Bootstrapping Theories with Four Supercharge

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Based on:

arXiv:1502.04124 with N. Bobev, D. Mazáč, and M. Paulos
arXiv:1503.02081 with N. Bobev, D. Mazáč, and M. Paulos

June 25, 2015

Strings 2015, Bengaluru

Why return to the (conformal) bootstrap?

1. Conformal symmetry powerful tool: strongly constrains dynamics *even in* $d > 2$.
2. Completely non-perturbative tool to study field theories
   - Does not require SUSY, large N, or weak coupling.
3. In $D = 2$ conformal symmetry enhanced to *Virasoro* symmetry
   - Allows us to *completely solve* some CFTs ($c < 1$).
4. Long term hope: generalize this to $d > 2$?

SUSY: "Bootstrapping" the Bootstrap

- SUSY provides additional non-perturbative constraints.
- Correlators of protected operators have a lot of structure but also depend on unprotected spectrum.

Results

- Universtal bounds on unprotected operators in 4-supercharge theories in $2 \leq d \leq 4$.
- Several "kinks/features" corresponding to one known and two unidentified theories.
- "Precision spectrometry" of 3d ($\mathcal{N} = 2$) analog of "Ising model".

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(Non-Susy) Bootstrap Refresher

Spectrum and OPE

CFT Background

CFT defined by specifying:

- Spectrum $\mathcal{S} = \{\mathcal{O}_i\}$ of primary operators dimensions, spins: (Δ_i, l_i)
- Operator Product Expansion (OPE)

$$\mathcal{O}_i(x) \cdot \mathcal{O}_j(0) \sim \sum_k C_{ij}^k D(x, \partial_x) \mathcal{O}_k(0)$$

$\mathcal{O}_i$ are primaries. Diff operator $D(x, \partial_x)$ encodes *descendent* contributions.

This data fixes all correlators (of local observables) in the CFT:

- 2-pt & 3-pt fixed:

$$\langle \mathcal{O}_i \mathcal{O}_j \rangle = \frac{\delta_{ij}}{x^{2\Delta_i}}, \qquad \langle \mathcal{O}_i \mathcal{O}_j \mathcal{O}_k \rangle \sim C_{ijk}$$

- Higher pt functions contain no new dynamical info:

$$\langle \underbrace{\underbrace{\mathcal{O}_1(x_1)\mathcal{O}_2(x_2)}_{\sum_k C_{12}^k D(x_{12}, \partial_{x_2}) \mathcal{O}_k(x_2)} \underbrace{\mathcal{O}_3(x_3)\mathcal{O}_4(x_4)}_{\sum_l C_{34}^l D(x_{34}, \partial_{x_4}) \mathcal{O}_l(x_4)}}_{\sum_{k,l} C_{12}^k C_{34}^l D(x_{12}, x_{34}, \partial_{x_2}, \partial_{x_4}) \langle \mathcal{O}_k(x_2) \mathcal{O}_l(x_4) \rangle} \rangle$$

- Associativity of OPE leads to crossing symmetry: $\langle \mathcal{O}_1 \mathcal{O}_2 \mathcal{O}_3 \mathcal{O}_4 \rangle$

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Crossing Symmetry Nuts and Bolts

Bootstrap

So how do we check crossing symmetry in practice?

Correlator of four *identical* scalars: $\langle \phi(x_1)\phi(x_2)\phi(x_3)\phi(x_4)\rangle$ $\dim(\phi) = \Delta_\phi$

Check crossing symmetry assuming some set of *possible* operators $\mathcal{S} = \{(\Delta_k, \ell_k)\}$.

Crossing symmetry:

$$\sum_{\mathcal{O}_k \in \mathcal{S}} (C^k_{\phi\phi})^2 \, G^{12;34}_{\Delta_k, l_k}(u,v) = \sum_{\mathcal{O}_k \in \mathcal{S}} (C^k_{\phi\phi})^2 \, G^{14;23}_{\Delta_k, l_k}(u,v) \tag{1}$$

Consider space, $\mathcal{M} \cong \mathbb{R}^N$, of diff operators: $\alpha = \sum_{m,n}^{N} \alpha_{m,n} \partial_u^m \partial_v^n$

- $\mathcal{S}$ defines a convex subspace, $\mathcal{M}_\mathcal{S}$ via constraints:

$$\alpha\left(F_{\Delta_k, l_k}(u,v)\right) \geq 0 \qquad \forall (\Delta_k, l_k) \in \mathcal{S}$$

- If $\mathcal{M}_\mathcal{S}$ non-empty then $\mathcal{S}$ is not a valid CFT spectrum.
  $\Rightarrow$ eqn. (1) cannot be satisfied because $(C_{ijk})^2 \geq 0$.
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Move both channels to LHS:

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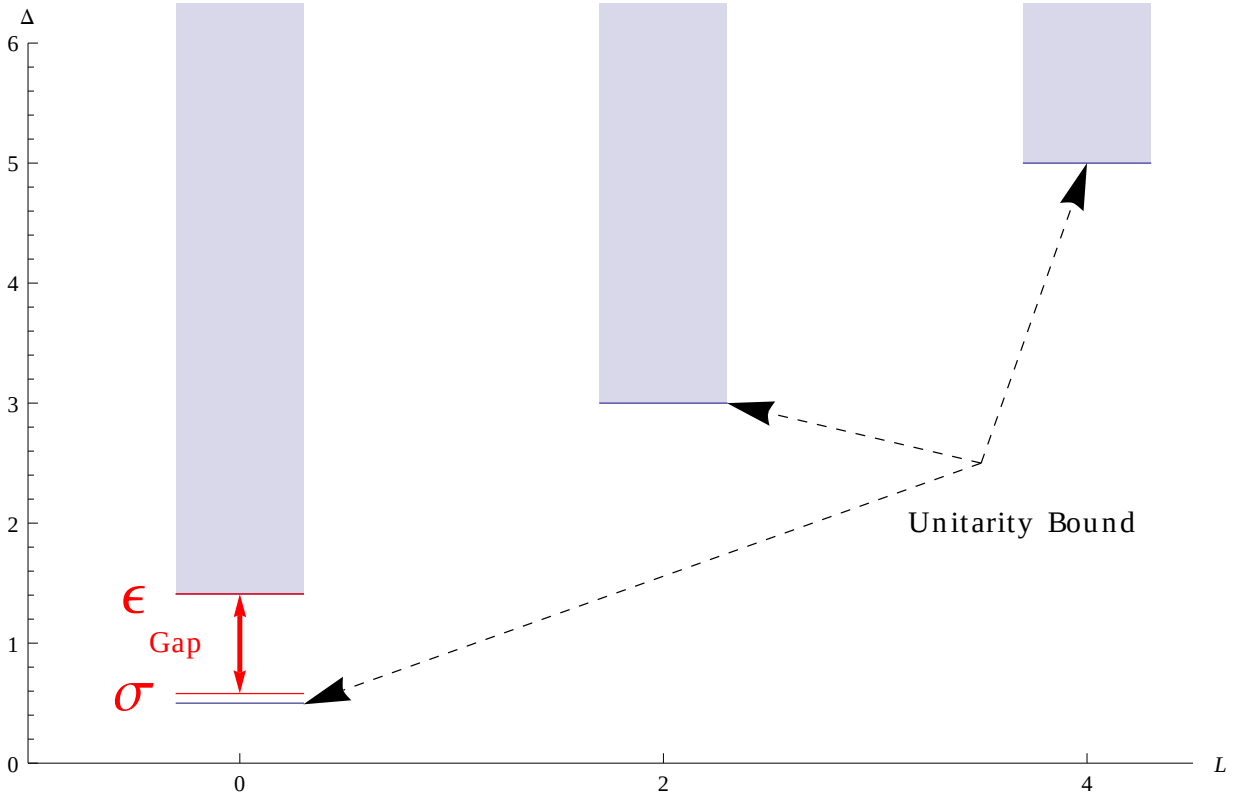
Constraining the spectrum

Figure : $\mathcal{S}$: a putative spectrum in $D = 3$



- Unitarity implies:

$$\Delta \geq \frac{d-2}{2} \quad (l = 0),$$
$$\Delta \geq l + d - 2 \quad (l \geq 0)$$

- "Carve" landscape of CFTs by imposing gap in scalar sector.
- Fix lightest scalar: σ.
- Vary next scalar: ϵ.
- Spectrum otherwise *unconstrained*: allow any other operators.

Constraining Spectrum using Crossing Symmetry

Is crossing symmetry consistent with a gap?

Is $\langle \sigma_1 \sigma_2 \sigma_3 \sigma_4 \rangle$ consistent with $\boxed{\sigma \times \sigma \sim 1 + \epsilon + \ldots}$??

Crossing symmetric values of σ-ϵ

- Certain values of σ, ϵ inconsistent with crossing symmetry.
- Solutions to crossing:
  1. white region $\Rightarrow$ 0 solutions.
  2. blue region $\Rightarrow$ ∞ solutions.
  3. boundary $\Rightarrow$ 1 solution (unique)!
- Can read off unique solution at boundary.
- Ising model special in two ways:
  1. On boundary of allowed region.
  2. At kink in boundary curve.

Blue = solution may exists.
White = No solution exists.

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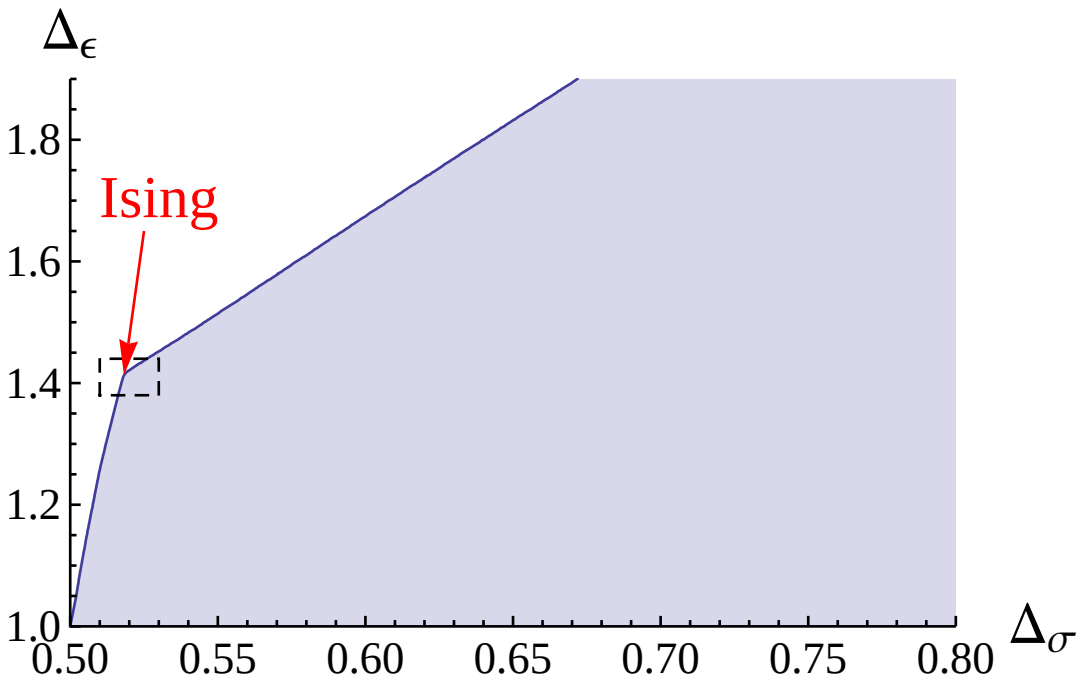

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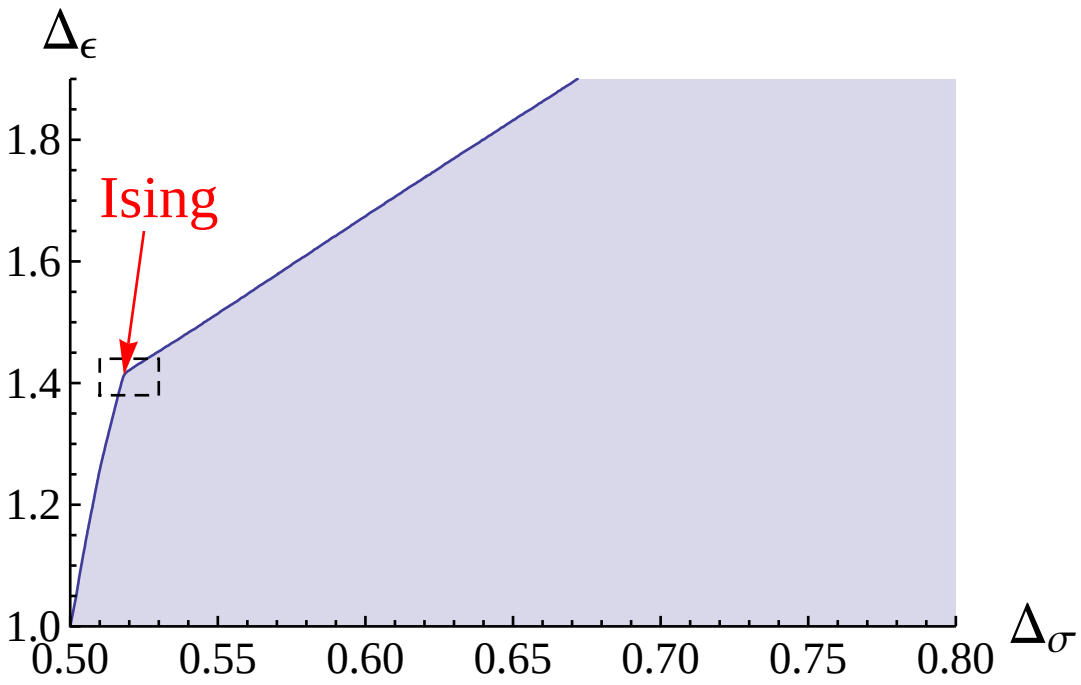

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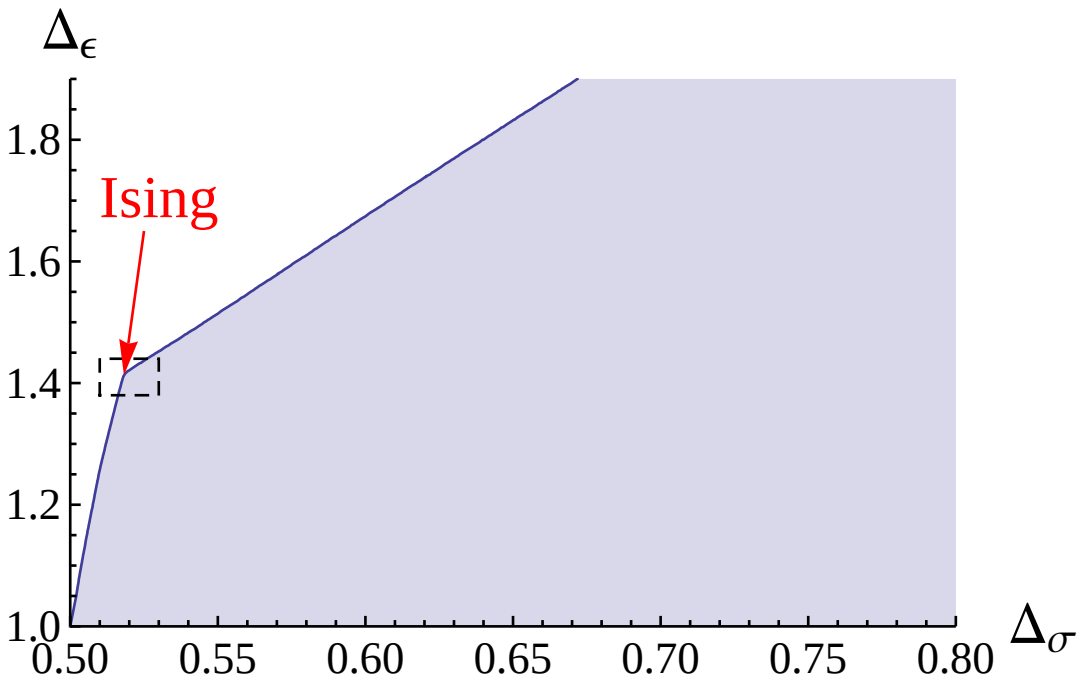

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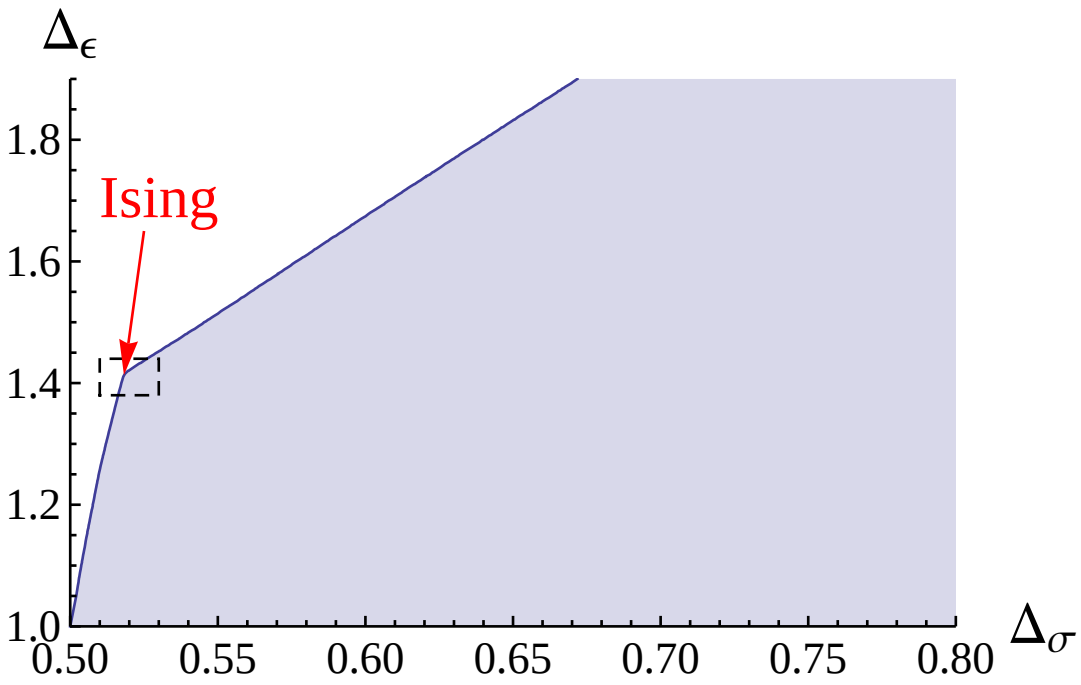

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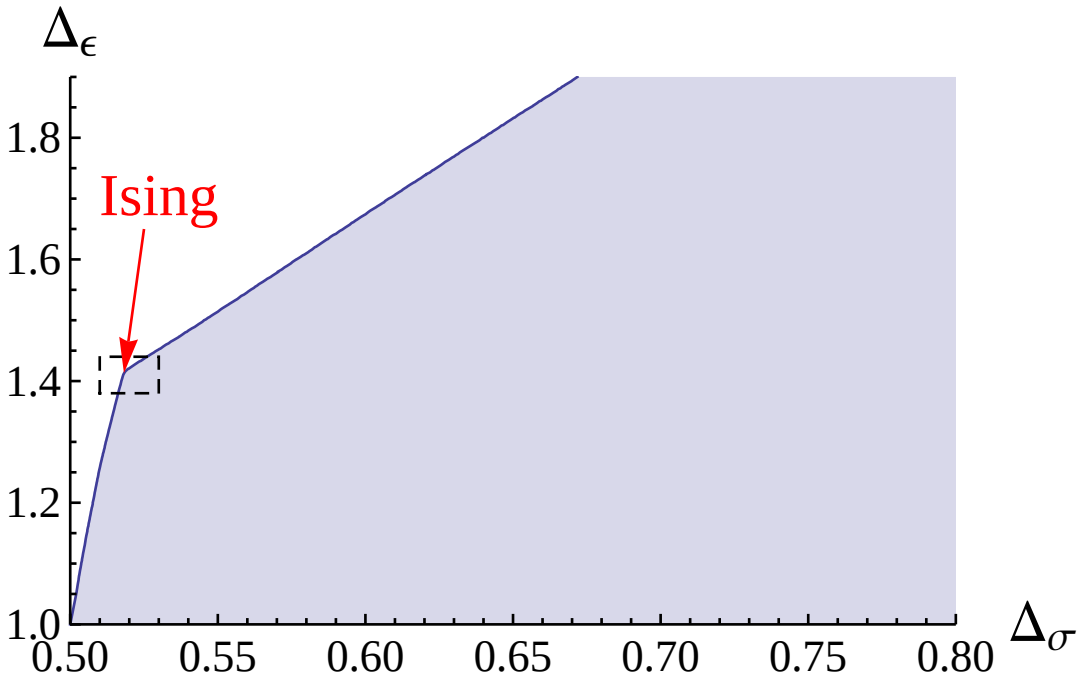

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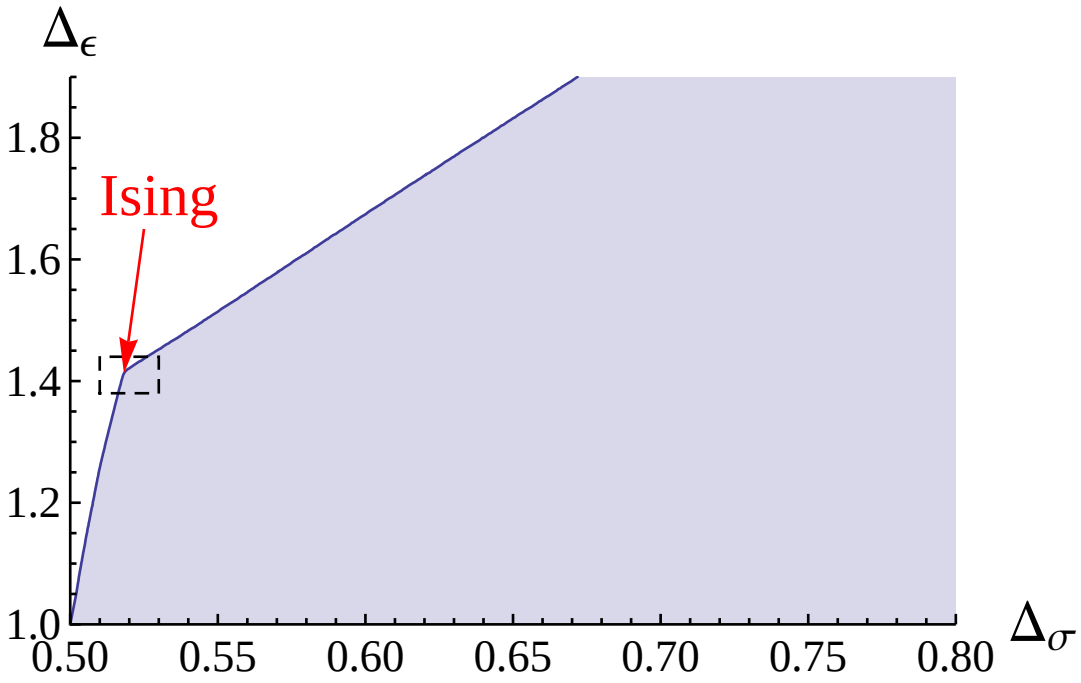

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Bootstrapping Theories with Four Supercharges ($2 \leq d \leq 4$)

Theories with Four Supercharges

Bootstrapping Theories with Four Supercharges

Consider "minimal" $\mathcal{N} = 1$ SUSY in $d = 4$ and its dim reduction.

- Gives $\mathcal{N} = (2, 2)$ in $d = 2$ or $\mathcal{N} = 2$ in $d = 3$.
- Relation via dim reduction means many shared features and universal treatment.

Defining SUSY in "fractional" d:

1. We will only consider scalar quantities (correlators, conf blocks).
2. Formally define super-conformal algebra in any $d \leq 4$:

$$P_i,\ K_i,\ D,\ M_{ij},\ Q^{\pm}_{\alpha/\dot{\alpha}},\ S^{\pm}_{\alpha/\dot{\alpha}},\ \hat{M}_{\hat{i}\hat{j}}$$

   with $\alpha/\dot{\alpha} = 1, 2$, $i = 1, \ldots, d$, and "transverse" $\hat{i} = d + 1, \ldots, 4$.
3. Imposing super-Jacobi identities at the level of traces fixed algebra.
4. Important to keep $\hat{M}_{\hat{i}\hat{j}}$ to satisfy super-Jacobis.
5. Using associated superconformal Casimir can determine superconformal blocks for continuous $2 \leq d \leq 4$ (in terms of conformal blocks).
6. Caveat: fractional dimensional theories have issues with unitarity but this is usually for high dim ops and does not seem to effect us.

[Hogervorst, Rychkov, van Rees].

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Some nice properties:

1. SUSY algebra contains $U(1)$ R-charge and this gives stronger unitarity bound. E.g. for scalars:

$$\Delta = \left(\frac{d-1}{2}\right)|R|, \qquad \Delta \geq \left(\frac{d-1}{2}\right)|R| + d - 2$$

2. Chiral operator is annihilated by half supercharges and saturates unitarity bound.
3. Superpotential has $R = 2$ so in simple cases (only one chiral field) can fix Δ:

$$W = \Phi^3$$

implies superfield Φ has $R = 2/3$ and $\Delta = \left(\frac{d-1}{3}\right)$.

4. If more than one field (e.g. XY^2) can use a- or F-maximization to compute R-charge.
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Results

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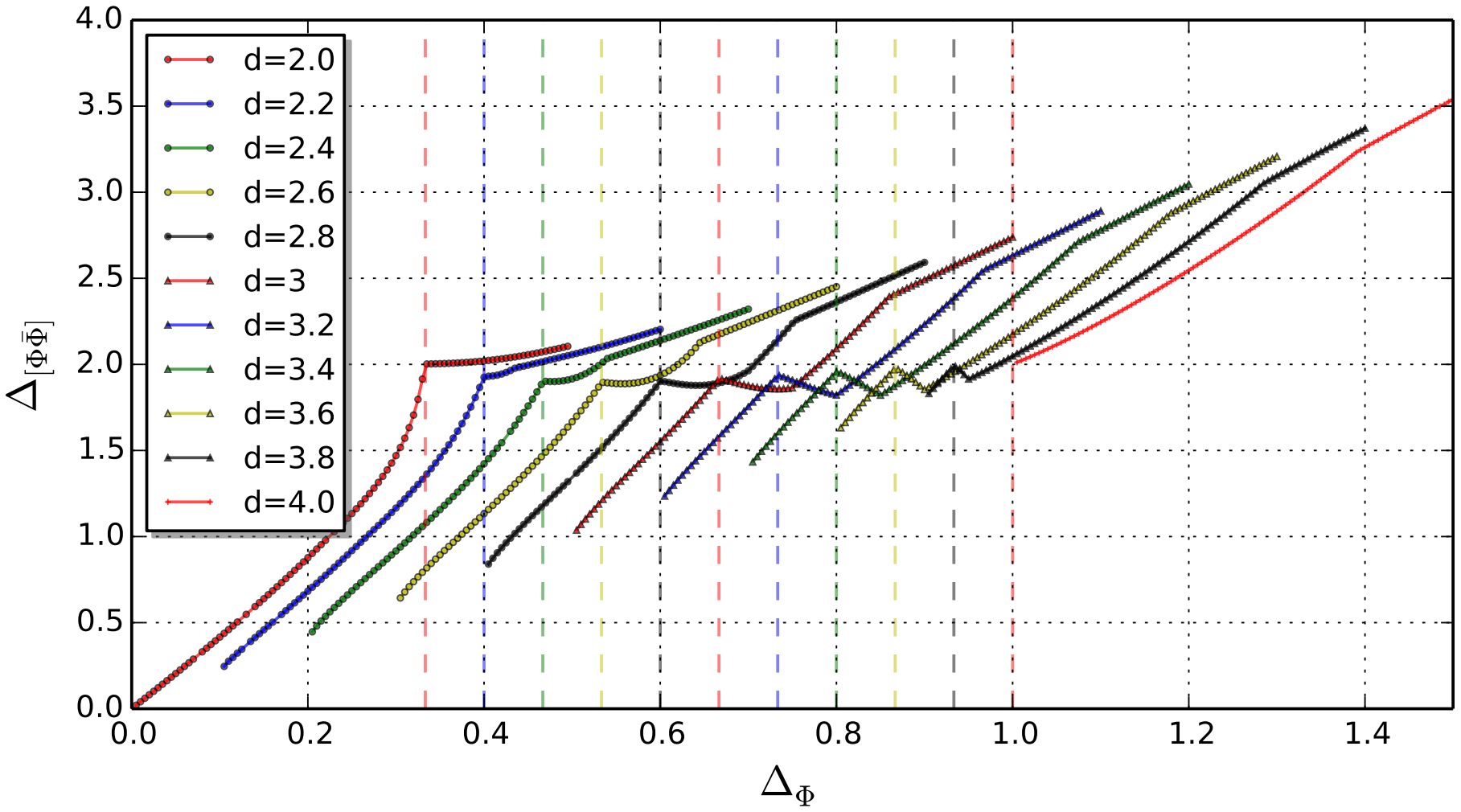
$\Delta_{[\Phi\bar{\Phi}]}$ Bound
d=2.0
d=2.2
d=2.4
d=2.6
d=2.8
d=3
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$\Delta_{[\Phi\bar{\Phi}]}$
Δ_{Φ}
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0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
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0.4
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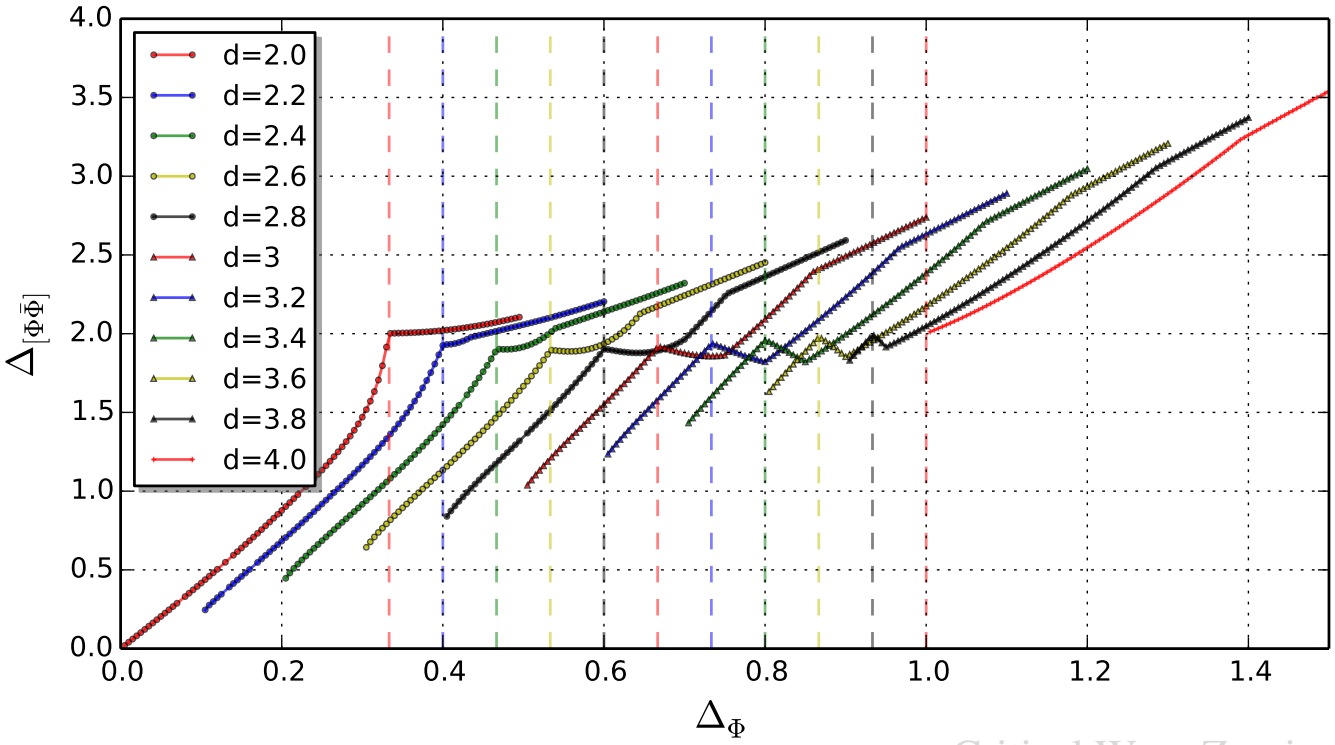

- Bounds for $d = 2 - 4$ (color coded).
- Can also minimize central charge, c.
- Multiple kinks!! in all d.
- Three kinks in $d = 3$: $\Delta_\Phi \sim \frac{2}{3}, \frac{3}{4}, 0.86$.
- Kink coincides with min of c.

Critical Wess-Zumino model

- Horizontal dashed line: Δ_Φ in WZ model
- Δ_Φ fixed because superpotential

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has $R = 2$ so $\Delta_\Phi = \frac{d-1}{3}$ $\left(\Rightarrow \frac{2}{3} \text{ in } d = 3\right)$.

- This is SUSY version of ϕ^4 theory!

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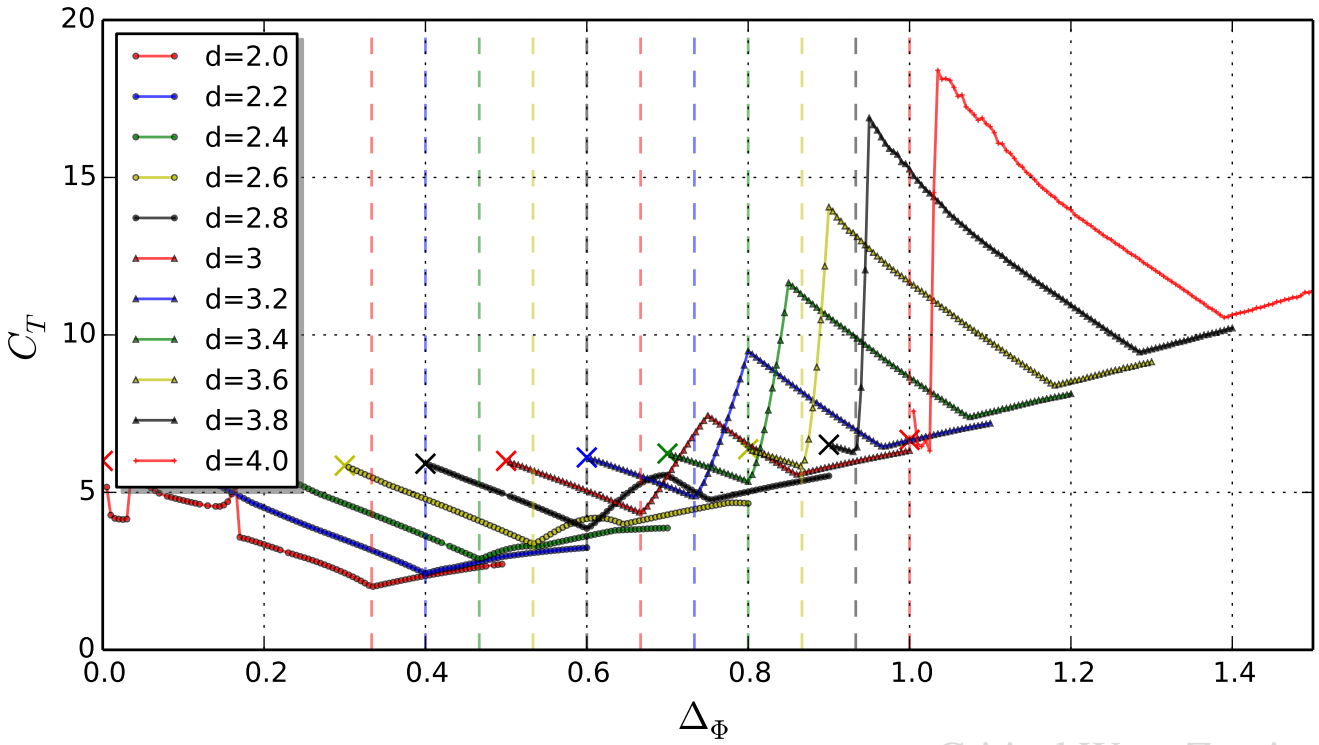
Central Charge Bound



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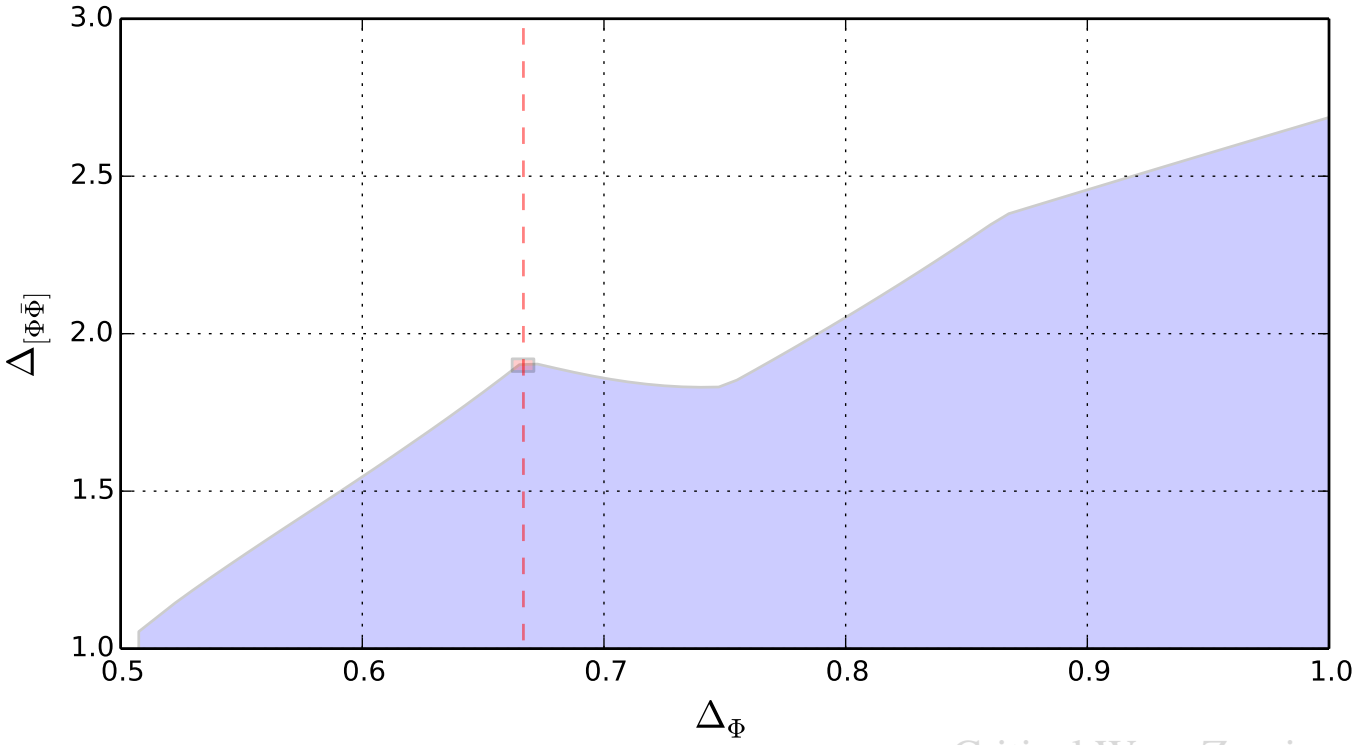

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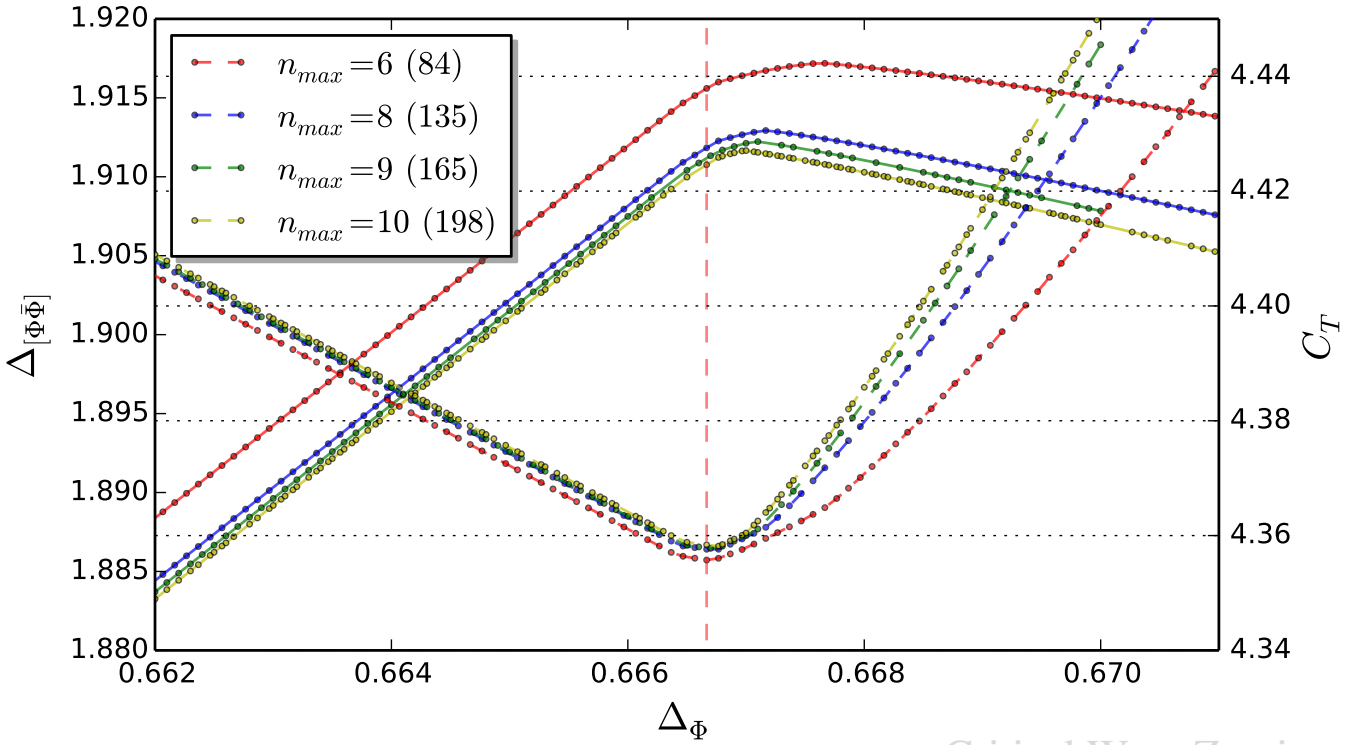

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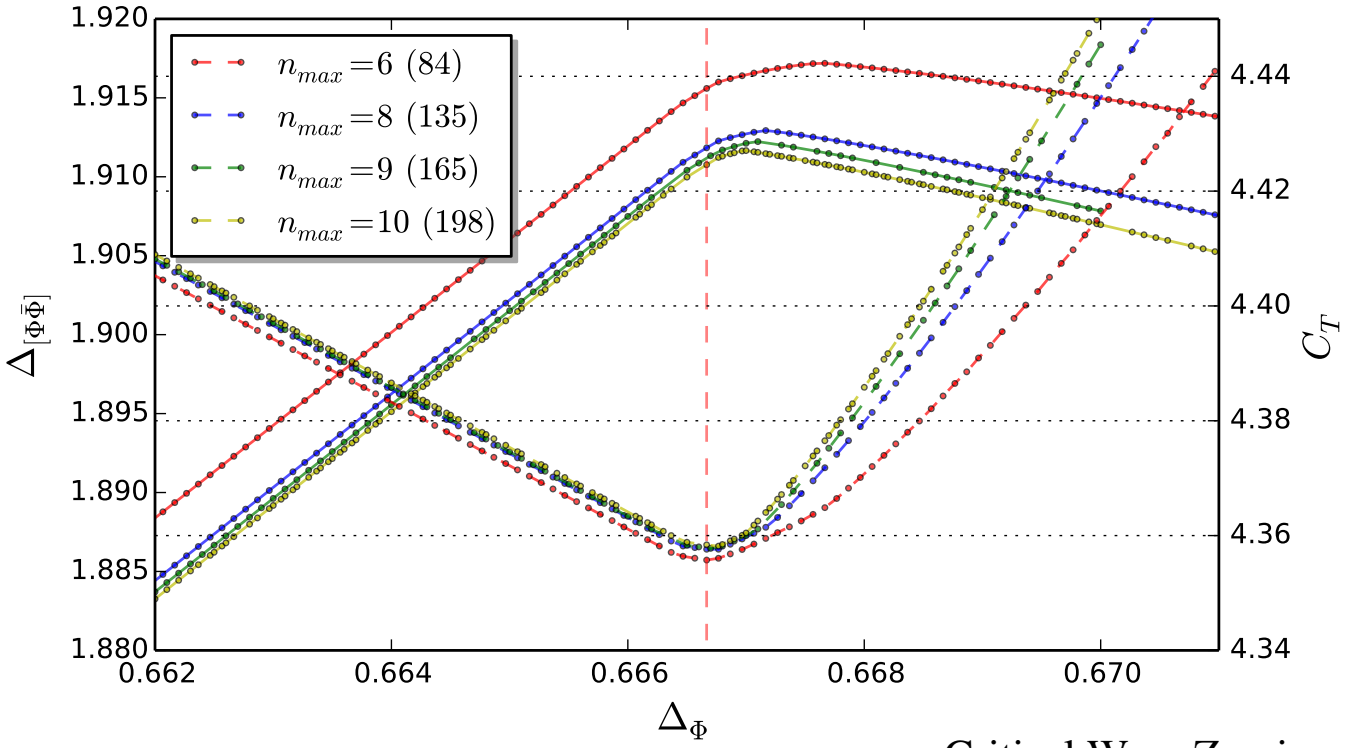

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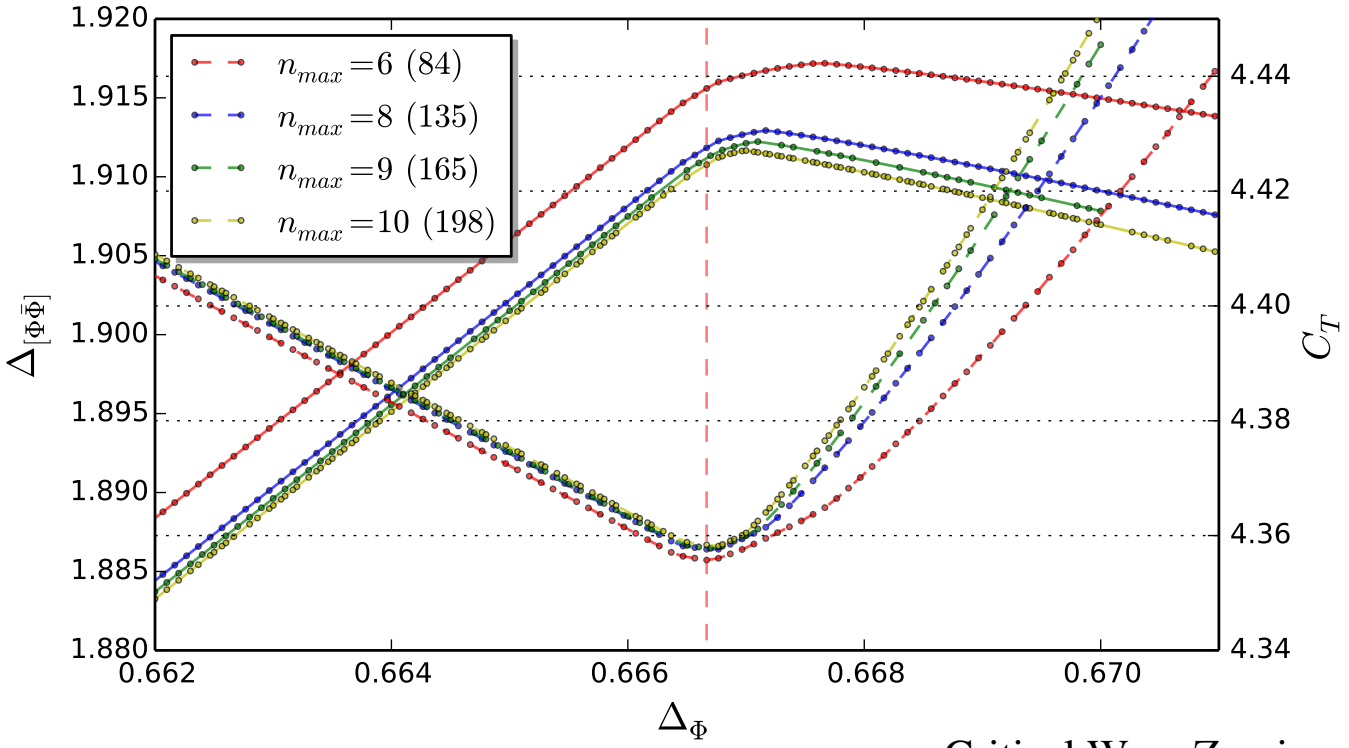

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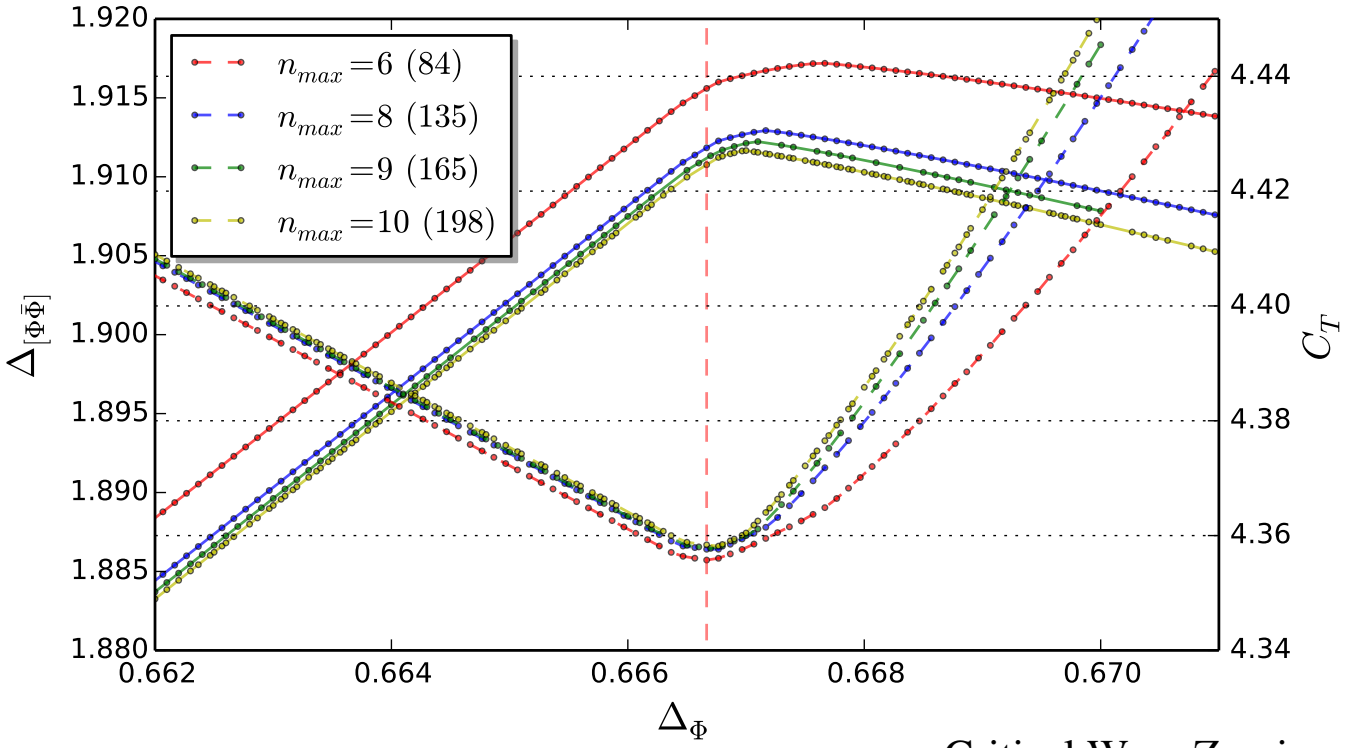

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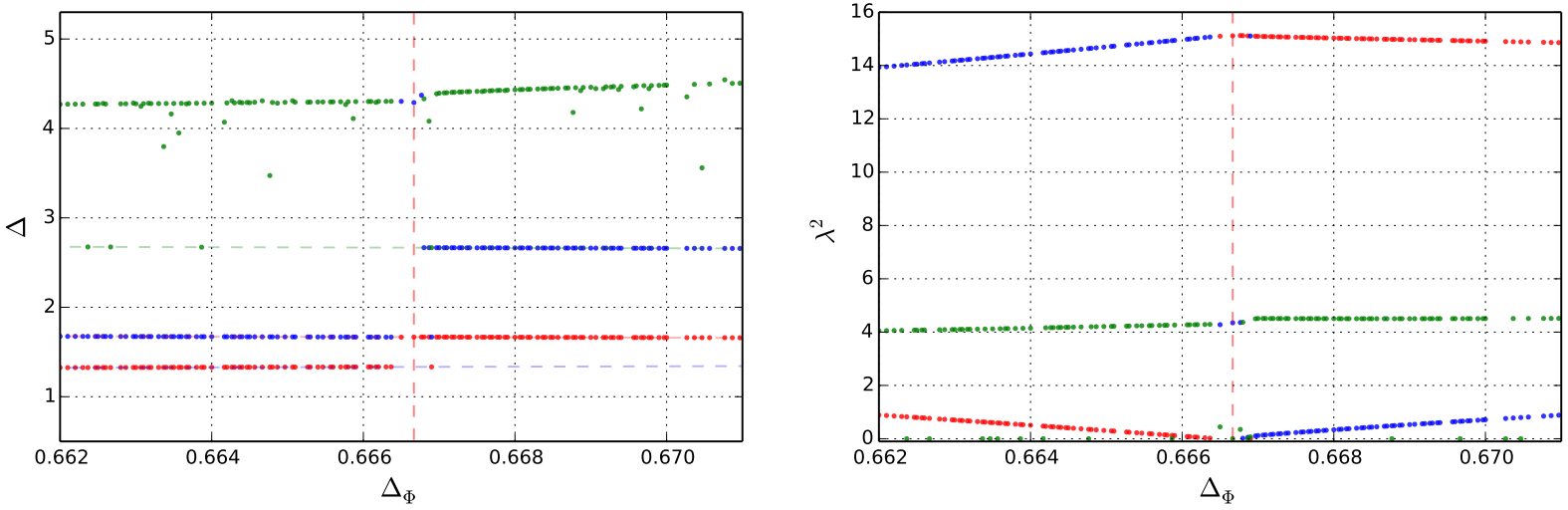
SUSY also imposes interesting dynamical constraints on theory

- WZ model: chiral superfield $X = \Phi + \ldots$ with cubic superpotential: $\boxed{W = X^3}$
- SUSY eqns $\frac{\partial W}{\partial X} = 0$ implies Φ^2 should decouple in theory.

R-charged scalar spectrum (left) and OPE (right)



C_T exactly computable (in $d = 3$) via localization of squashed-sphere partition function:

$$C_T / C_T^{(\text{free})} \simeq 0.7268 \qquad \text{(localization)}$$

$$C_T / C_T^{(\text{free})} \simeq 0.72652(33) \qquad \text{(numerics)}$$

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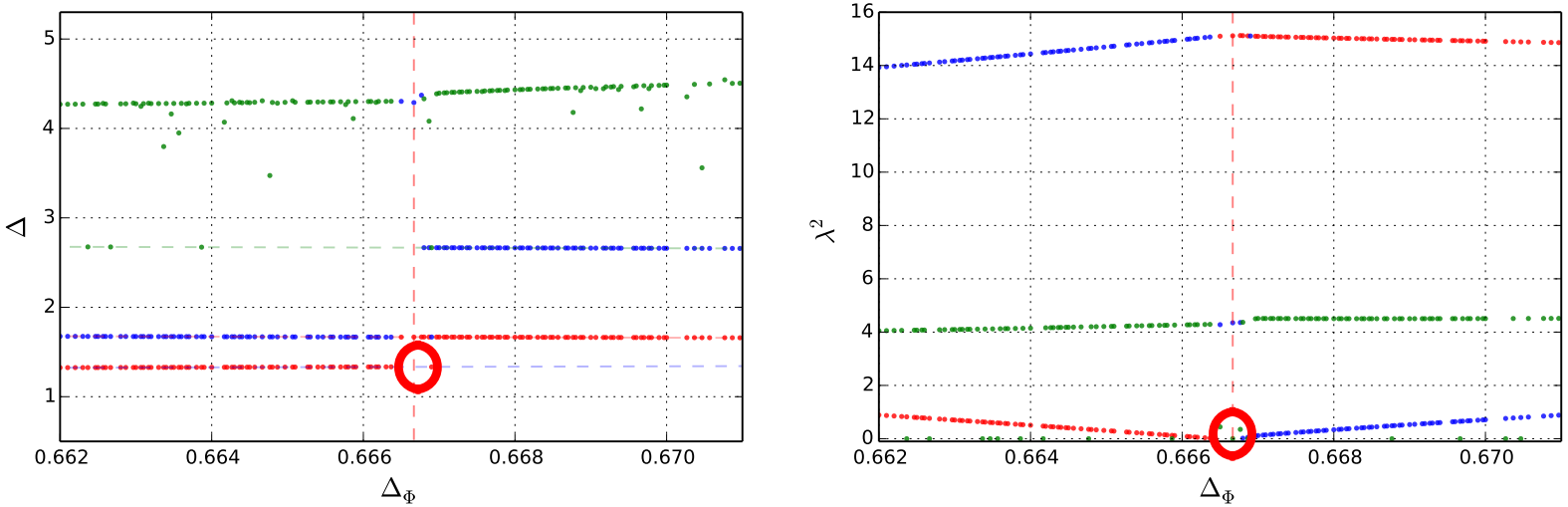

C_T exactly computable (in $d = 3$) via localization of squashed-sphere partition function:

$$C_T / C_T^{(\text{free})} \simeq 0.7268 \qquad \text{(localization)}$$

$$C_T / C_T^{(\text{free})} \simeq 0.72652(33) \qquad \text{(numerics)}$$

Results/Checks

SUSY also imposes interesting dynamical constraints on theory

- WZ model: chiral superfield $X = \Phi + \ldots$ with cubic superpotential: $\boxed{W = X^3}$
- SUSY eqns $\frac{\partial W}{\partial X} = 0$ implies Φ^2 should decouple in theory.

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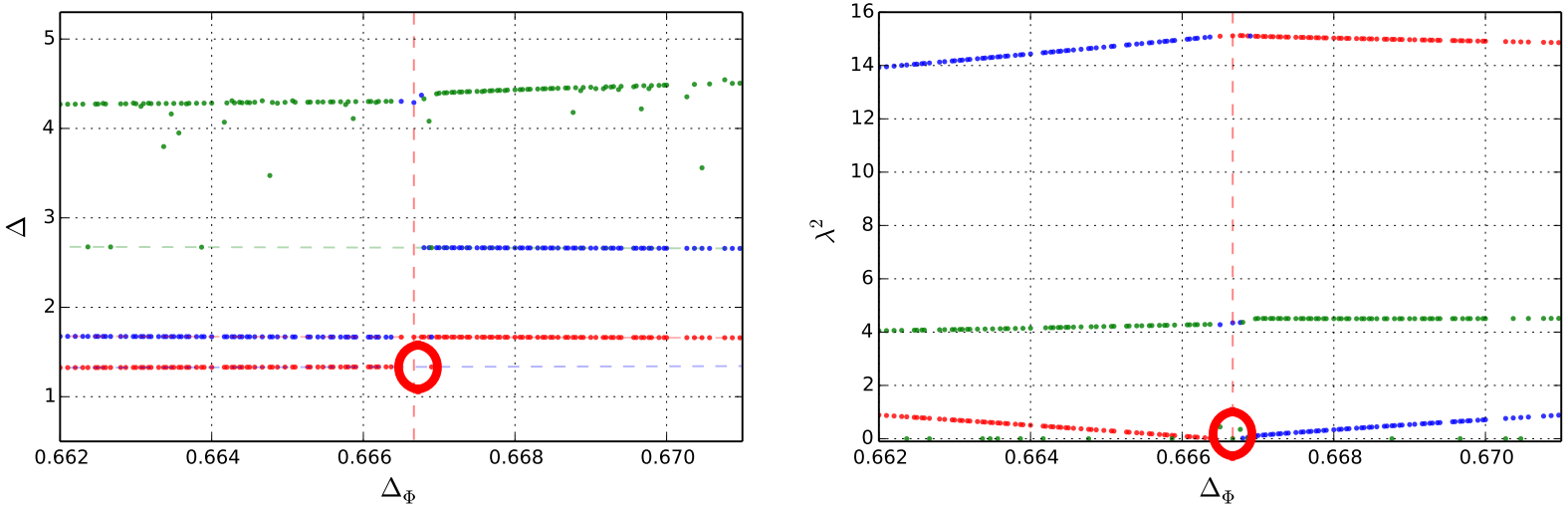

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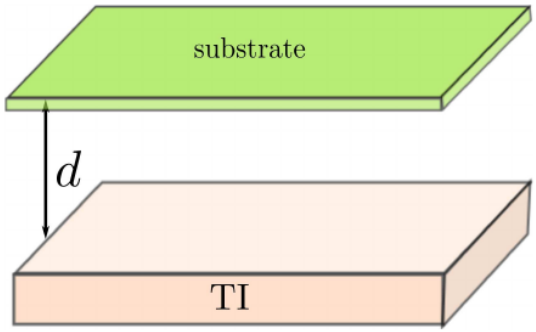

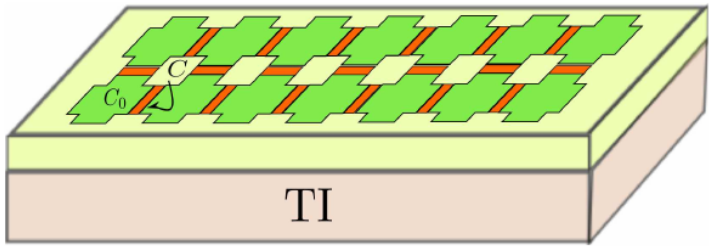

Figure : Substrate over topological insulator (left), Josephson junction on topological insulator (right). [Ponte, Lee]

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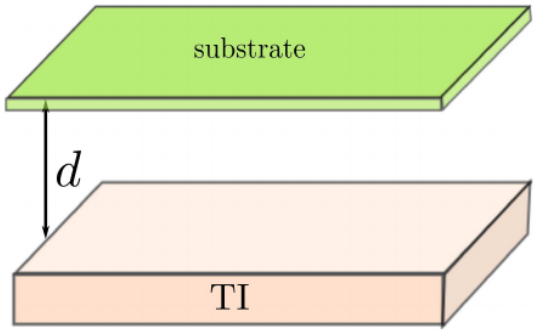

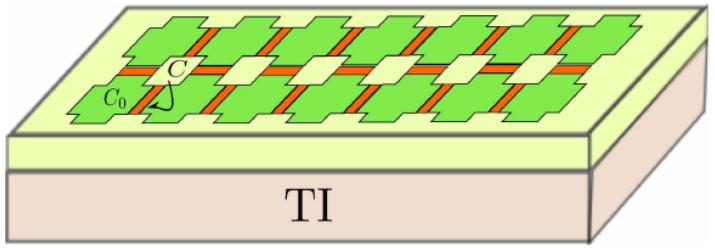

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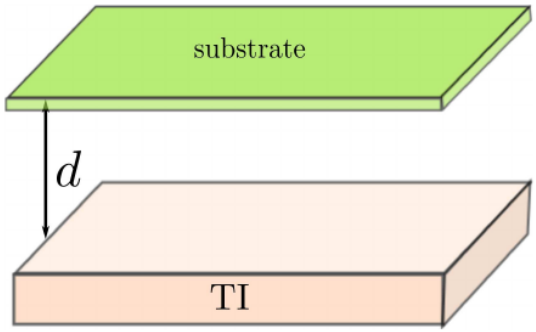

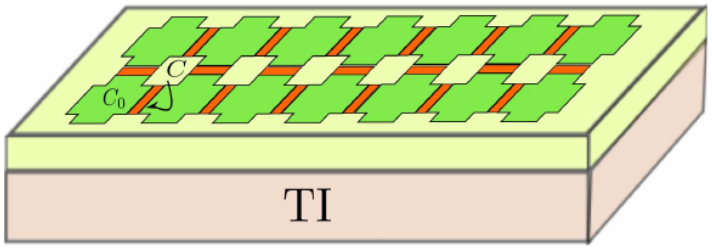

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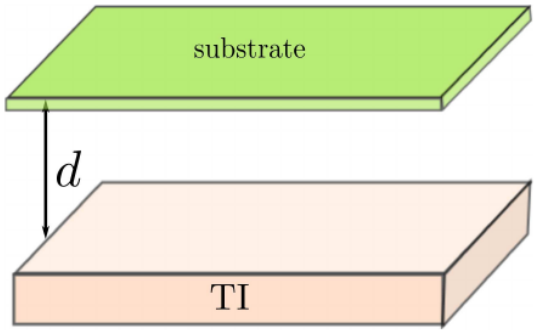

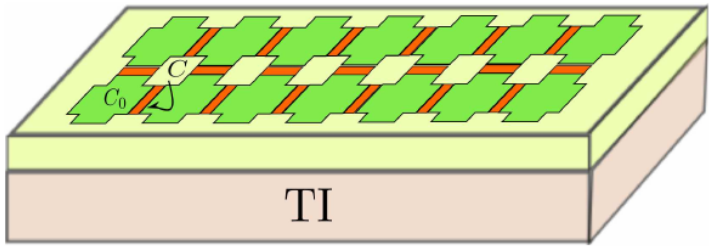

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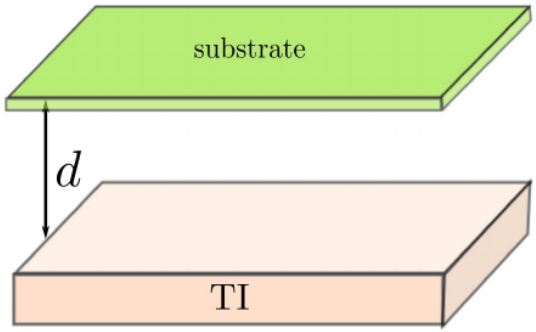

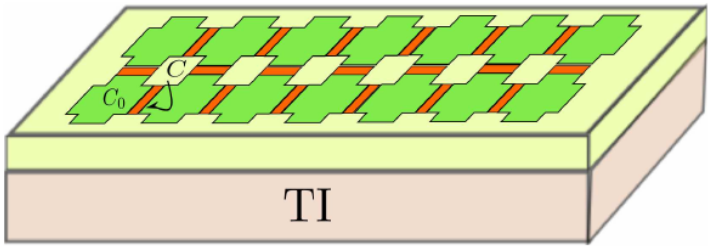

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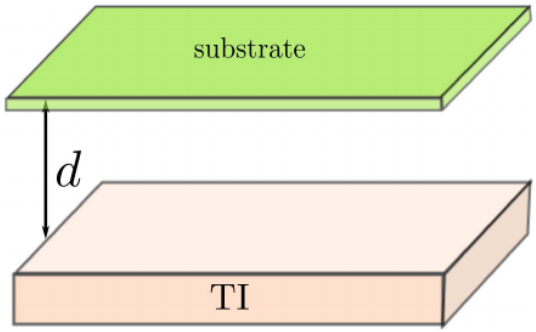

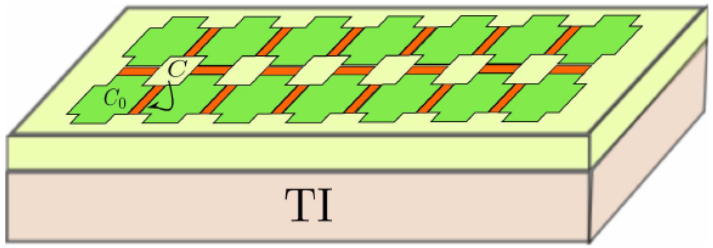

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New Theories?

In addition to kink at $\Delta_\Phi = \frac{d-1}{3}$ there seem to be ~ 2 more kinks.

Second Kink

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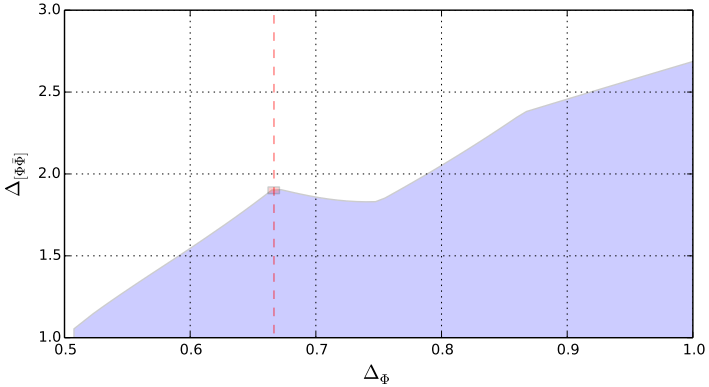

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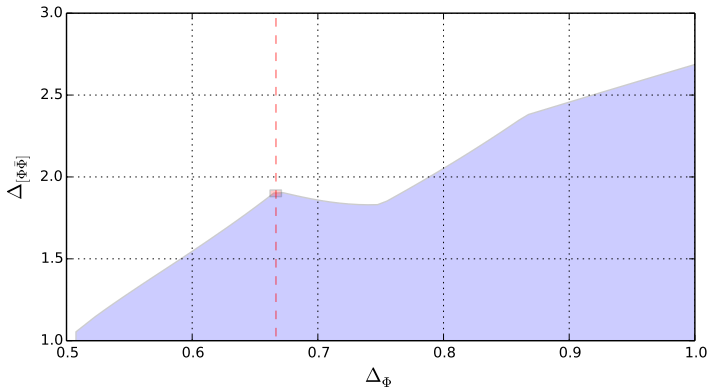

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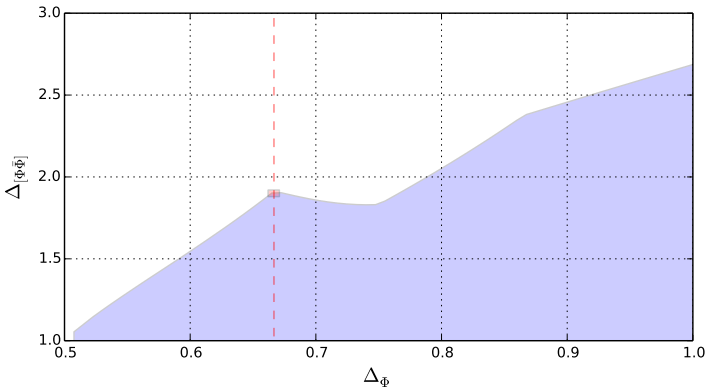

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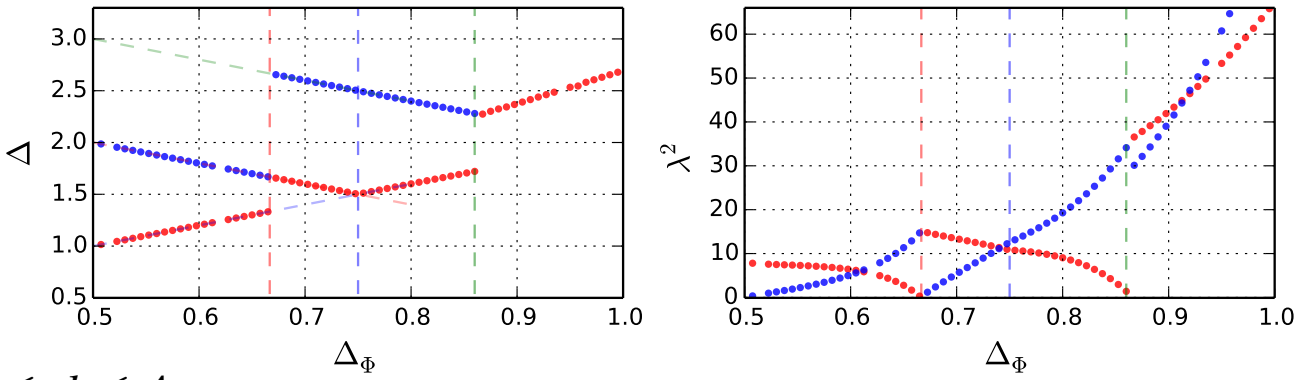

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(time allowing)

Superconformal Blocks

Bootstrapping Theories with Four Supercharges

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SUSY Bootstrap Details

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Let Φ be complex chiral scalar field so $\Delta = \left(\frac{d-1}{2}\right) R$ and consider

$$\langle \Phi(x_1)\bar{\Phi}(x_2)\Phi(x_3)\bar{\Phi}(x_4)\rangle$$

- Φ carries R-charge so can decompose OPE in reps of R-charge:

  (12, 34), (14, 23) channels: $\Phi \times \bar{\Phi} \sim \sum_{\text{even } \ell} (\mathcal{O}_{R=0} + \dots) + \sum_{\text{odd } \ell} (\mathcal{O}_{R=0} + \dots),$

  (13, 24) channel: $\Phi \times \Phi \sim \sum_{\text{even } \ell} \mathcal{O}_{R=2}$

- In (13,24) channel contracting identical operators so only even spin (and $R = 2$).
- (12,34) & (14,23) channels differ only in sign of odd spin blocks.
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Let Φ be complex chiral scalar field so $\Delta = \left(\frac{d-1}{2}\right) R$ and consider

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Comments, Experimental realization, etc...

- Four supercharge bootstrap should allow us to "solve" critical WZ-model.
- This theory is conjectured to describe a superconducting quantum critical point on $2+1d$ surface of $3d$ topological insulator.
  [Ponte-Lee, Grover-Sheng-Vishwanath]
- We also found two additional features for $2 \leq d \leq 4$ which may correspond to physically interesting theories.
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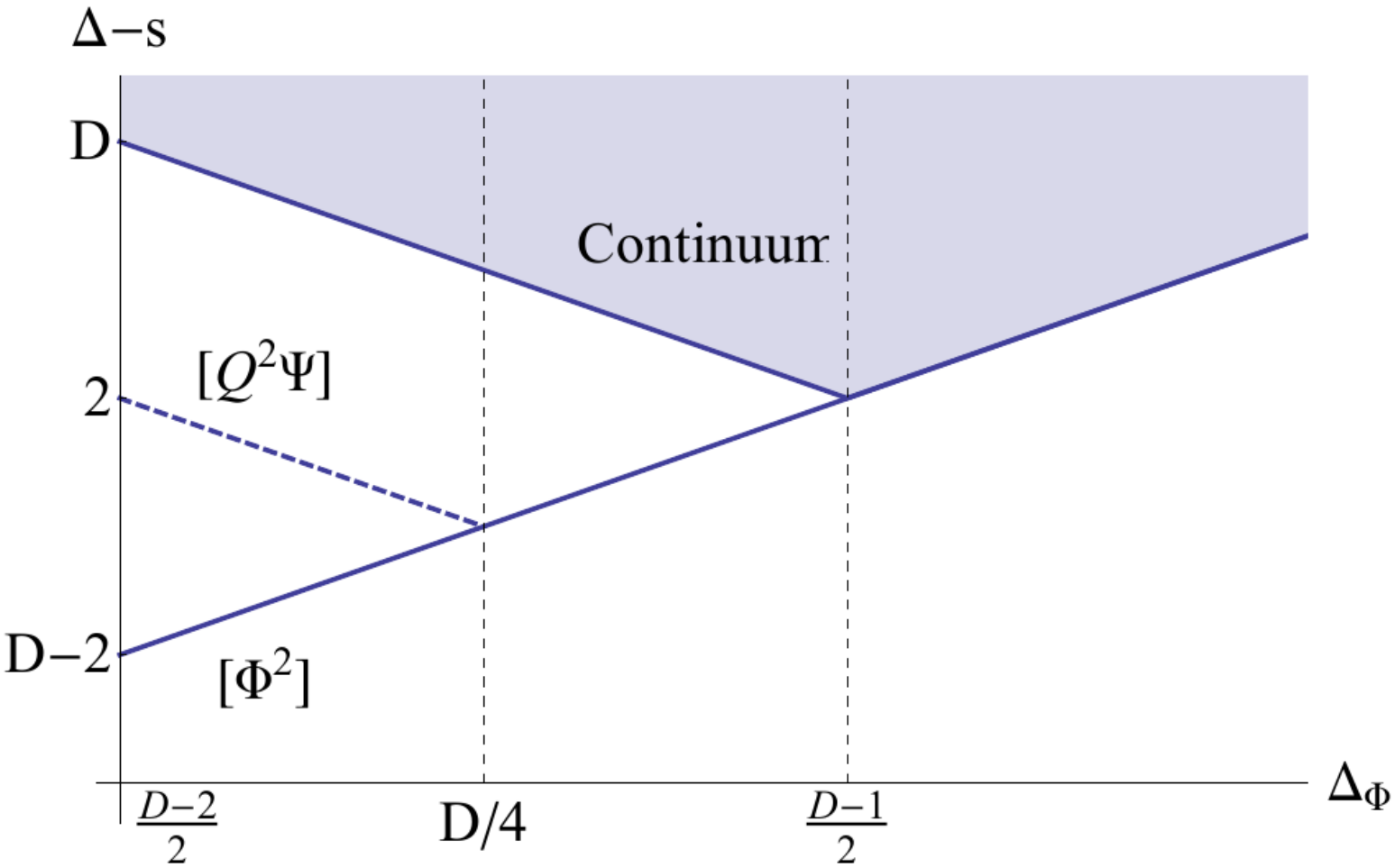
Operators we allow in $\Phi \times \Phi$ OPE
(dashed line only in scalar channel)



Origin of Kinks?

Null States?

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What is special about Ising model in $d = 2$?

- In $d = 2$ Virasoro strongly constrains spectrum.
- Minimal models ($c < 1$) have few (Virasoro) primaries in short representations of Virasoro.
- Ising model has only two Virasoro primaries: $|\sigma\rangle$ and $|\epsilon\rangle$.
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- Note for 2d Ising: $\Delta_\sigma = \frac{1}{8}$ and $\Delta_\epsilon = 1$ so $\Delta_{T'} = 3$.
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Spin 2 spectrum in $d = 2, 3$.

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To study this lets return to non-SUSY bootstrap.

Recall can extract spectrum (as a function of Δ_σ) for *all points on the boundary*.

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Is d=3 kink also related to a null state decoupling?

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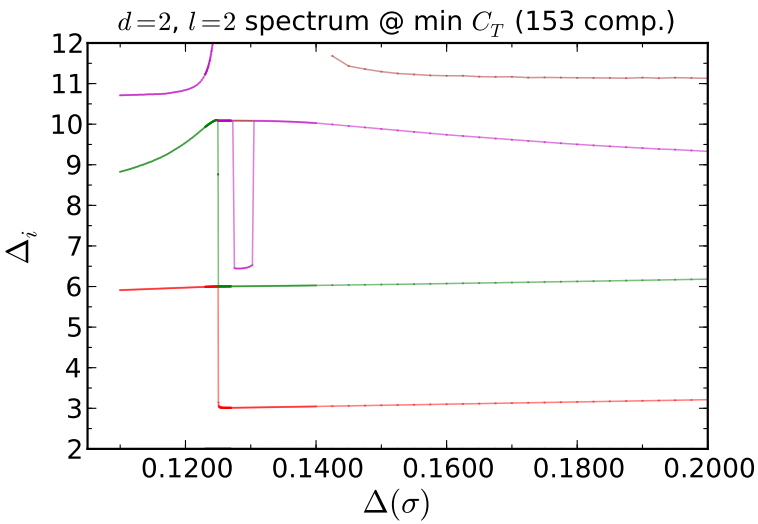

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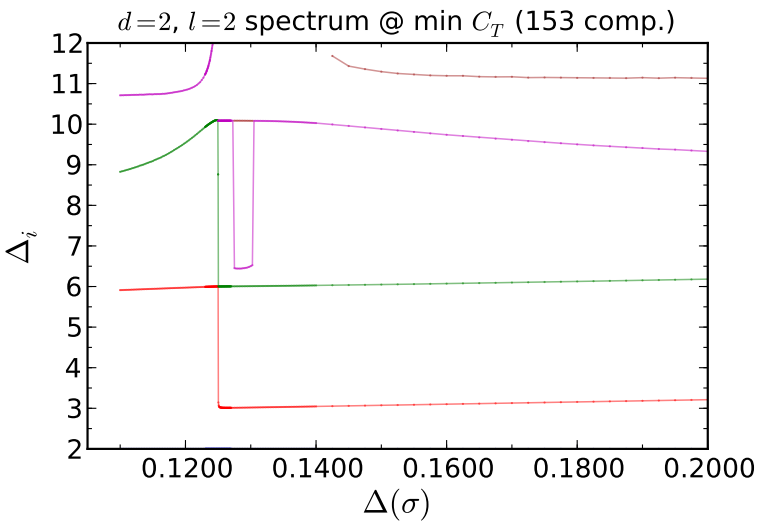

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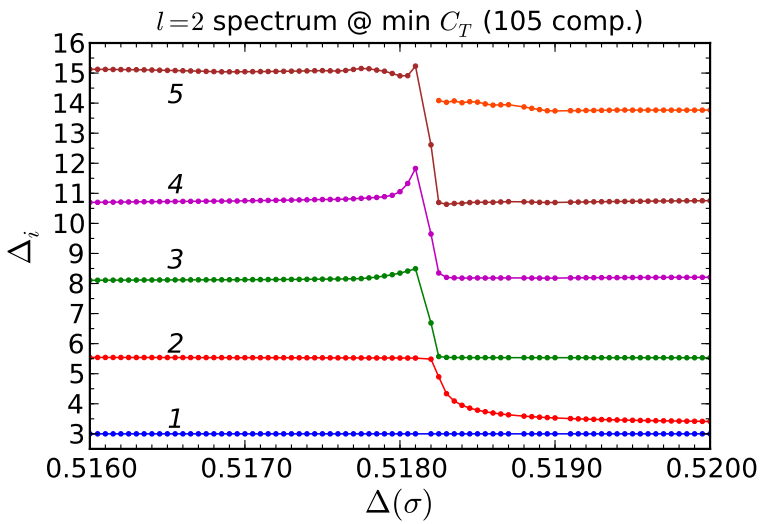

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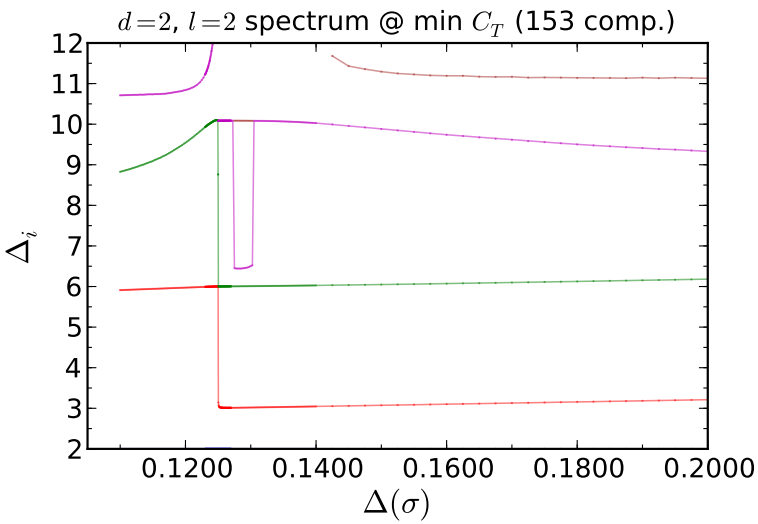

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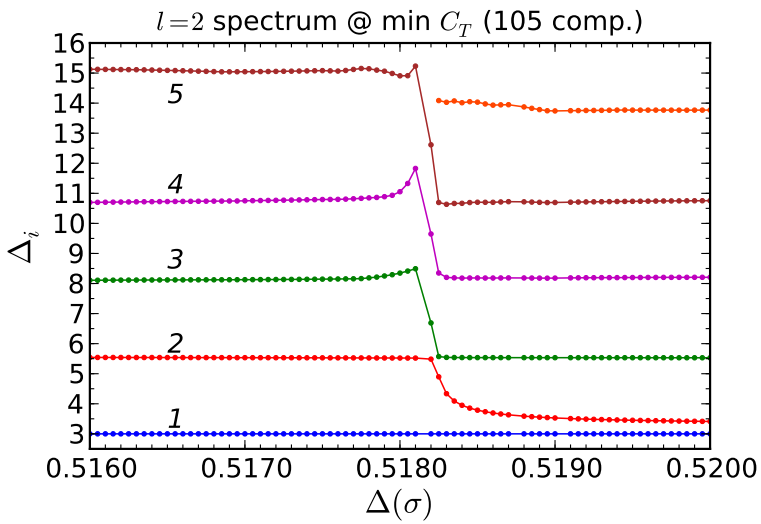

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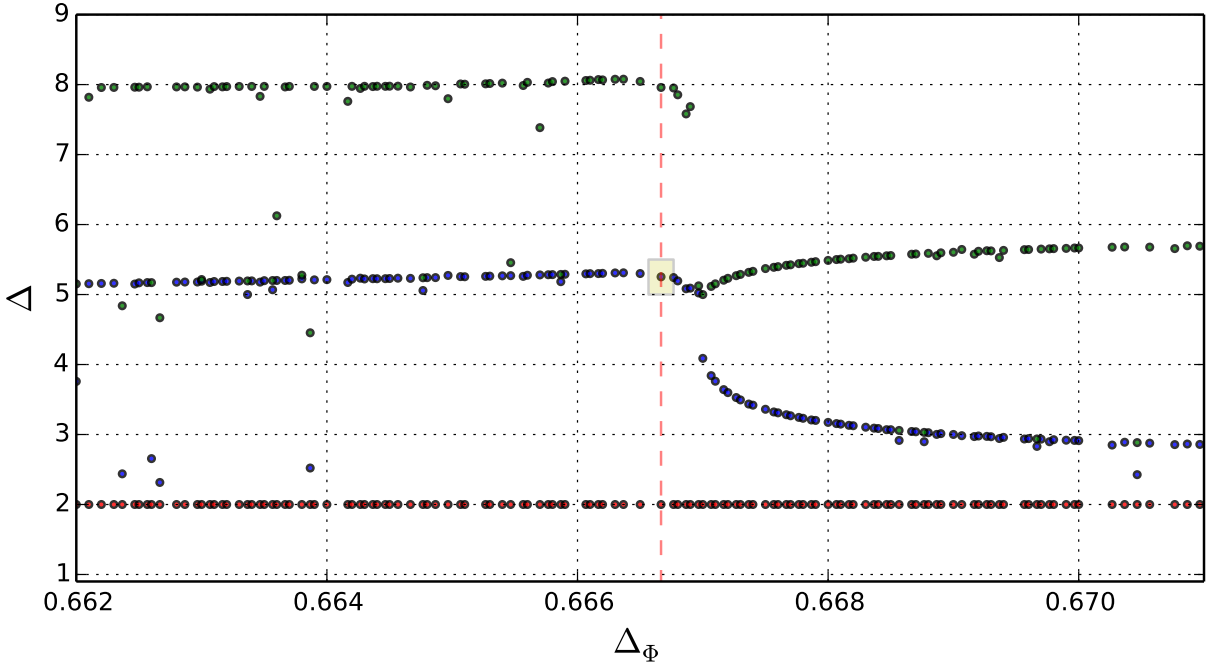
In non-SUSY 3d Ising found interesting (surprising) kinematical structure.

What about SUSY case?

Bounds on Spin 1*



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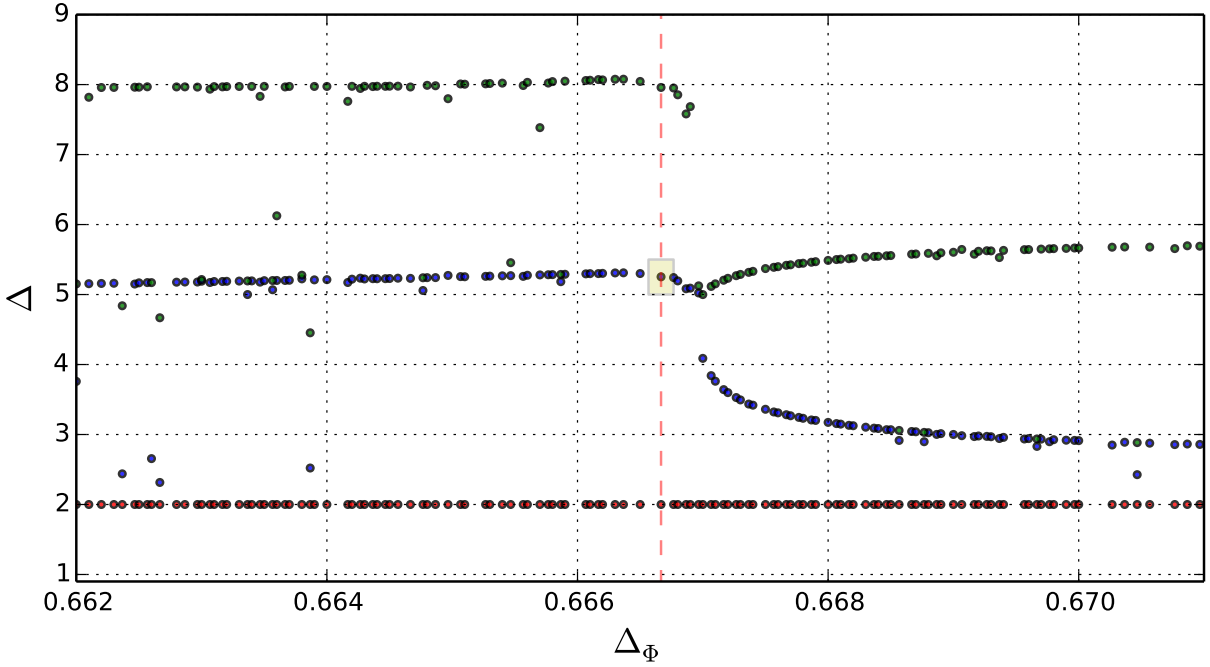

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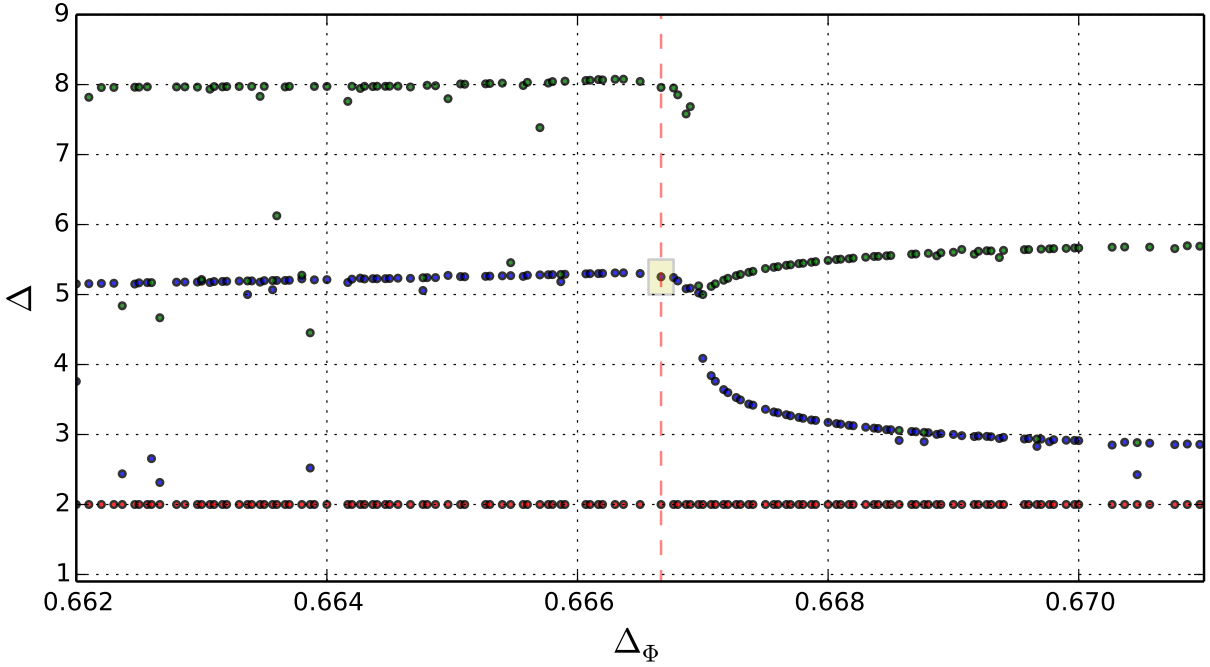

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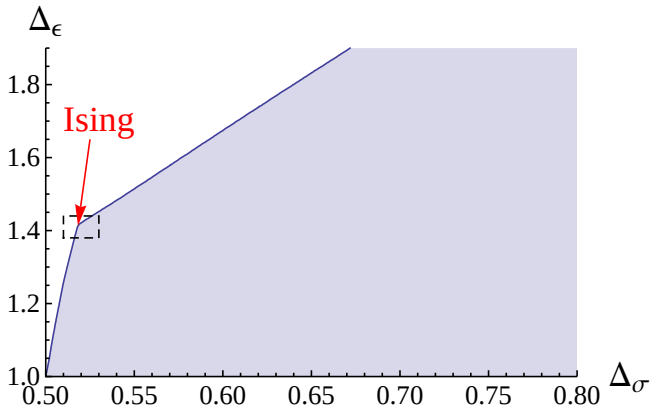
solving cfts on the boundary via crossing

extremal functional method

- study crossing symmetry of $\langle \sigma\sigma\sigma\sigma \rangle$ correlator.
- impose gap in scalar spectrum (no other assumptions).
- find that ising model corresponds to maximal allowed gap

$\rightarrow$ unique solution to crossing!

- extract spectrum & ope coefficients of ising model.
- note: this can be used with any cft on boundary.



OPE Bounds & central charge minimization

What else can we bound?

Bootstrap allows us to:

- Consider arbitrary CFT data $\mathcal{S} = \{(\Delta_i, \ell_i), C_{ijk}\}$.
- Check if this $\mathcal{S}$ is consistent with crossing sym of $\langle\sigma\sigma\sigma\sigma\rangle$.

We can additionally impose:

- Global symmetries e.g. $O(N), \ldots$
- SUSY when form of superconformal blocks constrained.

Kinds of bounds we can place on $\mathcal{S}$:

$$\sigma \times \sigma \sim 1 + C^{\epsilon}_{\sigma\sigma} \boxed{\epsilon} + \cdots + C^{T}_{\sigma\sigma} T_{\mu\nu} + \ldots$$

Can bound dimension of first scalar on Δ_ϵ (or any ℓ).

Any time a bound is saturated can compute full OPE.

OPE Bounds & central charge minimization

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Kinds of bounds we can place on $\mathcal{S}$:

$$\sigma \times \sigma \sim 1 + C^{\epsilon}_{\sigma\sigma}\,\epsilon + \cdots + \boxed{C^{T}_{\sigma\sigma}}\,T_{\mu\nu} + \ldots$$

Can bound (maximize) OPE coefficient of any operator.

Any time a bound is saturated can compute full OPE.

OPE Bounds & central charge minimization

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Kinds of bounds we can place on $\mathcal{S}$:

$$\sigma \times \sigma \sim 1 + C^{\epsilon}_{\sigma\sigma}\,\epsilon + \cdots + \left(\frac{\Delta_\sigma}{\sqrt{c}}\right) T_{\mu\nu} + \ldots$$

If operator e.g. $T_{\mu\nu}$ get *lower bound* on c.

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OPE Bounds & central charge minimization

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