Generalized Global Symmetries

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Global Symmetries

- Ordinary global symmetries
  - Act on local operators
  - The charged states are particles
- Generalized global symmetries
  - The charged operators are lines, surfaces, etc.
  - The charged objects are strings, domain walls, etc.
- It is intuitively clear and many people will feel that they have known it. We will make it more precise and more systematic.
- We will repeat all the things that are always done with ordinary symmetries.
- The gauged version of these are common in physics and in mathematics.

Ordinary global symmetries

- Generated by operators associated with co-dimension one manifolds M

$$U_g(M)$$

  $g \in G$ a group element
- The correlation functions of $U_g(M)$ are topological!
- Group multiplication $U_{g_1}(M)U_{g_2}(M) = U_{g_2 g_1}(M)$
- Local operators $O(p)$ are in representations of G

$$U_g(M)O_i(p) = R_i^j(g)O_j(p)$$

  where M surrounds p (Ward identity)
- If the symmetry is continuous,

$$U_g(M) = e^{i\int j(g)}$$

$j(g)$ is a closed form current (its dual is a conserved current).

q-form global symmetries

- Generated by operators associated with co-dimension $q+1$ manifolds M (ordinary global symmetry has $q=0$)
$$U_g(M)$$
$g \in G$ a group element
- The correlation functions of $U_g(M)$ are topological!
- Group multiplication $U_{g_1}(M)U_{g_2}(M) = U_{g_2 g_1}(M)$. Because of the high co-dimension the order does not matter and G is Abelian.
- The charged operators $\mathrm{V}(L)$ are on dimension q manifolds L. Representations of G – Ward identity
$$U_g(M)V(L) = R(g)V(L)$$
where M surrounds L and $R(g)$ is a phase.

q-form global symmetries

If the symmetry is continuous,

$$U_g(M) = e^{i\int j(g)}$$

$j(g)$ is a closed form current (its dual is a conserved current).

Compactifying on a circle, a q-form symmetry leads to a q-form symmetry and a $q-1$-form symmetry in the lower dimensional theory.

For example, compactifying a one-form symmetry leads to an ordinary symmetry in the lower dimensional theory.

No need for Lagrangian

- Exists abstractly, also in theories without a Lagrangian
- Useful in dualities

$4d\ U(1)$ gauge theory

Two global $U(1)$ one-form symmetries:

- Electric symmetry
  - Closed form currents: $\frac{2}{g^2} * F$ (measures the electric flux)
  - Shifts the gauge field A by a flat gauge field
- Magnetic symmetry
  - Closed form currents: $\frac{1}{2\pi} F$ (measures the magnetic flux)
  - Shifts the magnetic gauge field by a flat gauge field

The charged objects are dyonic lines $W_n(L) H_m(L)$

($W_n(L)$ are Wilson lines and $H_m(L)$ are 't Hooft lines) with global symmetry charges n and m.

$4d\ SU(N)$ gauge theory

- Electric $\boldsymbol{Z}_N$ one-form symmetry
  - A Gukov-Witten surface operator depends on a conjugacy class in $SU(N)$. When this class is in the center of $SU(N)$ the surface operator is topological. This is our U.
  - It shifts the gauge field by a flat $\boldsymbol{Z}_N$ gauge field.
  - It acts on the Wilson lines according to their representation under the $\boldsymbol{Z}_N \in SU(N)$ center.
- No magnetic one-form symmetry
  - No genuine 't Hooft lines
  - Their definition needs a choice of a surface.

Applications

As with ordinary symmetries:

- Selection rules on amplitudes
- Couple to a background classical gauge field (twisted boundary conditions)
  - Interpret 't Hooft twisted boundary conditions as an observable in the untwisted theory
- Gauge by summing over twisted sectors (like orbifolds)
  - New parameters in *4d* gauge theories – discrete θ-parameters (like discrete torsion) [Aharony, NS, Tachikawa]
- Dual theories often have different gauge symmetries. But the global symmetries must be the same.
  - Non-trivial tests of duality including non-BPS operators

Characterizing phases

- In a confining phase the electric one-form symmetry is unbroken.
  - The confining strings are charged and are classified by the unbroken symmetry.
  - Area law of Wilson loop – order parameter $\langle W \rangle$ vanishes when it is large – symmetry unbroken.
  - Ordinary global symmetry after compactification. It is unbroken [Polyakov, Susskind].

Characterizing phases

- In a Higgs or Coulomb phase the electric one-form symmetry is spontaneously broken.
  - Renormalizing the perimeter law to zero, the large size limit of $\langle W \rangle$ is nonzero – vev “breaks the symmetry.”
  - No strings
  - Upon compactification, ordinary global symmetry. It is broken [Polyakov, Susskind].

Low energy behavior

As with ordinary symmetries, low energy consequences when the symmetry is spontaneously broken:

- A continuous broken symmetry leads to a massless Nambu-Goldstone boson.
  - Example: a photon in a Coulomb phase (cf. [Kovner, Rosenstein, 1990][He, Mitra, Porfyriadis, Strominger, 2014])

$$\langle 0|F_{\mu\nu}|\epsilon,p\rangle = (\epsilon_\mu p_\nu - \epsilon_\nu p_\mu)e^{ipx}$$

- A discrete broken symmetry leads to a TQFT in the IR – long range topological order
  - Example: a spontaneously broken $\boldsymbol{Z}_n$ one-form symmetry leads to a $\boldsymbol{Z}_n$ gauge theory

Anomalies

The *4d* $U(1)$ gauge theory has two global one-form symmetries.

Gauge the electric $U(1)$ one-form symmetry

$$\frac{1}{g^2}\,F_{\mu\nu}^2 \quad\rightarrow\quad \frac{1}{g^2}\left(F_{\mu\nu} - \,B^E{}_{\mu\nu}\right)^2$$

Gauge the magnetic $U(1)$ one-form symmetry

$$\frac{1}{g^2}\,F_{\mu\nu}^2 \quad\rightarrow\quad \frac{1}{g^2}F_{\mu\nu}^2 \;-\; \epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}B^M{}_{\rho\sigma}$$

An anomaly prevents us from gauging both of them.

Similar to attempting to gauge both the ordinary (zero-form) momentum and winding symmetries in *2d*.

Higher Form SPT Phases

Consider a system with an unbroken symmetry with anomalies.

- ‘t Hooft anomaly matching forces excitations (perhaps only topological excitations) in the bulk, or only on the boundary.
- Symmetry Protected Topological Phase
- Domain walls between vacua in different SPT phases must have excitations.
- For examples, $\boldsymbol{N} = 1$ SUSY $SU(N)$ gauge theory has N vacua in different SPT phases (the relevant symmetry is the one-form $\boldsymbol{Z}_N$ symmetry) and hence there is $U(k)_N$ on the domain walls between them [Dierigl, Pritzel]. This $U(k)_N$ was originally found by [Acharya, Vafa] using string considerations.

Conclusions

- Higher form global symmetries are ubiquitous.
- They help classify
  - extended operators/defects (lines, surfaces, etc.)
  - extended objects (strings, domain walls, etc.)
- Global symmetries must be the same in dual theories.
- They extend Landau's characterization of phases based spontaneous global symmetry breaking and the corresponding order parameters
  - Rephrase the Wilson/'t Hooft classification in terms of broken or unbroken one-form global symmetries.

Conclusions

- Anomalies (as for ordinary global symmetries)
  - Obstruction to gauging
  - ‘t Hooft matching conditions
  - Anomaly inflow
  - Degrees of freedom on domain walls

Thank you for your attention