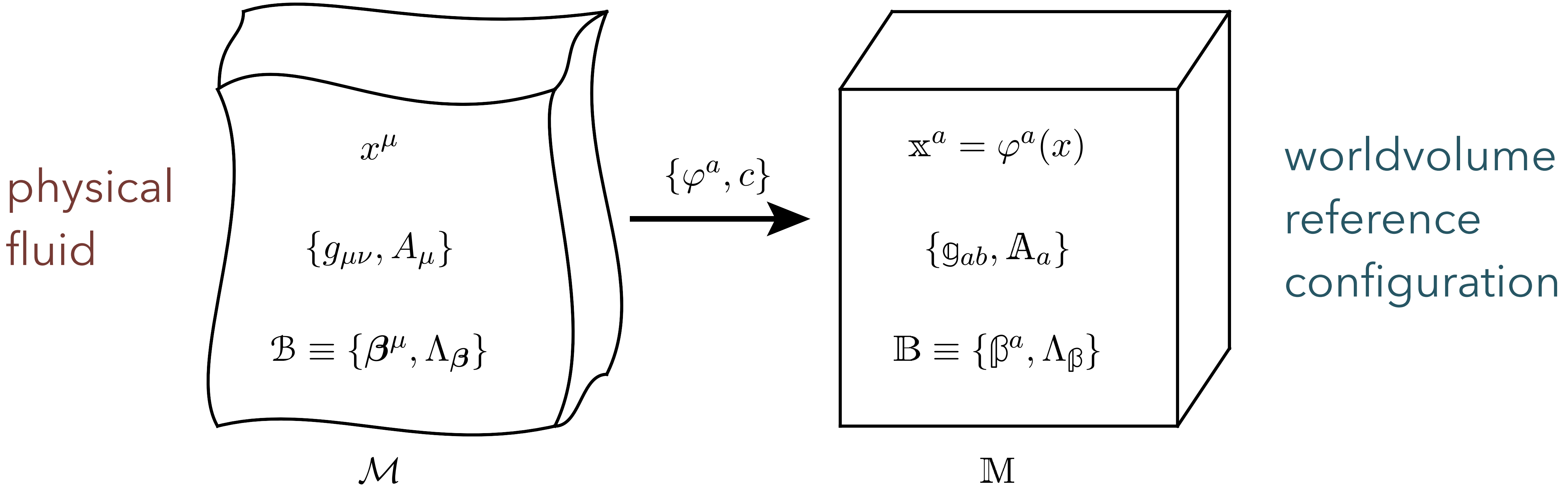
√̸ ???

B: Nicolis, Son ’11;
Haehl, MR ’13;
Geracie, Son ‘14

A: Dubovsky, Nicolis, Hui ’12;
Haehl, Loganayagam, MR ’13

Landau-Ginzburg sigma models

- Class L: effective action is just a sigma model parameterized by a scalar functional (free energy density) $\mathcal{L}[\Psi]$.
- *Adiabaticity equation:* Off-shell Bianchi identity from invariance under diffeomorphisms and flavour transformations.
- Dynamics: current conservation obtained from a constrained variational principle. Fix reference configuration & vary the pullback maps.



Symmetry from the eightfold way

- For the remaining 6 classes we took the microscopic Schwinger-Keldysh picture, and lessons from anomalous transport seriously.
- Empirically we stumbled upon a framework which captured all of the adiabatic transport in a single Lagrangian density (for the 7 classes).
- We however needed a symmetry principle to rule out H_F: *KMS invariance.*

the background sources	$\{g_{\mu\nu}, A_\mu\}$	
the fluid fields	$\{\boldsymbol{\beta}^\mu, \Lambda_{\boldsymbol{\beta}}\}$	
partners for the sources	$\{\tilde{g}_{\mu\nu}, \tilde{A}_\mu\}$	``Schwinger-Keldysh″ partners
KMS gauge field	$A^{(T)}_\mu$	ensures adiabaticity, forbids H_F

The Eightfold Lagrangian

- The Lagrangian density is actually very simple:

$$\mathcal{L}_T = \frac{1}{2}\, T^{\mu\nu}\, \tilde{g}_{\mu\nu} + J^{\mu} \cdot \tilde{A}_{\mu} - \frac{\mathcal{G}^{\sigma}}{T}\, A^{(T)}_{\sigma}$$

- It works to give precisely the desired seven classes and reduces in special cases to non-dissipative effective actions considered in the literature.

- The free energy current is the Noether current associated with the KMS flavour invariance.

- The linear couplings to the partners is highly suggestive of structures encountered in analysis of linear dissipative systems and topological sigma models.

- Take the symmetry seriously and attempt to work out a full theory including dissipation.

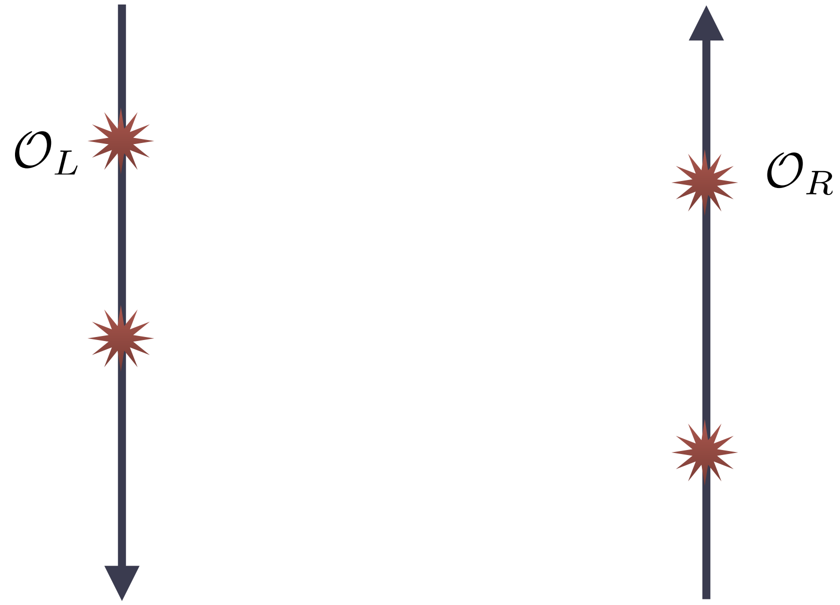
Schwinger-Keldysh doubling & symmetries: I

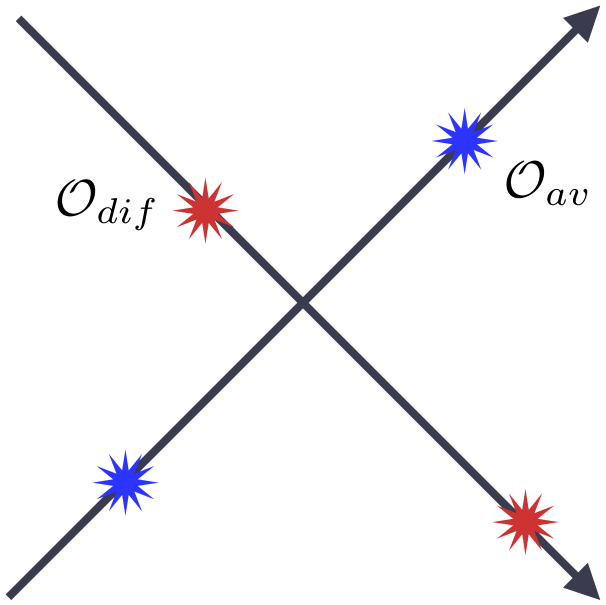
- Doubling fields (operators), sources, etc., implies some redundancy. Difference operators reside in a topological sector as their correlators vanish identically.

$$\langle \prod_I \mathcal{O}^I_{dif} \rangle = 0 \quad \Longrightarrow \quad \exists \;\; \mathcal{Q}_{SK} \, , \;\; \bar{\mathcal{Q}}_{SK}$$

$$\mathcal{O}_{av} = \frac{1}{2}\left(\mathcal{O}_R + \mathcal{O}_L\right)$$

$$\mathcal{O}_{dif} = \mathcal{O}_R - \mathcal{O}_L$$





$$\mathcal{O}_{dif} = \mathcal{Q}_{SK}(\cdots)$$

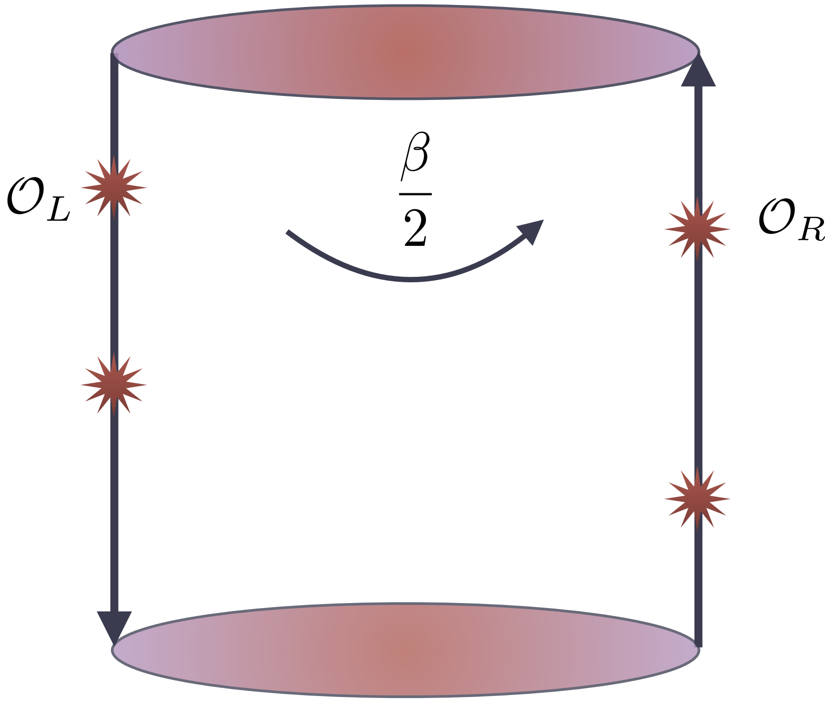
- This is a field redefinition symmetry ensuring that we get the correct time ordering prescription.

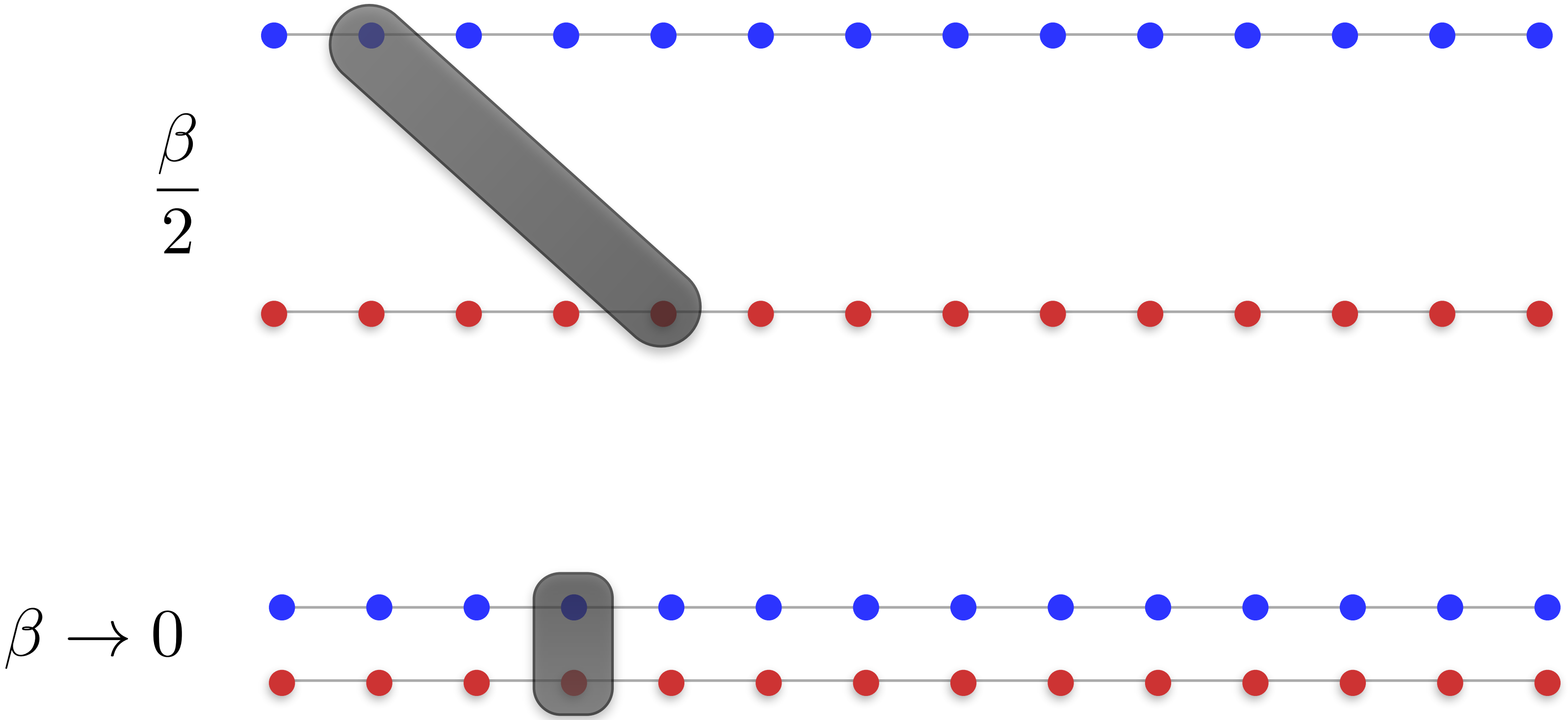
Schwinger-Keldysh doubling & symmetries: II

- A second topological symmetry arises from the KMS condition operating in equilibrium thermal systems (Euclidean periodicity)

$$\mathcal{O}_{ret} = (1+\mathfrak{f}_\beta)\, \mathcal{O}_R + \mathfrak{f}_\beta\, \mathcal{O}_L\,, \qquad \mathcal{O}_{adv} = \mathcal{O}_R - \mathcal{O}_L$$

- The symmetry is non-local, but approximate locality is attained in the high temperature limit.





$$\Longrightarrow \quad \exists\ \mathcal{Q}_{KMS}\,, \bar{\mathcal{Q}}_{KMS}$$

Brownian branes

- **Hydrodynamics:** low energy theory of spontaneously broken difference diffeomorphisms and flavour transformations, with emergent $U(1)_T$ symmetry.

- Brownian p-branes: Worldvolume dynamics captured by a gauged topological sigma model (balanced TQFT) incorporating:

Dijkgraaf, Moore '96

field redefinition supercharge $\mathcal{Q}_{SK}\,,\,\bar{\mathcal{Q}}_{SK}$

$\mathcal{Q}_{KMS}\,,\,\bar{\mathcal{Q}}_{KMS}$ KMS U(1) supercharge

- The equivariant cohomology construction for thermal diffeomorphisms and flavour transformations captures the topological sector of the theory.

Vafa, Witten '94

- Physical fluid observables can be constructed for space-filling Brownian branes by deforming the theory to include sources for the average fields.

Brownian particles

- A particularly simple case of the general set-up is a Brownian 0-brane, a particle, usually captured by the Langevin equation (with noise).

$$-\mathcal{E}_x \equiv m\,\ddot{x} + \eta\,\beta\,\dot{x} + \frac{\partial V}{\partial x} = \eta\,x_f\,, \qquad x_f \to \text{stochastic}$$

- This system is described by a Schwinger-Keldysh like an effective action

$$\begin{aligned}\mathcal{L}_{B_0} &= \tilde{x}\,\mathcal{E}_x + i\,\eta\,\tilde{x}^2 - i\,\bar{\psi}\,\frac{\partial\mathcal{E}_x}{\partial x}\,\psi \\ &= \frac{m}{2}\,\left(\dot{x}_R^2 - \dot{x}_L^2\right) + \mathcal{L}_{IF}(x_R, x_L) + \text{ghosts}\end{aligned}$$

Martin, Siggia, Rose '73

Mathai, Quillen '86

- This is a familiar topological field theory being described by the Morse theory supersymmetric quantum mechanics model.

Witten '82

- The general Bp-brane is more complicated but follows along similar lines.

Some consistency checks

- The gauge-fixed theory with BRST ghosts set to zero agrees with the eightfold effective action.
- The partner fields are Lagrange multiplier fields of BRST symmetry enforcing physical constitutive relations.
- For linear dissipative systems the macroscopic manifestation of KMS, viz., the fluctuation-dissipation theorem, is naturally incorporated in the Brownian brane theory (in BRST ghost kinetic terms).
- Supported by the picture of Brownian motion in AdS/CFT wherein one monitors the stochastic fluctuations of a quark in a hot plasma.

de Boer, Hubeny, MR, Shigemori ’08
Son, Teaney ‘09

Eightfold classification of physical fluids

- The stress tensor for a neutral conformal fluid in the eightfold basis is

$$\begin{aligned} T^{\mu\nu} = & \, p\,(d\,u^\mu u^\nu + g^{\mu\nu}) - \eta\,\sigma^{\mu\nu} \\ & + (\lambda_1 - \kappa)\,\sigma^{<\mu\alpha}\sigma_\alpha^{\ \nu>} + (\lambda_2 + 2\,\tau - 2\kappa)\;\sigma^{<\mu\alpha}\omega_\alpha^{\ \nu>} \\ & + \tau\left(u^\alpha \mathcal{D}^{\mathcal{W}}_\alpha \sigma^{\mu\nu} - 2\,\sigma^{<\mu\alpha}\omega_\alpha^{\ \nu>}\right) + \lambda_3\,\omega^{<\mu\alpha}\omega_\alpha^{\ \nu>} \\ & + \kappa\,\left(C^{\mu\alpha\nu\beta}\,u_\alpha\,u_\beta + \sigma^{<\mu\alpha}\sigma_\alpha^{\ \nu>} + 2\,\sigma^{<\mu\alpha}\omega_\alpha^{\ \nu>}\right). \end{aligned}$$

H_S

D

B

$\overline{H}_S$

- Explicit results for transport from holography, kinetic theory lend excellent support the eightfold classification.

Baier et. al.; Bhattacharyya et. al., '07

York, Moore '08

- Some interesting accidental(?) relations in Einstein gravity

$$\lambda_1 = \kappa\,, \qquad \lambda_2 = 2(\kappa - \tau)$$

Haack, Yarom '08

Holographic fluids

- Transport coefficients for holographic fluids (sans viscosity) up to second order can be obtained from a simple effective action:

$$\mathcal{L}^{\mathcal{W}} = c_{\text{eff}} \left(\frac{4\pi T}{d}\right)^{d} - c_{\text{eff}} \left(\frac{4\pi T}{d}\right)^{d-2} \left[\frac{{}^{\mathcal{W}}R}{(d-2)} + \frac{1}{2}\,\omega^2 + \frac{1}{d}\,\text{Harmonic}\left(\frac{2}{d}-1\right)\sigma^2\right]$$

- First principles derivation from gravitational dynamics?

Nickel, Son ‘10

de Boer et. al., Crossley et. al., ‘15

cf., Heemskerk, Polchinski; Faulkner, Liu, MR ’10

- *Optimum dissipation conjecture:* Holographic fluids not only attain the minimum allowed value of shear viscosity, but also ensure that the entropy production in any fluid flow is minimized.

Summary & Open Questions

- ✦ Basic story: a topological field theory with emergent gauge symmetry underlies the low energy dynamics of near-equilibrium states of a QFT.
- ✦ Construction aided in large part by a complete solution to the structural aspects of relativistic hydrodynamics.

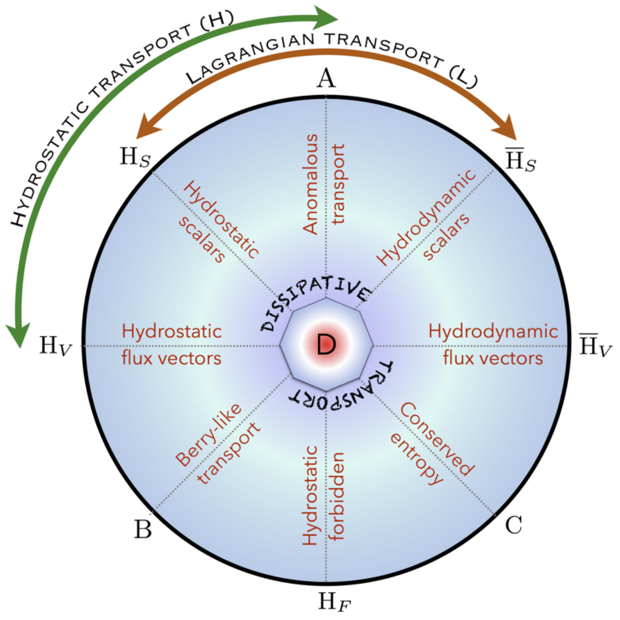
- ✤ Connections with generalized fluctuation-dissipation theorems?
- ✤ AdS/CFT derivation of the Brownian brane dynamics?
- ✤ Implications for black hole physics?
- ✤ Scrambling rates, equilibration, chaos?
- ✤ General principles for out-of-equilibrium dynamics?

More Qs: Section 19 of 1502.00636

It is rather fitting that in the centennial year of three theoretical milestones:

- ✳ Einstein’s theory of General Relativity
- ✳ Schwarzschild’s discovery of the first black hole solution
- ✳ Noether’s understanding of symmetries and conservation laws

we yet again encounter an interplay of some of these principles, which may pave the way for a better understanding of non-equilibrium dynamics of QFTs.



Thank you!