Conformal Bootstrap With Slightly Broken Higher Spin Symmetry

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ICTS-TIFR, Bengaluru

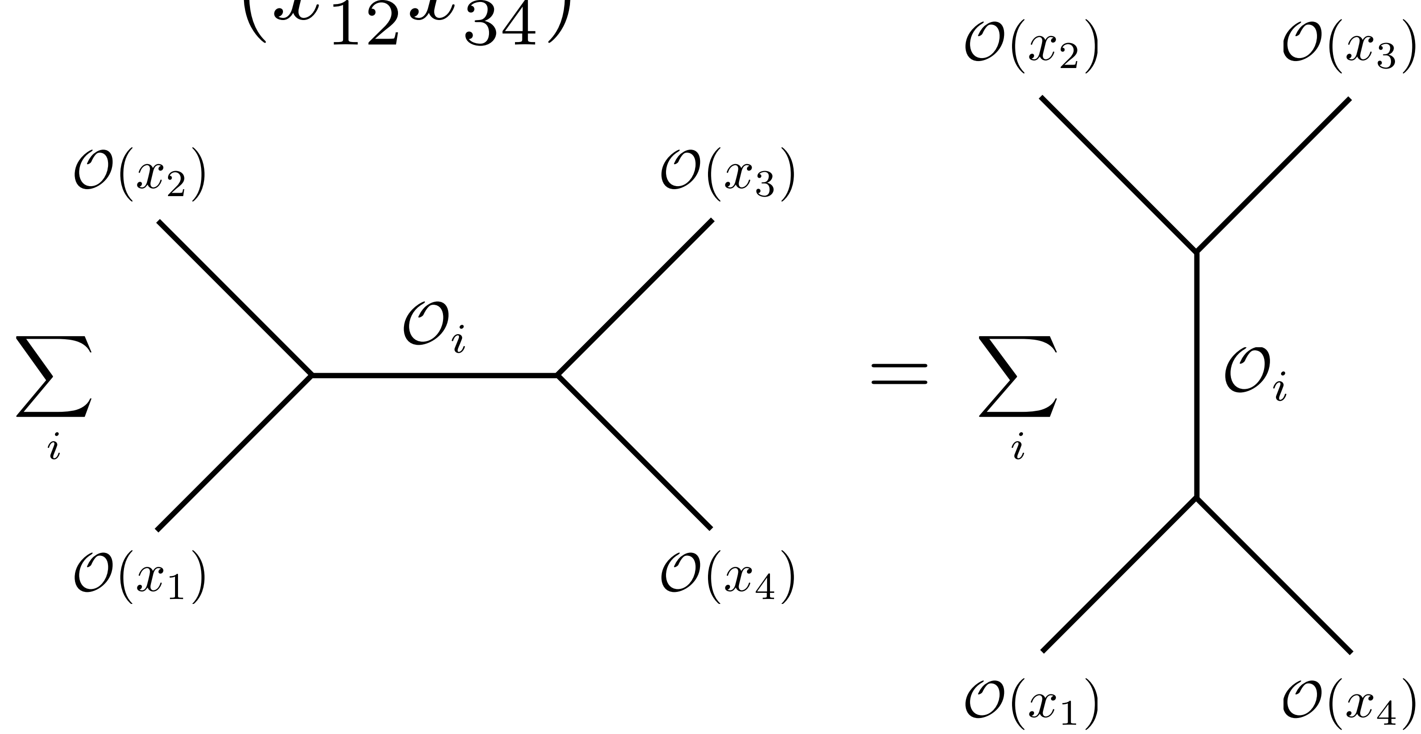
based on the work with Luis F. Alday

Conformal Bootstrap

[Ferrara, Gatto, Grillo ’74] [Polyakov ’74]

[Rattazzi, Rychkov, Tonni, Vichi ’08]

$$\langle \mathcal{O}\mathcal{O}\mathcal{O}\mathcal{O} \rangle = \frac{G(u,v)}{(x_{12}^2 x_{34}^2)^{\Delta}}$$



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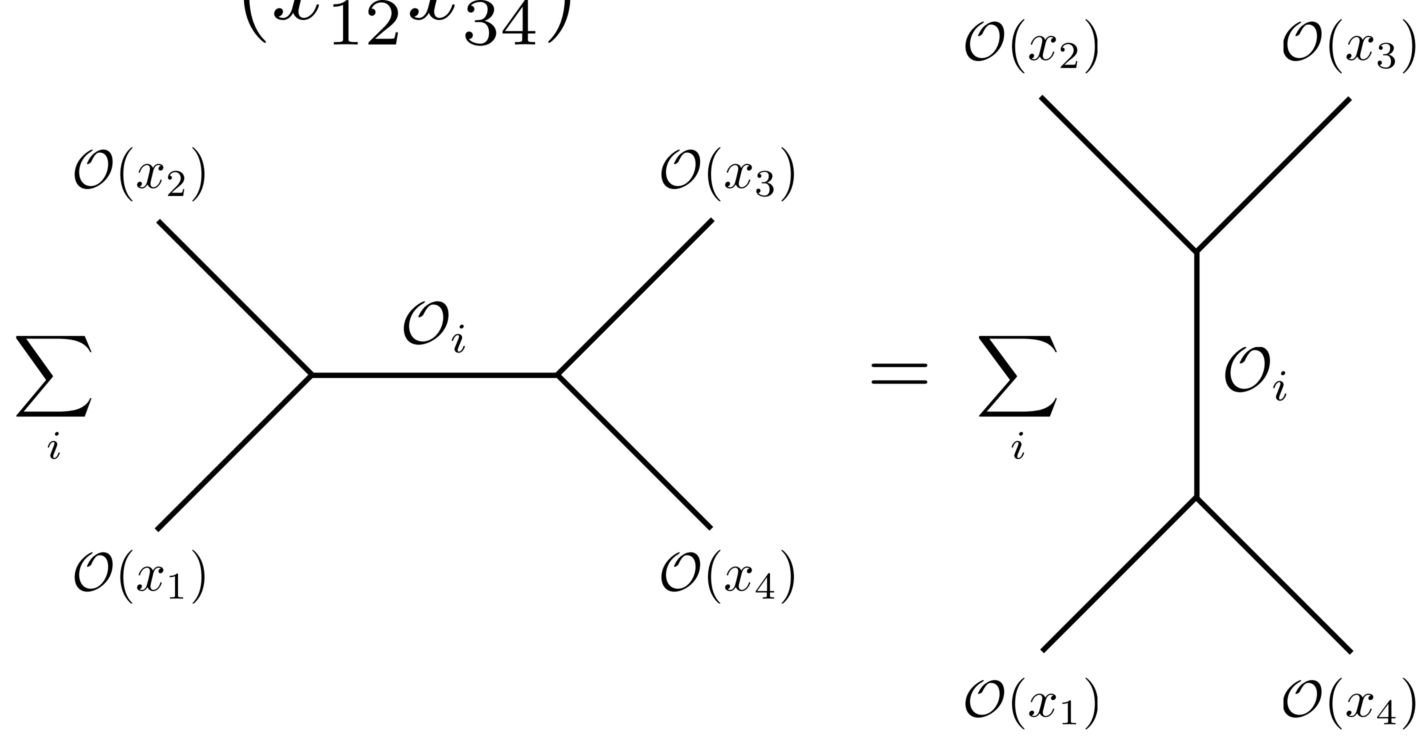

- Non-perturbative

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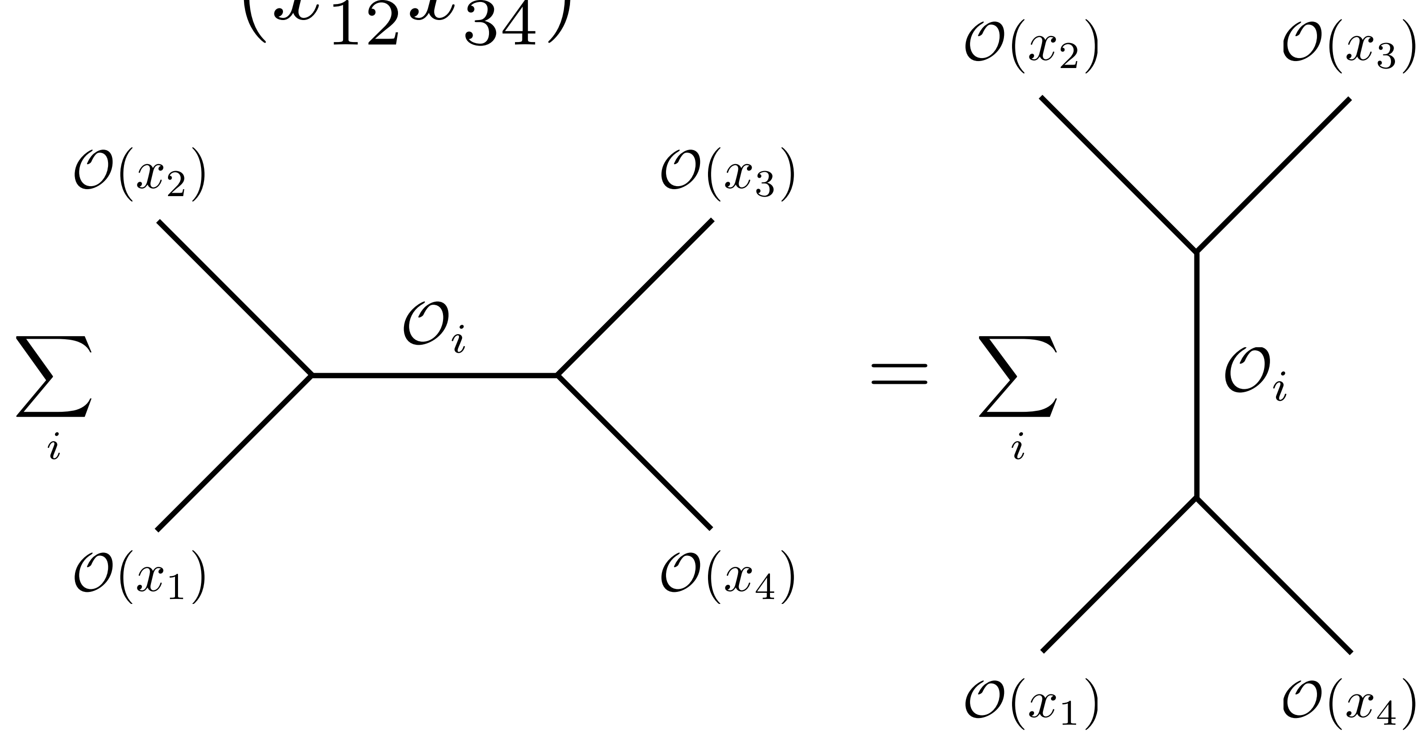

- Non-perturbative
- AdS/CFT

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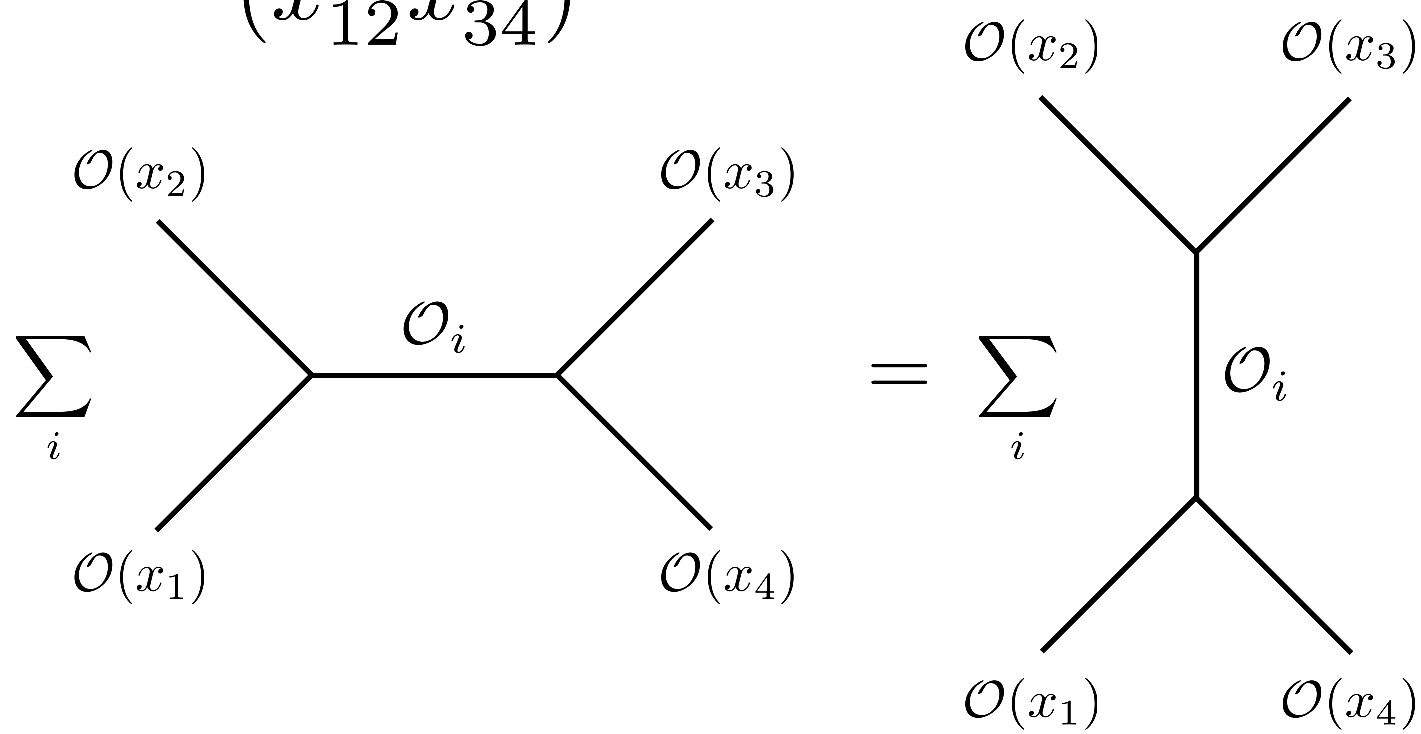

- Number of primary operators is infinite

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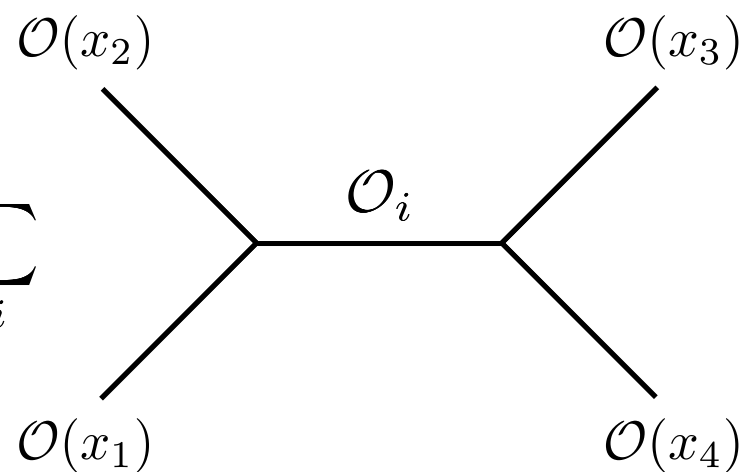

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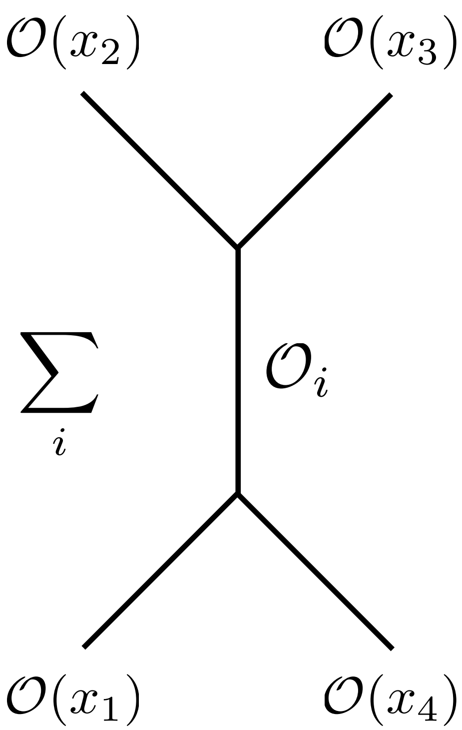

- Number of primary operators is infinite
- No simple map

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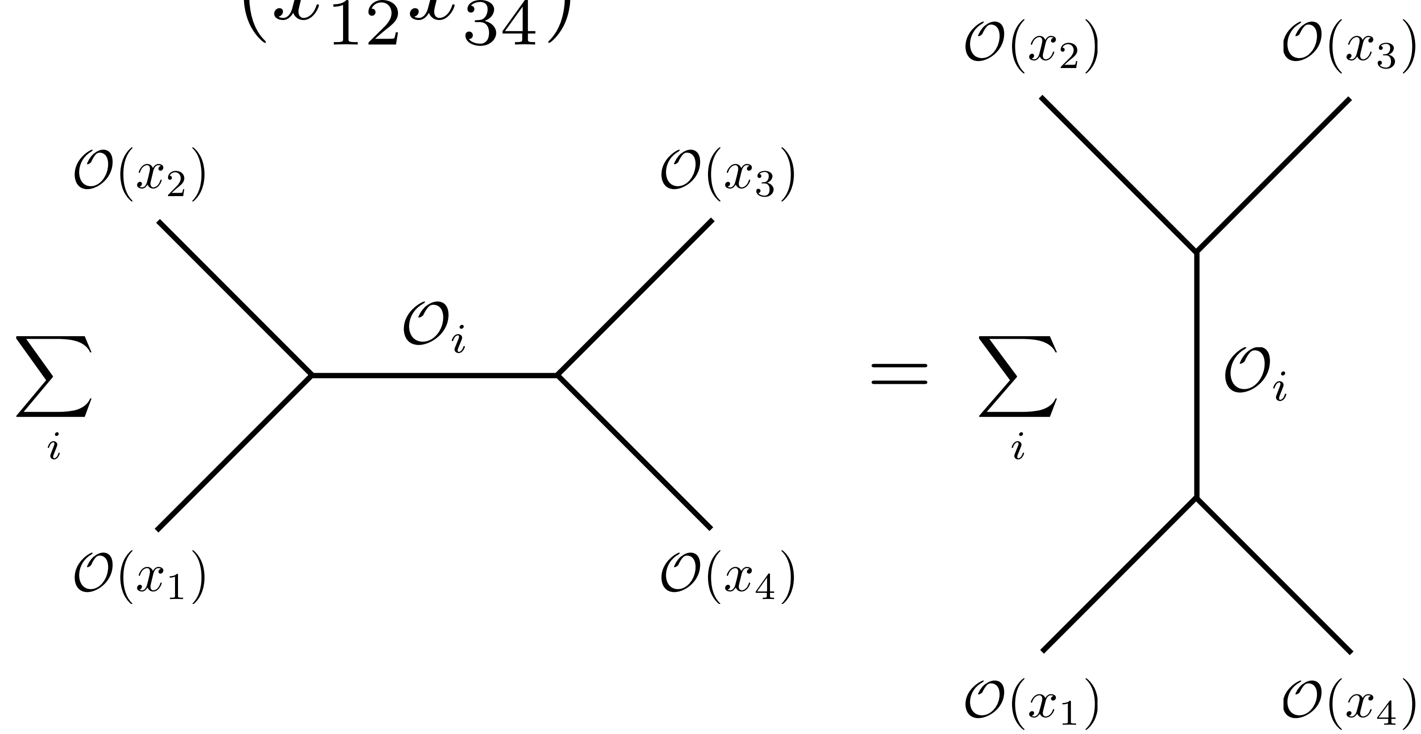

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What is the mechanism?

Conformal Bootstrap

$$\langle \mathcal{O}\mathcal{O}\mathcal{O}\mathcal{O} \rangle = \frac{G(u,v)}{(x_{12}^2 x_{34}^2)^{\Delta}}$$

$$u = z\bar{z} = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2}$$

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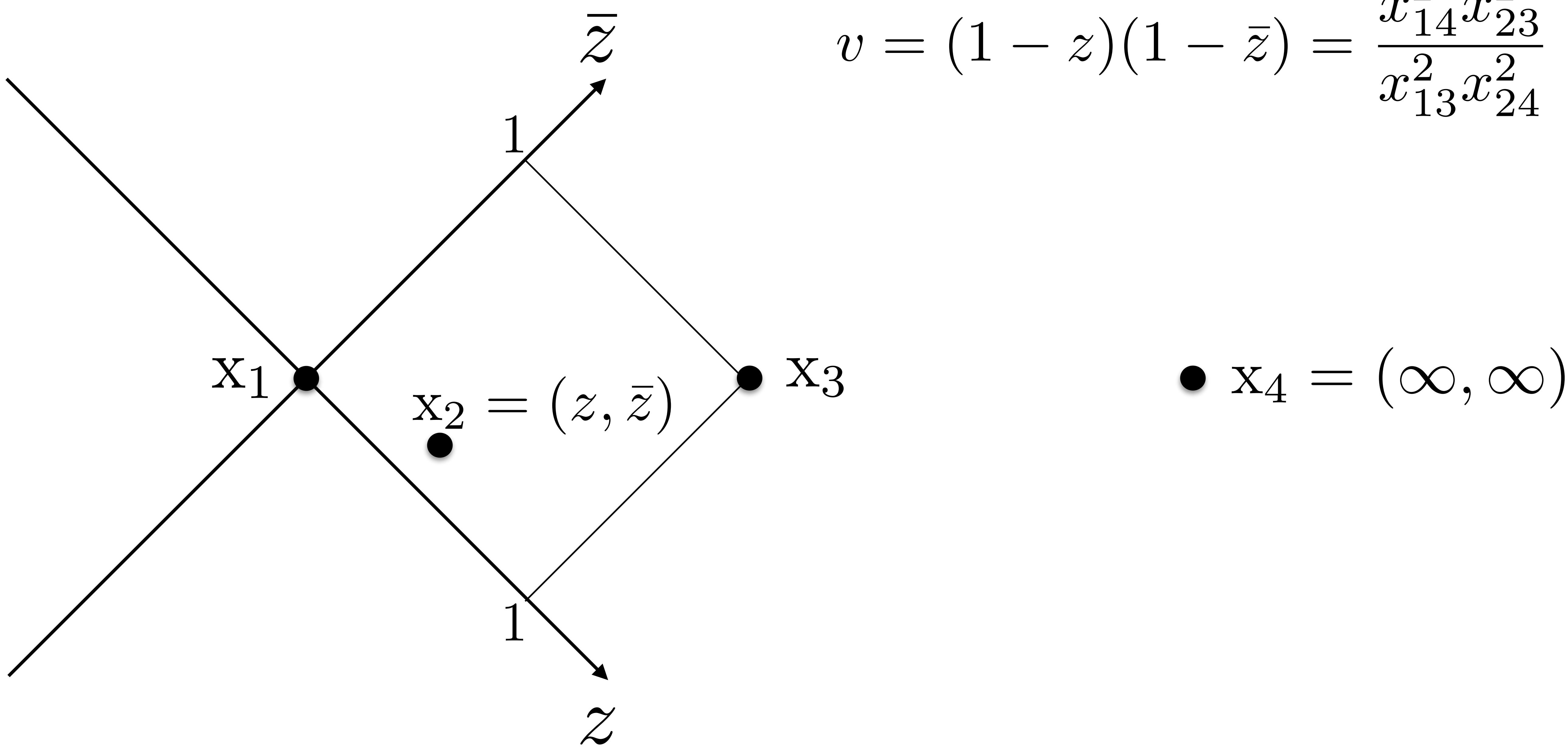

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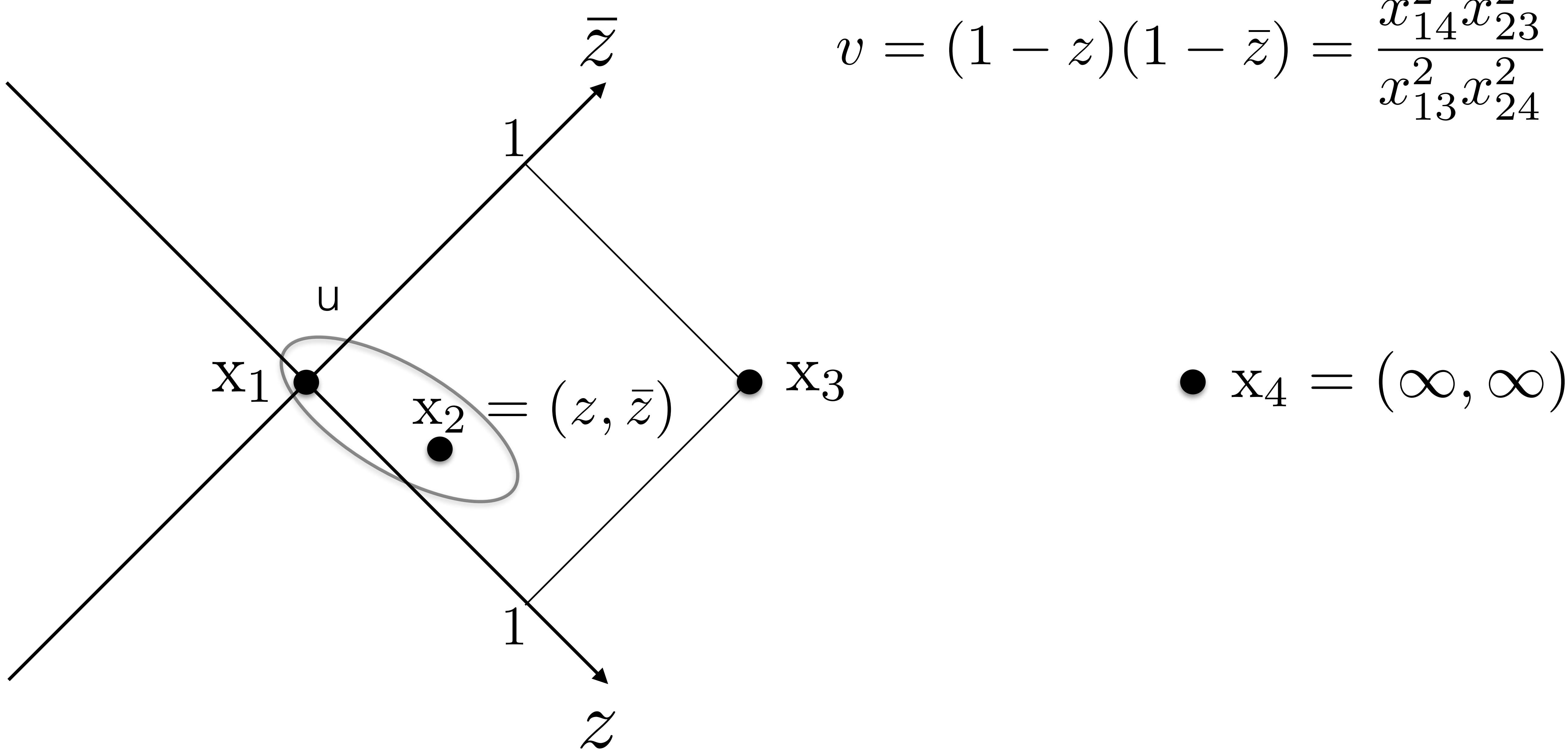

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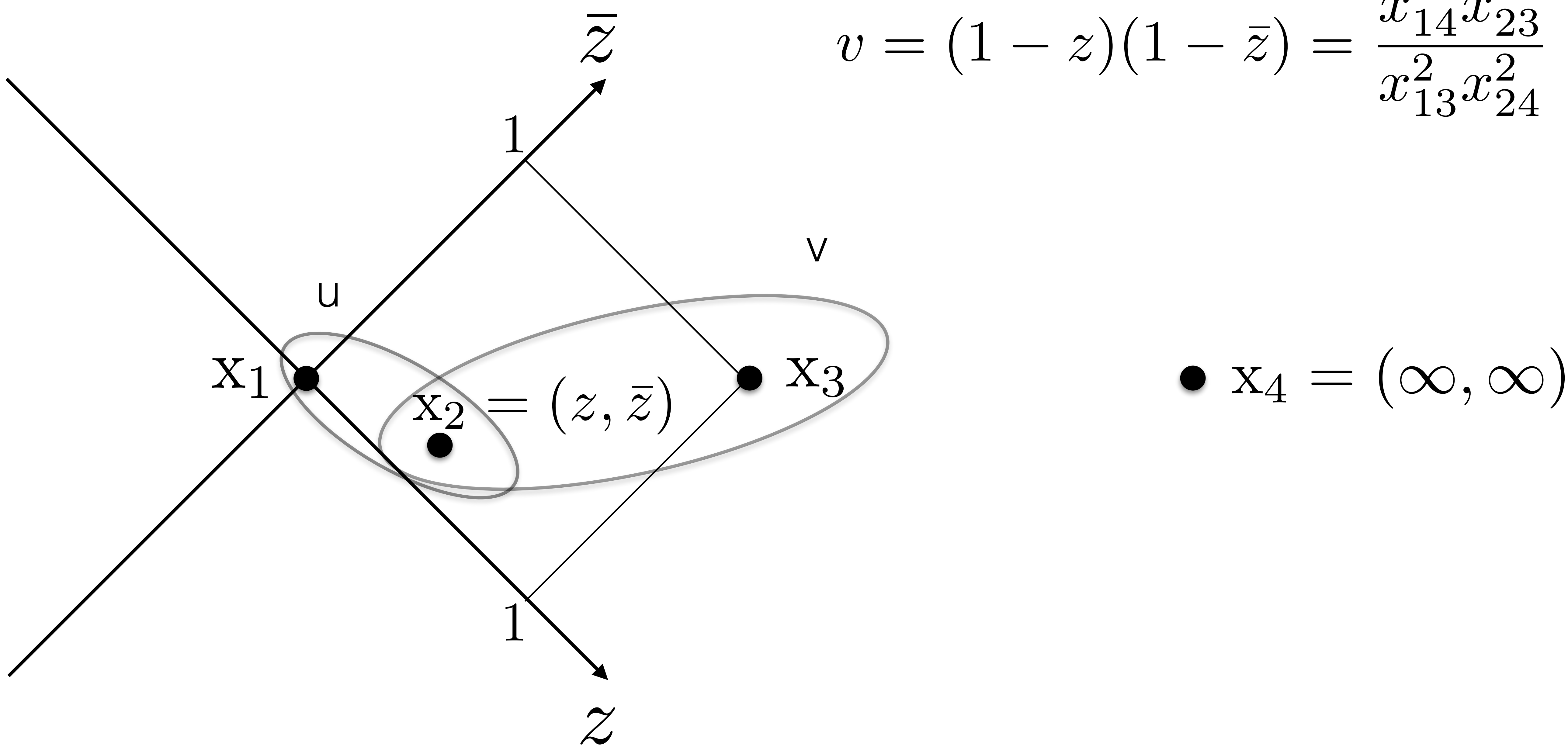

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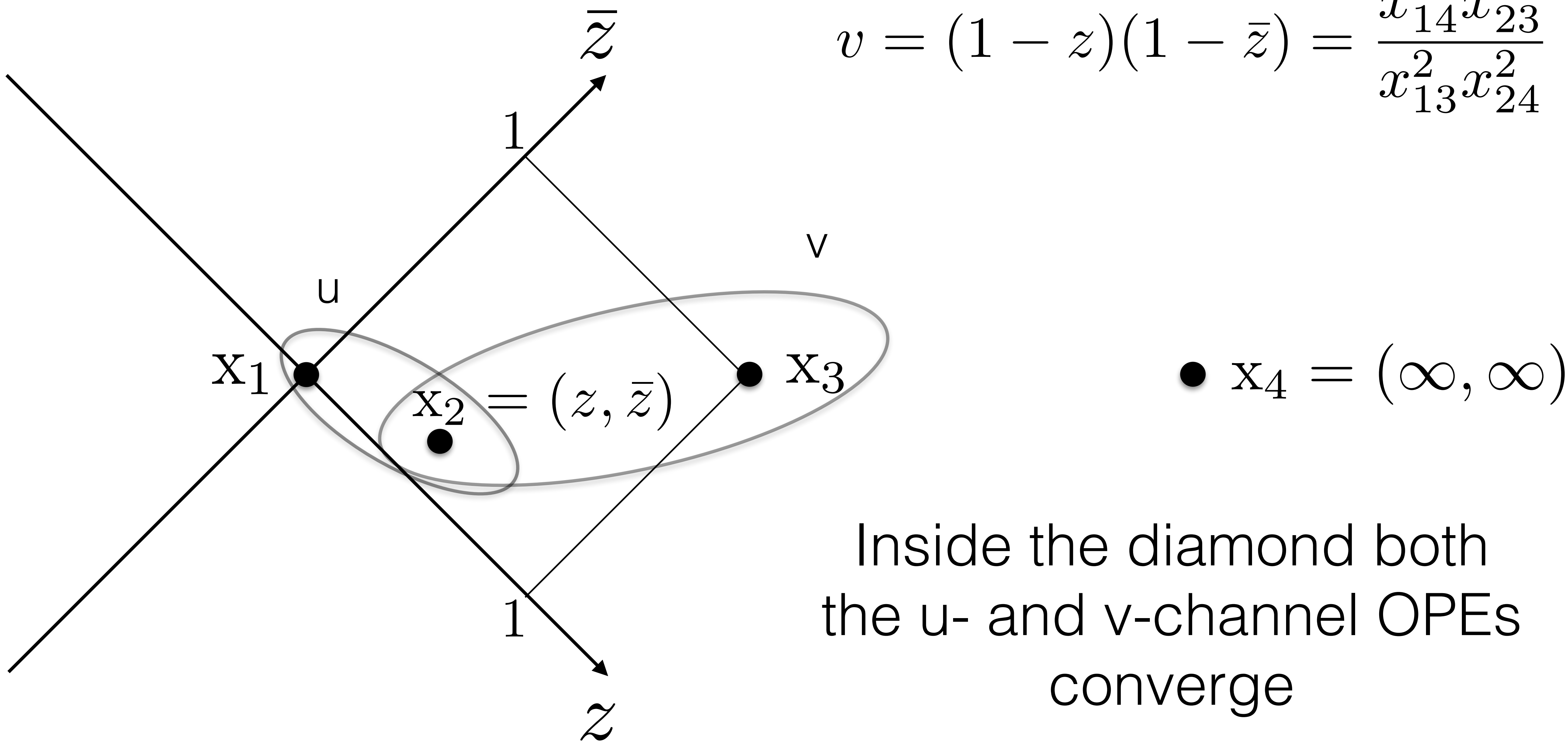

Inside the diamond both the u- and v-channel OPEs converge

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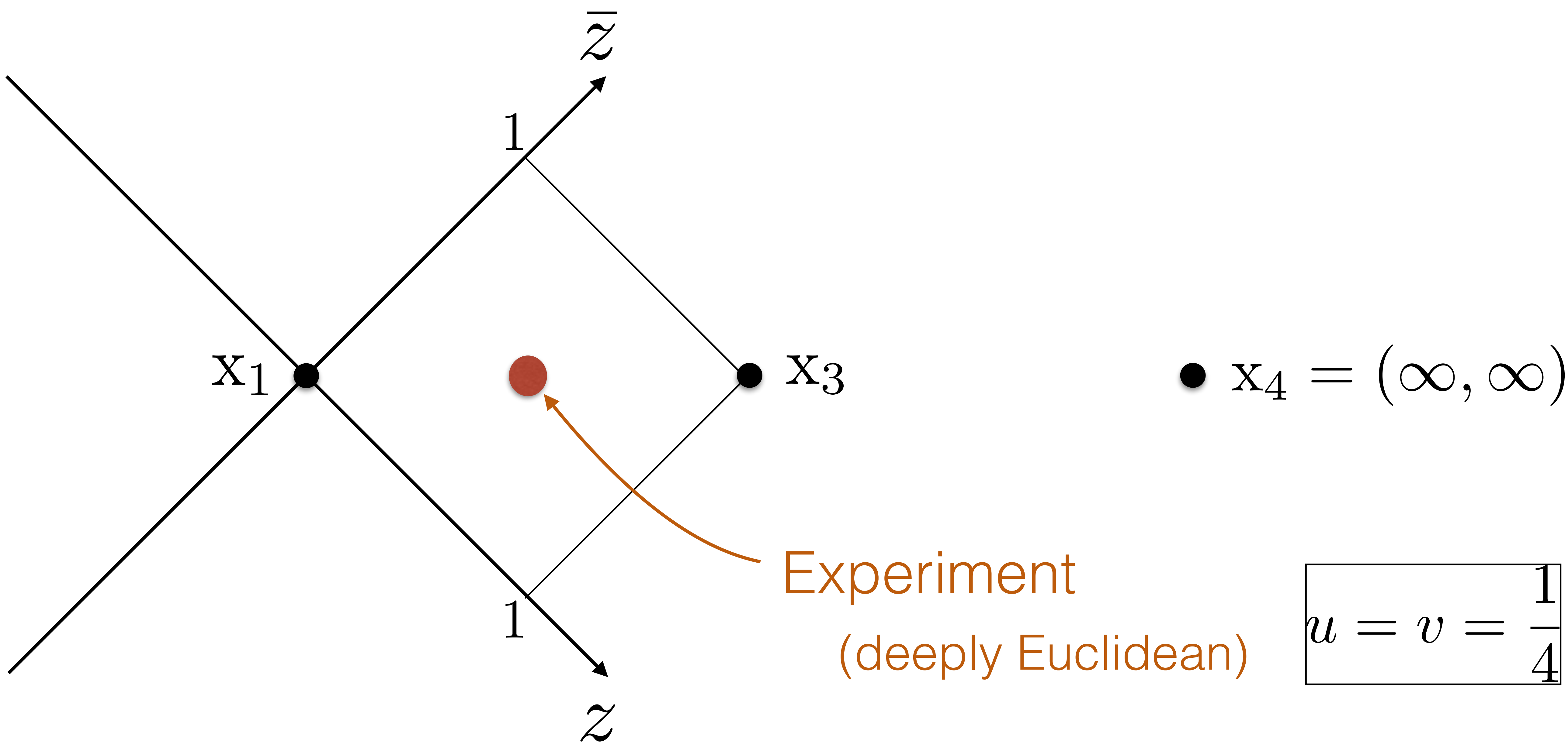

$$\boxed{u = v = \frac{1}{4}}$$

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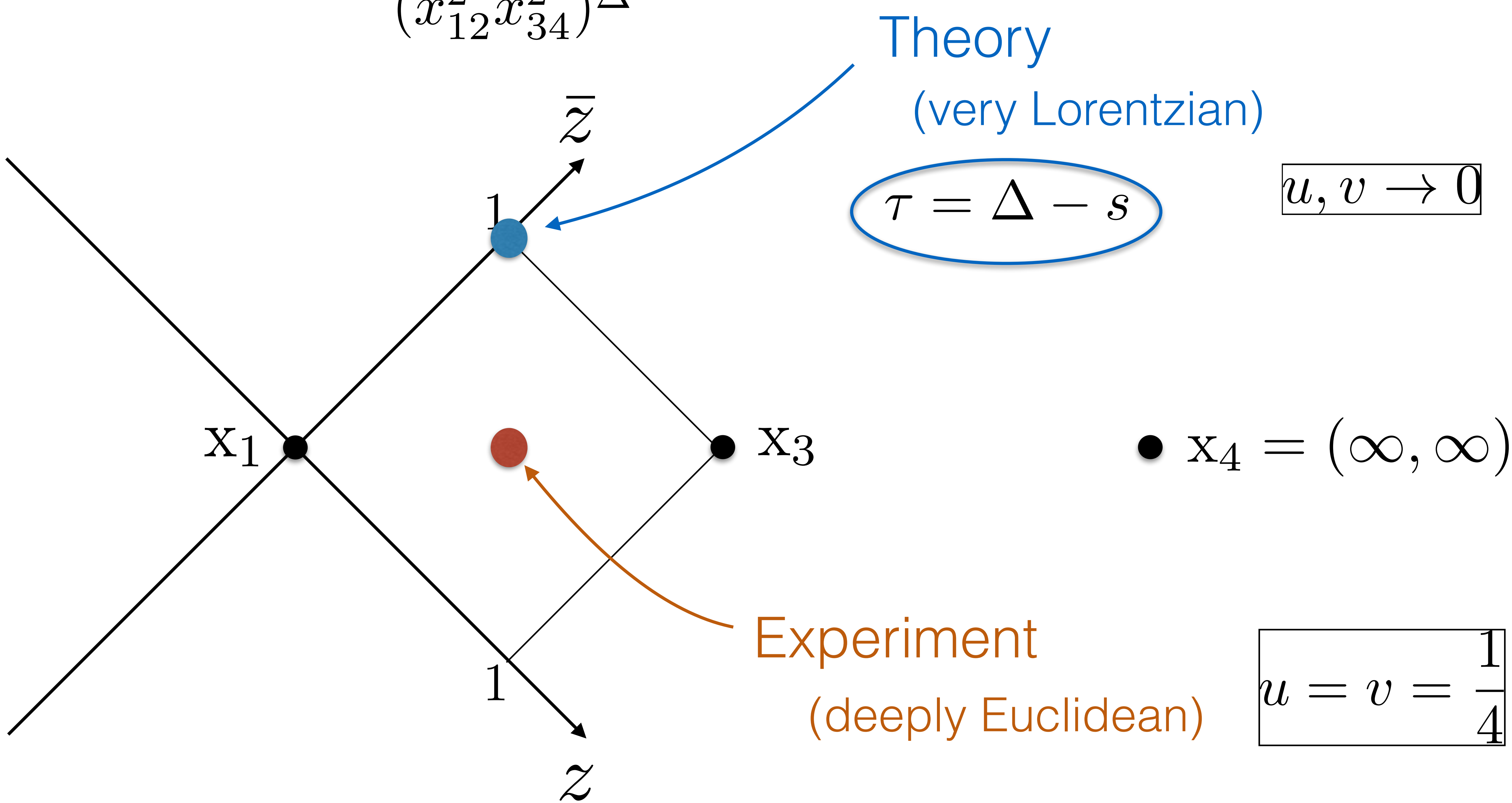

Analytic Bootstrap (d>2)

[Fitzpatrick, Kaplan, Poland, Simmons-Duffin ’12]

[Komargodski, AZ ’12]

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2. Consider the limit $v \ll u \ll 1$. Use the v-channel OPE

unit operator

$$G(u,v) = \left(\frac{u}{v}\right)^{\Delta} (1 + c_{\tau_{min},s_{min}} v^{\frac{\tau_{min}}{2}} f_{\tau_{min},s_{min}}(u) + \ldots)$$

minimal twist operator (stress tensor)

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3. Reproduce it using the u-channel OPE

$$\sum_{\tau,s} c_{\tau,s} u^{\frac{\tau}{2}} f_{\tau,s}(v) = \left(\frac{u}{v}\right)^{\Delta} \left(1 + c_{\tau_{min},s_{min}} v^{\frac{\tau_{min}}{2}} f_{\tau_{min},s_{min}}(u) + \ldots\right)$$

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$$f_{\tau,s}(v) = \ {}_2F_1(\frac{\tau}{2} + s, \frac{\tau}{2} + s, \tau + 2s, 1 - v) \sim \log v$$

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The mechanism is the following

$$\sum s^{\alpha-1} e^{-s\sqrt{v}} \sim \frac{1}{v^{\alpha}}$$

$$s \sim \frac{1}{\sqrt{v}}$$

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- Double trace-like operators with large spin are always present in the spectrum

$$\tau_s = 2\Delta - \alpha_d \frac{c_{\tau_{min}}}{s^{\tau_{min}}} \qquad (***)$$

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- (***) is expected be valid for $\delta\tau_{gap} \log s \gg 1$

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Example: 3d Ising model $\delta\tau_{gap} \simeq 0.02$ ~~HS~~

We would like to understand this case

Plan

Find anomalous dimensions of higher spin currents in theories with slightly broken higher spin symmetry using conformal bootstrap

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(In some interesting cases s=4 is already large)

Double Light-Cone Limit

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Double light-cone limit smoothly interpolates between the u-channel and v-channel OPE.

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$$f(u,v)^{disc} = u^{\Delta} + v^{\Delta}$$

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$$f(u,v)^{HS} = u^{\frac{d-2}{2}} v^{\frac{d-2}{2}}$$

- Higher spin currents are self-dual under crossing

Double Light-Cone Limit

When we turn on the coupling g the correlator becomes (perturbatively)

$$f(u,v)=\sum_{m,n} c_{mn}(\log u,\log v)u^{\frac{m}{2}}v^{\frac{n}{2}},\quad c_{mn}(\log u,\log v)=c_{nm}(\log v,\log u).$$

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tree-level twists

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To first order

$$c_{mn}=c^{(0)}_{mn}+g\delta c_{mn}\ ,$$

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anomalous dimensions

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new operators

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Whereas at L-th order we have

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Generically, we think of $g \sim \gamma_s \ll 1$

(light higher spin currents)

Self-duality of Higher Spin Currents

Let us consider a situation when higher spin currents are the lowest twist operators that appear in the OPE

fixed by the microscopic theory

[Alday, Bissi '13]

$$f(u,v) = u^{\frac{d-2}{2}} v^{\frac{d-2}{2}} h(\log u, \log v), \quad h(\log u, \log v) = h(\log v, \log u)$$

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(microscopically: gauge theories)

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  (microscopically: gauge theories)

- Case 2: $\mathcal{O} = \phi, \quad \Delta_{ext} = \dfrac{d-2}{2}$

  (microscopically: critical O(N), 3d Ising)

Log(S) From Bootstrap

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Consider external operators $\mathcal{O} = \phi^2, \quad \Delta_{ext} = d - 2$

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- Power-like divergences can only come from
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- The relevant spins are $\quad s = \frac{h}{\sqrt{v}}$

Log(S) From Bootstrap

This becomes an equation for anomalous dimensions (and 3pt functions) of higher spin currents

$$\Delta_s = d - 2 + s + \gamma_s, \quad \gamma_s \ll 1.$$

$$\frac{4}{\Gamma(\frac{d}{2}-1)^2} \int_0^\infty dh\ h^{d-3} u^{\frac{1}{2}\gamma_{\frac{h}{\sqrt{v}}}} \left(\frac{a_{\frac{h}{\sqrt{v}}}}{a^{(0)}_{\frac{h}{\sqrt{v}}}} \right) K_0(2h) = h(\log u, \log v)$$

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$$\Delta_s = d - 2 + s + \gamma_s, \quad \gamma_s \ll 1.$$

sum over spins

$$\frac{4}{\Gamma(\frac{d}{2}-1)^2} \int_0^\infty dh\ h^{d-3} u^{\frac{1}{2}\gamma_{\frac{h}{\sqrt{v}}}} \left(\frac{a_{\frac{h}{\sqrt{v}}}}{a^{(0)}_{\frac{h}{\sqrt{v}}}} \right) K_0(2h) = h(\log u, \log v)$$

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The consistent form of the correction is

$$\gamma_s = \gamma^{(1)} \log s + \gamma^{(2)} \log^2 s + \gamma^{(3)} \log^3 s + \ldots,$$

$$\frac{a_s}{a_s^{(0)}} = 1 + a^{(1)} \log s + a^{(2)} \log^2 s + a^{(3)} \log^3 s + \ldots .$$

Log(S) From Bootstrap

[Alday, Maldacena '07]
[Alday, Bissi '13]

The solution is

$$\gamma_s = \gamma^{(1)}(g) \log s,$$

$$\frac{a_s}{a_s^{(0)}} = \frac{\Gamma(\frac{d}{2} - 1 - \frac{\gamma_s}{2})^2}{\Gamma(\frac{d}{2} - 1)^2}$$

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It implies the following form of the corrected correlator

[Alday, Eden, Korchemsky, Maldacena, Sokatchev '10]

$$f(u,v) = u^{\frac{d-2}{2}} v^{\frac{d-2}{2}} e^{-\frac{f(g)}{4} \log u \log v}$$

Z_2-preserving Theory

Consider external operators $\mathcal{O} = \phi, \Delta_{ext} = \dfrac{d-2}{2}$

$Z_2 : \phi \to -\phi$

$$\sum_{\tau,s} u^{\frac{\tau}{2}} c_{\tau,s} f_{\tau,s} = \frac{f(u,v)}{v^{\frac{d-2}{2}}} = u^{\frac{d-2}{2}} h(\log u, \log v)$$

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Let us act with the Casimir operator on both sides of the sum rule. We get

$$\sum_{\tau,s} u^{\frac{\tau}{2}} c_{\tau,s} (s^2 - \frac{1}{4}) f_{\tau,s}(v) = \mathcal{D}\left(u^{\frac{d-2}{2}} h(\log u, \log v)\right)$$

Z_2-preserving Theory

The most singular terms in the small v limit take the following form

$$\mathcal{D}\left(u^{\frac{d-2}{2}}\log u(\log v)^k\right) \approx \frac{k(k-1)u^{\frac{d-2}{2}}\log u\ (\log v)^{k-2}}{v}$$

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The sum rule takes the form

$$\frac{1}{2}\frac{4}{\Gamma(d/2-1)^2}\int_0^\infty dh\ h^{d-3}\left(\frac{h^2}{v}\right)K_0(2h)\gamma(\frac{h}{\sqrt{v}}) = (\log v)^{k-2}v^{\frac{d-4}{2}}.$$

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The sum rule requires anomalous dimensions
of the higher spin currents to have the following structure

$$\gamma_s = \frac{\alpha_0(g) + \alpha_1(g)\log s + \alpha_2(g)\left(\log s\right)^2 + \dots}{s^{d-2}} \ ,$$

$$\alpha_0(g) \sim g^2, \quad \alpha_1(g) \sim g^3, \quad \alpha_2(g) \sim g^4$$

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For $1 \ll s^{4-d} \ll N$ the $\frac{1}{s^2}$ term dominates!

3d Ising Model

- Conformal invariance and Z_2 invariance

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- Contains in the spectrum a scalar operator σ

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$$\Delta_\sigma = \frac{1}{2} + \gamma_\sigma \qquad \gamma_\sigma \simeq 0.018$$

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From this it follows that the theory contains an infinite set of light higher spin currents

$$\Delta_s = 1 + s + \gamma_s \qquad s = 2, 4, 6, ...$$

$$0 \leq \gamma_s < 2\gamma_\sigma \ll 1$$

[Nachtmann '73]
[Callan, Gross '73]

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- The leading correction to γ_s at large spin

comes from current themselves (critical O(N))

$$\mathrm{HS} = \mathrm{HS}$$

As we argued above in this case we get

$$\gamma_s = \frac{c(\log s)\gamma_\sigma^2}{s} = \frac{\gamma_\sigma^2}{s}(c_0 + c_1 \log s + \ldots), \quad \gamma_\sigma^2 \simeq 3 \cdot 10^{-4}$$

$$c(\infty) \simeq 8.5 \qquad\qquad c_0 = ?$$

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- The next operator in the spectrum is $\mathcal{E}$

$$\Delta_{\mathcal{E}} \simeq 1.41$$ (strongly coupled)

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Comparing $\frac{f_{\sigma\sigma\varepsilon}^2}{s^{\Delta_{\varepsilon}}}$ with $\frac{\gamma_{\sigma}^2}{s}$ we find that for

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$2 \leq s \leq 10^4$ $\frac{f_{\sigma\sigma\varepsilon}^2}{s^{\Delta_\varepsilon}} > \frac{\gamma_\sigma^2}{s}$

Thus, we expect the higher spin currents to be irrelevant for small spins (which are accessible experimentally).

[similar to the O(N) case]

3d Ising Model

Moreover, we can treat the contribution of ε exactly!

The result is

$$\gamma_s \simeq 2\gamma_\sigma - \frac{2\Gamma(\Delta_\varepsilon)}{\Gamma(\frac{\Delta_\varepsilon}{2})^2} \frac{\Gamma(\Delta_\sigma)^2}{\Gamma(\Delta_\sigma - \frac{\Delta_\varepsilon}{2})^2} \frac{f_{\sigma\sigma\varepsilon}^2}{s^{\Delta_\varepsilon}}$$

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(not a ``precise photography'', but a ``very good caricature'')

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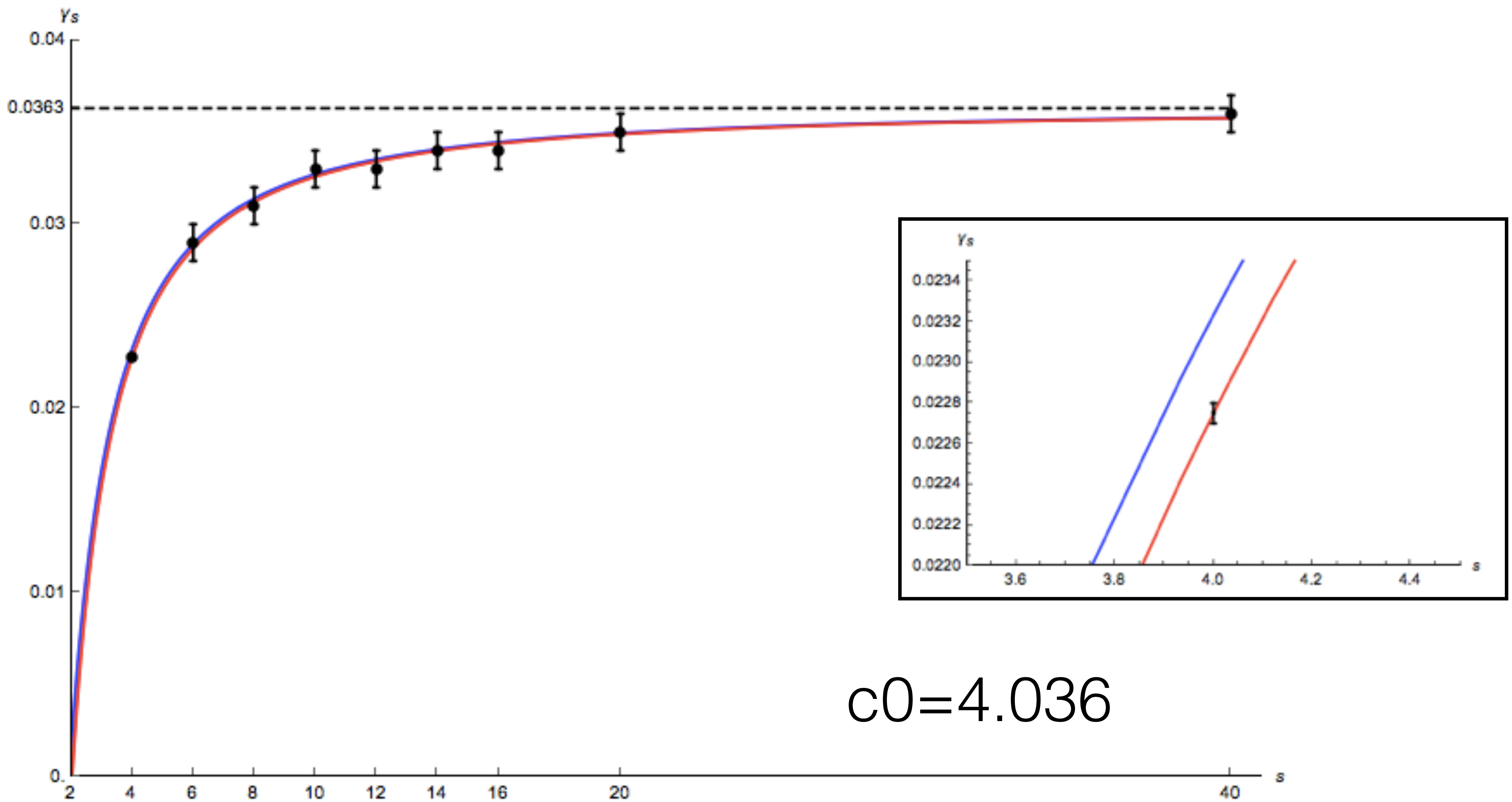
$$\gamma_s \simeq 0.0363 - \frac{0.0926}{s^{1.4126}} + \frac{0.0012}{s^{2.4126}} - \frac{0.0220}{s^{3.4126}} - \frac{0.0003c_0}{s}$$

3d Ising Model

We can determine c0 from spin-4 anomalous dimension.

[Numerical bootstrap predictions, unpublished]
(3d Ising collaboration: S. El-Showk, M. Paulos, D. Poland, S. Rychkov, D. Simmons-Duffin, A. Vichi)



c0=4.036

3d Ising Model

Or we can construct c0-independent combinations

$$\left(\gamma_6 - \frac{2}{3}\gamma_4\right)^{theory} = 0.0135,$$

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- Sometimes for low enough spins not the smallest twist operators are the most relevant ones (critical O(N), 3d Ising)

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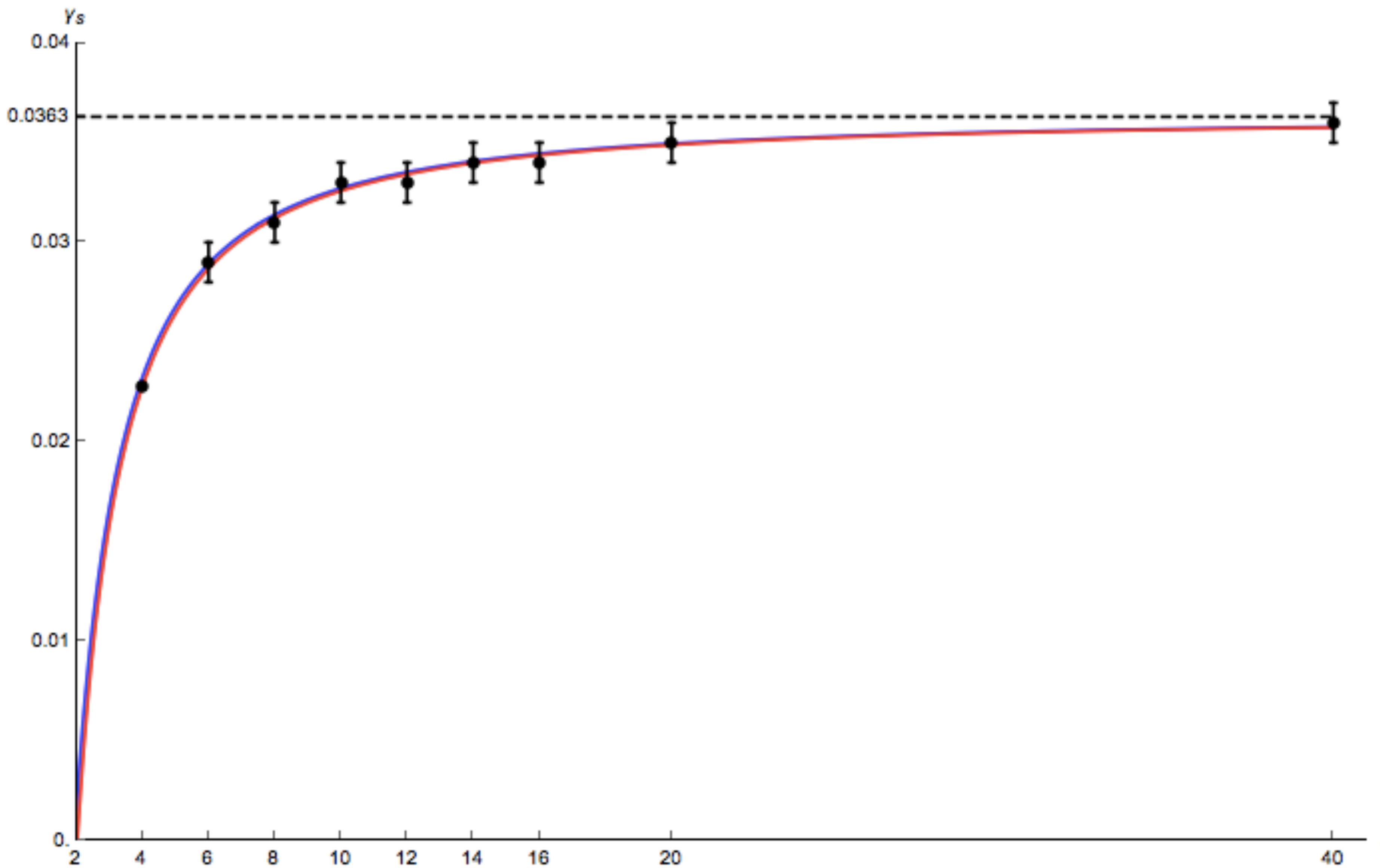
Can all perturbative solutions of crossing be classified? (Mellin amplitudes)

Thank you for the attention!



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Back Up

Operators With High Twist

Consider operators made of n fields. We can ask what is the number of primary operators of this type exist. There is sharp transition

$$N(n,s) \sim \frac{s^{n-2}}{\Gamma(n-1)\Gamma(n+1)}$$

- Low twist operators live on finite number of Regge trajectories

- The number of high twist operators grows with spin

Anomalous Dimension of External Operator

When the external operator receives anomalous dimension we get

$$v^{\Delta_0+\gamma_{ext}} G(u,v) = u^{\Delta_0+\gamma_{ext}} G(v,u)$$

$$v^{\Delta_0} u^{\frac{d-2}{2}} \log u \sum_s \frac{\gamma_s - 2\gamma_{ext}}{2} a_s^{(0)} \; f_s(v)$$

$$= u^{\Delta_0} \left(\sum_{\tau_i^{(0)}} v^{\frac{\tau_i^{(0)}}{2}} \delta F_{\tau_i}^{(0)}(u) + \log v \sum_{\tau_i^{(0)},s} v^{\frac{\tau_i^{(0)}}{2}} \frac{\gamma_{\tau_i^{(0)},s} - \gamma_{ext}}{2} F_{\tau_i}^{(0)}(u) \right)$$

Thus, the trivial effect is

$$\gamma_s \to \gamma_s - 2\gamma_{ext}$$