Entanglement Dynamics in 2d CFT

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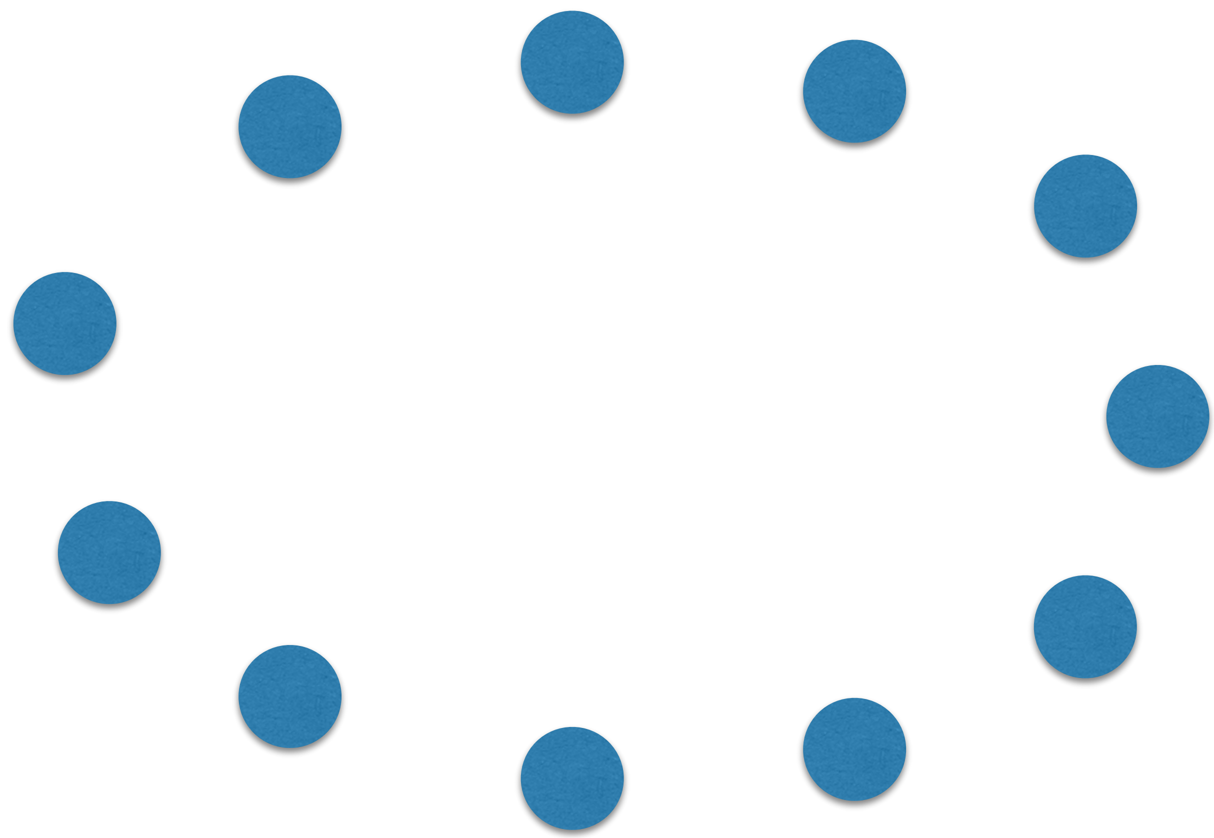
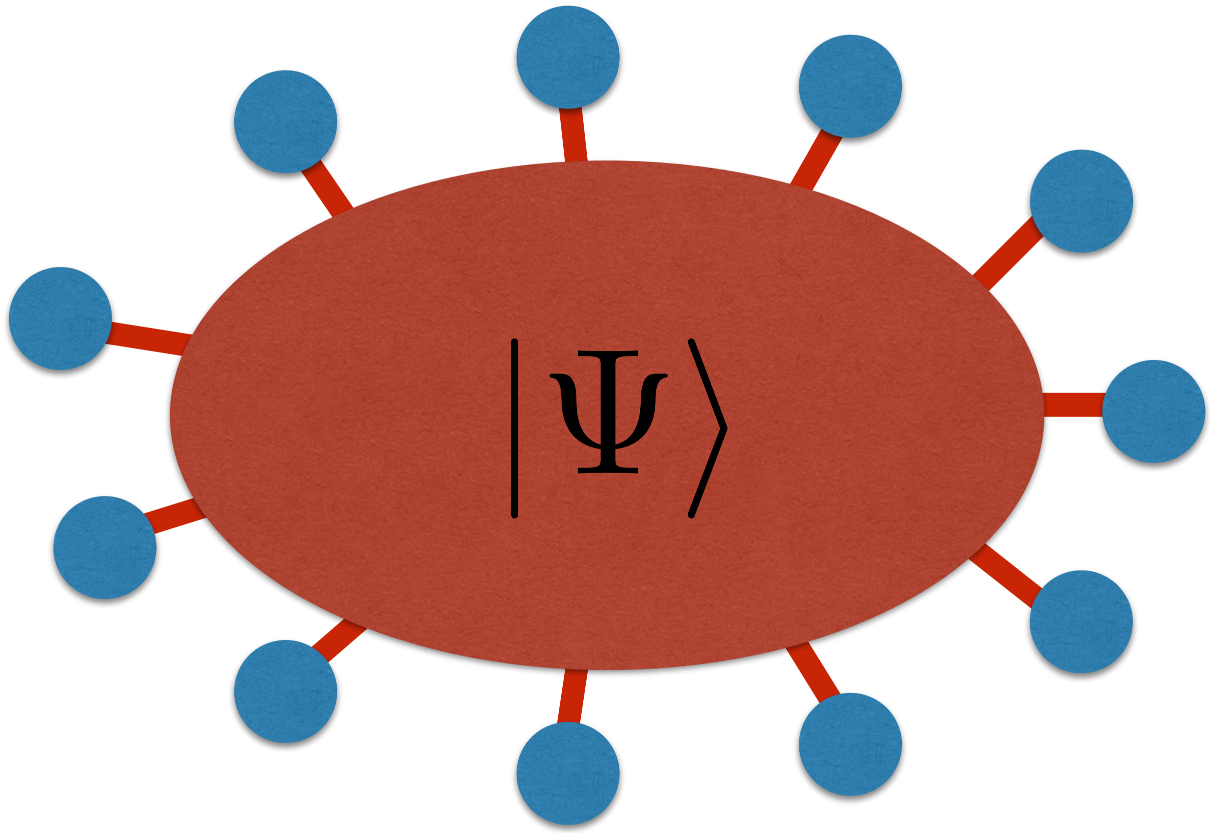
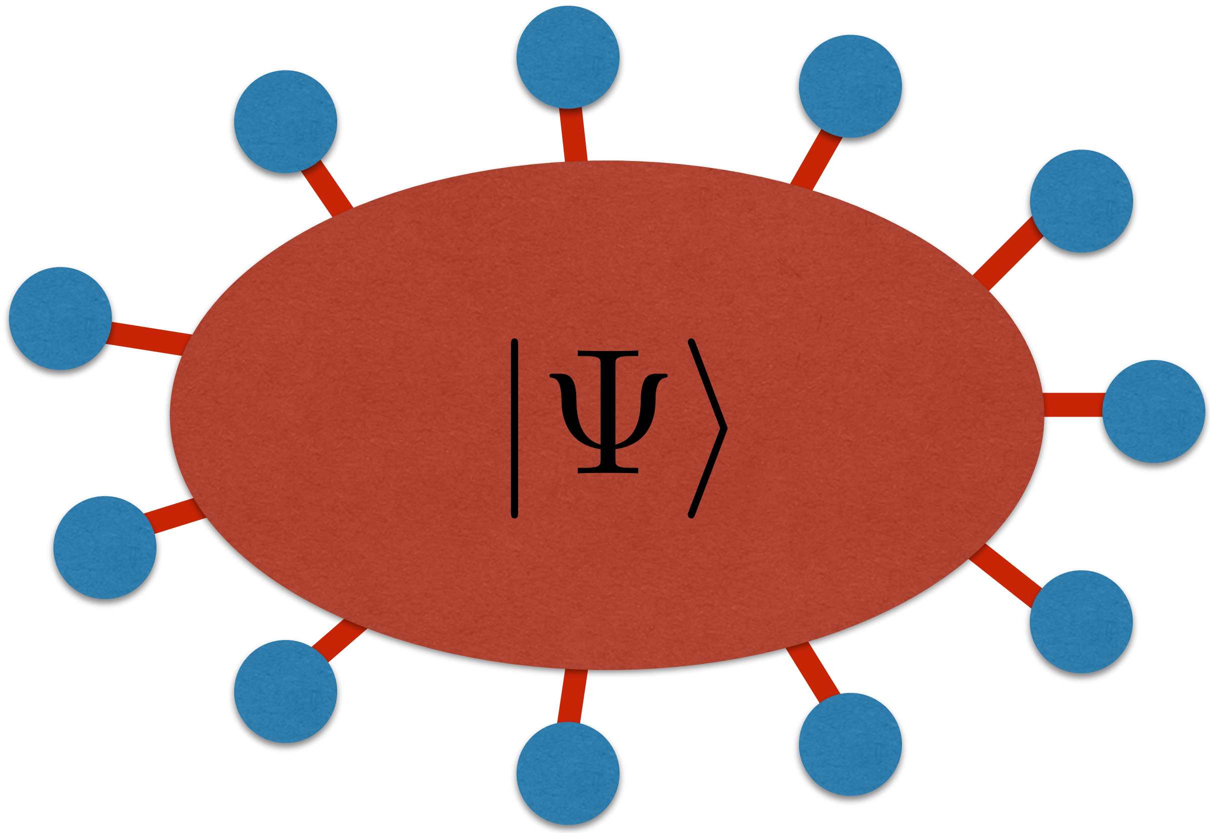
STRINGS 2015 ✦ ICTS-TIFR ✦ BANGALORE

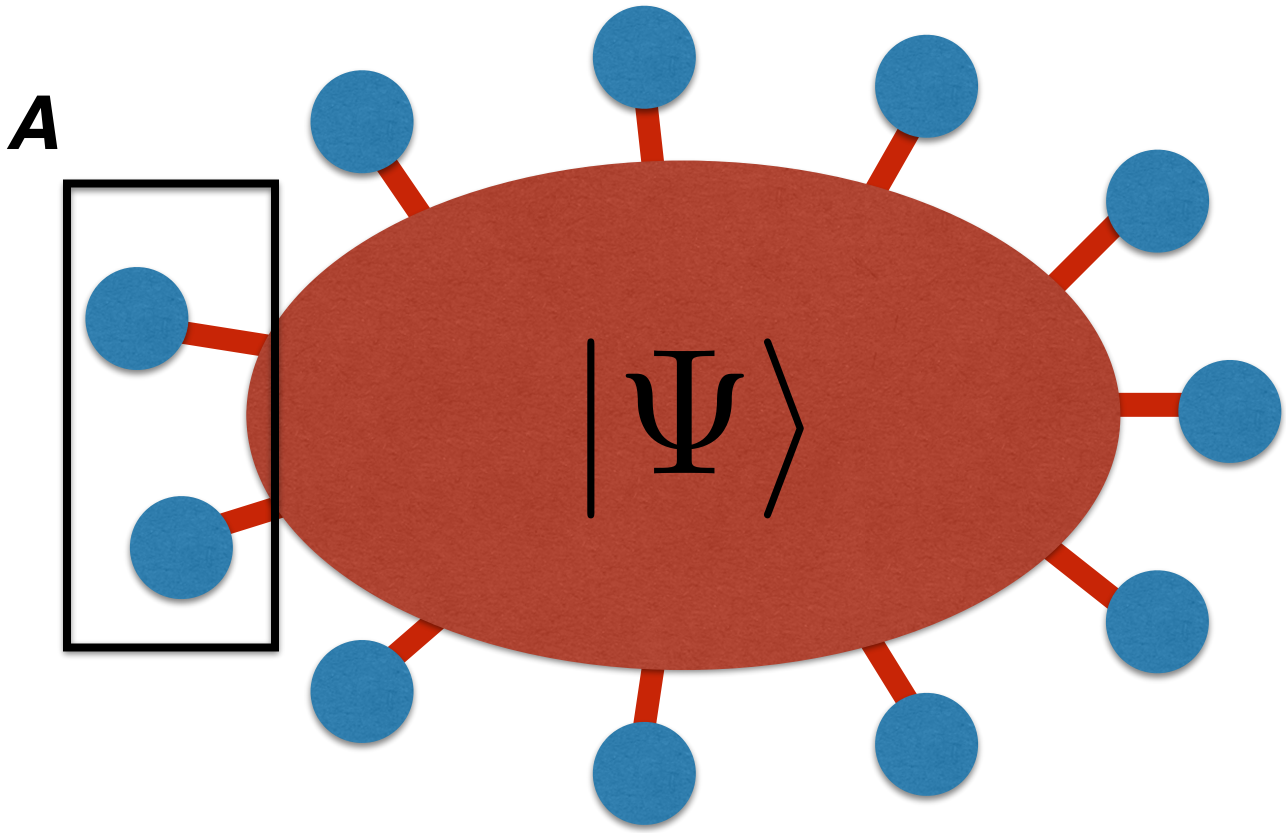



$|\Psi\rangle$

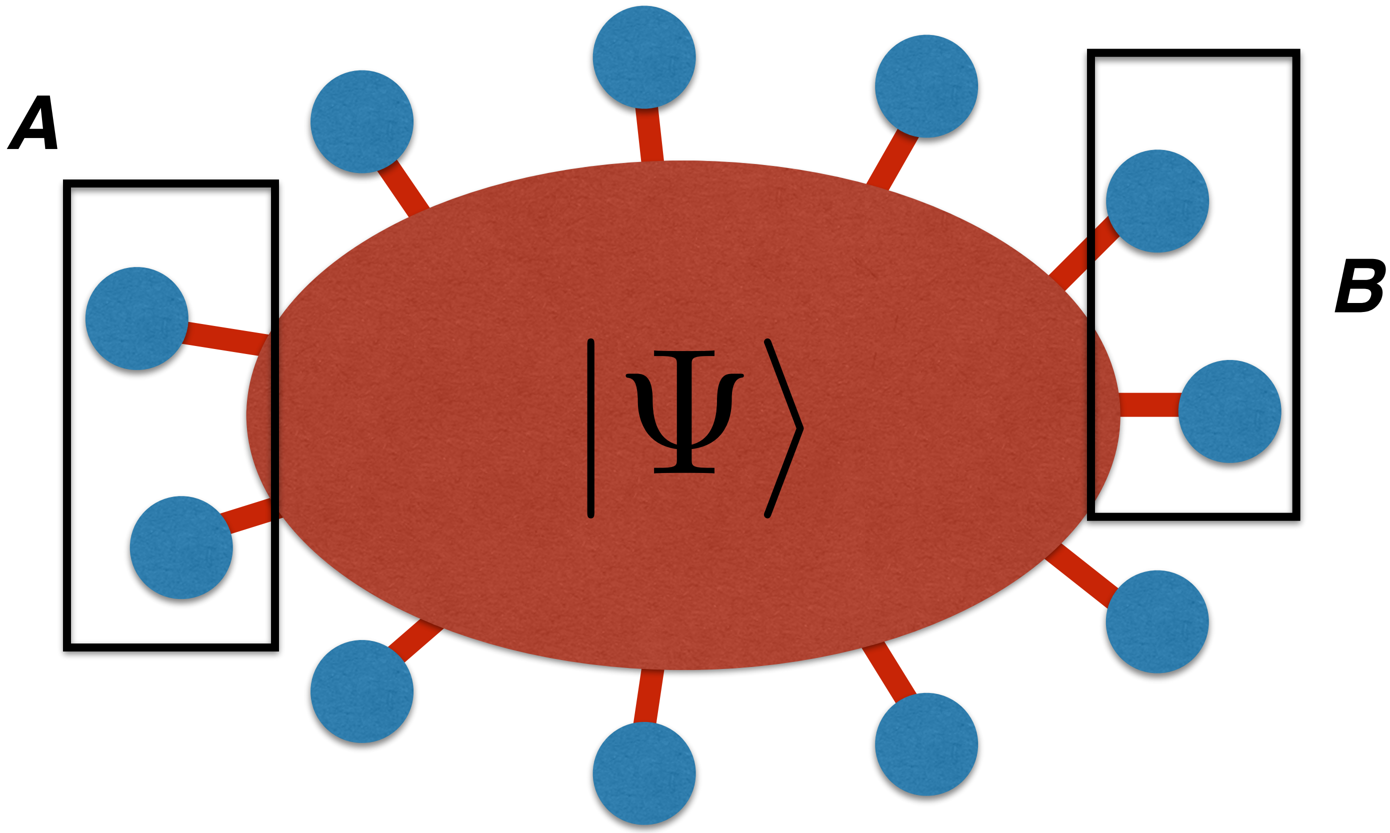


How is quantum information organized in:

- Ground states?
- Energy eigenstates?
- Thermal / thermalizing states?



$$S_A = -\mathrm{tr}\rho_A \log \rho_A$$



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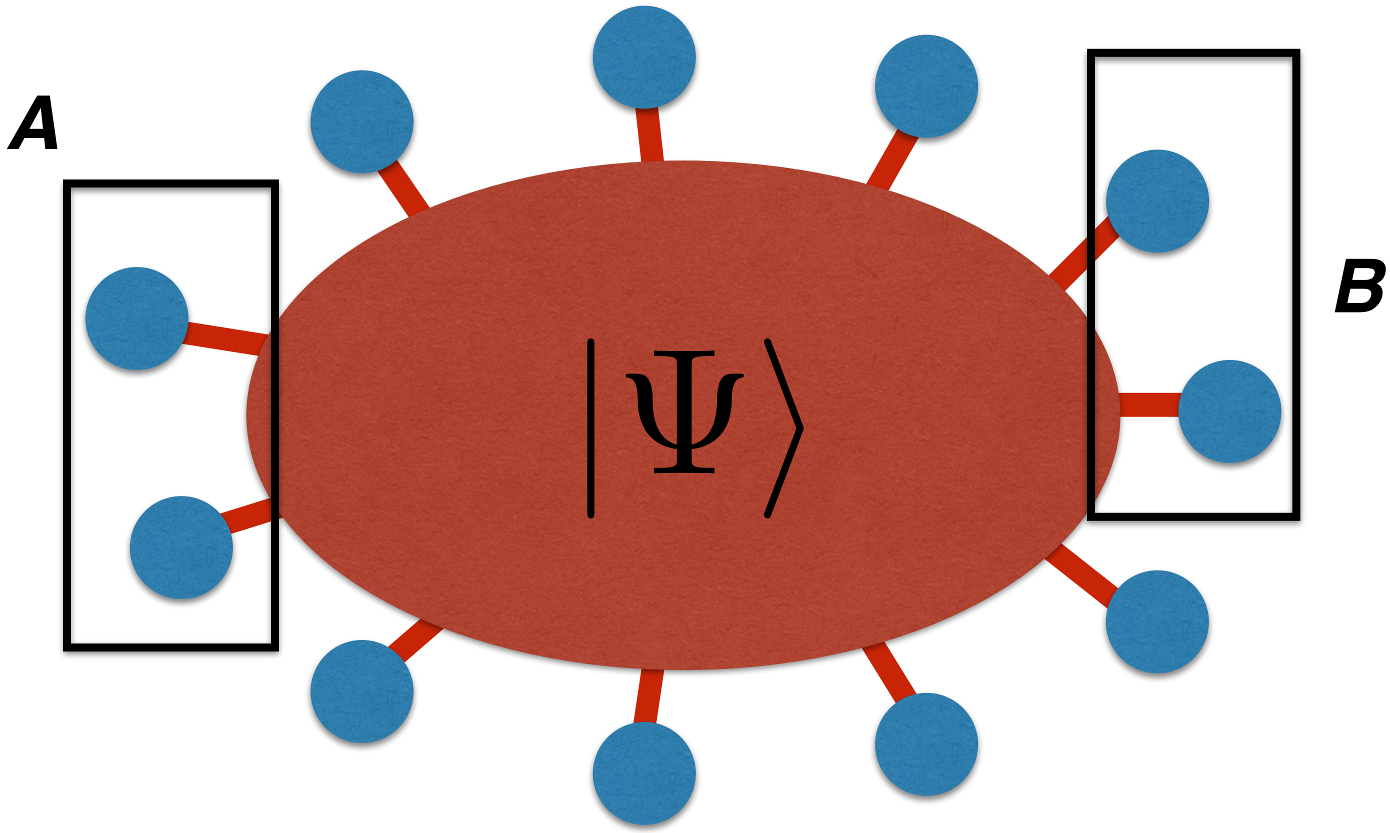

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Entanglement entropy of every possible subsystem “maps out” the storage of quantum information.

In local systems, this entanglement map and its dynamics have an intricate structure.

This structure is not something we directly measure in experiment — but it is the background arena for all of the more detailed questions about the system (correlation functions).

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In holographic QFTs, it is closely related to emergent geometry:

- Thermofield double for eternal black holes
  Maldacena '01

- Holographic entanglement entropy formula
  Ryu, Takayanagi '06; Hubeny, Rangamani, Takayanagi '07; Lewkowycz, Maldacena '13; etc.

This suggests that

Gravity <—> Entanglement Dynamics

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Point of this talk:

- In several interesting situations, we can derive nonlinear *3d gravity* directly from the entanglement dynamics of *2d CFT*.

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  TH ‘13;
  Asplund, Bernamonti, Galli, TH ‘14

- Similar methods lead to new results in non-holographic 2d CFTs.

  Asplund, Bernamonti, Galli, TH ’15

Mostly based on 1410.1392 and 1506.03772, with:

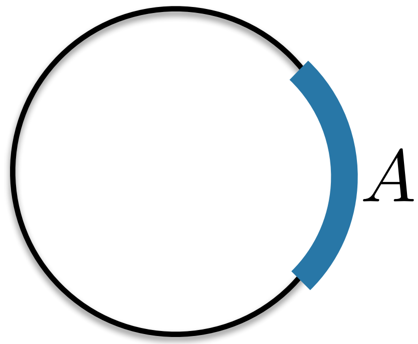
Curtis Asplund

Alice Bernamonti

Federico Galli.

We are in 1+1d, so space is a line and subsystem ***A*** consists of one or more intervals:

Or, we can put the theory on a circle:



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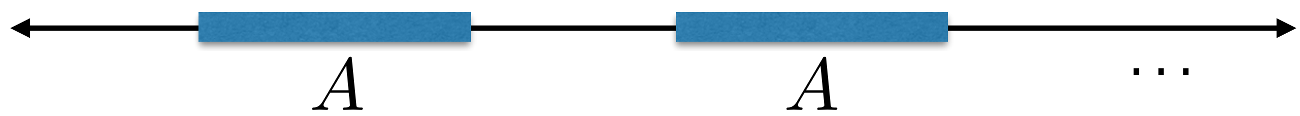

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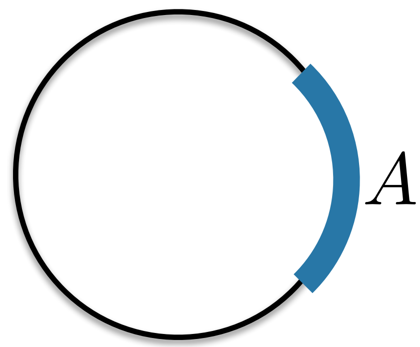

The full system is in a pure state:

$$\rho = |\Psi\rangle\langle\Psi|$$

and we want to compute the entanglement entropy

$$S_A = -\mathrm{Tr}\rho_A \log \rho_A , \qquad \rho_A \equiv \mathrm{Tr}_{A^C}\rho$$

Calculating Entanglement Entropy

[Calabrese, Cardy '04]

$$S_A = -\lim_{\epsilon\to 0} \log \mathrm{Tr}\rho_A^{1+\epsilon}$$

Entanglement entropy is computed from a correlation function of infinitesimal conical defects:

$$S_A \sim -\lim_{\epsilon\to 0} \frac{1}{\epsilon} \log\langle\Psi|\sigma_\epsilon(0)\bar{\sigma}_\epsilon(x)|\Psi\rangle$$

The "twist operator" σ_ϵ has conformal dimension

$$\Delta = \frac{c}{12}\epsilon$$

In general, these correlators cannot be computed. They depend on all the details of the CFT.

But in holographic CFTs:

- Large central charge $c >> 1$
- Sparse spectrum of low-dimension operators

Dominant contribution comes from stress-tensor exchange and can be computed analytically in several interesting states.

—> Universality

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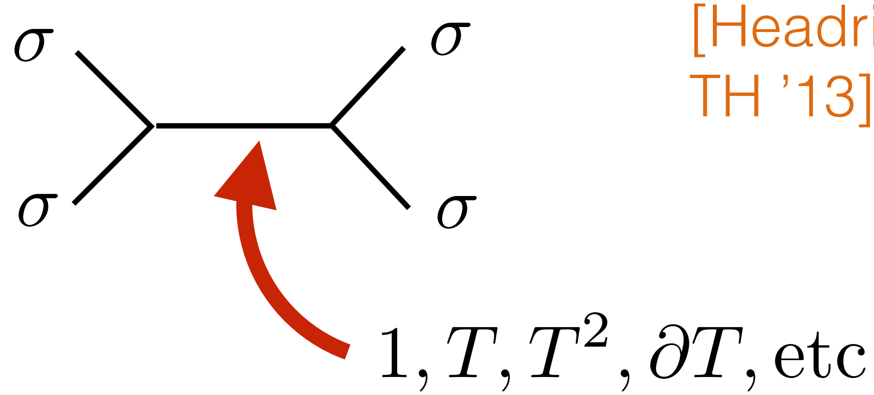
*with caveats

Two intervals in vacuum:

[Headrick '10; TH '13]

$$\langle 0|\sigma\sigma\sigma\sigma|0\rangle \sim \sum$$



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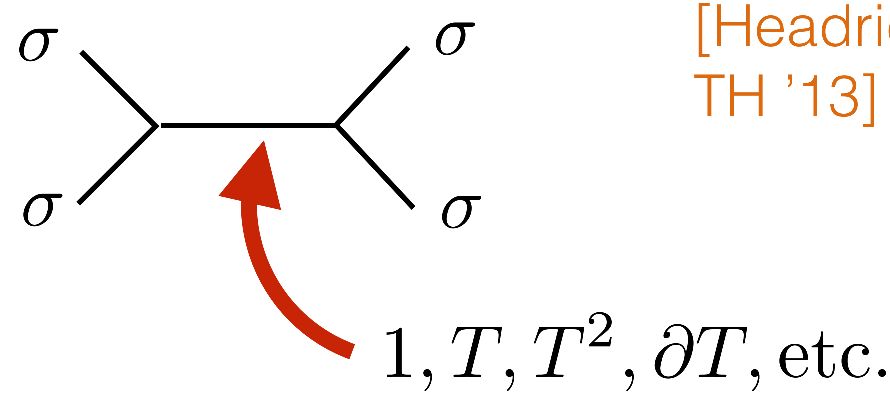
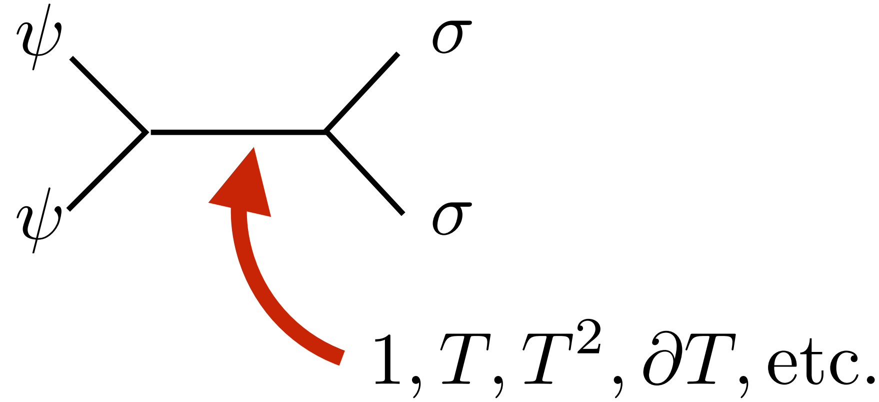


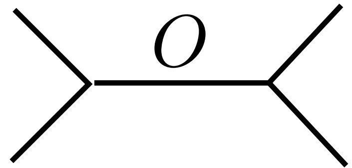
Single interval in an excited state:

$$\langle 0|\psi\sigma\sigma\psi|0\rangle \sim \sum$$



[Fitzpatrick, Kaplan, Walters '14; Asplund, Bernamonti, Galli, TH '14]

The sum of stress-tensor exchanges,

$$\sum_{1, T, T^2, \partial T, \text{etc.}}$$ 

is, by definition, the Virasoro conformal block for the vacuum representation.

$$S_A \leftrightarrow \log \mathcal{F}_{\text{vac}}(z)\mathcal{F}_{\text{vac}}(\bar{z})$$

Virasoro block

This function is not known in general; but can be computed in the relevant limit (large *c*, vanishing defect) using a method of Zamolodchikov.

Zamolodchikov's method to calculate $\mathcal{F}$ (rephrased slightly):

- Find a flat SL(2,C) connection with defects at the interval endpoints
- Impose trivial holonomy around region ***A***
- *…etc…*

This is identical to constructing a 3d hyperbolic geometry, and evaluating the on-shell Einstein action!

Therefore: geodesic lengths are encoded in the Virasoro vacuum block.

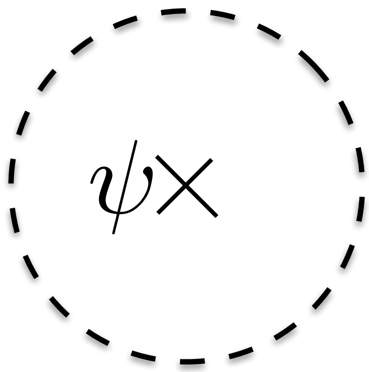
[TH '13; Faulkner '13]

Results

Application 1: Black hole Microstates

[Asplund, Bernamonti, Galli, TH'14; Fitzpatrick, Kaplan, Walters '14]

Insert a high-dimension operator at the center of a disk:



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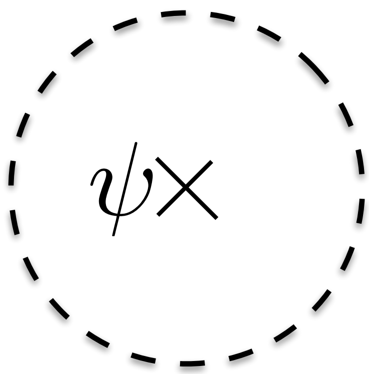

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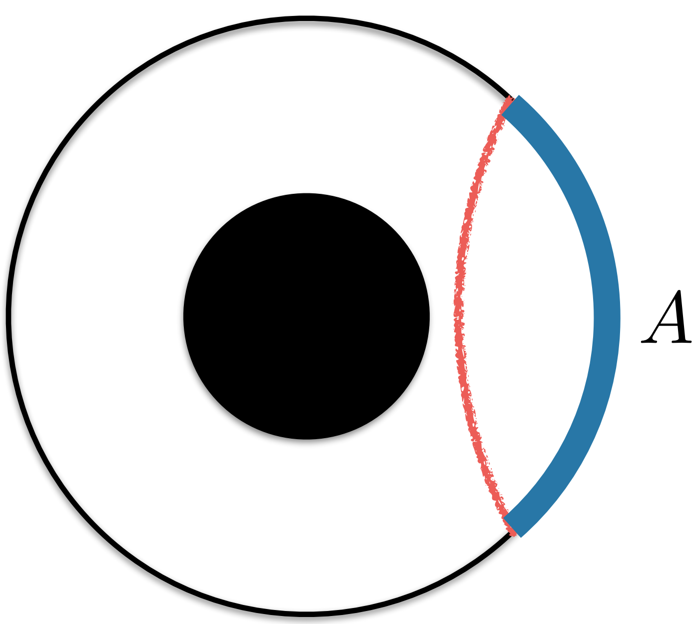

Entanglement from RT:

$$S_A = \frac{\text{area}}{4}$$

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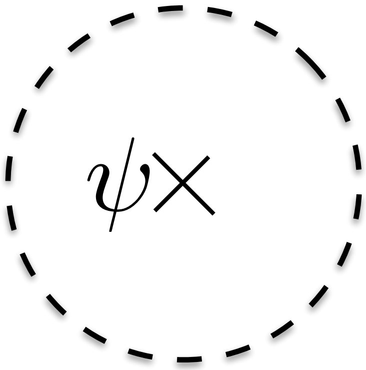

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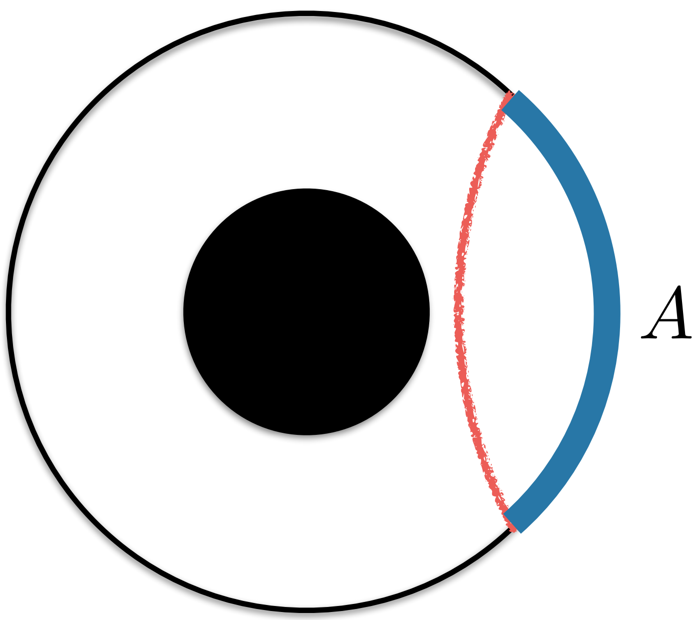

Entanglement from RT:

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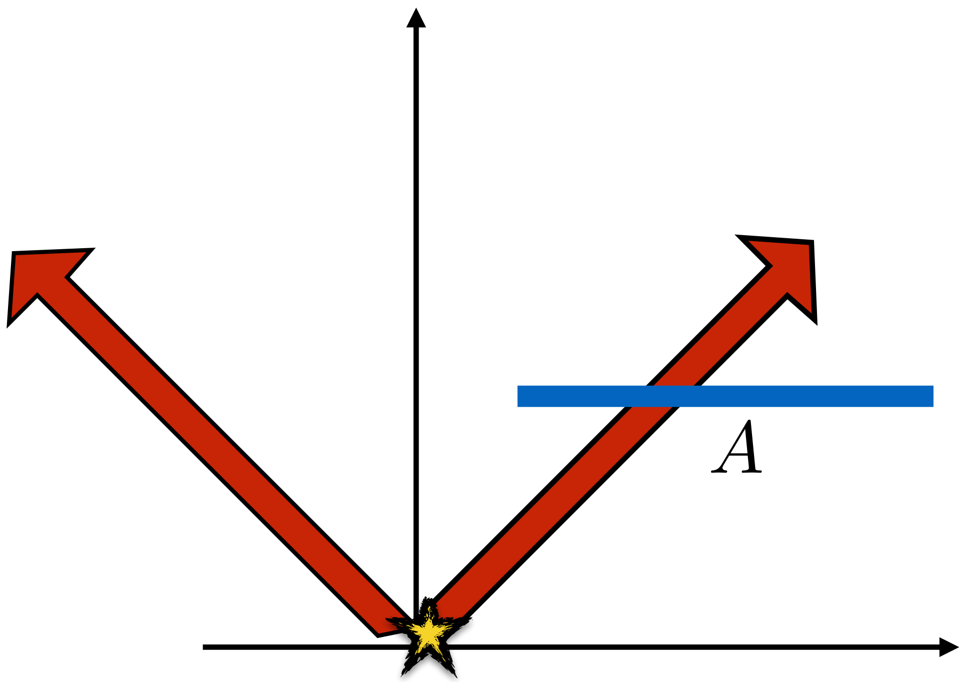
This is reproduced by the Virasoro block.

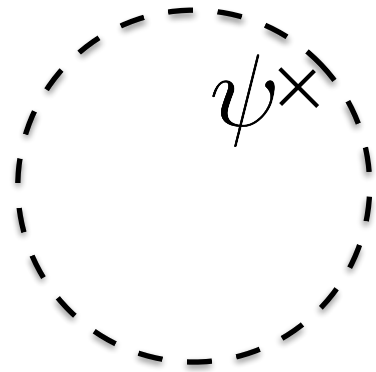
Application 2: Local operator quench

Inject a large *O(c)* amount of energy into a CFT at the origin.

How does entanglement evolve with time?



State is created by operator inserted near the unit disk:

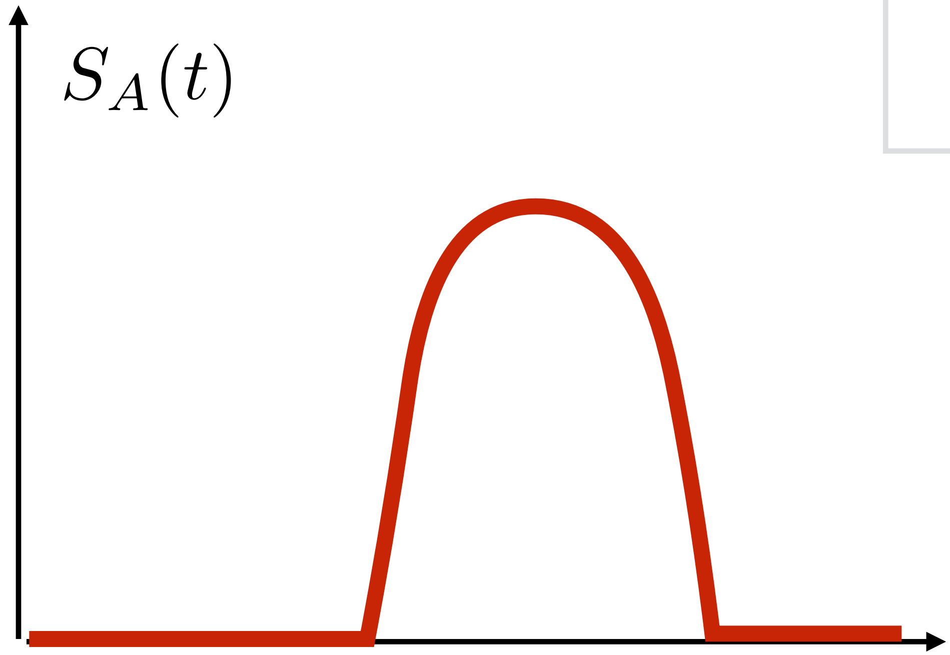


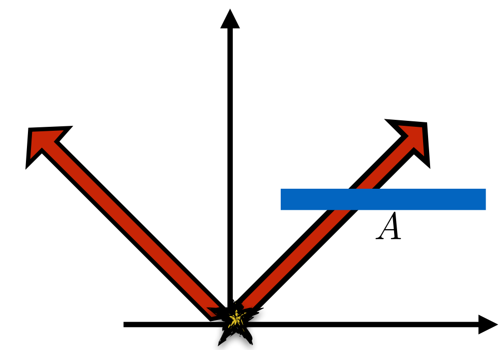
Almost the same problem as the black hole, but need to be careful with operator ordering in Lorentzian signature.

When the dust settles, entanglement is computed by a monodromy of the Virasoro vacuum block:

$$\mathcal{F}_{\text{vac}}(z)\Big|_{z \circlearrowleft 1,\, z\to 0}$$

This gives





which agrees with the holographic formula applied to a boosted particle or black hole.

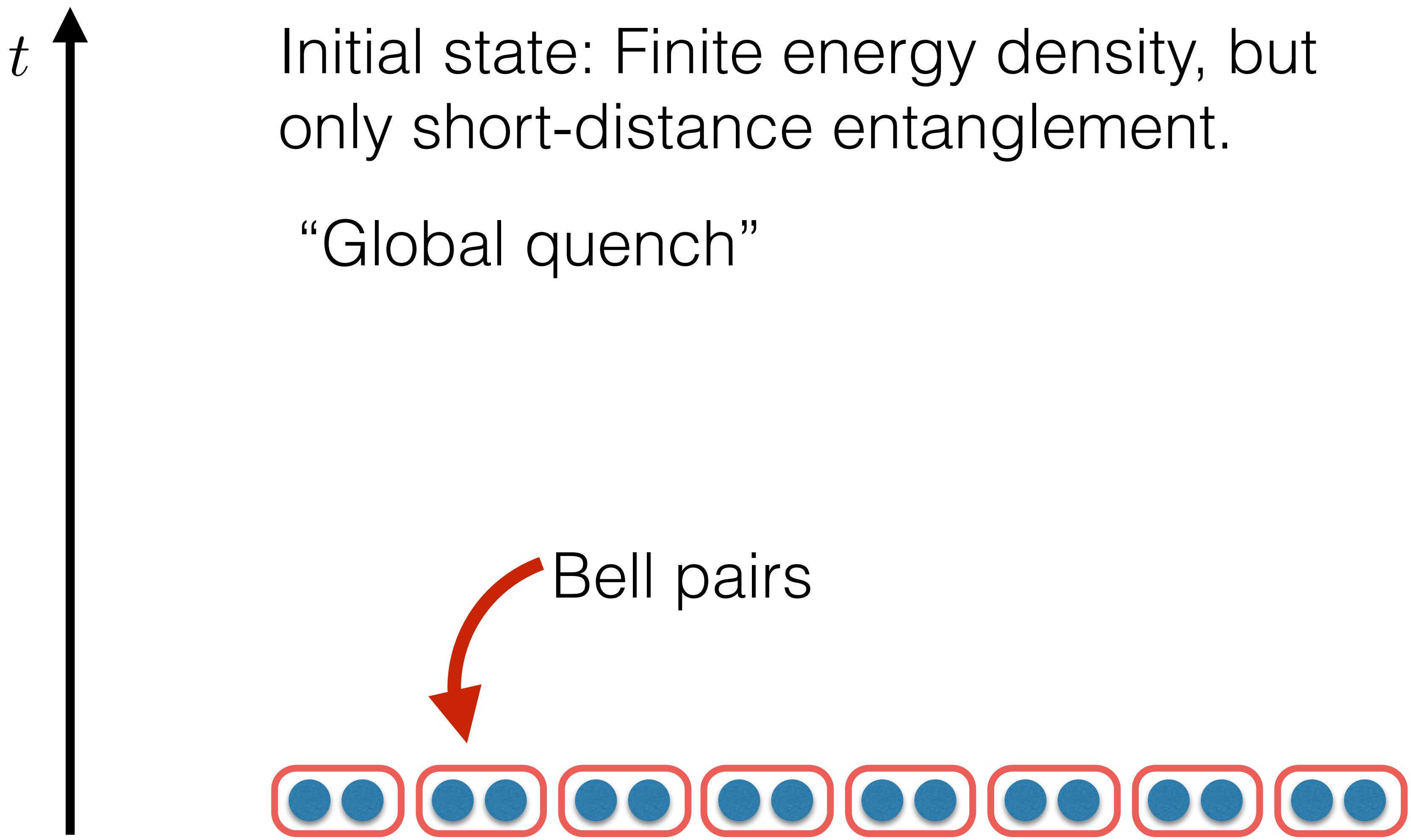
Gravity: Nozaki, Numasawa, Takayanagi ’13
CFT: Asplund, Bernamonti, Galli, TH’14

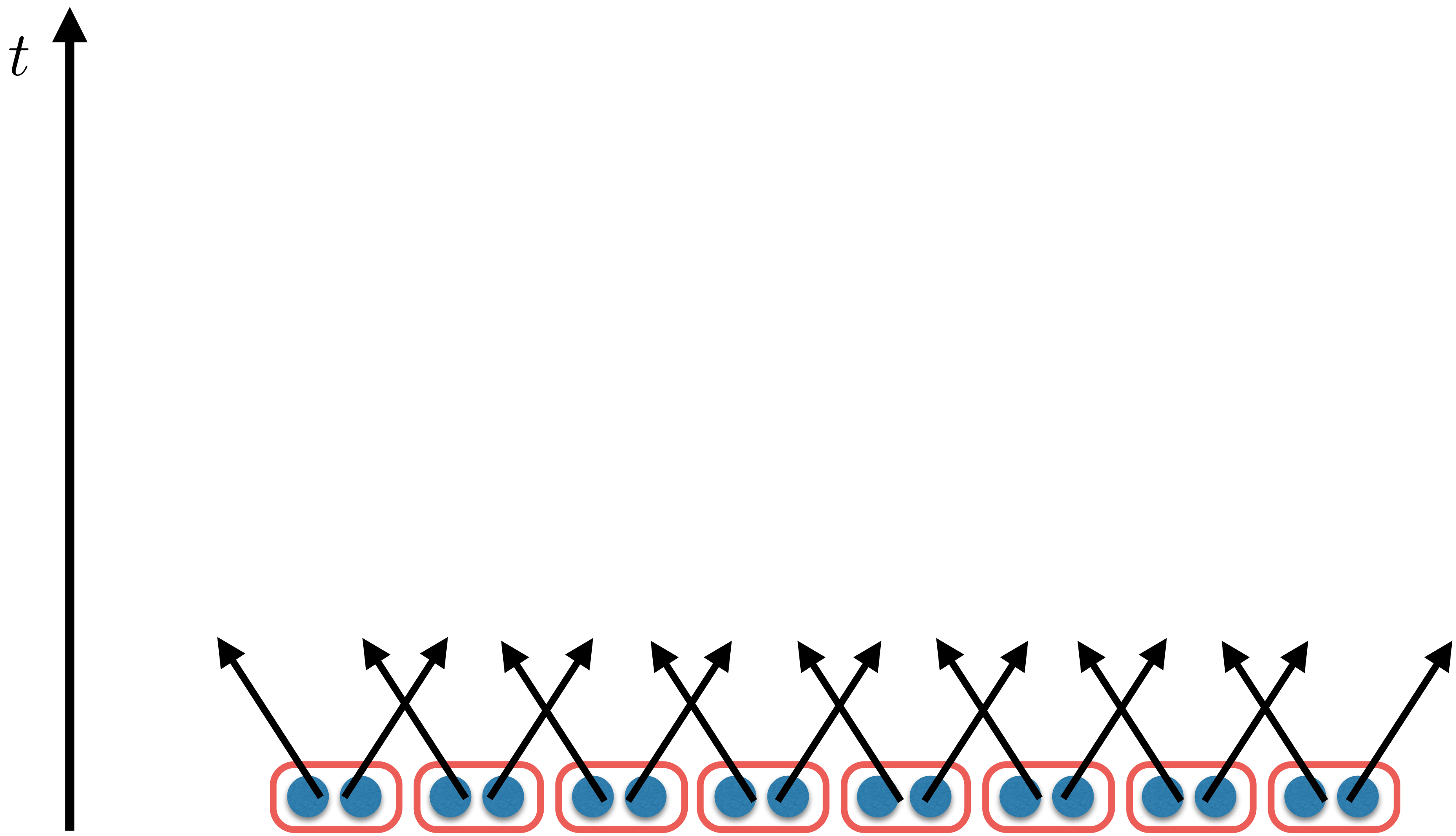
Application 3: Thermalization

Asplund, Bernamonti, Galli, TH'15

Initial state: Finite energy density, but only short-distance entanglement.

“Global quench”

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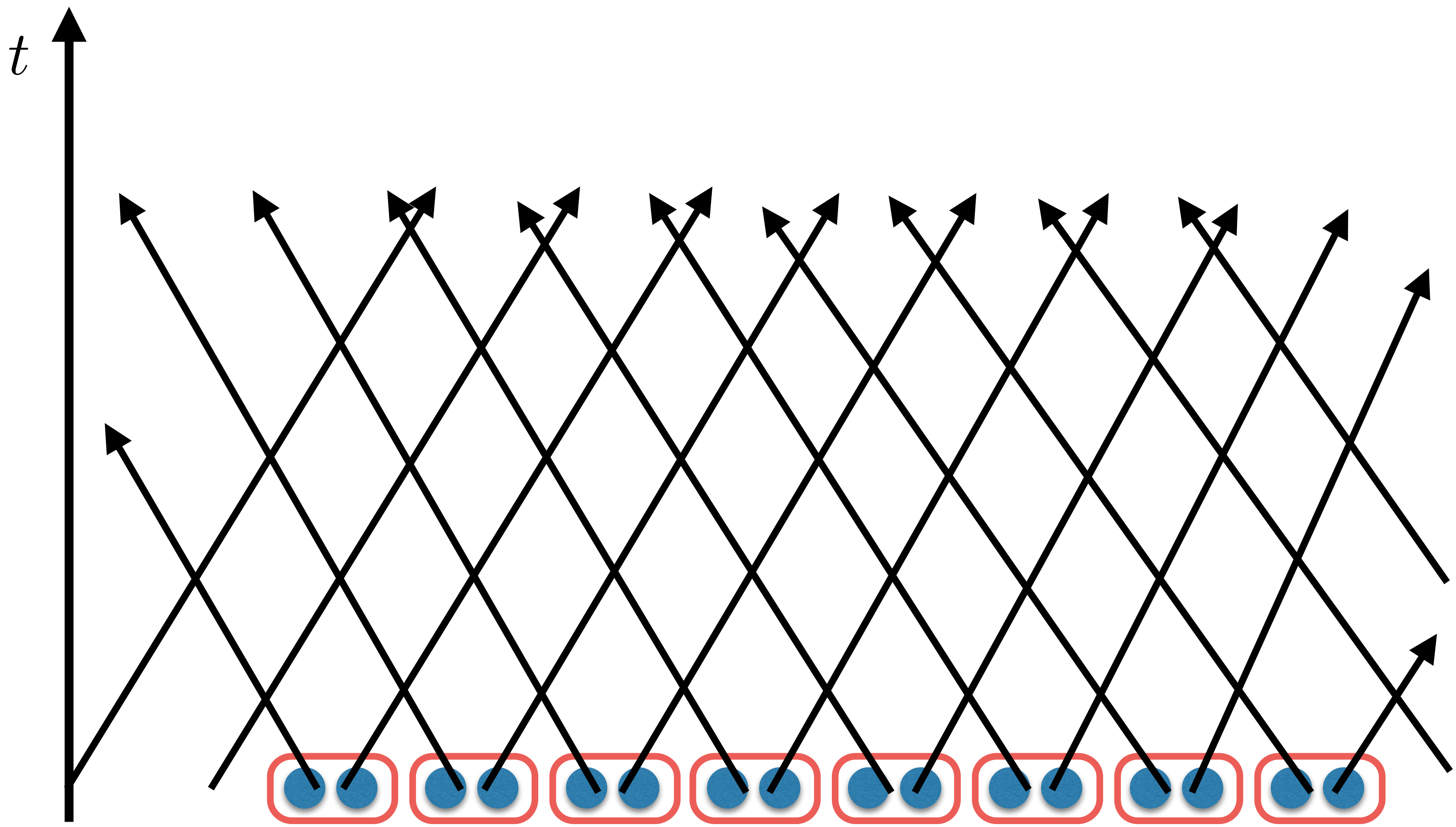
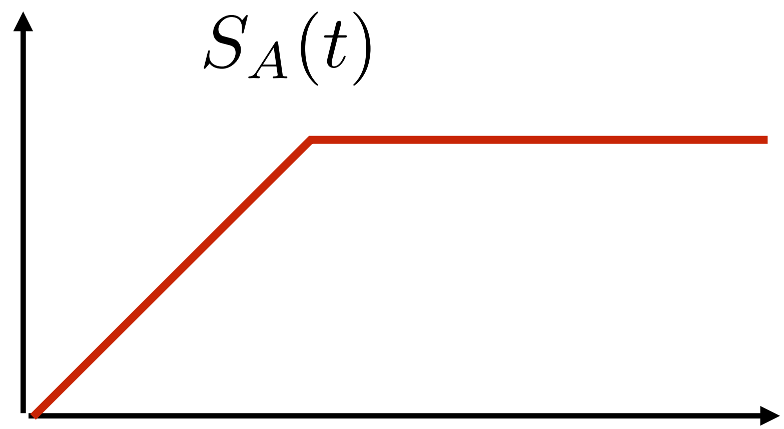


How does this state thermalize?

For a single interval:



- Entanglement grows linearly, then saturates at the thermal value.

- in any CFT; holographic or not

[CFT]
Calabrese, Cardy ‘05
[Holography]
Abajo-Arriasta et al ’10
Balasubramanian et al ’11
TH, Maldacena ’13

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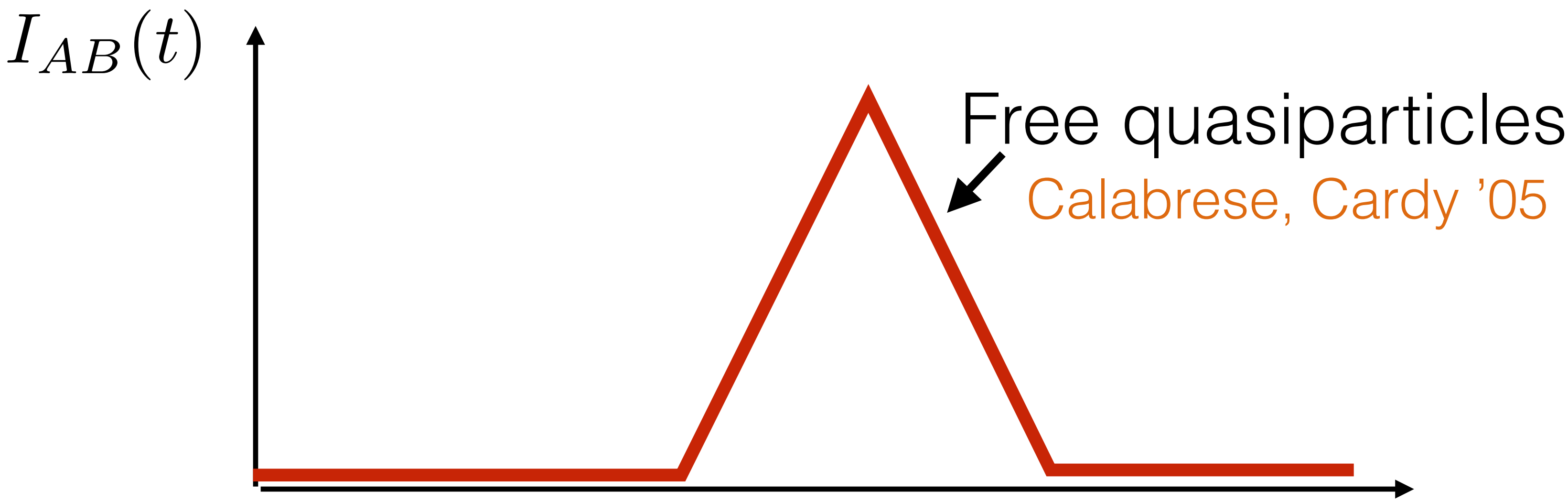

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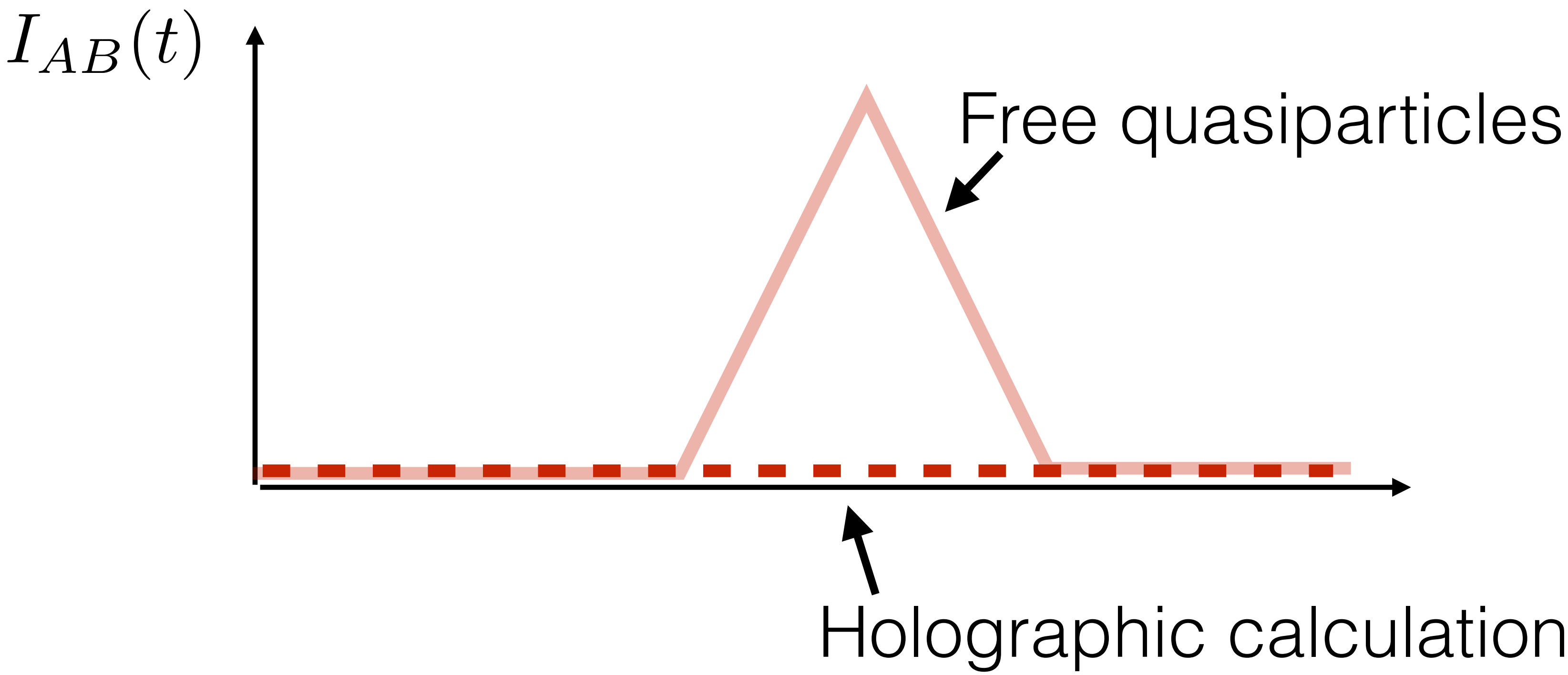

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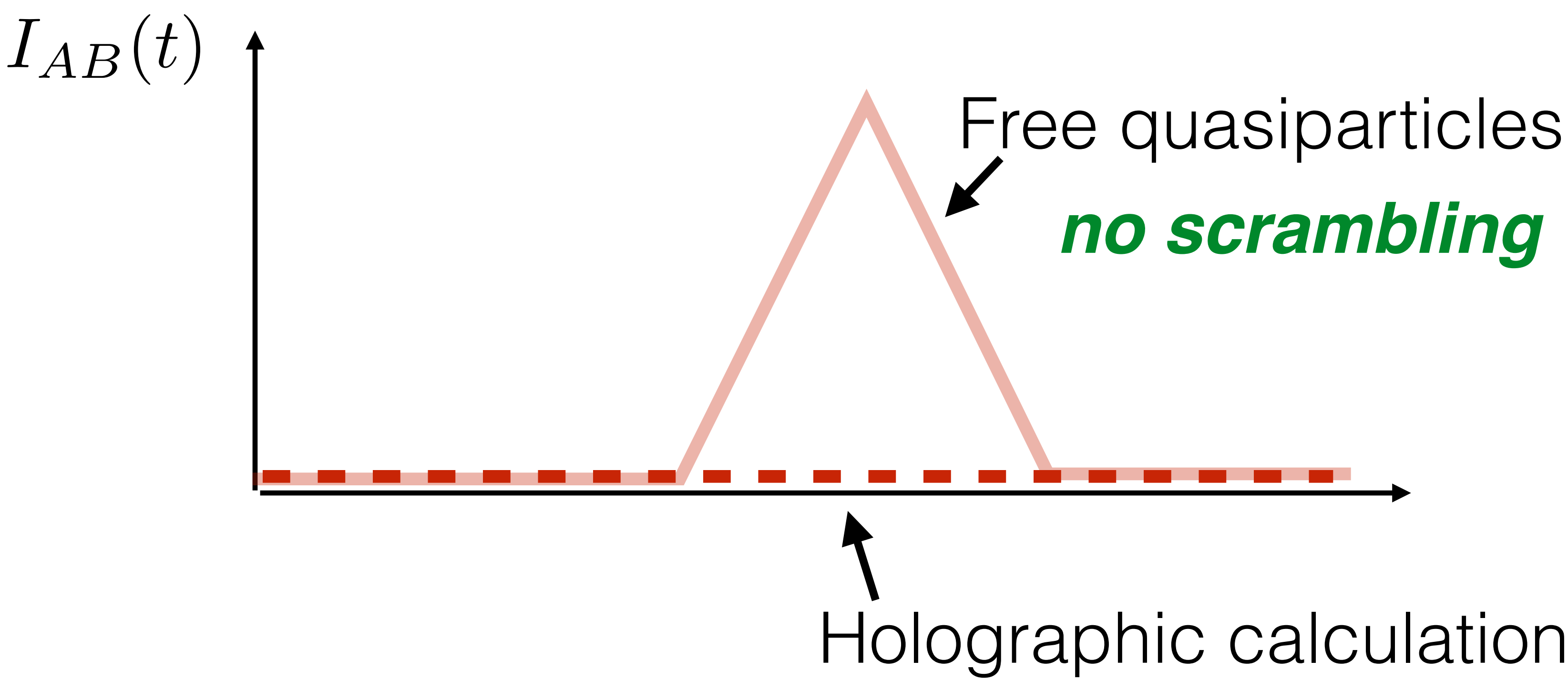

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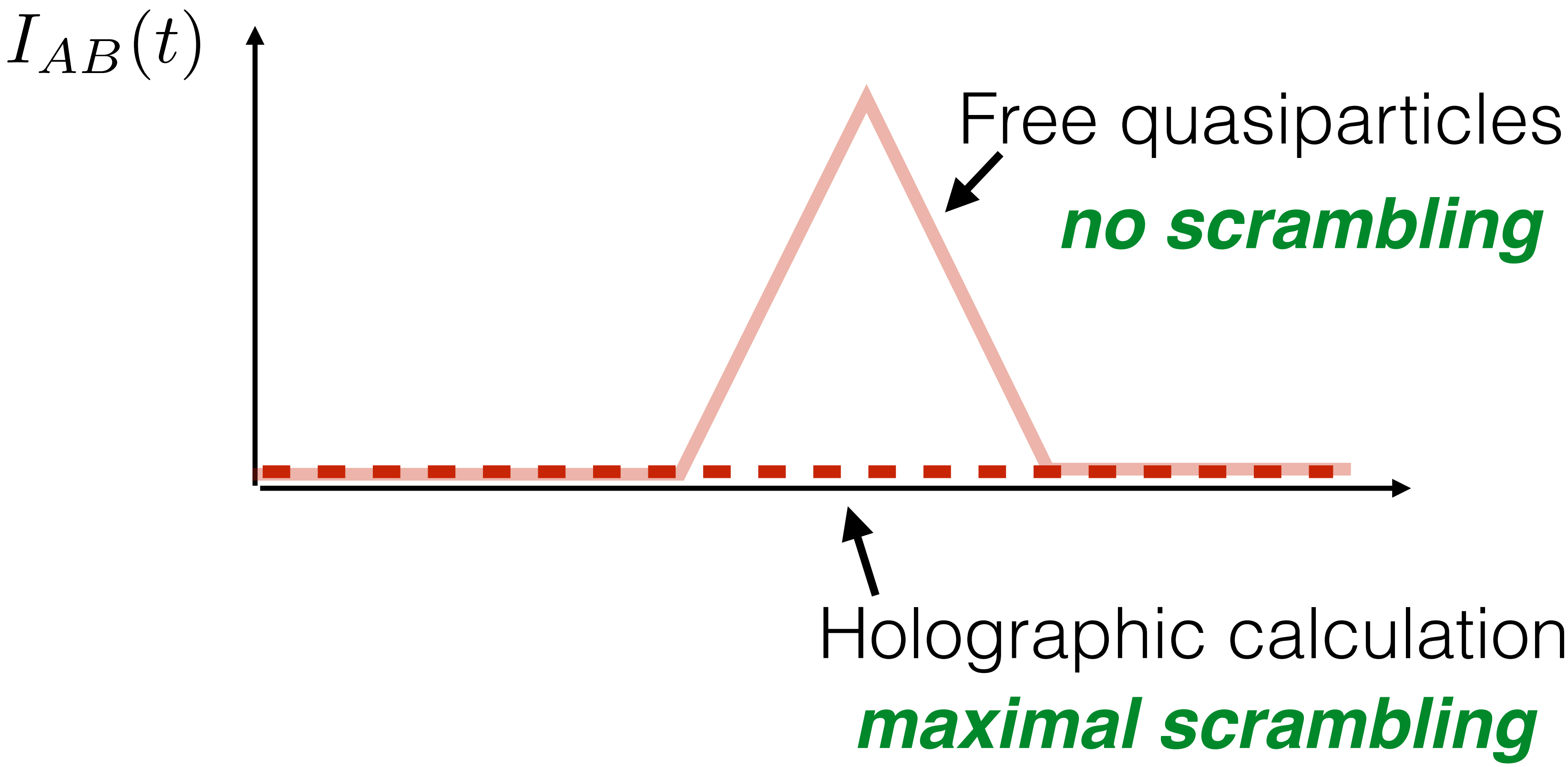

Asplund, Bernamonti, Galli, TH'15

General CFT results

"Bump" comes from singularity in

$$\mathcal{F}_{\text{vac}}(z)\Big|_{z\to 1}$$

The strength of this singularity depends on the central charge compared to # of conserved currents:

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- $c \leq N_{currents}$: free quasiparticle answer
- $c \to \infty$ and sparseness: holographic answer

Entanglement scrambles maximally in holographic 2d CFTs

Asplund, Bernamonti, Galli, TH'15

For moderate $c > 1$:

- Entanglement scrambles
- This was not thought to be possible in 2d CFT, due to "factorization" of left and right-movers
- Condensed matter incarnation?

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- Ultimately, the most interesting question is how/when geometry breaks down in quantum gravity — *eg,* info loss.
- This question can be asked in 3d; presumably the answer is similar to higher dimensions.
- To address this in AdS/CFT we need to understand where semiclassical geometry came from in the first place; then we can ask how this approximation breaks down.