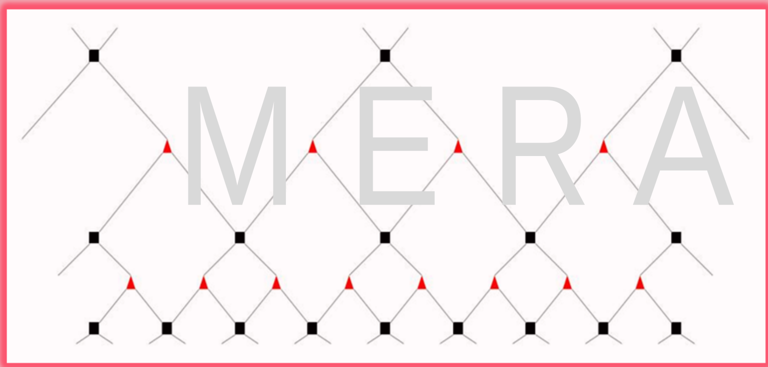
Tensor Networks $\longrightarrow$ Holography

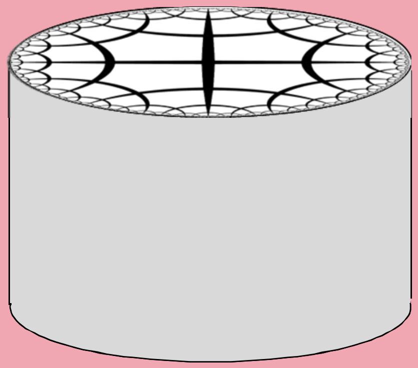
?

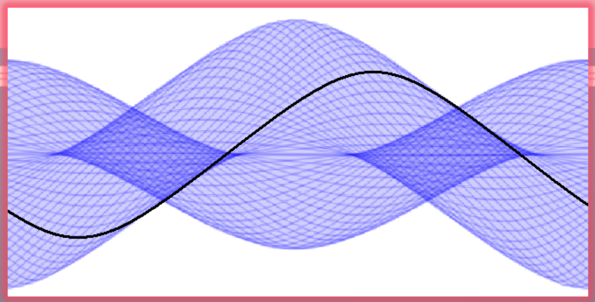
via

$\longrightarrow$

Integral Geometry







Bartłomiej Czech

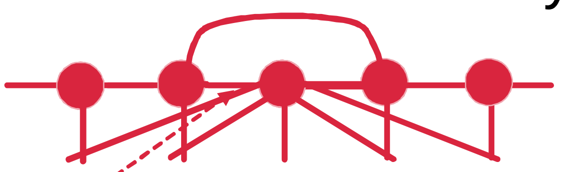
with Lampros Lamprou, Sam McCandlish, James Sully

Stanford University

Strings '15, Bengaluru, 25 June 2015

What are Tensor Networks?

- A tool in condensed matter theory
- useful for efficiently representing many-body wavefunctions:



$$|\Psi\rangle = \sum_{i_1 i_2 \ldots i_N} \Psi_{i_1 i_2 \ldots i_N} |i_1 i_2 \ldots i_N\rangle$$

a wavefunction

a tensor

a vector

in some α-dimensional vector space

O(N) parameters

α - “bond dimension”

$O(\#^N)$ parameters

- This class of states does not cover the whole Hilbert space

efficient representation

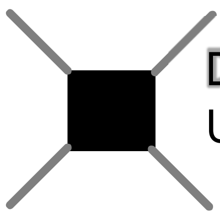
- The art is to define a class of tensor network states with desired physical properties

- For understanding the holographic architecture of AdS_3, use Multi-scale Entanglement Renormalization Ansatz: (Vidal, 2006)

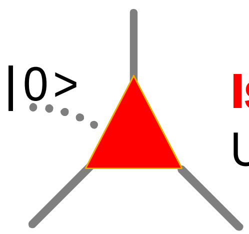
MERA

What is MERA?

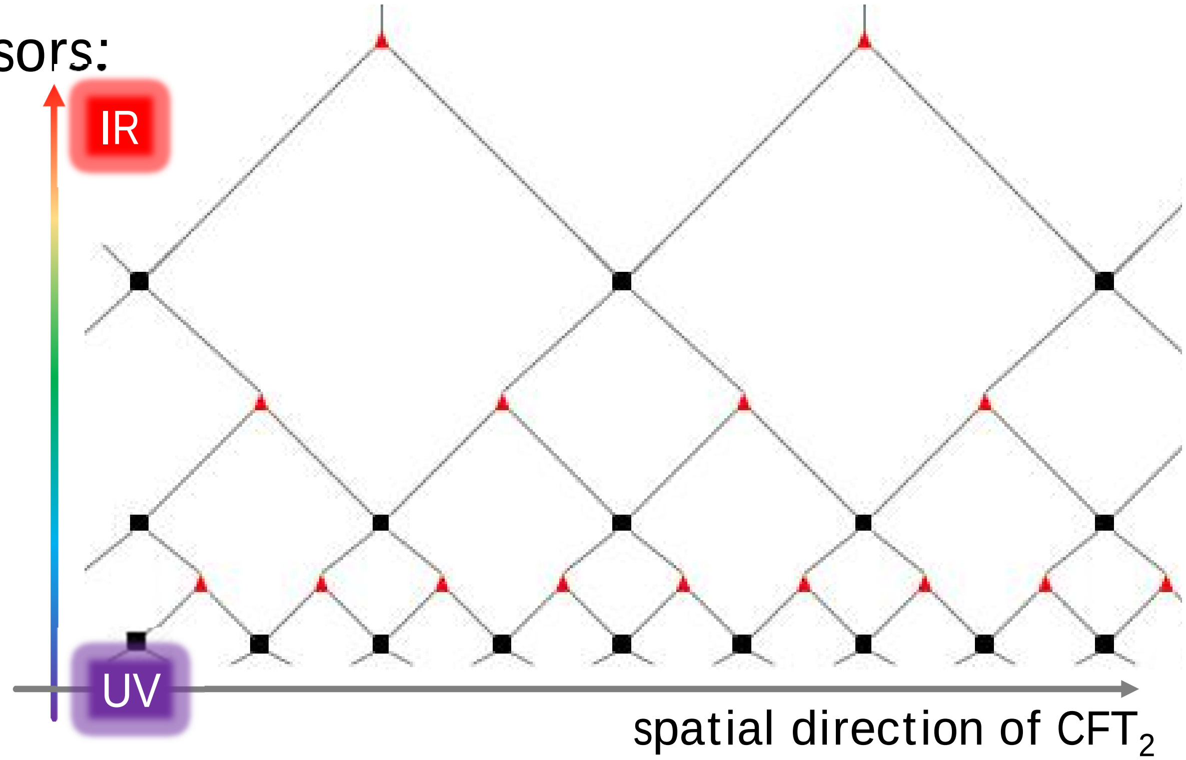
- Two types of unitary tensors:



Disentanglers remove UV entanglement



Isometries remove UV degrees of freedom

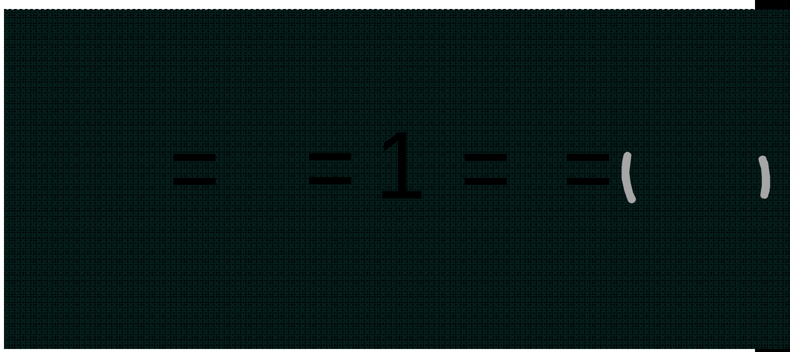


- Implements real space coarse-graining (renormalization group)
- A successful variational ansatz for finding ground states of 1+1-dimensional critical systems (e.g. Ising model)
- Manifests compression (entanglement entropies are apparent)

a working model of CFT_2

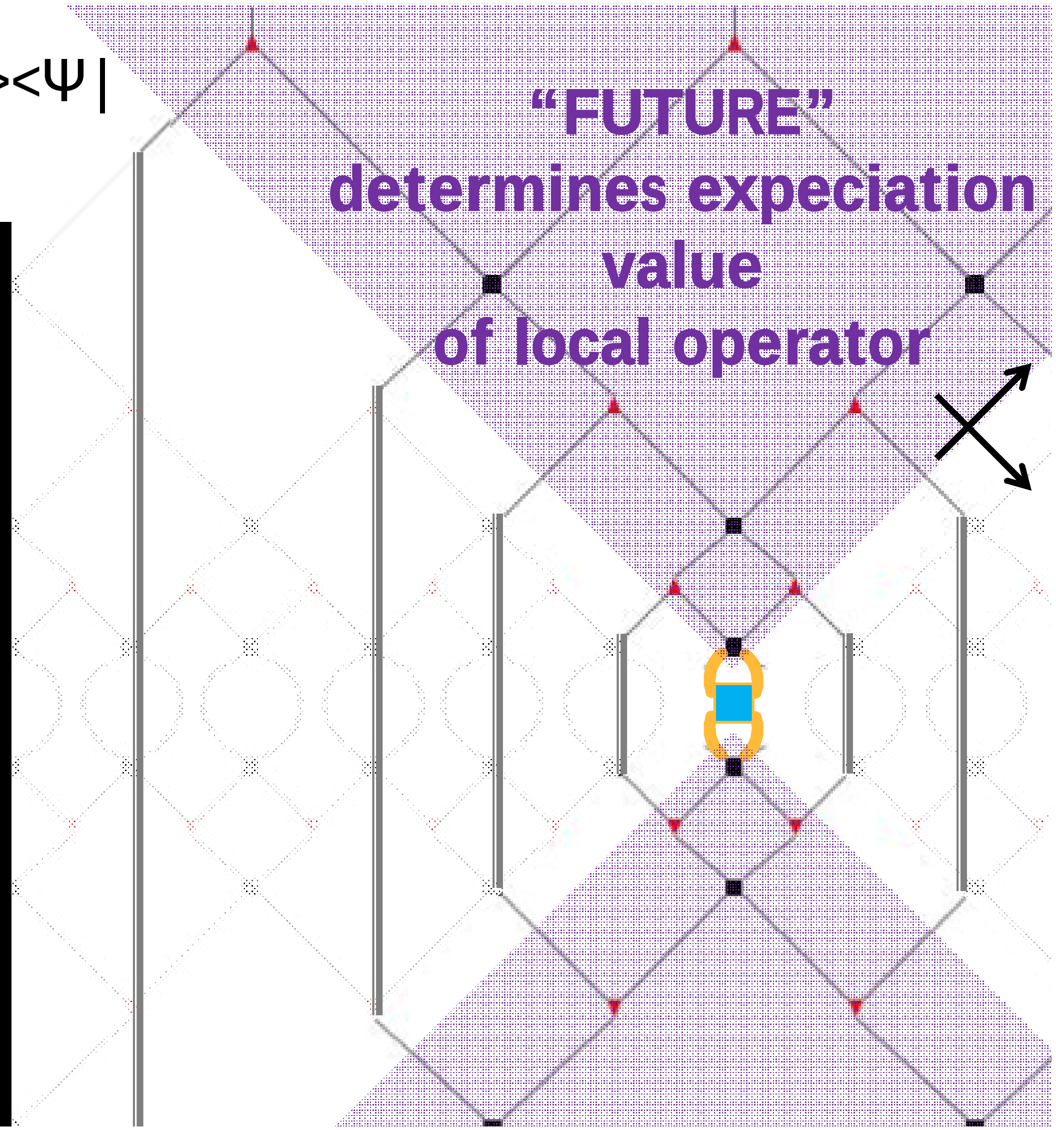
Causal structure and locality in MERA

- Compute <Ψ|**O**|Ψ> = Tr **O**|Ψ><Ψ|
- Unitarity of tensors implies:



Causal Structure

(in auxiliary time ↔ scale)

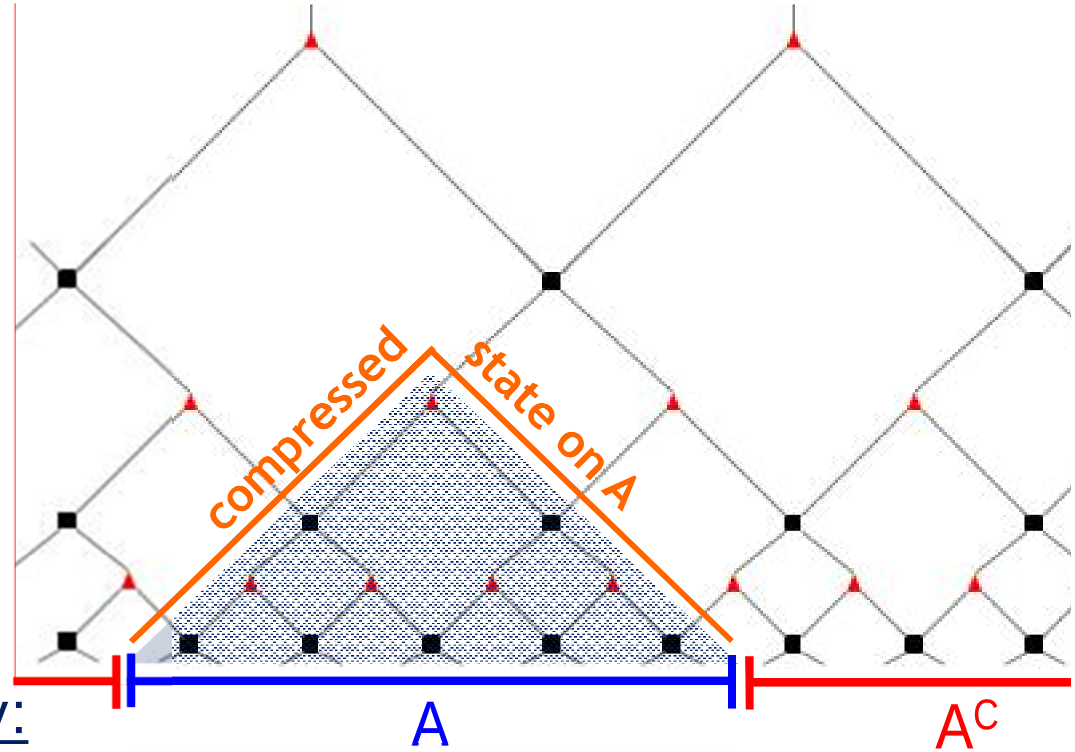


Minimal cuts and entanglement entropy

- The causal structure determines which tensors affect which expectation values
- The state on top of a ‘causal diamond’ (on a **minimal cut**) is a compressed state on **A**
- This gives an upper bound for the entanglement entropy: S(A) ≤ #(**cuts**)
- I will assume that:

$$\boxed{S(A) \sim \#(\textbf{cuts})}$$

- This reproduces S(A) ~ log|A|



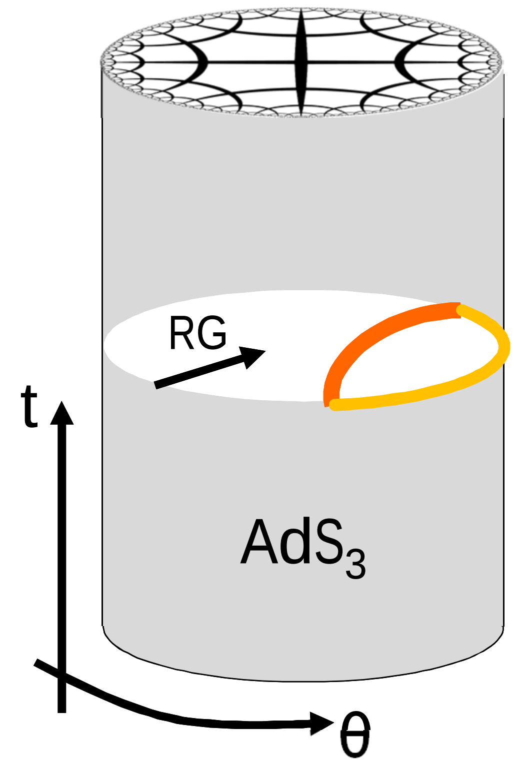
- these tensors do not affect expectation values of operators acting on A^C
- they form an isometry that acts within H_A

What is holographic duality?

- An auxiliary representation of a conformal field theory as a gravity theory
- which manifests scale transformations (RG) with an additional dimension

  spacelike!
- and which manifests **entanglement entropies** as spacelike geodesics

  (Ryu-Takayanagi, 2006)

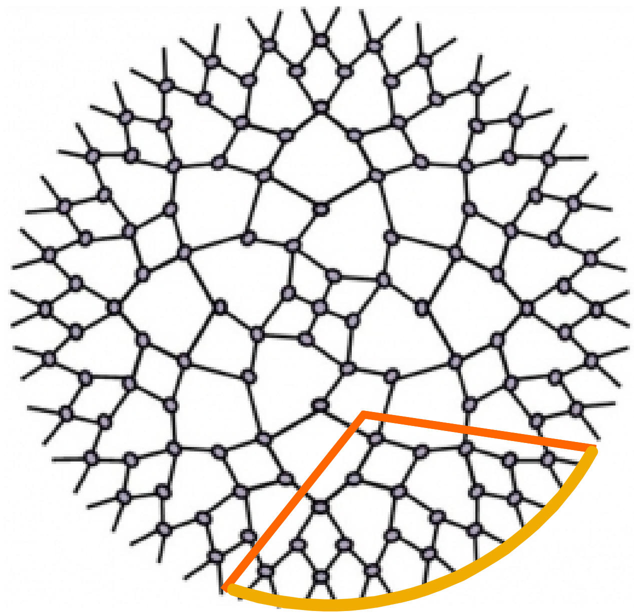


1+1-dimensional CFT

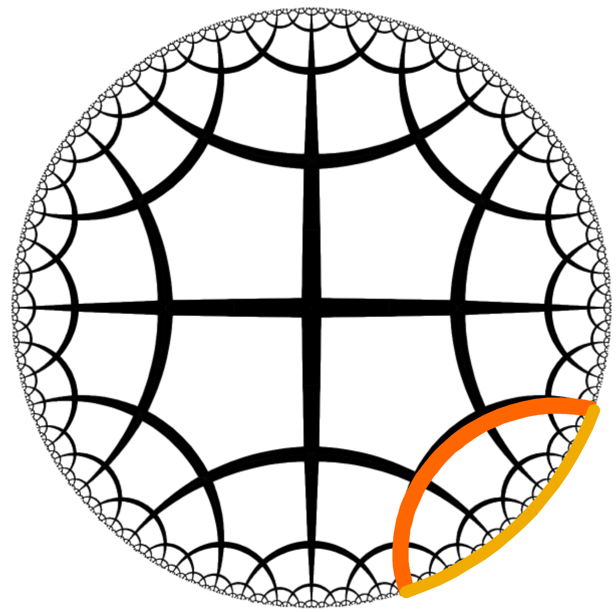
Sounds just like MERA!

Two descriptions of a state

MERA network



spatial slice
of holographic geometry



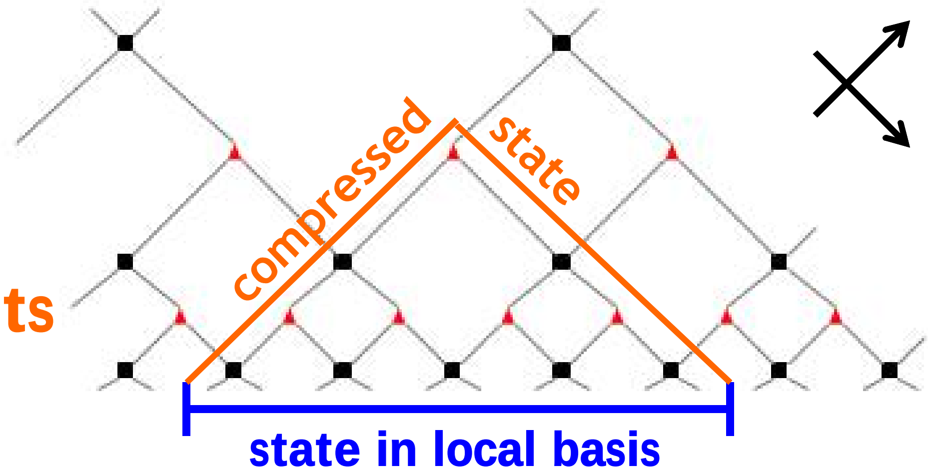
- an additional dimension timelike vs spacelike
- which encodes RG transformations
- entanglement entropies represented by **minimal cuts** ↔ **geodesics**
  (Ryu-Takayanagi, 2006)

Swingle, 2009

Goal: Quantify this relation

Ingredients:

- the causal structure of MERA
- the minimal cut prescription
- entanglement entropy ⟷ **#cuts**



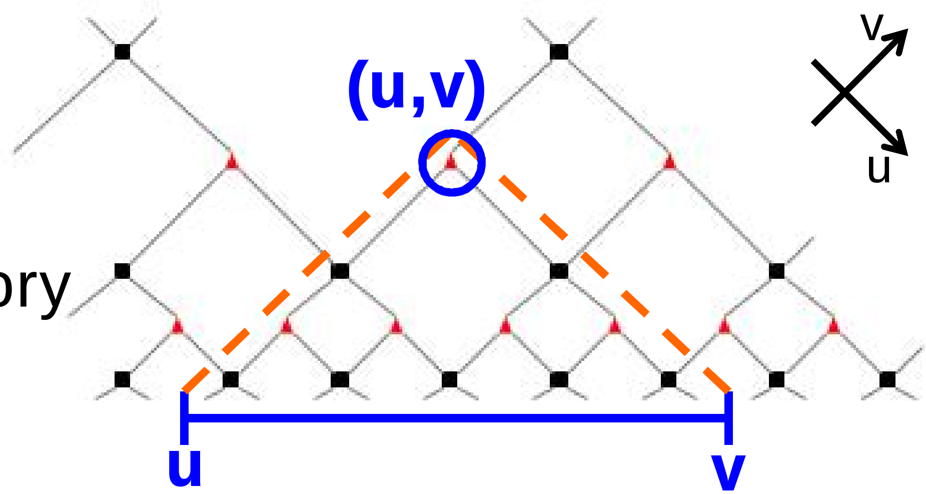
Strategy:

- construct a **metric** that captures this structure of MERA
- relate that **metric** to the holographic geometry

Kinematic Metric

Choosing a metric: causal structure

- identify null coordinates
- point **(u,v)** sits on top of a unique minimal cut,
- which identifies a field theory interval **(u,v)**
- the isometry at **(u,v)** completes the compression of the state on interval **(u,v)**

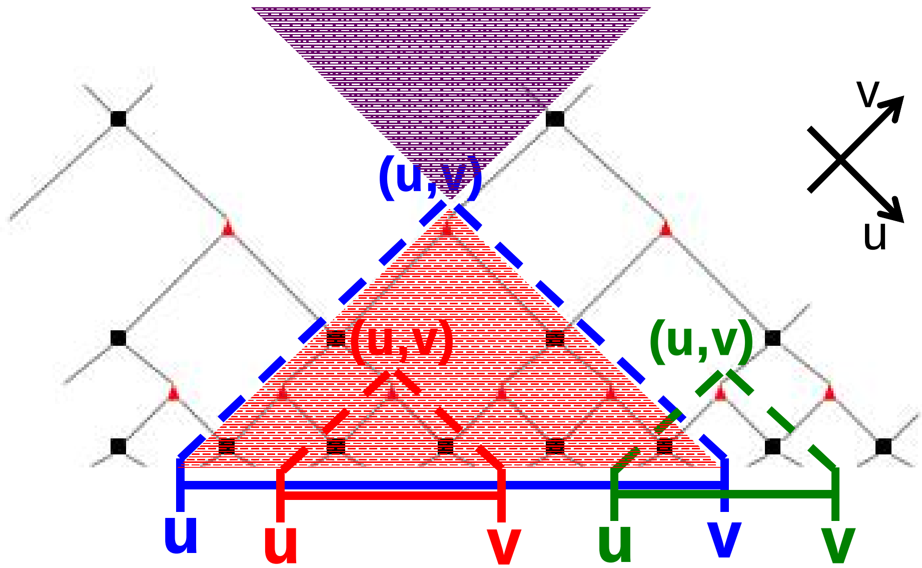


Kinematic Metric:

$$ds^2 = (...)\, du dv$$

Why is the metric Lorentzian?

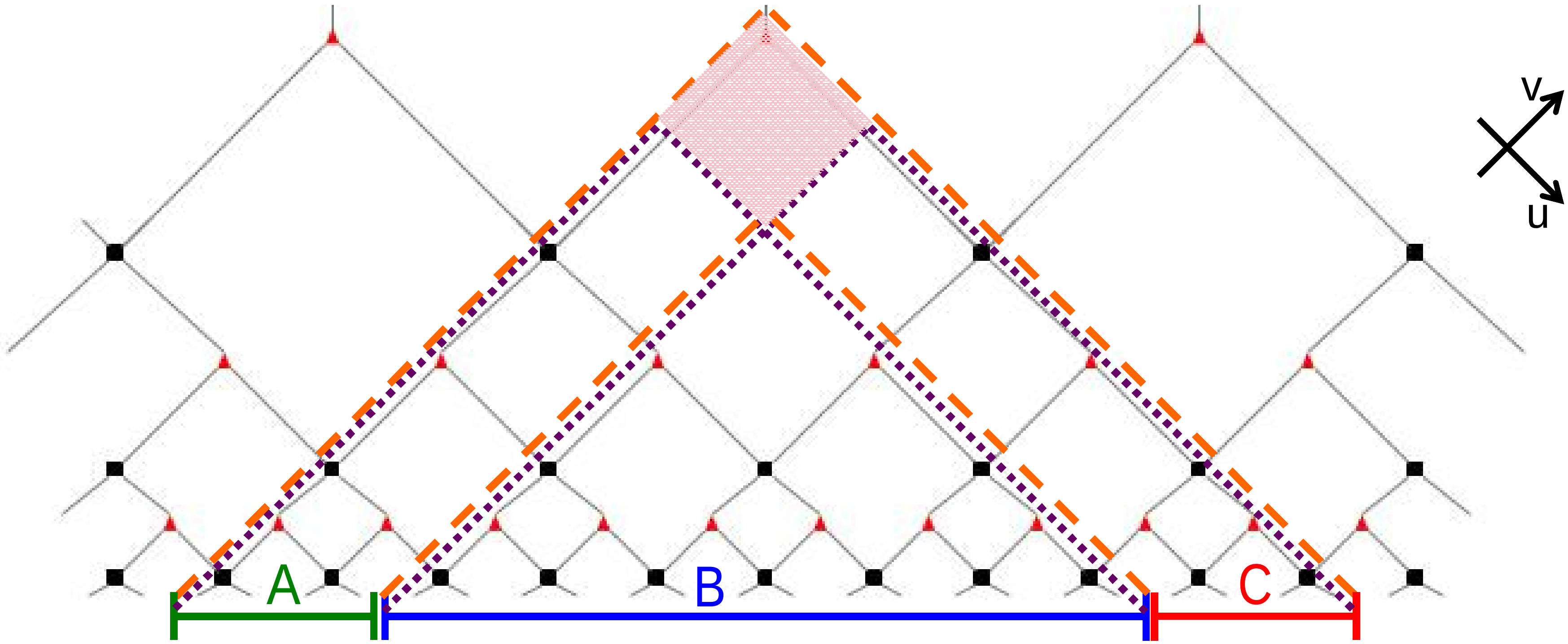
- timelike separated (u,v):
  (u,v) contains (u,v)
- spacelike separated (u,v):
  neither contains the other
- null separated:
  common endpoint
  left (**u** = **u**) or right (**v** = **v**)



- **Past:** all intervals contained in **(u,v)**
- **Future:** all intervals containing **(u,v)**

Kinematic Metric: $ds^2 = (\ldots)\, du dv$

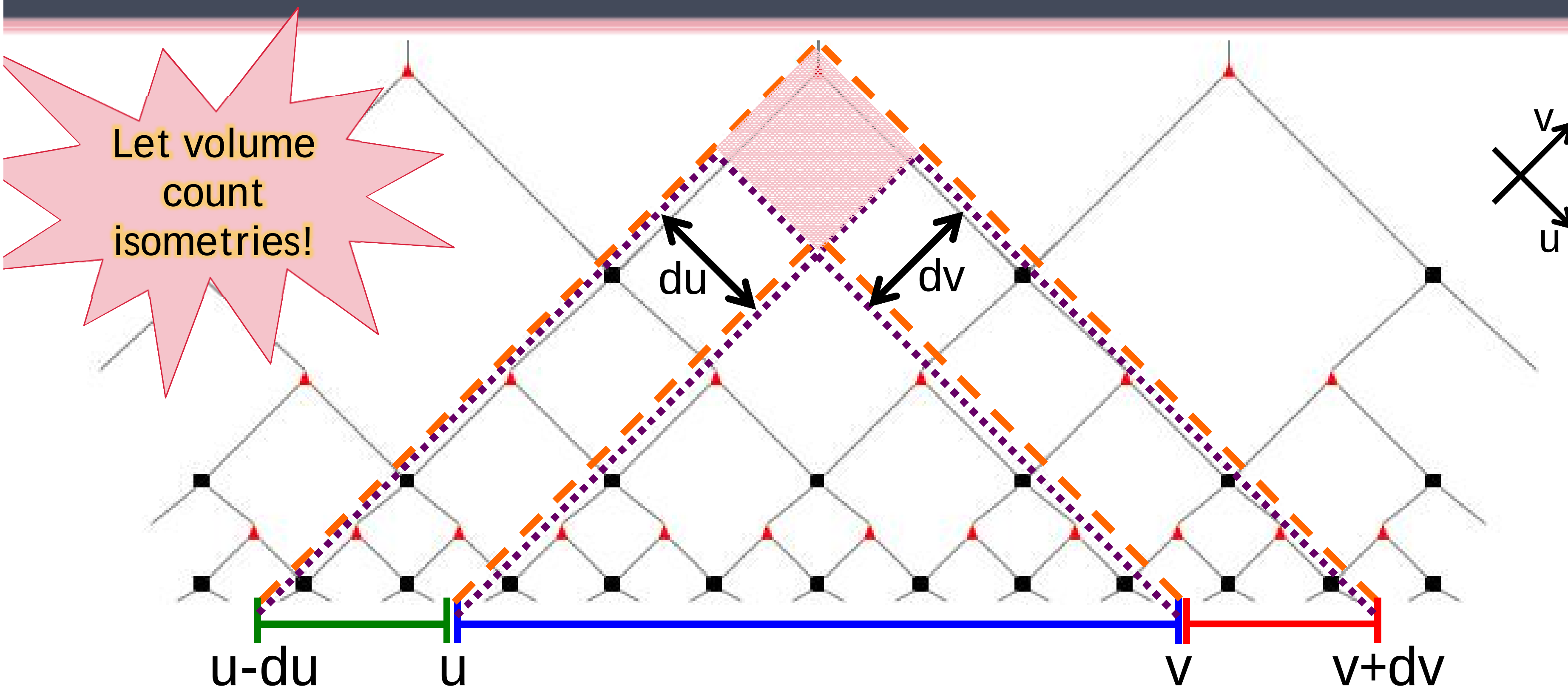
Conditional mutual information



$$I(A,C|B) = S(AB) + S(BC) - S(ABC) - S(B)$$

- strong subadditivity of entropy: $I(A,C|B) \geq 0$ → $\#(\Delta) \geq 0$
- because of cancellations, this quantity localizes in the network
- it counts the number of isometries in a causal diamond

Choosing a metric: volume element

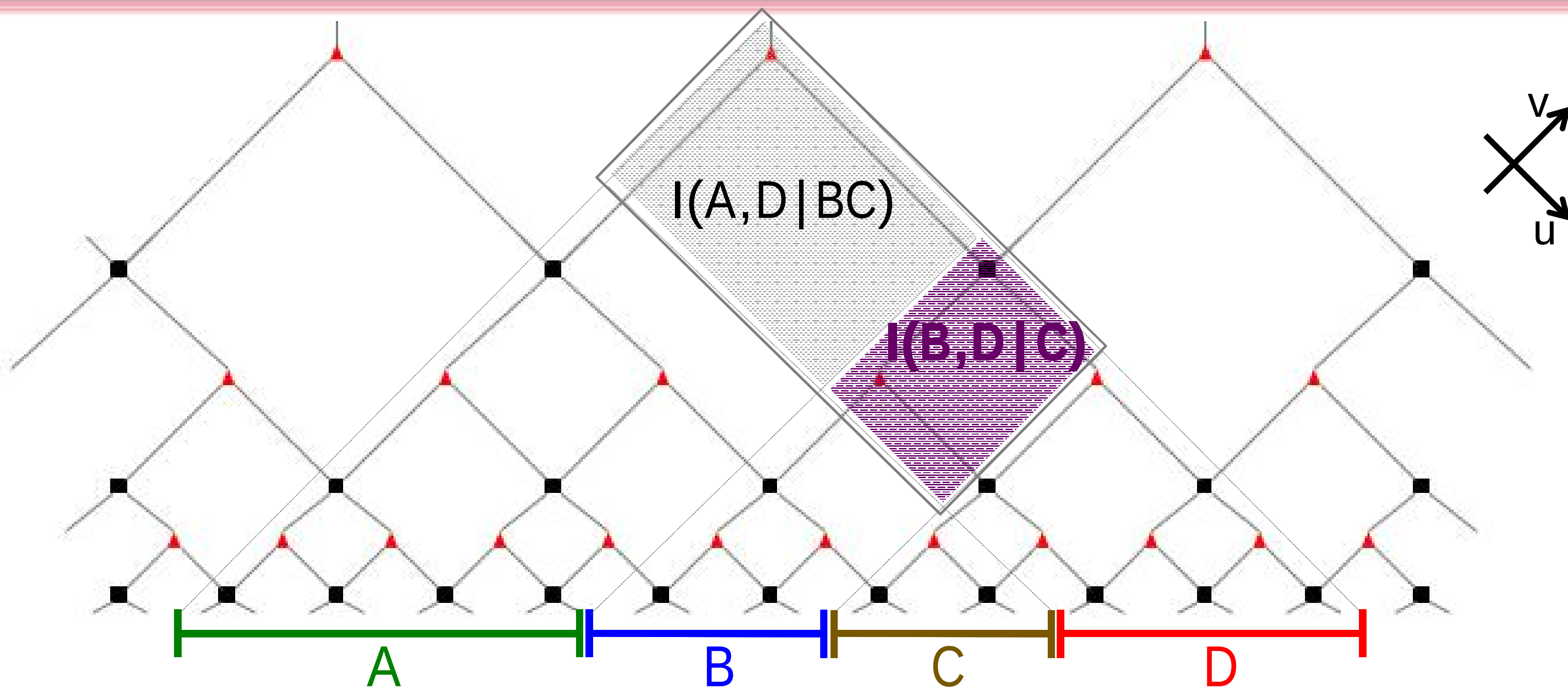


$$I(A,C|B) = S(AB) + S(BC) - S(ABC) - S(B) \rightarrow \frac{\partial^2 S(u,v)}{\partial u\, \partial v}$$

Kinematic Metric:

$$ds^2 = \frac{\partial^2 S_{\text{ent}}(\ldots)}{\partial u\, \partial v}\, du\, dv$$

Conditional mutual information as volume

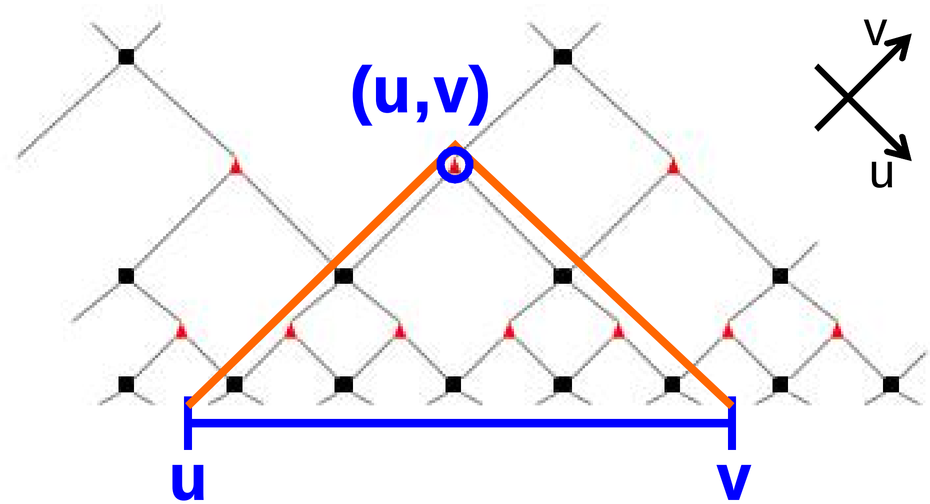


- Because volume is additive, consistency requires:

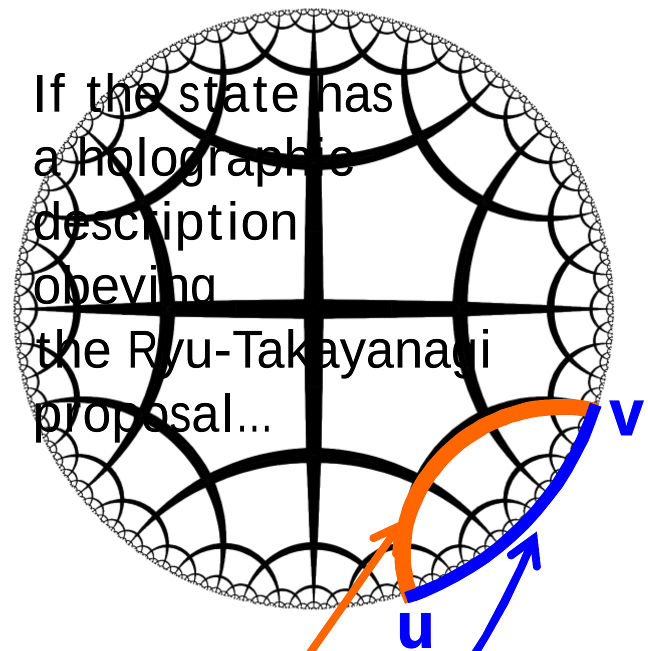
$$\boxed{I(A,D|BC) + I(B,D|C) = I(AB,D|C)}$$

- This is an identity: the chain rule for conditional mutual information

Kinematic Space ⟶ Holographic Geometry



If the state has a holographic description obeying the Ryu-Takayanagi proposal...



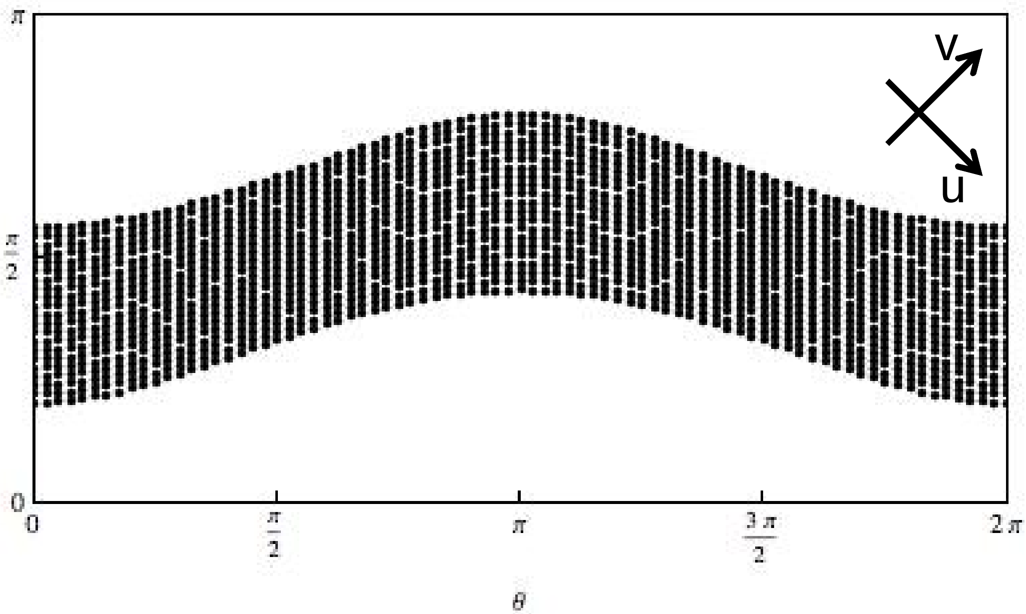
$$ds^2 = \frac{\partial^2 S_{\text{ent}}}{\partial u\, \partial v}\, du dv$$

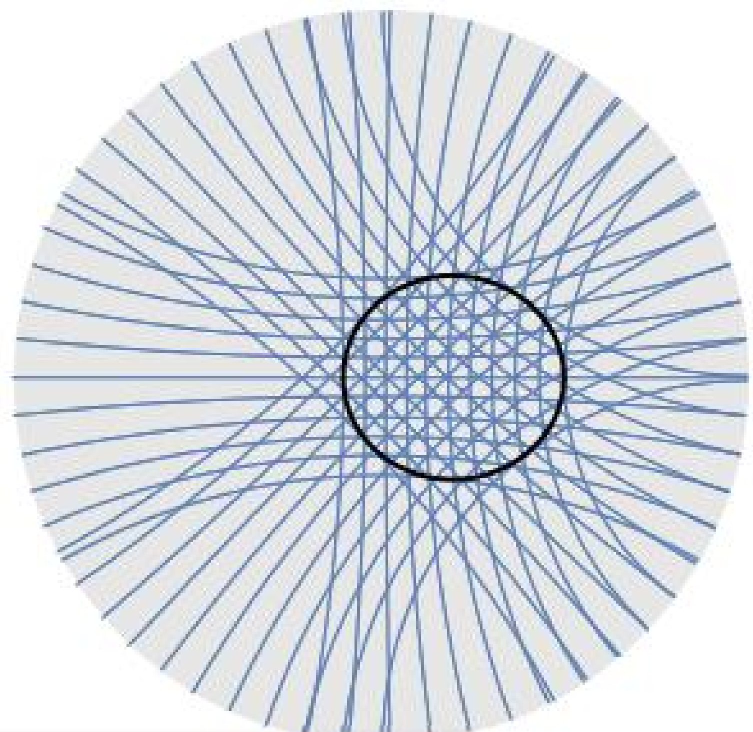
Point **(u,v)** uniquely selects:

- **a minimal cut** ⟷ **an oriented geodesic**
  (dropped along the causal cone)
- a boundary interval **(u,v)**

Kinematic Space is the space of oriented geodesics ⟷ boundary intervals!

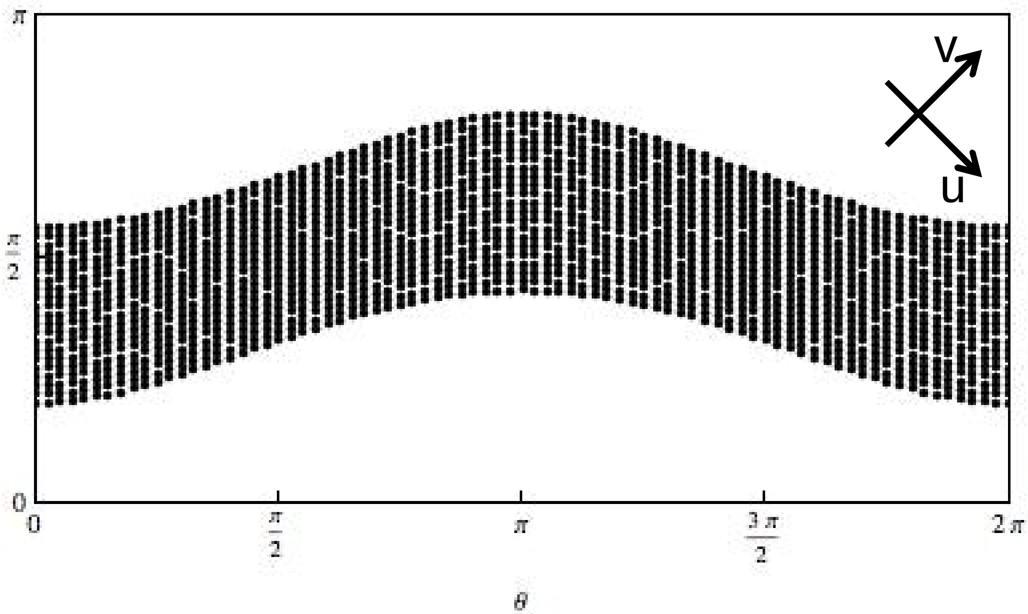
Convex curves are kinematic regions

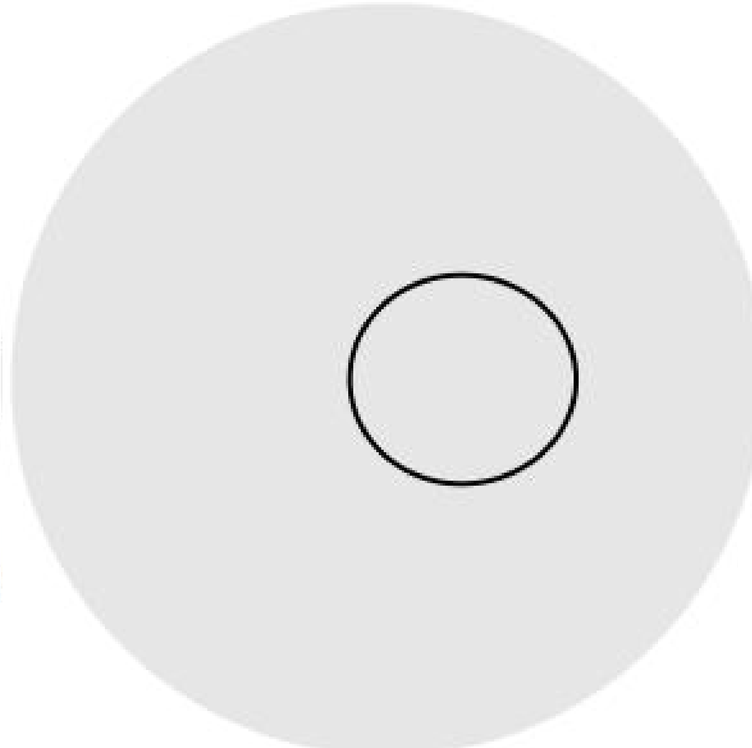




$$\frac{\text{circumference}}{4G} = \int_{\text{intersect}} \frac{\partial^2 S(u,v)}{\partial u\, \partial v}\, du\, dv \;=\; \text{volume in kinematic space}$$

Reading a curve from a region

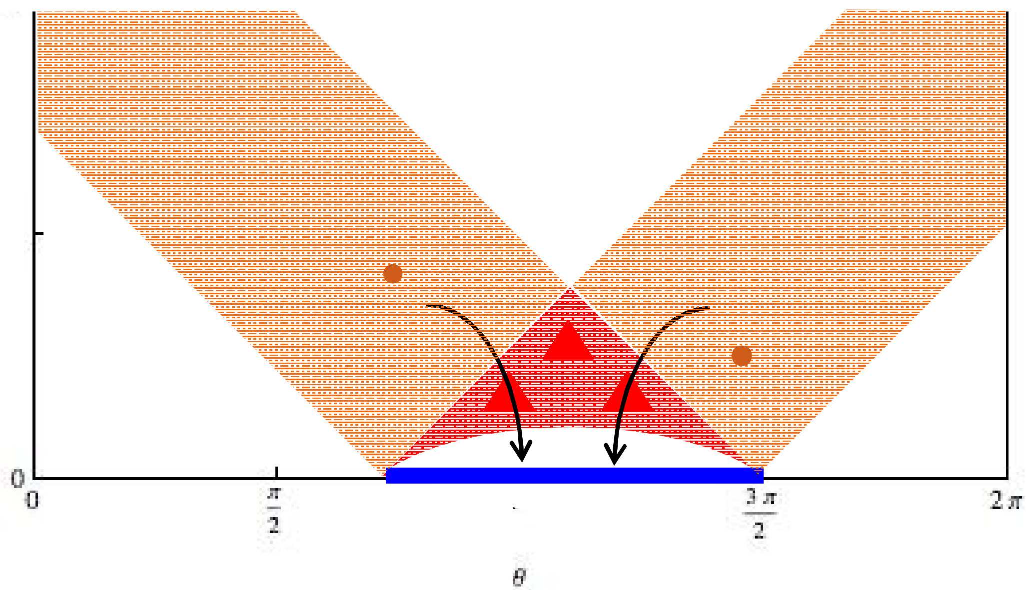


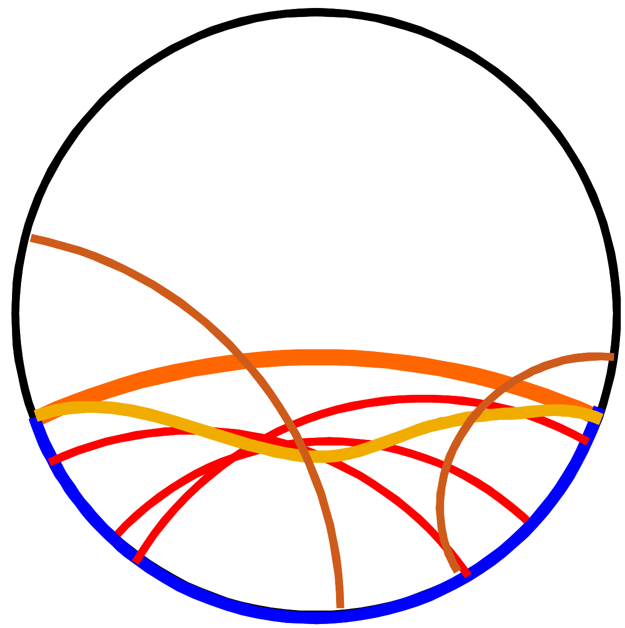


$$\frac{\text{circumference}}{4G} = \int_{\text{intersect}} \frac{\partial^2 S(u,v)}{\partial u\, \partial v}\, du\, dv \; = - \int du\, \frac{\partial S(u,v)}{\partial u}\bigg|_{v=v(u)}$$

works in every geometry
that obeys the Ryu-Takayanagi proposal

Curves with common endpoints





- length = volume of intersecting geodesics
- every geodesic intersecting **orange** also intersects yellow
- **kinematic volume** ≤ kinematic volume ⟷ **orange curve** is a geodesic
- length - **length** = **volume** of geodesics that intersect yellow but not **orange**

length - **length** = # isometries in this isometric embedding of states

Kinematic space of vacuum is de Sitter

$$ds^2 = \frac{\partial^2 S_{\rm ent}}{\partial u\, \partial v}\, du dv$$

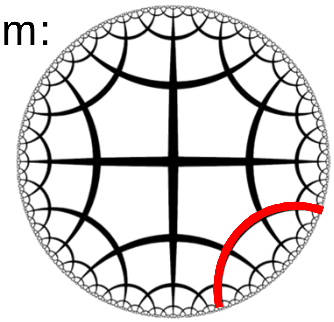
Plug in: $S_{\rm ent} = \frac{c}{3} \log \frac{\sin(v-u)/2}{\mu}$

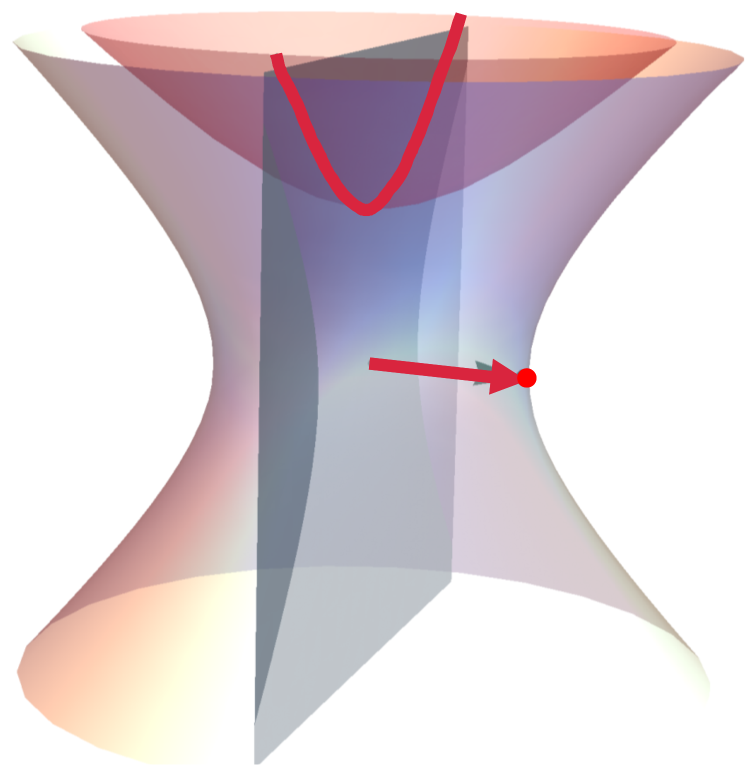
$$ds^2 = \frac{c}{12} \frac{du\, dv}{\sin^2(v-u)/2} \quad \to \quad \frac{c}{12} \cdot \frac{-dt^2 + d\theta^2}{\sin^2 t}$$

de Sitter geometry

We show that MERA corresponds to a discretization of de Sitter space.
Beny, 2011

dual to vacuum:





Summary

- A quantitative connection between tensor networks (MERA) and a holographic geometry, mediated by integral geometry
- A blueprint for constructing a tensor network for any holographic geometry:

$$\boxed{\frac{\partial^2 S_{\text{ent}}}{\partial u\, \partial v}\, du dv} \longrightarrow \boxed{\text{“density of isometries”}}$$

<u>Assumptions:</u>

- Geometry: the Ryu-Takayanagi proposal
- Tensor networks: #cuts in MERA ➔ EE

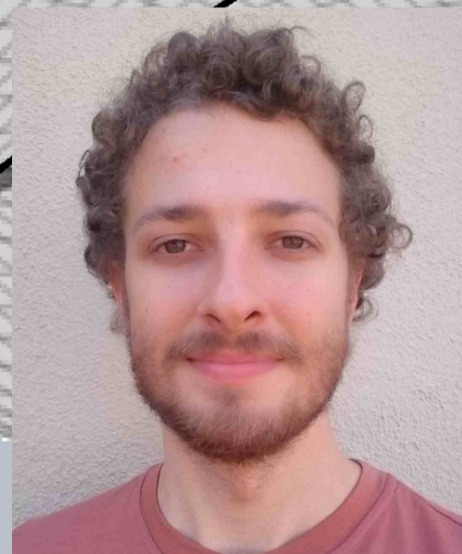
role of disentanglers

Questions

- Understand the kinematic space directly in the language of the path integral
  (cf. Tensor Network Renormalization, Evenbly-Vidal 2014)
- Kinematic space versus c-MERA
  (Haegeman-Osborne-Verschelde-Verstraete, 2011)
- Include time evolution
- Describe near-horizon regions
  (cf. entwinement, Balasubramanian-BC-et al., 2014)
- Construct bulk operators
  (cf. Daniel Harlow's talk)
- Higher dimensions?
- Higher spin theories?

My collaborators:

Lampros Lamprou



Sam McCandlish

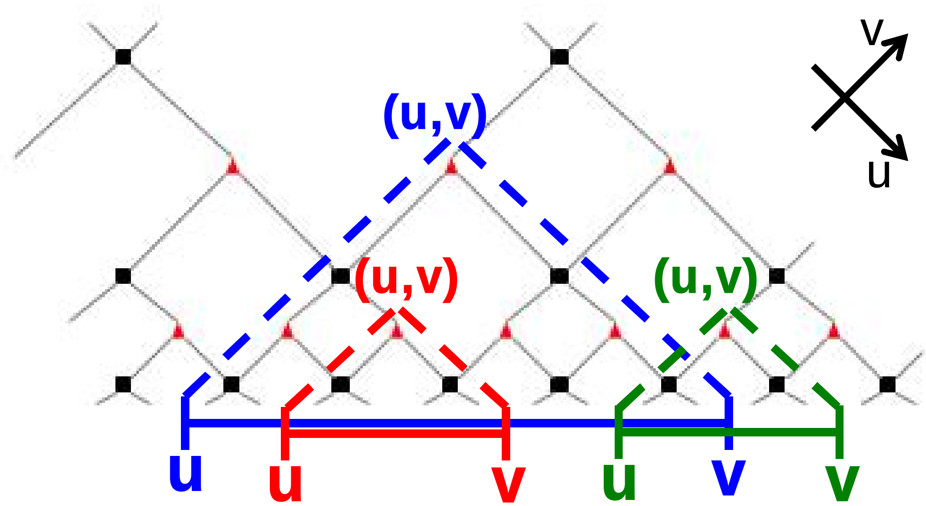
James Sully

arXiv:1505.05515
arXiv:1506.0xxxx

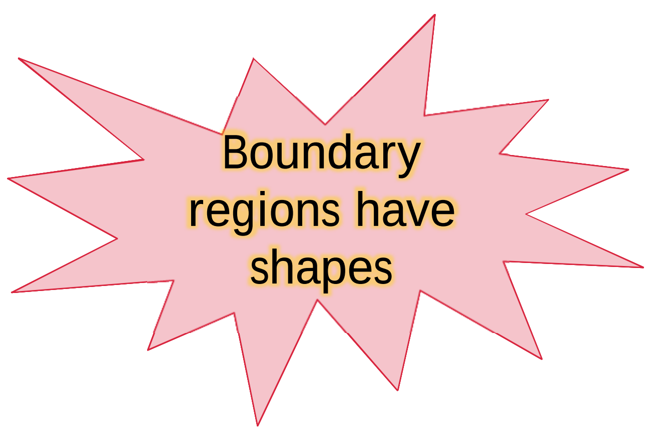
THANK YOU!

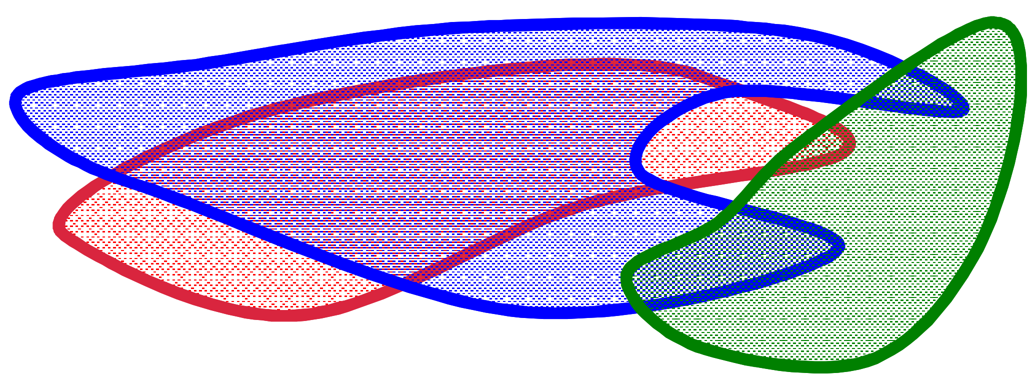
Kinematic Space in higher dimensions?

- 1+1-dimensional CFT:

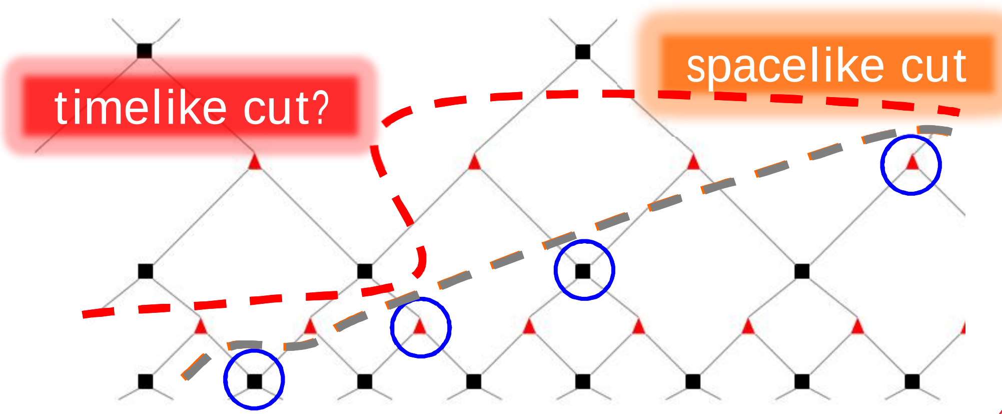


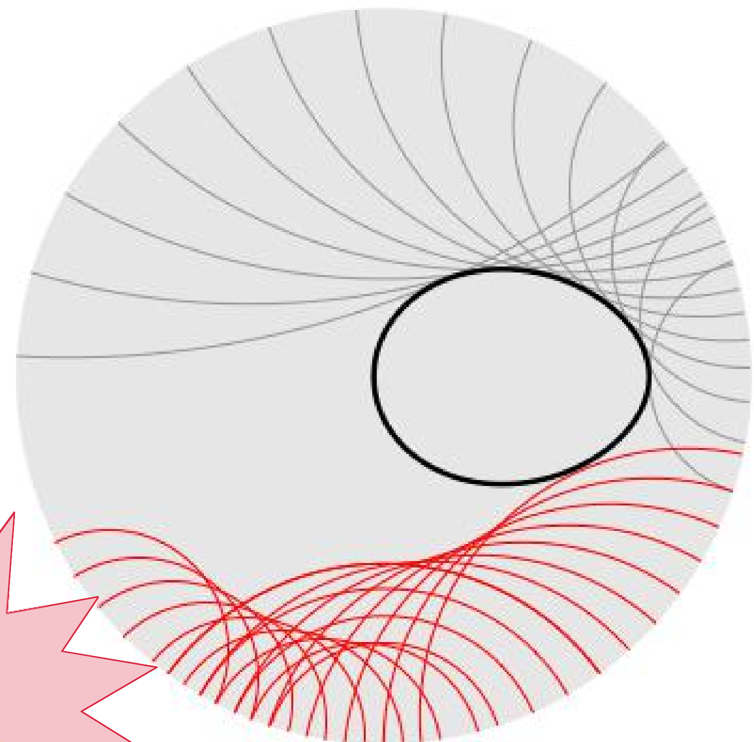
- d+1-dimensional CFT:



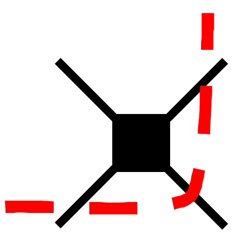


Why only spacelike cuts are allowed





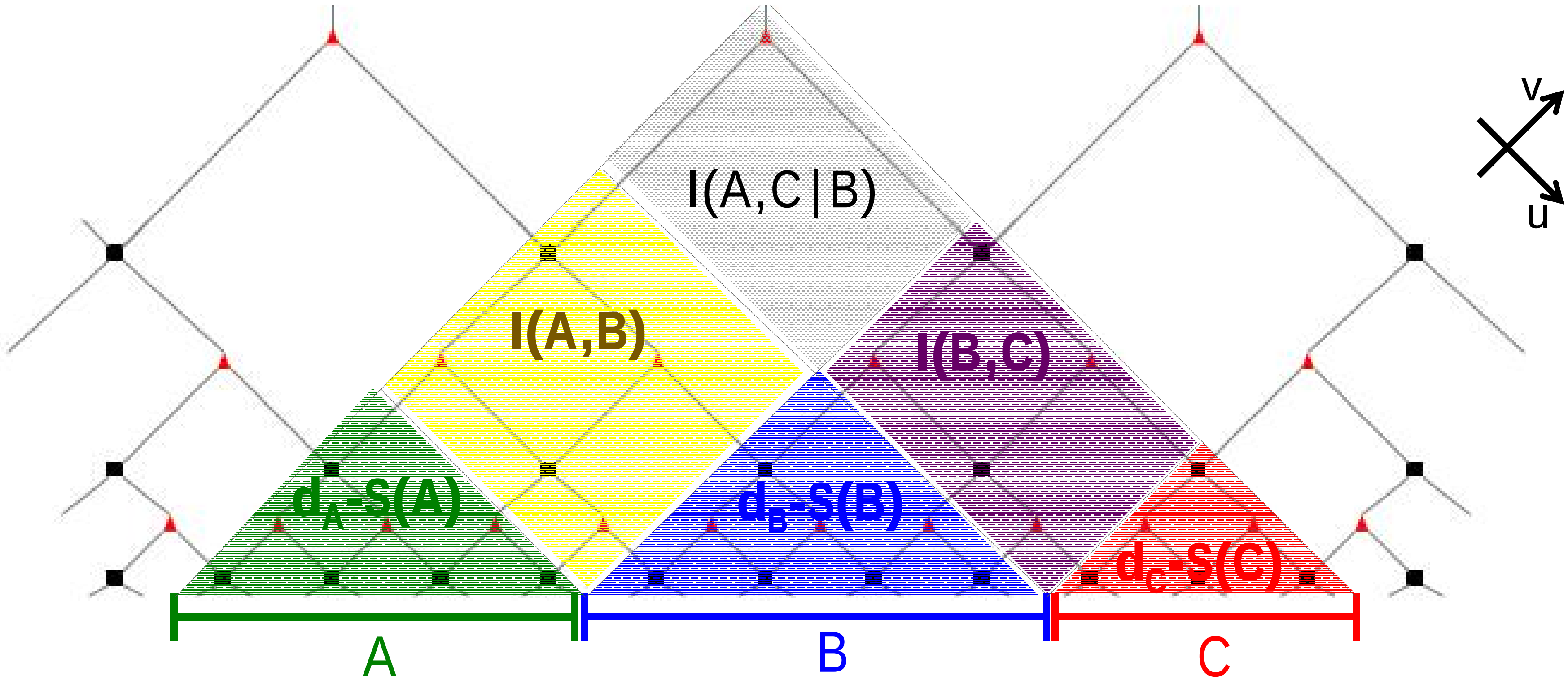
What goes wrong?



- Timelike cuts do not effect coarse-graining
- They introduce spurious degrees of freedom

- Timelike cuts fail to define bulk curves

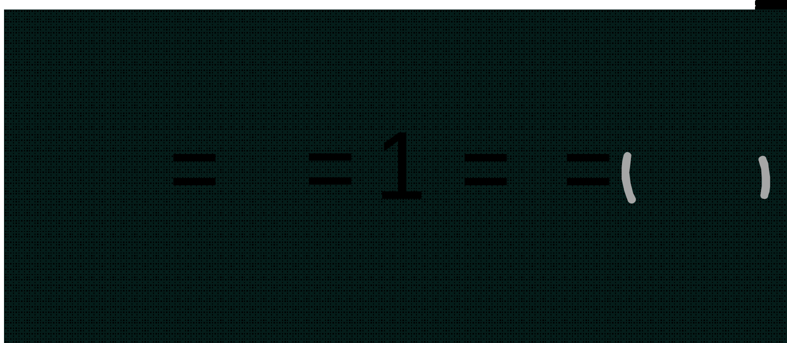
Structure of kinematic space

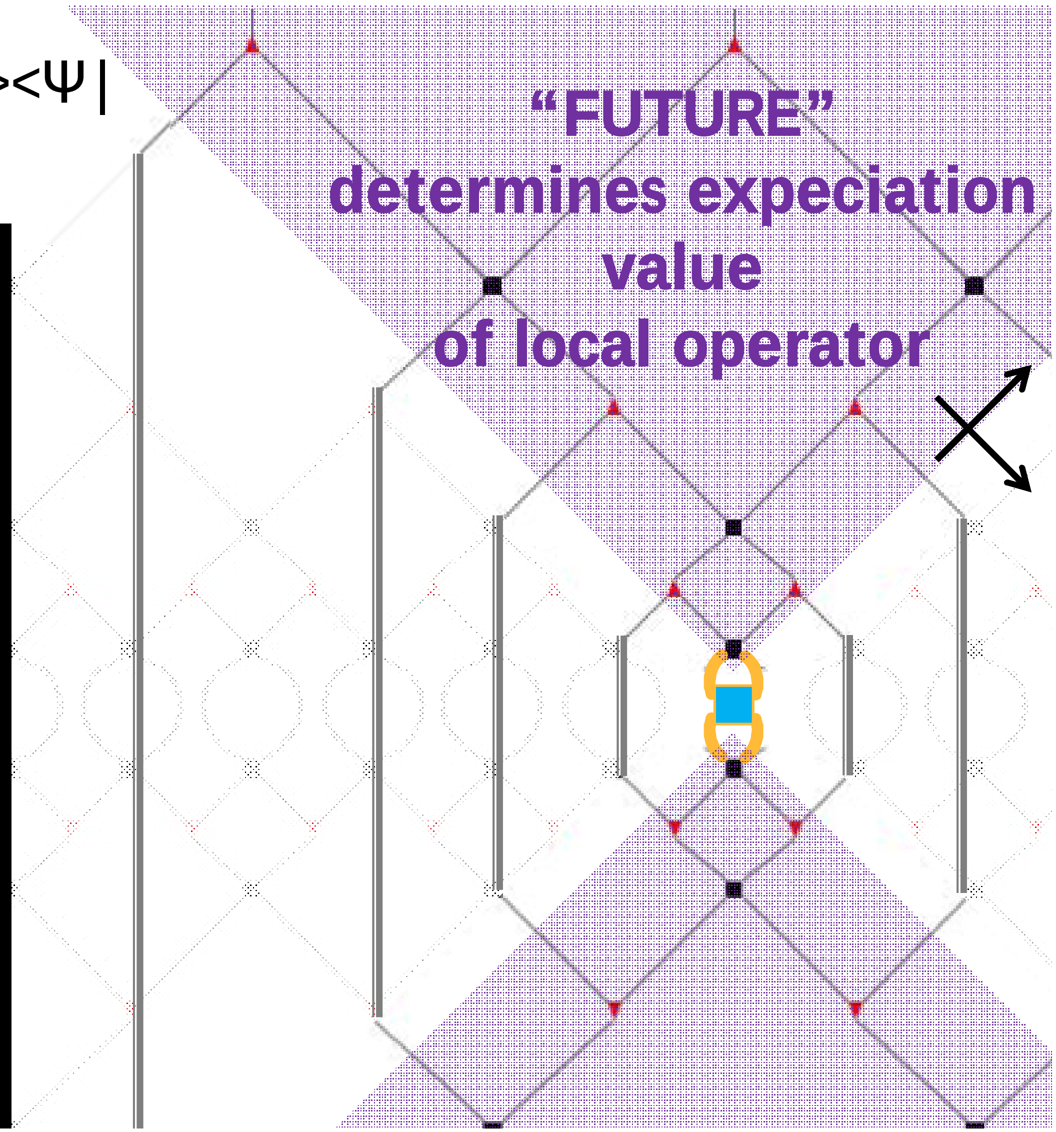


- Causal diamonds are conditional mutual informations
- Diamonds that extend all the way to the bottom are mutual informations
- Past causal diamonds of kinematic points characterize the isometric embedding of a compressed state in the Hilbert space

Causal structure and locality in MERA

- Compute <Ψ|**O**|Ψ> = Tr **O**|Ψ><Ψ|
- Unitarity of tensors implies:





Causal Structure

(in <u>auxiliary</u> time ↔ scale)