Cluster Algebras
and
Scattering Amplitudes

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Introduction

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What exactly would it mean to say that the problem of computing scattering amplitudes is "solved"? When would we be content that we have found the "solution", and what might it look like?

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Since SYM theory is unique, there should exist some collection of principles, or properties, both physical and mathematical, which ought to determine amplitudes uniquely.

For the moment, this is a question which we approach "experimentally", by seeking out and exploiting hidden symmetries and mathematical structures, with great progress in recent years coming from a variety of complementary approaches, including

- twistor string theory,
- Grassmannia and amplituhedra,
- the amplitude bootstrap,
- the flux tube OPE,
- scattering equations,
- and others ...

Cluster Algebras and Scattering Amplitudes

Today I will review a series of papers

	James M. Drummond
[1305.1617: JG, AG, MS, CV, AV]	John Golden
[1306.1833: JG, MS]	Alexander Goncharov
[1401.6446: JG, MFP, MS, AV]	Georgios Papathanasiou
[1406.2055: JG, MS]	Daniel Parker
[1411.3289: JG, MS]	Miguel F. Paulos
[1412.3764: JMD, GP, MS]	Adam Scherlis
[15??.????: DP, AS, MS, AV]	Cristian Vergu
	Anastasia Volovich

which explore an apparently deep connection between (multi-loop) scattering amplitudes in SYM theory and cluster algebras.

Cluster Algebras and Scattering Amplitudes

- What is a cluster algebra?
- In what way do amplitudes manifest “cluster structure”?
- To what extent might they be determined by this structure?

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What, exactly, do I mean by "cluster structure"?

I'm not sure! I'll show some examples of how it manifests itself, but undoubtedly we are far from understanding the full story.

The A_2 Cluster Algebra

Suppose you have a 2-dimensional Poisson manifold and a pair of log-canonical functions x_1, x_2 on that manifold, which we'll call "coordinates"

$$\{\log x_1, \log x_2\} = 1$$

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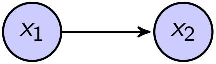
Interestingly, this relation is periodic, with period 5:

$$x_1, \quad x_2, \quad \frac{1+x_2}{x_1}, \quad \frac{1+x_1+x_2}{x_1 x_2}, \quad \frac{1+x_1}{x_2}, \quad x_1, \quad \ldots$$

The A_2 Cluster Algebra

The five x_i constitute the cluster coordinates of the A_2 cluster algebra, so named because the Poisson bracket may be represented by the quiver:



Cluster Poisson Varieties

[math/0311245: Fock, Goncharov]

More generally, a cluster on a cluster Poisson variety of dimension d is a collection of d log-canonical cluster coordinates x_i

$$\{\log x_i, \log x_j\} = B_{ij} \in \mathbb{Z}$$

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(which can be represented by a quiver, with B_{ij} arrows from node i to node j), and there is a simple operation called mutation on x_j which generates a new cluster according to

$$x_i' = \begin{cases} 1/x_j & i = j \\ x_i \left(1 + x_j^{\operatorname{sgn} B_{ij}}\right)^{B_{ij}} & i \neq j \end{cases}$$

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The cluster algebra associated to this Poisson variety is generated by the union of the cluster coordinates contained in all possible clusters reachable by arbitrary sequences of mutations.

The Kinematic Domain for $\mathcal{N} = 4$ SYM

What is the "cluster Poisson variety" relevant to $\mathcal{N} = 4$ SYM?

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What is the "cluster Poisson variety" relevant to $\mathcal{N} = 4$ SYM?

Nothing mysterious!

Let's first review: what is an *n*-particle amplitude a function of?

The Kinematic Domain for $\mathcal{N} = 4$ SYM

A collection of n ordered null vectors in Minkowski space $\mathbb{R}^{1,3}$, subject to overall momentum conservation, may be represented in terms of n momentum twistors

$$\begin{pmatrix} | & | & \cdots & | \\ Z_1 & Z_2 & \cdots & Z_n \\ | & | & \cdots & | \end{pmatrix}, \qquad Z_i \in \mathbb{P}^3$$

[0905.1473: Hodges]

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A very special property of SYM theory is dual conformal symmetry [Drummond, Henn, Korchemsky, Sokatchev, 2007], which corresponds to the left-action of $SL(4)$.

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Amplitudes in SYM theory are functions on the quotient space,

$$\mathrm{Gr}(4,n)/(\mathbb{C}^*)^{n-1} \simeq \mathrm{Conf}_n(\mathbb{P}^3)$$

which is a cluster Poisson variety!

Example: Six-Particle Scattering

For six particles, the space of kinematic configurations, modulo dual conformal invariance, is 3-dimensional.

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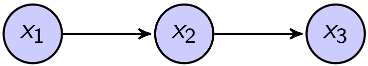
As coordinates on this space we may choose cross-ratios, e.g.

$$x_1 = \frac{\langle 1234\rangle\langle 3456\rangle}{\langle 1346\rangle\langle 2345\rangle}, \quad x_2 = \frac{\langle 1456\rangle\langle 2346\rangle}{\langle 1246\rangle\langle 3456\rangle}, \quad x_3 = \frac{\langle 1346\rangle\langle 1256\rangle}{\langle 1236\rangle\langle 1456\rangle},$$

(where $\langle ijkl\rangle = \det(Z_i Z_j Z_k Z_l)$) which have Poisson bracket



and constitute a cluster of the A_3 cluster algebra.

Example: Six-Particle Scattering

Via mutations one can generate the 12 other cluster coordinates:

$$(1+x_1)x_2 \qquad \frac{(1+x_2)(1+x_3+x_2x_3+x_1x_2x_3)}{x_1x_2}$$

$$(1+x_2)x_3 \qquad \frac{1+x_3+x_2x_3+x_1x_2x_3}{x_1(1+x_3)}$$

$$(1+x_2+x_1x_2)x_3 \qquad \frac{1+x_3+x_2x_3+x_1x_2x_3}{x_2(1+x_1)}$$

$$\frac{1+x_2}{x_1x_2} \qquad \frac{1+x_3+x_2x_3}{x_2}$$

$$\frac{1+x_3}{x_2x_3} \qquad \frac{1+x_3+x_2x_3}{x_1x_2x_3}$$

$$\frac{1+x_2+x_1x_2}{x_1} \qquad \frac{(1+x_1)x_2x_3}{1+x_3}$$

[See https://en.wikipedia.org/wiki/Cluster_algebra but beware; the variables listed there are a different type, called cluster $\mathcal{A}$-coordinates!]

Example: Six-Particle Scattering

or, if you prefer, the 15 cluster coordinates may be expressed as

$$\frac{\langle 1246\rangle\langle 1345\rangle}{\langle 1234\rangle\langle 1456\rangle}, \quad \frac{\langle 1235\rangle\langle 2456\rangle}{\langle 1256\rangle\langle 2345\rangle}, \quad \frac{\langle 1356\rangle\langle 2346\rangle}{\langle 1236\rangle\langle 3456\rangle},$$

$$\frac{\langle 1456\rangle\langle 2356\rangle}{\langle 1256\rangle\langle 3456\rangle}, \quad \frac{\langle 1346\rangle\langle 2345\rangle}{\langle 1234\rangle\langle 3456\rangle}, \quad \frac{\langle 1236\rangle\langle 1245\rangle}{\langle 1234\rangle\langle 1256\rangle},$$

$$\frac{\langle 1234\rangle\langle 2356\rangle}{\langle 1236\rangle\langle 2345\rangle}, \quad \frac{\langle 1256\rangle\langle 1346\rangle}{\langle 1236\rangle\langle 1456\rangle}, \quad \frac{\langle 1245\rangle\langle 3456\rangle}{\langle 1456\rangle\langle 2345\rangle},$$

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Example: Six-Particle Scattering

Let's look at some "experimental data" — the 2-loop six-particle MHV amplitude, expressed in [1006.3703: AG, MS, CV, AV] as

$$R_6^{(2)} = \sum_{\text{cyclic}} \text{Li}_4\left(-\frac{\langle 1234\rangle\langle 2356\rangle}{\langle 1236\rangle\langle 2345\rangle}\right) - \frac{1}{4}\,\text{Li}_4\left(-\frac{\langle 1246\rangle\langle 1345\rangle}{\langle 1234\rangle\langle 1456\rangle}\right)$$

$+$ products of $\text{Li}_k(-x)$ functions of lower weight with the same set of arguments,

where

$$\text{Li}_k(z) = \int_0^z \frac{dt}{t}\,\text{Li}_{k-1}(t), \qquad \text{Li}_1(z) = -\log(1-z)$$

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Observation: the argument of every polylog is a cluster coordinate, never some random cross-ratio (or function of cross-ratios).

Quiz

Which of these cross-ratios might you encounter in a two-loop MHV amplitude?

$$R_1 = \frac{\langle 1237\rangle\langle 1258\rangle\langle 2456\rangle\langle 5678\rangle}{\langle 1256\rangle\langle 2578\rangle\langle 78(123)\cap(456)\rangle}$$

$$R_2 = \frac{\langle 1235\rangle\langle 1278\rangle\langle 2456\rangle\langle 5678\rangle}{\langle 1256\rangle\langle 2578\rangle\langle 78(123)\cap(456)\rangle}$$

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Answer: only R_1; the other two are not cluster coordinates; you'll never see them!

Polylogarithm Functions

More sophisticated mathematical machinery is required to analyze more general amplitudes.

A large class of amplitudes may be expressed in terms of a zoo of generalized polylogarithm functions of "weight" $2L$, at L loops.

An operational definition of a weight-k function is that it is one whose total derivative takes the form

$$df_k = \sum_i f_{k-1}^{(i)} \, d \log R_i$$

where the R_i are rational functions.

The Symbol of Polylogarithm Functions

By iterating,

$$df_{k-1}^{(i)} = \sum_j f_{k-2}^{(i,j)} \, d\log R_j$$

we can break such a function up into a collection of rational numbers and rational functions, which are assembled to form the symbol

$$symbol(f_k) = \sum_{i_1, i_2, \ldots, i_k} f_0^{(i_1, i_2, \ldots, i_k)} \, R_{i_1} \otimes R_{i_2} \otimes \cdots \otimes R_{i_k}.$$

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The collection of R_i which appear in the symbol of a given function is called its symbol alphabet.

Cluster Structure Manifestation #1

The cluster coordinates on the $\mathrm{Gr}(4, n)$ cluster algebra provide a basis for the symbol alphabet of all n-particle amplitudes (of generalized polylogarithm type).

This hypothesis is supported by all “experimental” evidence available to date, including heroic computations by

[1003.1702: Del Duca, Duhr, Smirnov]
[1105.5606: Caron-Huot]
[1111.1704: Dixon, Drummond, Henn]
[1112.1060: Caron-Huot, He]
[1308.2276: Dixon, Drummond, von Hippel, Pennington]
[1402.3300: Dixon, Drummond, Duhr, Pennington]
[1408.1505: Dixon, von Hippel]

A Curious (?) Observation

For $n \geq 8$ the $\mathrm{Gr}(4, n)$ cluster algebra has infinitely many cluster coordinates, but only finitely many of them are needed to write the symbol of an amplitude at any finite loop order, if it is of generalized polylog type.

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So it is possible that the principle "symbol alphabet = cluster coordinates" might extend to **all** amplitudes, once properly interpreted for infinite algebras and infinitely long symbols.

Cluster Structure Manifestation #2

A much stronger connection between amplitudes and cluster algebras is known for the special case of 2-loop MHV amplitudes, where we have a lot of data thanks to [1105.5605: Caron-Huot].

The cobracket of any 2-loop MHV amplitude has components which can be written as linear combinations of

$$\mathrm{Li}_2(-x_i) \wedge \mathrm{Li}_2(-x_j), \qquad \mathrm{Li}_3(-x_k) \wedge \log(x_l)$$

for pairs of cluster coordinates always having Poisson bracket

$$\{\log x_i, \log x_j\} = 0, \qquad \{\log x_k, \log x_l\} = 1.$$

[1305.1617: JG, AG, MS, CV, AV], [1411.3289: JG, MS]

Sorry, no time to define the cobracket [Goncharov]!

The Cluster Bootstrap

We still have no idea “why” these properties hold, nor how, precisely, the Poisson bracket is encoded in more general amplitudes — this question must still be explored experimentally.

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We still have no idea "why" these properties hold, nor how, precisely, the Poisson bracket is encoded in more general amplitudes — this question must still be explored experimentally.

We can turn things around and ask: suppose we adopt, as a working hypothesis, that amplitudes must have cluster structure.

Can we carry out new, previously impossible calculations by building in the expected cluster structure from the outset?

(And subsequently check the consistency of results obtained in this way, to provide evidence for or against the hypothesis...)

Cluster Bootstrap Application #1

Suppose we knew nothing more about 2-loop MHV amplitudes than that they must obey the following rather basic physical properties:

- consistent collinear limits,
- only physical branch cuts,
- dihedral invariance in the particle labels,
- a certain condition on their differential, a consequence of SUSY [1105.5606: Caron-Huot],

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but somehow learn, from some as of yet unknown principle, that they must be polylogarithm functions

- whose symbol alphabet consists of the cluster coordinates on $\mathrm{Gr}(4, n)$,
- with the Poisson structure imprinted on the coproduct as indicated above.

Cluster Bootstrap Application #1

These properties completely determine all 2-loop MHV amplitudes, modulo terms which are products of functions of lower weight.

[1411.3289: JG, MS]

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[1411.3289: JG, MS]

Note: for $n = 6, 7$ the proviso may be dropped; I believe this is likely true for higher n but the function spaces become large very quickly, making this a hard "experiment" to perform (that means, a difficult calculation).

Cluster Bootstrap Application #2

The symbol of the 3-loop seven-particle MHV amplitude is uniquely determined by assuming it is a weight-6 polylogarithm function

- whose symbol alphabet is the set of cluster coordinates on the $\mathrm{Gr}(4,7)$ (also called E_6) cluster algebra,
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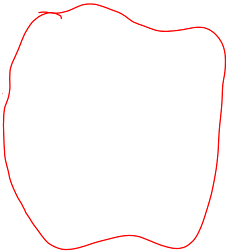
This is an example of a general phenomenon that there is a sense in which the scattering amplitudes of $\mathcal{N}=4$ SYM theory "barely exist", given the tightly interlocking physical and mathematical constraints they evidently satisfy.

(I would like to say "must satisfy", but don't know why!)

The Cluster Bootstrap

The power of the “cluster bootstrap” comes from the apparent fact that physical collinear limit constraints and mathematical cluster constraints are “nearly orthogonal”.

Consider the set of hexagon functions (six-point cluster functions with physical singularities, using the terminology of Dixon et. al.)



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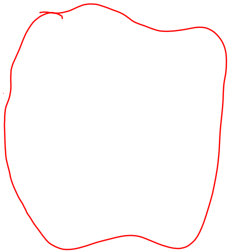

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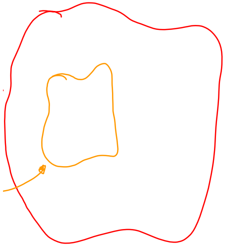

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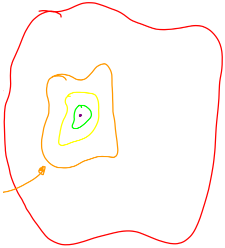

An actual six-point amplitude must sit at the base of an infinite tower of consistent nonagon functions, decagon functions, etc.

Conclusion

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In conclusion,

- Evidently, cluster algebras are (part of) the language in which amplitudes should be written.
- Scattering amplitude functions “barely exist”: what is the mathematical problem to which the scattering amplitudes of SYM theory are the unique solution?
- Is there an alternative formulation of SYM theory in which these physically mysterious cluster properties, in particular their relation to the Poisson structure on the kinematic domain, are manifest?