Mellin Amplitudes: the Scattering Amplitudes of CFT

João Penedones
Universidade do Porto & CERN

Collaborators: Costa, Drummond, Fitzpatrick, Gonçalves, Kaplan, Raju, Trevisani, van Rees

Strings 2015, Bengaluru, India, June 26th, 2015

Outline

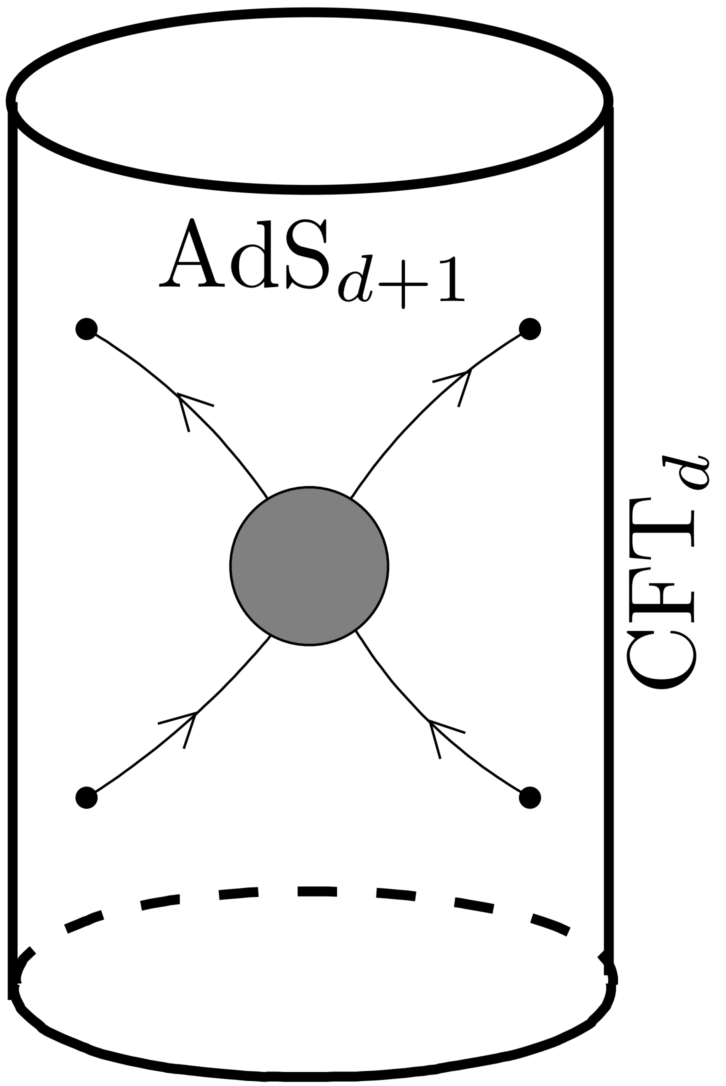
- Introduction
- CFT
  - Definition of Mellin amplitudes
  - OPE & Factorization
- AdS/CFT
  - Witten diagrams
  - Flat space limit of AdS
  - Bulk locality
- Problems for the future

Introduction

CFT correlation functions can be thought of as AdS scattering amplitudes.

Idea: explore this analogy to learn about CFT and AdS/CFT

Best language: Mellin amplitudes



CFT

Mellin Amplitudes

[Mack '09]

Correlation function of scalar primary operators

$$A(x_i) = \langle \mathcal{O}_1(x_1) \dots \mathcal{O}_n(x_n) \rangle$$

Mellin Amplitudes

[Mack '09]

Correlation function of scalar primary operators

$$A(x_i) = \langle \mathcal{O}_1(x_1) \dots \mathcal{O}_n(x_n) \rangle$$

$$A(x_i) = \int [d\gamma]\, M(\gamma_{ij}) \prod_{i<j}^{n} \Gamma(\gamma_{ij}) \left(x_{ij}^2\right)^{-\gamma_{ij}}$$

Mellin Amplitudes

[Mack '09]

Correlation function of scalar primary operators

$$A(x_i) = \langle \mathcal{O}_1(x_1) \dots \mathcal{O}_n(x_n) \rangle$$

$$A(x_i) = \int [d\gamma] \boxed{M(\gamma_{ij})} \prod_{i<j}^{n} \Gamma(\gamma_{ij}) \left(x_{ij}^2\right)^{-\gamma_{ij}}$$

Mellin Amplitudes

[Mack '09]

Correlation function of scalar primary operators

$$A(x_i) = \langle \mathcal{O}_1(x_1) \dots \mathcal{O}_n(x_n) \rangle$$

$$A(x_i) = \int [d\gamma] \boxed{M(\gamma_{ij})} \prod_{i<j}^{n} \Gamma(\gamma_{ij}) \left(x_{ij}^2\right)^{-\gamma_{ij}}$$

Constraints:

$$\sum_{j=1}^{n} \gamma_{ij} = 0 \ , \qquad \gamma_{ii} = -\Delta_i = -\dim[\mathcal{O}_i] \ , \qquad \gamma_{ij} = \gamma_{ji}$$

Mellin Amplitudes

[Mack '09]

Correlation function of scalar primary operators

$$A(x_i) = \langle \mathcal{O}_1(x_1) \dots \mathcal{O}_n(x_n) \rangle$$

$$A(x_i) = \int [d\gamma] \boxed{M(\gamma_{ij})} \prod_{i<j}^{n} \Gamma(\gamma_{ij}) \left(x_{ij}^2\right)^{-\gamma_{ij}}$$

Constraints:

$$\sum_{j=1}^{n} \gamma_{ij} = 0 \; , \qquad \gamma_{ii} = -\Delta_i = -\dim[\mathcal{O}_i] \; , \qquad \gamma_{ij} = \gamma_{ji}$$

integration variables = # ind. cross-ratios = $\dfrac{n(n-3)}{2}$

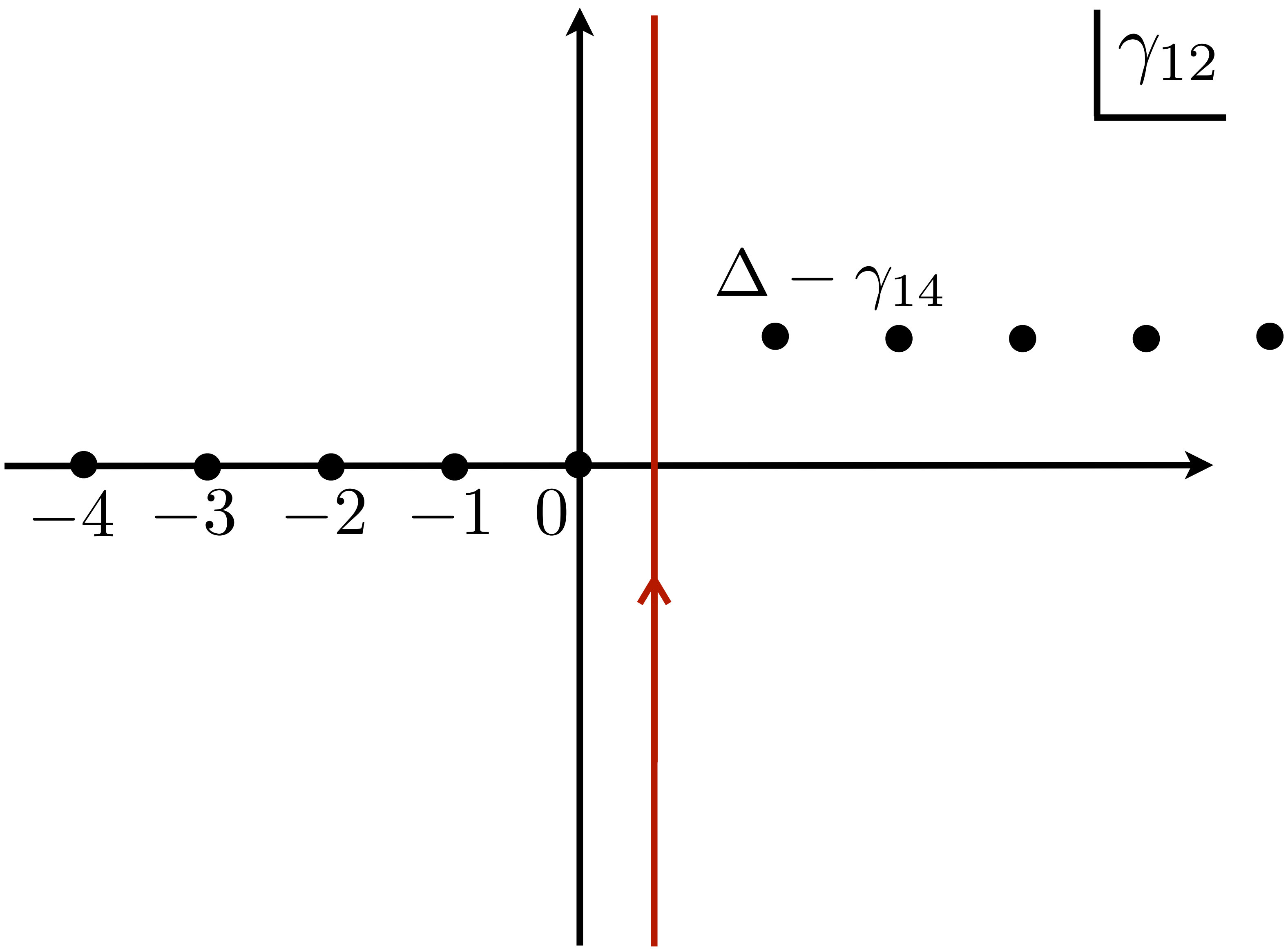
Example: $n = 4 \qquad \Delta_i = \Delta$

$$A(x_i) = \frac{1}{(x_{13}^2 x_{24}^2)^{\Delta}} \int_{-i\infty}^{i\infty} \frac{d\gamma_{12} d\gamma_{14}}{(2\pi i)^2} \left(\frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2} \right)^{-\gamma_{12}} \left(\frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2} \right)^{-\gamma_{14}} \times$$

$$\times M(\gamma_{12}, \gamma_{14}) \Gamma^2(\gamma_{12}) \Gamma^2(\gamma_{14}) \Gamma^2(\Delta - \gamma_{12} - \gamma_{14})$$

Example: $n = 4 \qquad \Delta_i = \Delta$

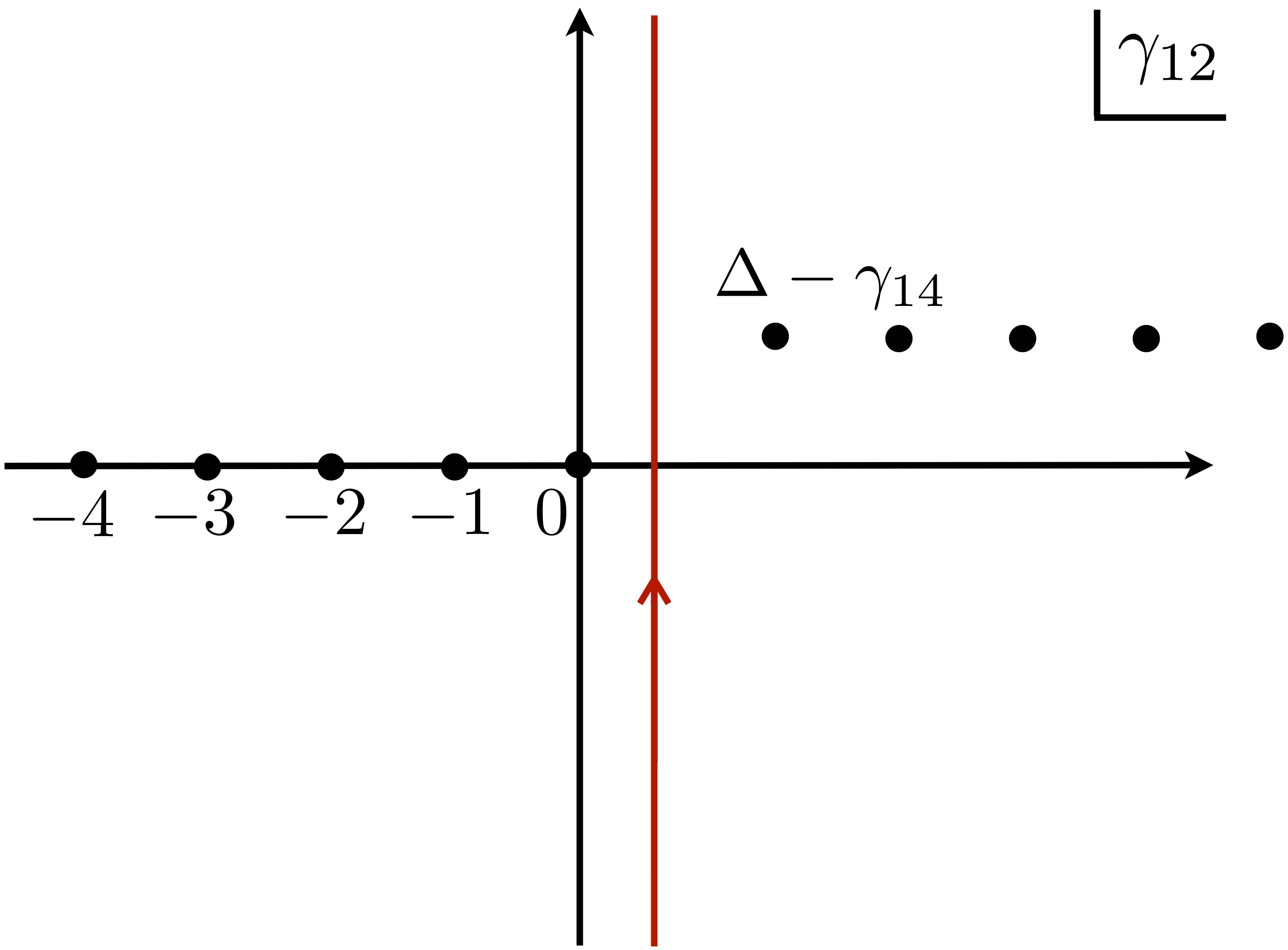
$$A(x_i) = \frac{1}{(x_{13}^2 x_{24}^2)^\Delta} \int_{-i\infty}^{i\infty} \frac{d\gamma_{12} d\gamma_{14}}{(2\pi i)^2} \left(\frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2}\right)^{-\gamma_{12}} \left(\frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2}\right)^{-\gamma_{14}} \times$$
$$\times M(\gamma_{12}, \gamma_{14}) \Gamma^2(\gamma_{12}) \Gamma^2(\gamma_{14}) \Gamma^2(\Delta - \gamma_{12} - \gamma_{14})$$



Example: $n = 4 \qquad \Delta_i = \Delta$

$$A(x_i) = \frac{1}{(x_{13}^2 x_{24}^2)^{\Delta}} \int_{-i\infty}^{i\infty} \frac{d\gamma_{12} d\gamma_{14}}{(2\pi i)^2} \left(\frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2}\right)^{-\gamma_{12}} \left(\frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2}\right)^{-\gamma_{14}} \times$$

$$\times M(\gamma_{12}, \gamma_{14}) \Gamma^2(\gamma_{12}) \Gamma^2(\gamma_{14}) \Gamma^2(\Delta - \gamma_{12} - \gamma_{14})$$



Crossing symmetry

$$M(\gamma_{12}, \gamma_{13}, \gamma_{14})$$

$$\gamma_{12} + \gamma_{13} + \gamma_{14} = \Delta$$

Operator Product Expansion

$$\mathcal{O}_1(x_1)\mathcal{O}_2(x_2) = \sum_k \frac{C_{12k}}{(x_{12}^2)^{\frac{1}{2}(\Delta_1+\Delta_2-\Delta_k)}} \left[\mathcal{O}_k(x_2) + \text{descendants}\right]$$

Operator Product Expansion

$$\mathcal{O}_1(x_1)\mathcal{O}_2(x_2) = \sum_k \frac{C_{12k}}{(x_{12}^2)^{\frac{1}{2}(\Delta_1+\Delta_2-\Delta_k)}} \left[\mathcal{O}_k(x_2) + \text{descendants}\right]$$

$$\langle \mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\ldots\rangle = \int d\gamma_{12} \left(x_{12}^2\right)^{-\gamma_{12}} \ldots M(\gamma_{12},\ldots)$$

Operator Product Expansion

$$\mathcal{O}_1(x_1)\mathcal{O}_2(x_2) = \sum_k \frac{C_{12k}}{(x_{12}^2)^{\frac{1}{2}(\Delta_1+\Delta_2-\Delta_k)}} \left[\mathcal{O}_k(x_2) + \text{descendants}\right]$$

$$\langle \mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\ldots\rangle = \int d\gamma_{12}\,\left(x_{12}^2\right)^{-\gamma_{12}} \ldots M(\gamma_{12},\ldots)$$

$\Rightarrow$ M has simple poles at

$$\gamma_{12} = \frac{\Delta_1+\Delta_2-\Delta_k-2m}{2}\,, \qquad m = 0, \underbrace{1, 2, \ldots}_{\text{descendants}}$$

with residue $\sim C_{12k}\langle \mathcal{O}_k\mathcal{O}_3\ldots\mathcal{O}_n\rangle$

Operator Product Expansion

$$\mathcal{O}_1(x_1)\mathcal{O}_2(x_2) = \sum_k \frac{C_{12k}}{(x_{12}^2)^{\frac{1}{2}(\Delta_1+\Delta_2-\Delta_k)}} \left[\mathcal{O}_k(x_2) + \text{descendants}\right]$$

$$\langle \mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\dots\rangle = \int d\gamma_{12} \left(x_{12}^2\right)^{-\gamma_{12}} \dots M(\gamma_{12},\dots)$$

$\Rightarrow M$ has simple poles at

$$\gamma_{12} = \frac{\Delta_1 + \Delta_2 - \Delta_k - 2m}{2} \,, \qquad m = 0, \underbrace{1, 2, \dots}_{\text{descendants}}$$

with residue $\sim C_{12k}\langle \mathcal{O}_k \mathcal{O}_3 \dots \mathcal{O}_n\rangle$

Discrete spectrum of scaling dimensions implies that the Mellin amplitude is meromorphic.

OPE $\Rightarrow$ Factorization

[Mack '09]

Introduce p_i such that $\gamma_{ij} = p_i \cdot p_j \quad (p_i^2 = -\Delta_i)$ and $\sum_{i=1}^{n} p_i = 0$. Then γ_{ij} automatically solves the constraints.

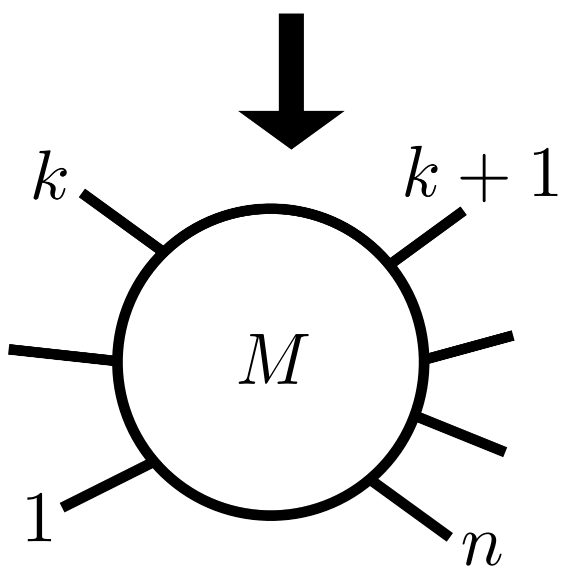
OPE $\Rightarrow$ Factorization

[Mack '09]

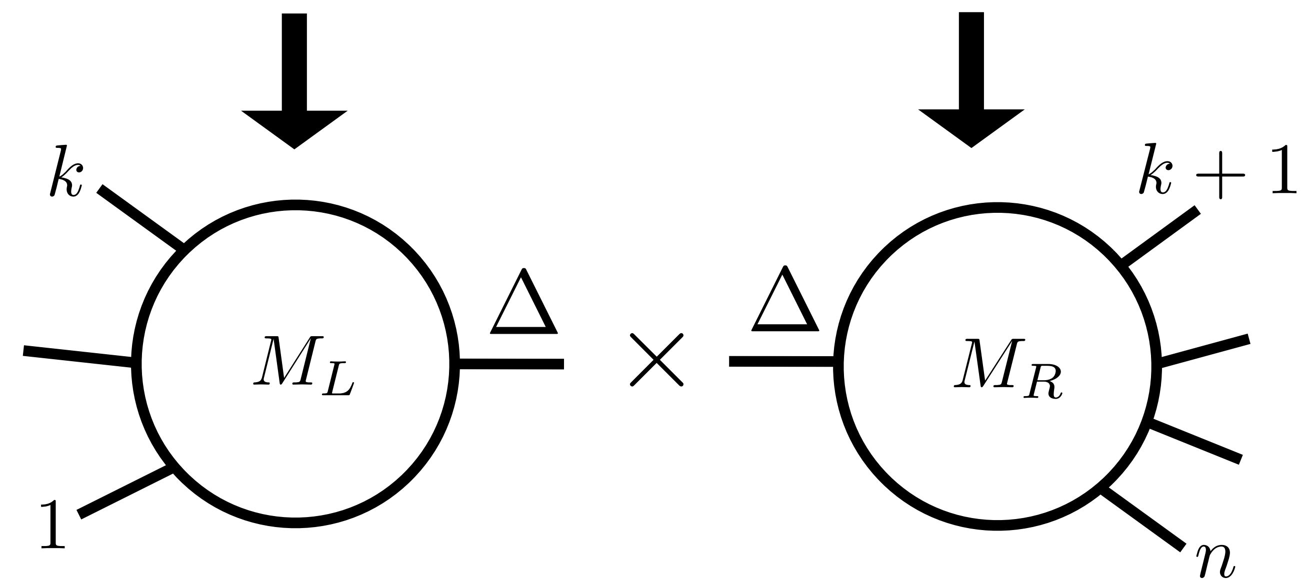
Introduce p_i such that $\gamma_{ij} = p_i \cdot p_j \quad (p_i^2 = -\Delta_i)$ and $\sum_{i=1}^n p_i = 0$. Then γ_{ij} automatically solves the constraints.

The OPE implies poles and factorization of residues.

$\langle \mathcal{O}_1 \ldots \mathcal{O}_n \rangle$ $\qquad\qquad$ $\langle \mathcal{O}_1 \ldots \mathcal{O}_k \mathcal{O}_\Delta \rangle \, \langle \mathcal{O}_\Delta \mathcal{O}_{k+1} \ldots \mathcal{O}_n \rangle$



$$\sim \frac{1}{\underbrace{(p_1 + \cdots + p_k)^2}_{\sum_{i,j=1}^k \gamma_{ij}} + \Delta}$$



[Fitzpatrick, Kaplan, JP, Raju, van Rees '11]
[Fitzpatrick, Kaplan '12]
[Gonçalves, JP, Trevisani, '14]

OPE $\Rightarrow$ Factorization

$$M_n \approx \frac{Q_m(\gamma_{ij})}{(p_1 + \cdots + p_k)^2 + \underbrace{\Delta - l + 2m}_{\text{twist}}} \qquad m = 0, \underbrace{1, 2, \ldots}_{\text{descendants}}$$

$$\langle \mathcal{O}_1 \ldots \mathcal{O}_k \mathcal{O}_{\Delta,l} \rangle \to M_L \qquad \langle \mathcal{O}_{\Delta,l} \mathcal{O}_{k+1} \ldots \mathcal{O}_n \rangle \to M_R$$

Q_m determined from M_L and M_R [Gonçalves, JP, Trevisani, '14]

OPE $\Rightarrow$ Factorization

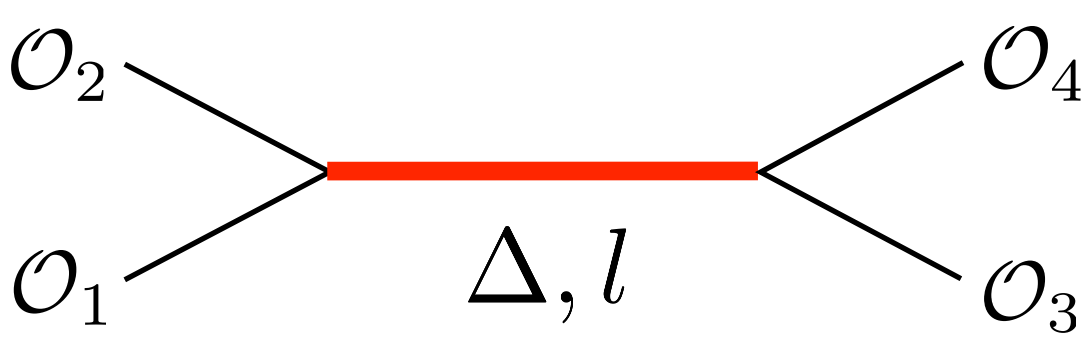
$$M_n \approx \frac{Q_m(\gamma_{ij})}{(p_1 + \cdots + p_k)^2 + \underbrace{\Delta - l + 2m}_{\text{twist}}} \qquad m = 0, \underbrace{1, 2, \ldots}_{\text{descendants}}$$

$$\langle \mathcal{O}_1 \ldots \mathcal{O}_k \mathcal{O}_{\Delta,l} \rangle \to M_L \qquad \langle \mathcal{O}_{\Delta,l} \mathcal{O}_{k+1} \ldots \mathcal{O}_n \rangle \to M_R$$

Q_m determined from M_L and M_R [Gonçalves, JP, Trevisani, '14]

For $n = 4$ and $k = 2$:

$$Q_m(\gamma_{ij}) = C_{12\mathcal{O}} C_{34\mathcal{O}} P_{l,m}(\gamma_{13})$$

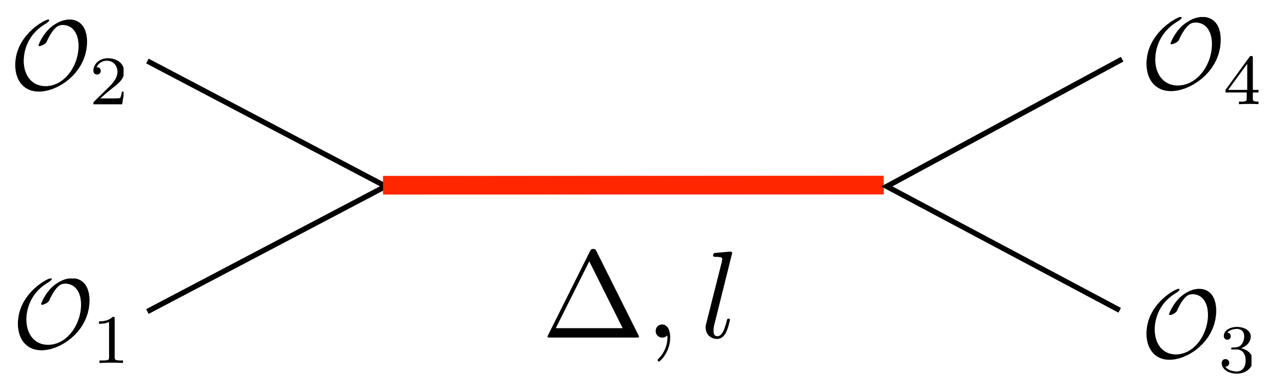


Mack polynomial
Kinematical polynomial of degree l

[Mack '09]

Large N CFT

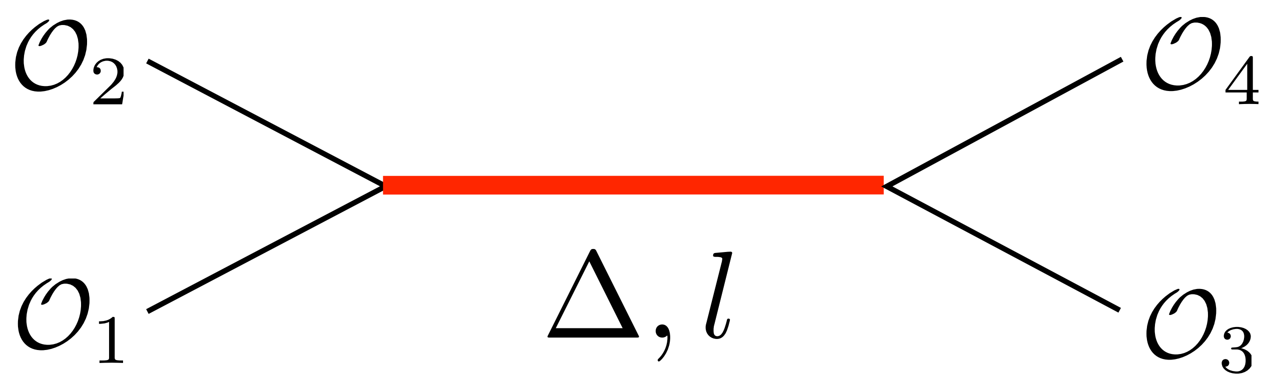
In planar correlators of large N CFT



$$\Delta = \Delta_1 + \Delta_2 + l + 2n + O\left(\frac{1}{N^2}\right)$$

Large N CFT

In planar correlators of large N CFT



$$\Delta = \Delta_1 + \Delta_2 + l + 2n + O\left(\frac{1}{N^2}\right)$$

$\Gamma(\gamma_{12}) \rightarrow$ OPE contributions of $\mathcal{O}_1 \partial \ldots \partial \mathcal{O}_2$

Only single-trace operators give rise to poles in planar Mellin amplitudes.

AdS/CFT

Witten diagrams

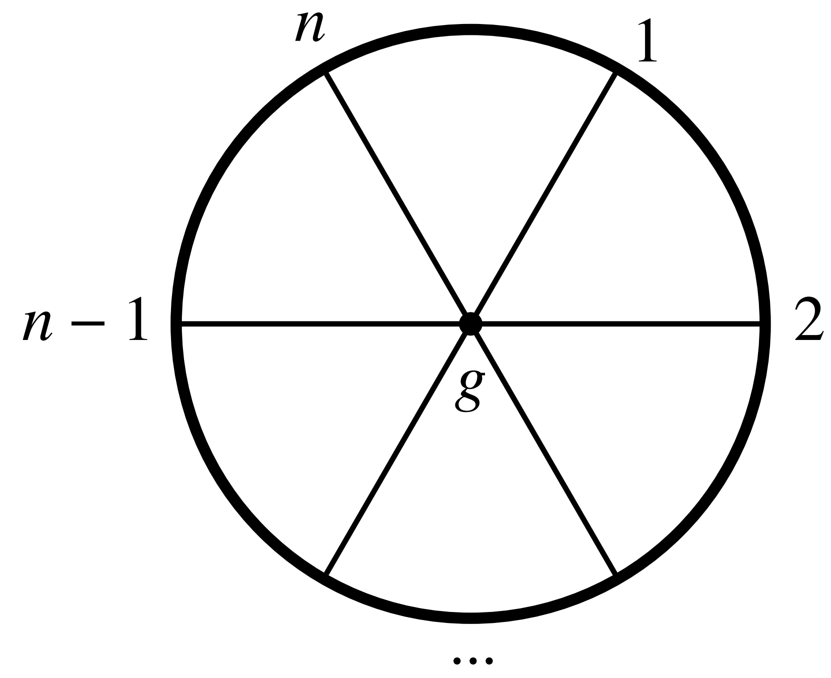
Bulk contact interaction

[JP '10]

$$g\phi_1\phi_2\ldots\phi_n$$

$$\Downarrow$$

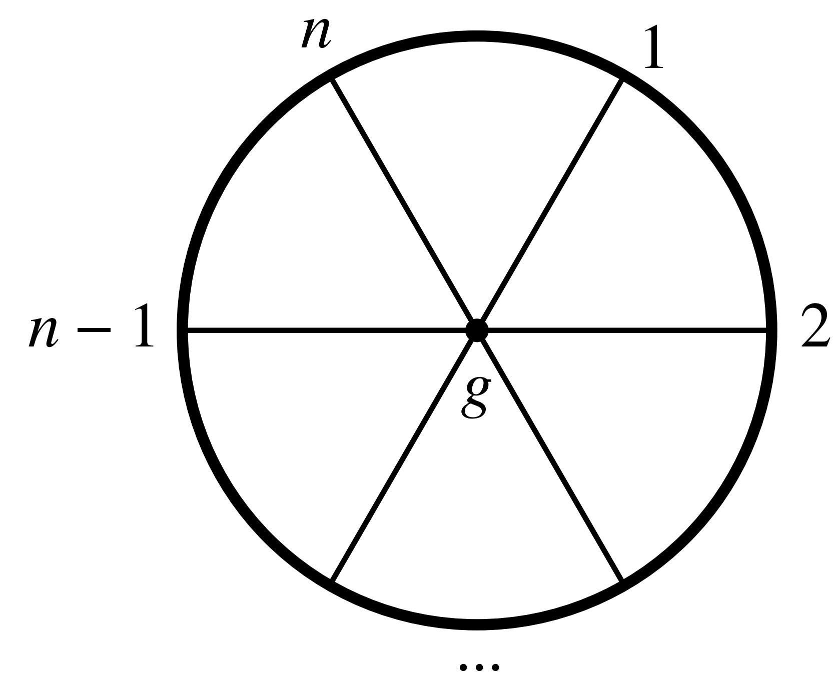
$$M(\gamma_{ij}) = g$$



Witten diagrams

Bulk contact interaction

[JP '10]



$$g\phi_1\phi_2\ldots\phi_n$$

$$\Downarrow$$

$$M(\gamma_{ij}) = g$$

$$g\nabla\ldots\nabla\phi_1\ldots\nabla\ldots\nabla\phi_n$$

$$\Downarrow$$

$$M(\gamma_{ij}) = g\,\text{Polynomial}(\gamma_{ij})$$

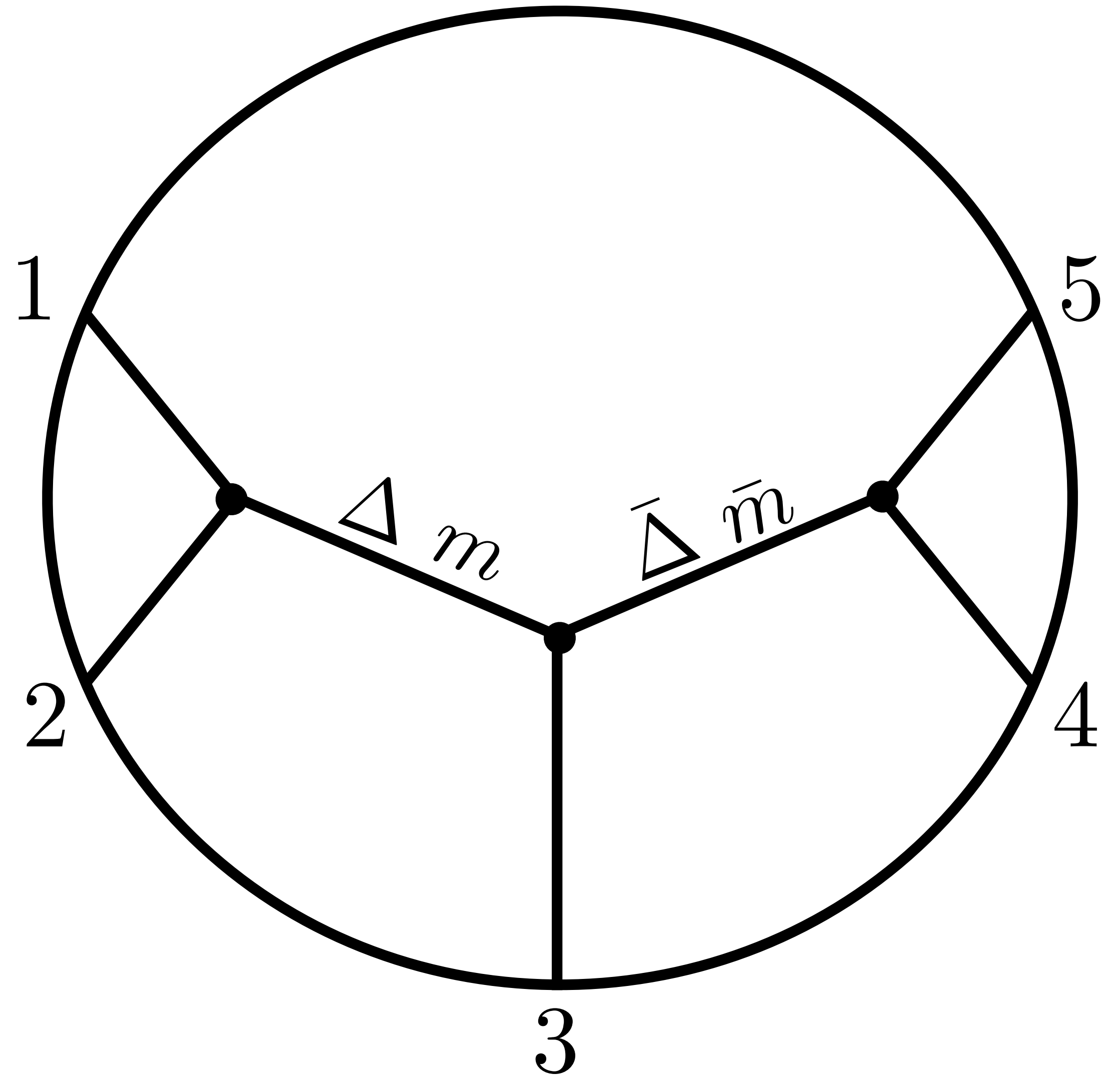
$$\text{degree} = \frac{1}{2}(\#\ \text{derivatives})$$

Contact diagrams in AdS give polynomial Mellin amplitudes

Witten diagrams

[Fitzpatrick, Kaplan, JP, Raju, van Rees '11]
[Paulos '11]

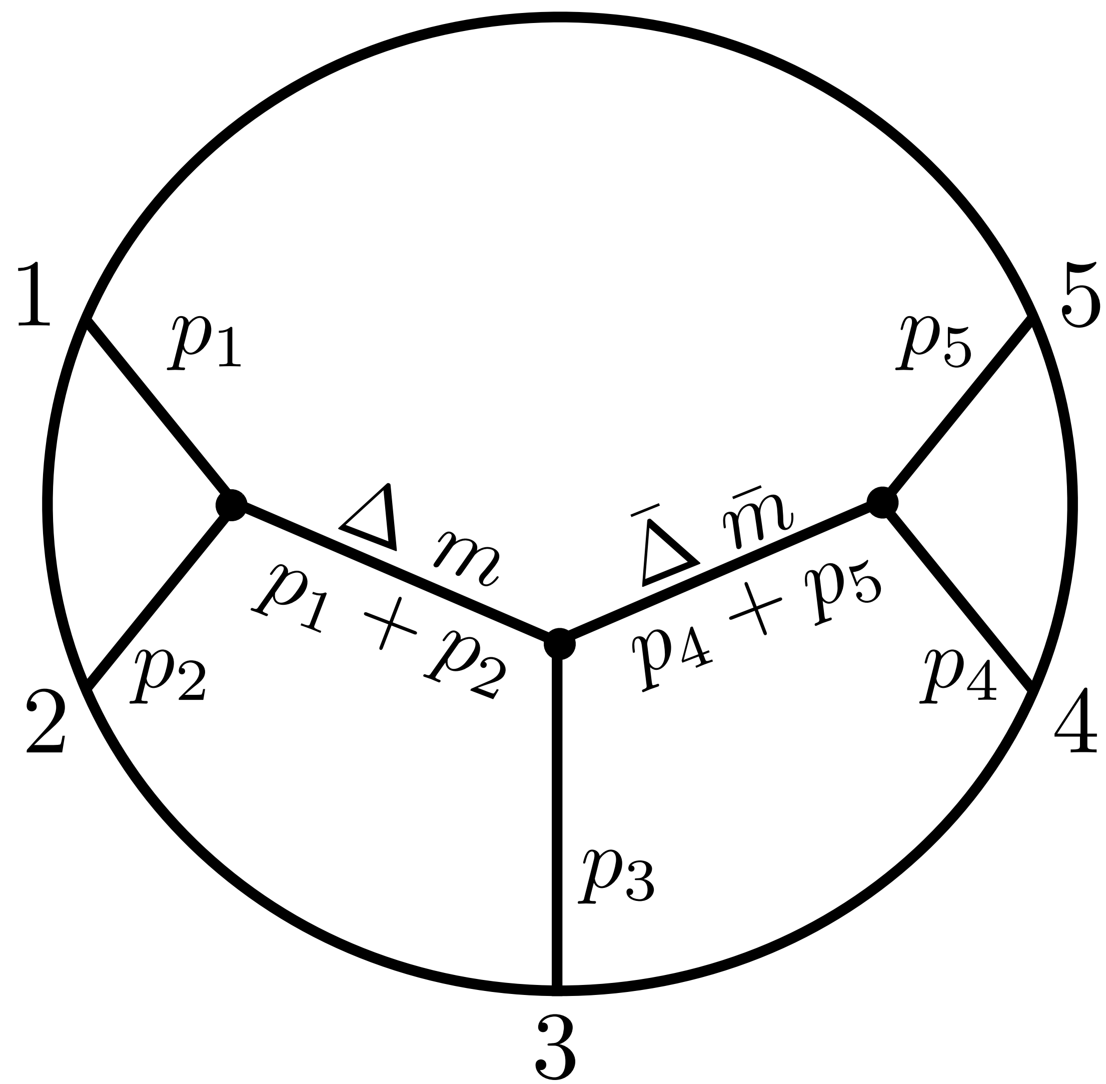
Feynman rules for scalar exchanges



Witten diagrams

[Fitzpatrick, Kaplan, JP, Raju, van Rees '11]
[Paulos '11]

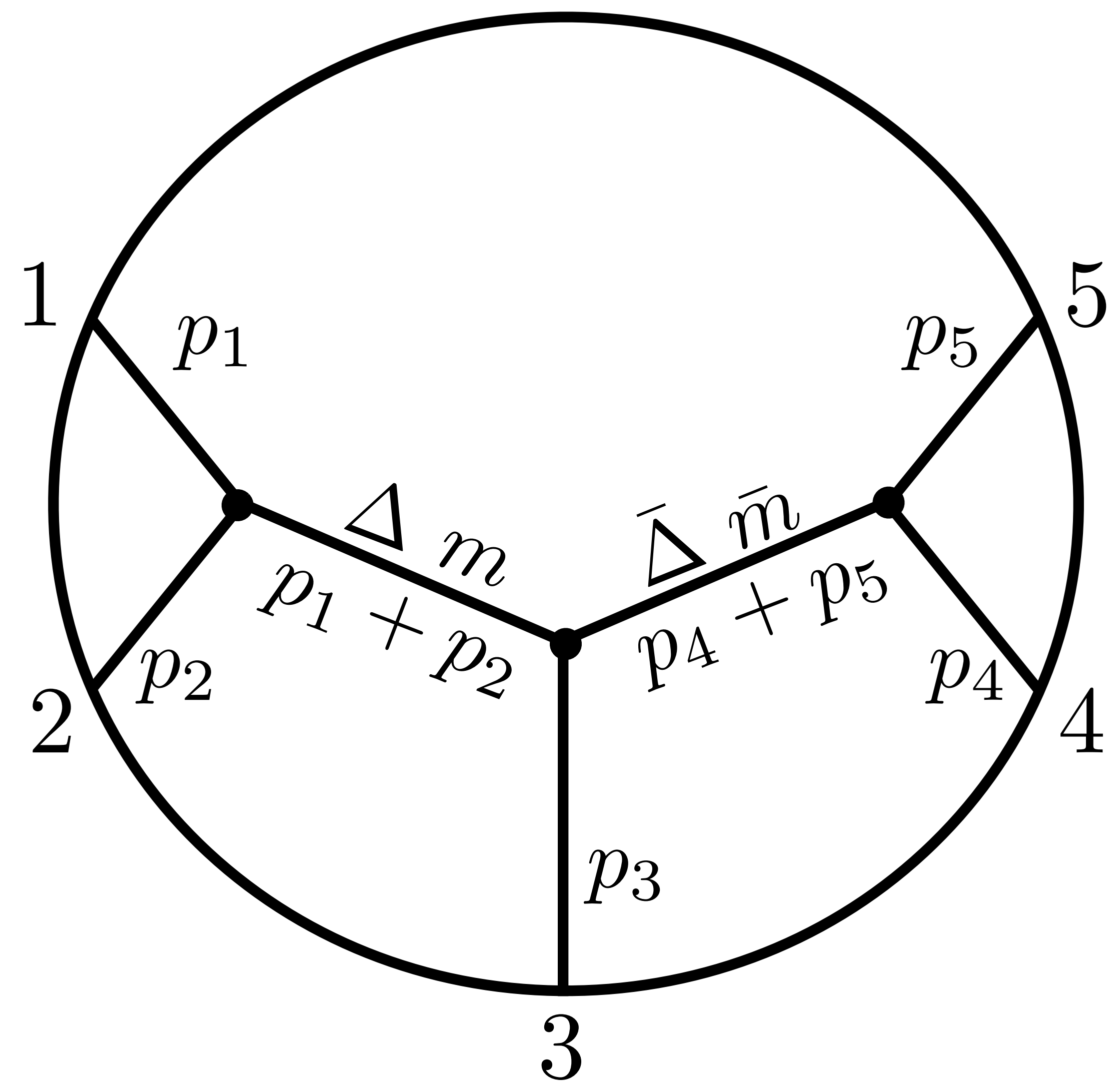
Feynman rules for scalar exchanges



Witten diagrams

[Fitzpatrick, Kaplan, JP, Raju, van Rees '11]
[Paulos '11]

Feynman rules for scalar exchanges

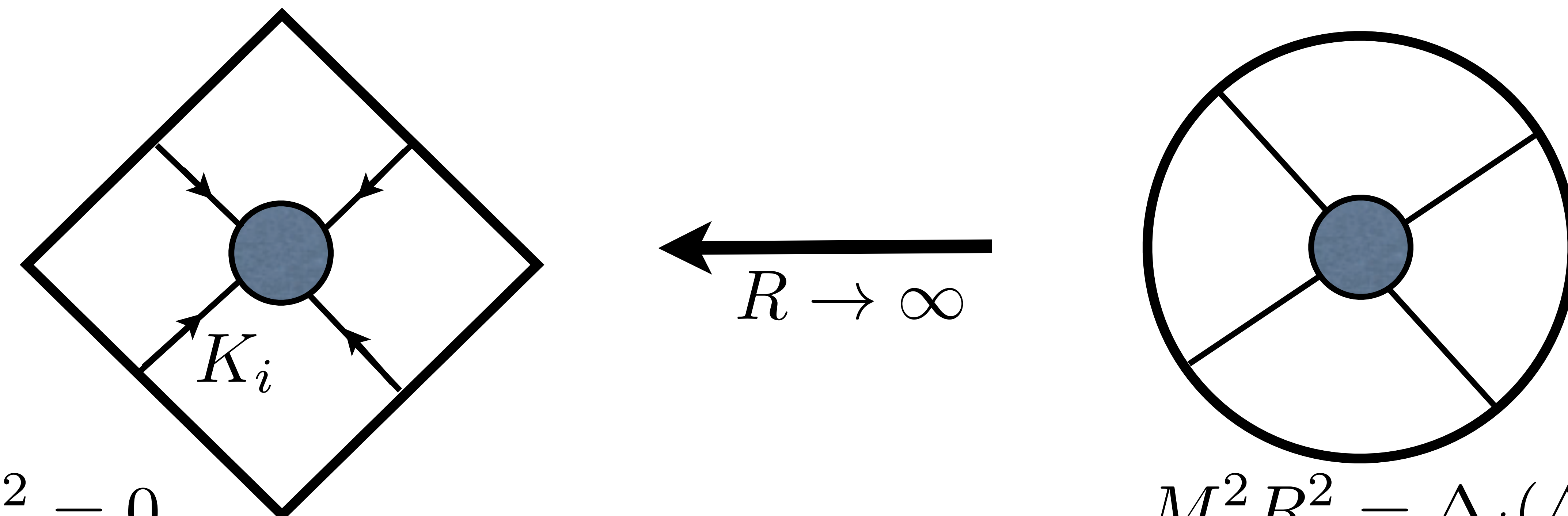


$$M = \sum_{m,\bar{m}=0}^{\infty} V\begin{bmatrix} \Delta_1 & \Delta_2 & \Delta \\ 0 & 0 & m \end{bmatrix} \frac{S_m(\Delta)}{(p_1+p_2)^2+\Delta+2m} V\begin{bmatrix} \Delta & \bar{\Delta} & \Delta_3 \\ m & \bar{m} & 0 \end{bmatrix} \frac{S_{\bar{m}}(\bar{\Delta})}{(p_4+p_5)^2+\bar{\Delta}+2\bar{m}} V\begin{bmatrix} \bar{\Delta} & \Delta_4 & \Delta_5 \\ \bar{m} & 0 & 0 \end{bmatrix}$$

Flat space limit of AdS

[Susskind '99] [Polchinski '99]
[Gary, Giddings, JP '09] [Okuda, JP '10]
[JP '10] [Fitzpatrick, Kaplan '12]

Minkowski | Anti-de Sitter

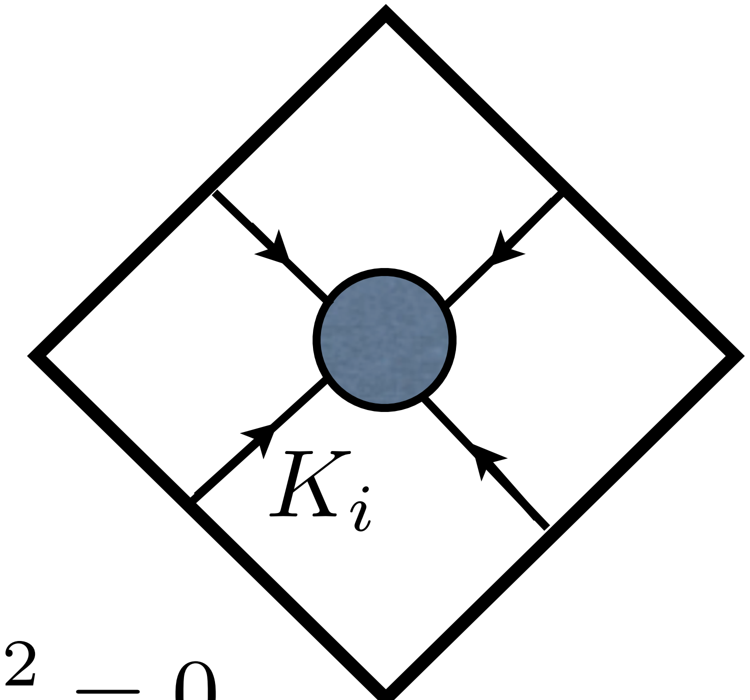


$K_i^2 = 0$ | $M_i^2 R^2 = \Delta_i(\Delta_i - d)$

Flat space limit of AdS

[Susskind '99] [Polchinski '99]
[Gary, Giddings, JP '09] [Okuda, JP '10]
[JP '10] [Fitzpatrick, Kaplan '12]

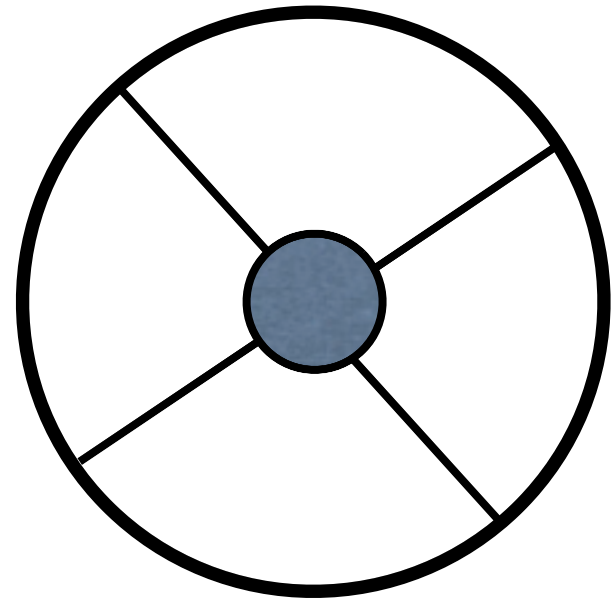
Minkowski



$K_i^2 = 0$

$R \to \infty$

Anti-de Sitter



$M_i^2 R^2 = \Delta_i(\Delta_i - d)$

$$T(K_i) = \lim_{R\to\infty} \mathcal{N} \int_{-i\infty}^{i\infty} \frac{d\alpha}{2\pi i} e^{\alpha} \alpha^{\frac{d-\sum \Delta_i}{2}} M\left(\gamma_{ij} = \frac{R^2 K_i \cdot K_j}{2\alpha}\right)$$

Scattering amplitude

Mellin amplitude

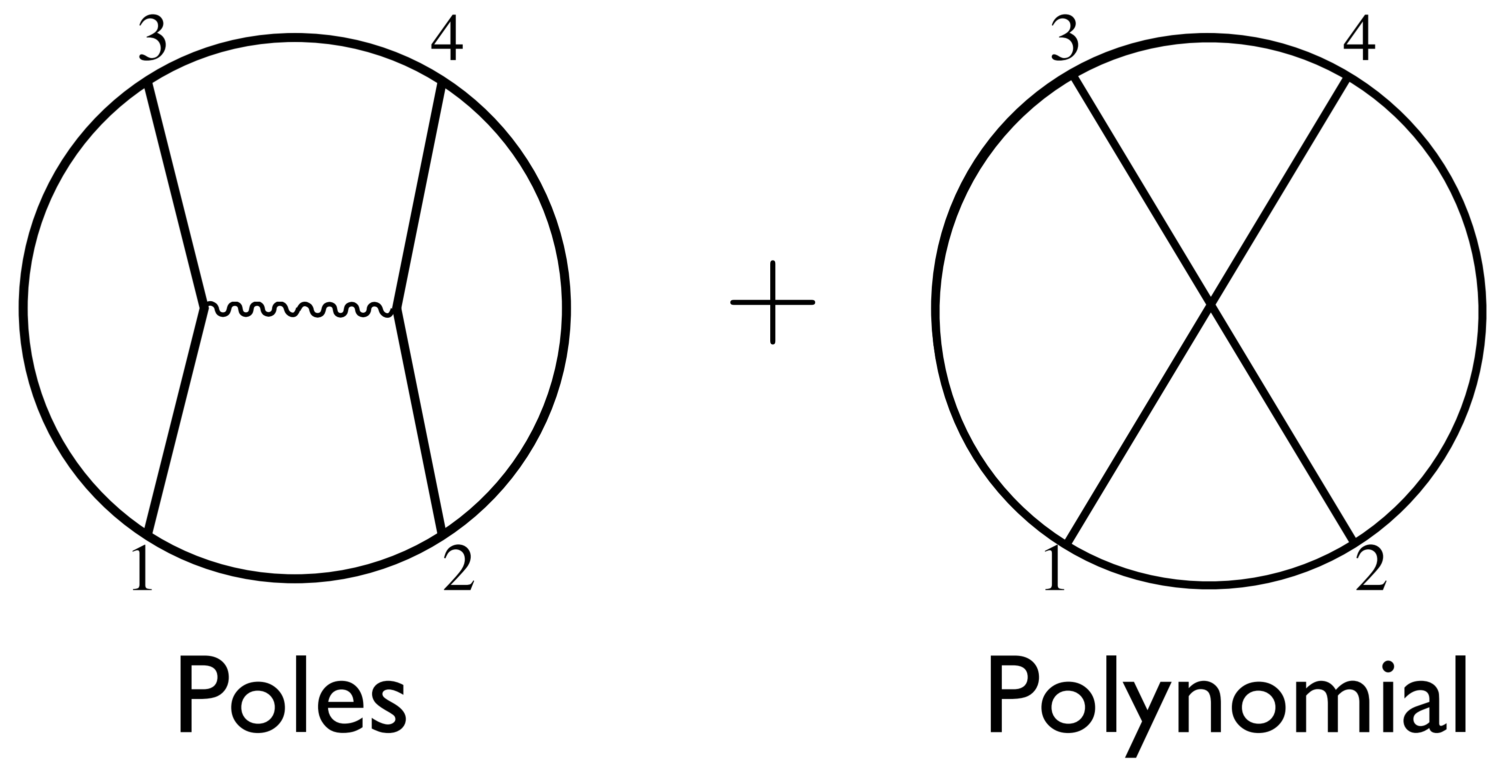
Bulk Locality from CFT

[Heemskerk, JP, Polchinski, Sully '09]
[Heemskerk, Sully '10] [JP '10]

- Large N + finite number of low dimension single-trace operators

$\Delta_{gap} \to \infty \quad \Rightarrow$ **Finite # of exchange diagrams**

$$\langle \mathcal{O}_1 \ldots \mathcal{O}_4 \rangle_{\text{planar}} =$$



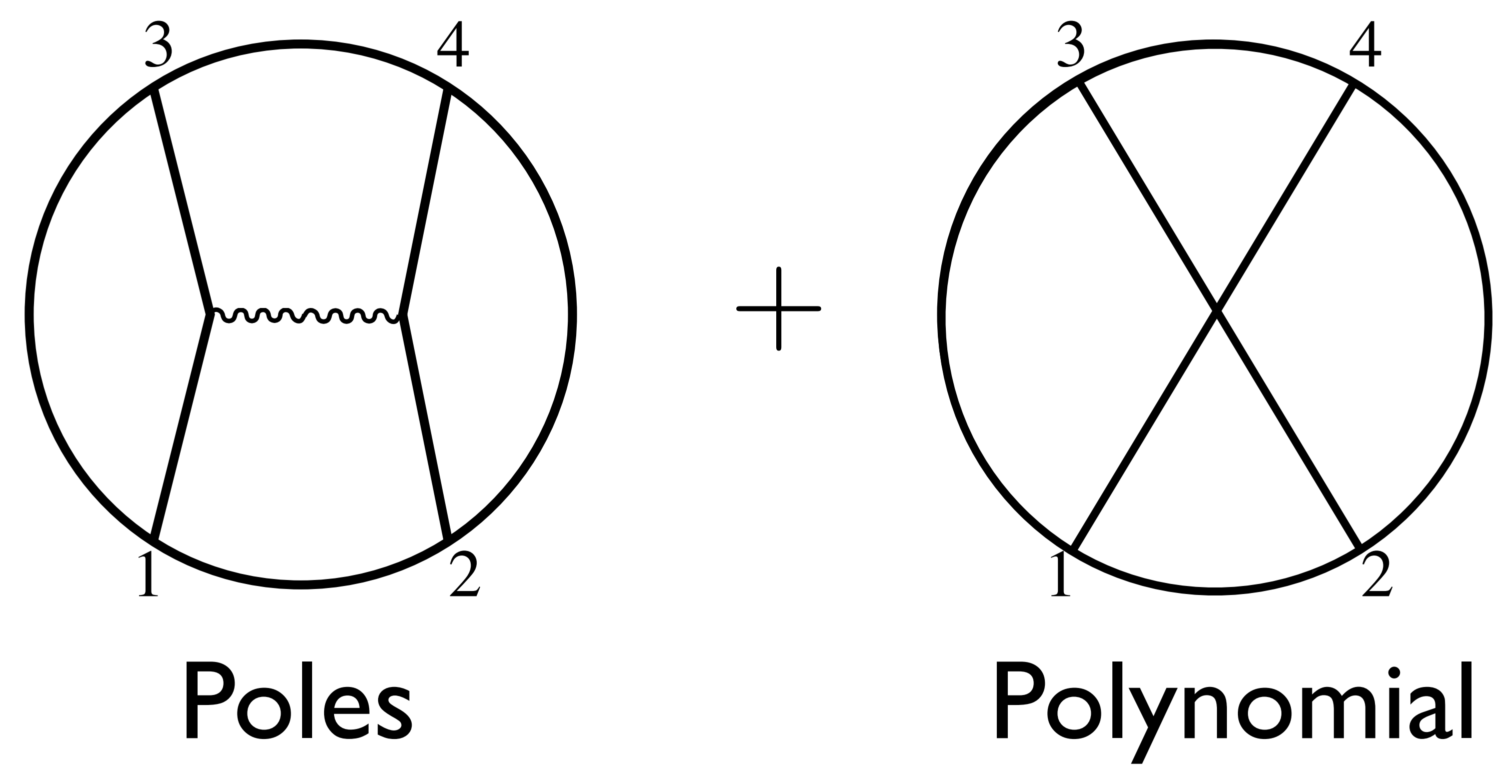
Bulk Locality from CFT

[Heemskerk, JP, Polchinski, Sully '09]
[Heemskerk, Sully '10] [JP '10]

- Large N + finite number of low dimension single-trace operators

$\Delta_{gap} \to \infty \quad \Rightarrow$ **Finite # of exchange diagrams**

$$\langle \mathcal{O}_1 \dots \mathcal{O}_4 \rangle_{\text{planar}} =$$



- Regge limit: $s \to \infty \; , \quad t \text{ fixed}$

[Cornalba '07] [Costa, Gonçalves, JP '12]
[Maldacena, Shenker, Stanford '15]

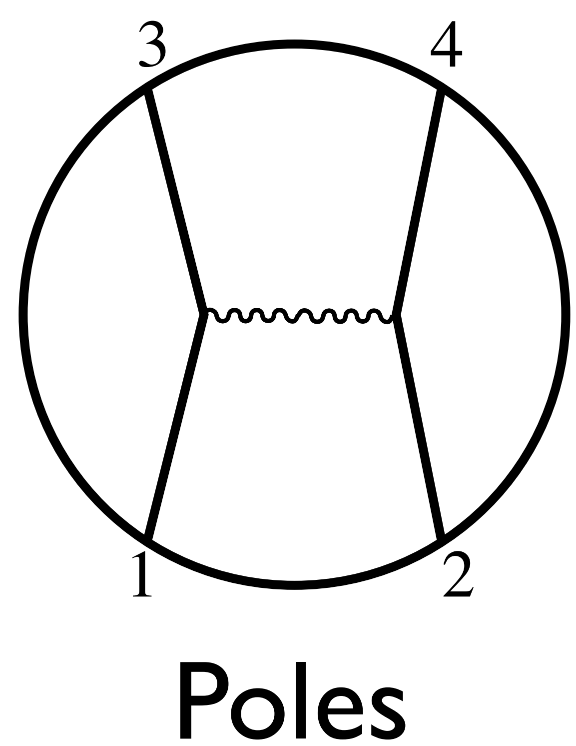
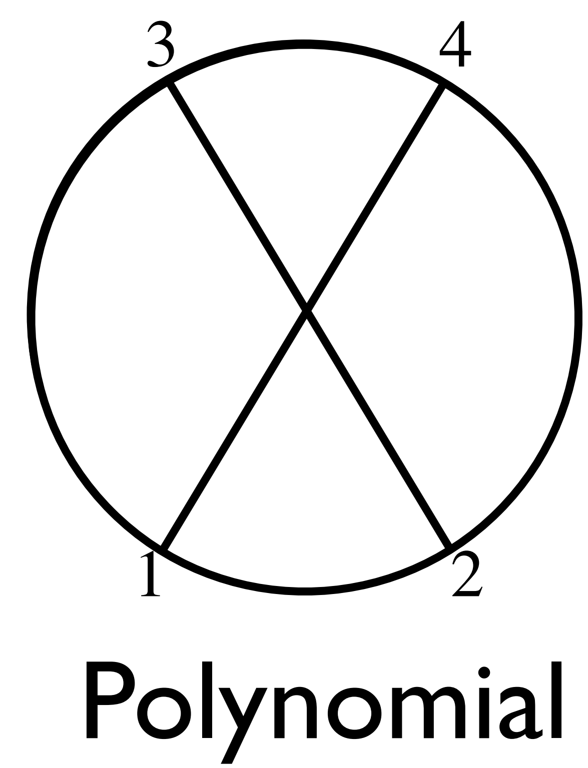
$M(s,t) \sim s^{j_0} \qquad j_0 \leq 2$

Bulk Locality from CFT

[Heemskerk, JP, Polchinski, Sully '09]
[Heemskerk, Sully '10] [JP '10]

- Large N + finite number of low dimension single-trace operators

$\Delta_{gap} \to \infty \quad \Longrightarrow$ **Finite # of exchange diagrams**

$$\langle \mathcal{O}_1 \ldots \mathcal{O}_4 \rangle_{\text{planar}} =$$  $+$ 

- Regge limit: $s \to \infty \; , \quad t$ fixed

[Cornalba '07] [Costa, Gonçalves, JP '12]
[Maldacena, Shenker, Stanford '15]

$M(s,t) \sim s^{j_0} \qquad j_0 \leq 2 \quad \Longrightarrow$ **Finite # of contact diagrams**

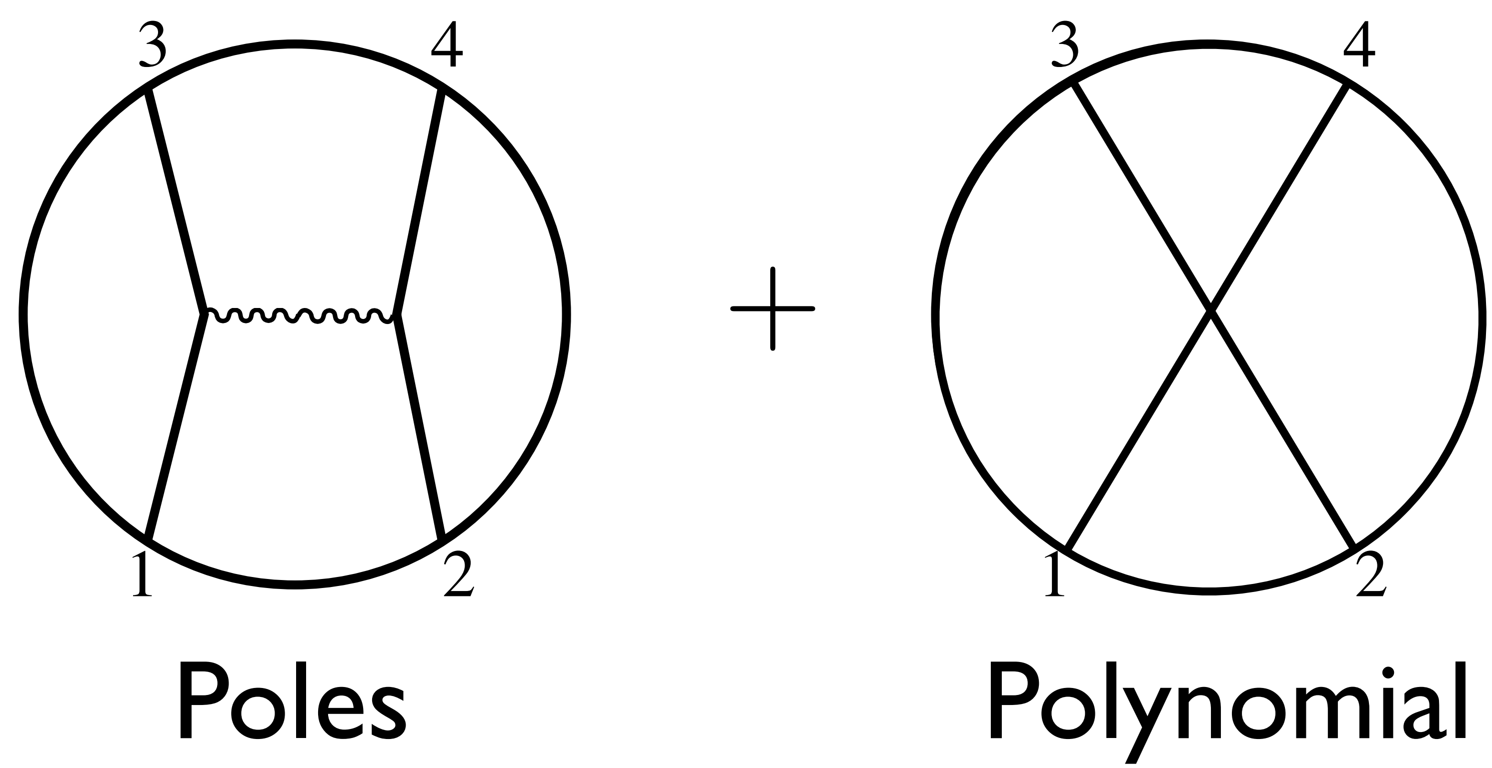
Bulk Locality from CFT

[Heemskerk, JP, Polchinski, Sully '09]
[Heemskerk, Sully '10] [JP '10]

- Large N + finite number of low dimension single-trace operators

$\Delta_{gap} \to \infty \qquad \Longrightarrow$ **Finite # of exchange diagrams**

$$\langle \mathcal{O}_1 \ldots \mathcal{O}_4 \rangle_{\text{planar}} =$$



- Regge limit: $s \to \infty \;, \quad t$ fixed

[Cornalba '07] [Costa, Gonçalves, JP '12]
[Maldacena, Shenker, Stanford '15]

$M(s,t) \sim s^{j_0} \qquad j_0 \leq 2 \;\; \Longrightarrow$ **Finite # of contact diagrams**

Generalization of causality constraints for $\langle T_{\mu\nu} T_{\alpha\beta} T_{\rho\sigma} \rangle$

[Camanho, Edelstein, Maldacena, Zhiboedov '14]

Problems for the future

- Extend factorization formulas to general spin
- AdS tree-level n-point Mellin amplitudes a la [Cachazo, He, Yuan '14]
- When is Conformal Regge Theory valid?
- Simplify the OPE in large N CFTs
- Understand scaling of higher derivative corrections in AdS with $\Delta_{gap} \gg 1$ [Camanho, Edelstein, Maldacena, Zhiboedov '14]
- Mellin amplitude of Virasoro blocks (minimal models) [Alday, Zhiboedov '15]
- Study boundary correlators of QFT in AdS
  from BCFT correlators (UV) to scattering amplitudes (IR)

Problems for the future

- Extend factorization formulas to general spin
- AdS tree-level n-point Mellin amplitudes a la [Cachazo, He, Yuan '14]
- When is Conformal Regge Theory valid?
- Simplify the OPE in large N CFTs
- Understand scaling of higher derivative corrections in AdS with $\Delta_{gap} \gg 1$ [Camanho, Edelstein, Maldacena, Zhiboedov '14]
- Mellin amplitude of Virasoro blocks (minimal models) [Alday, Zhiboedov '15]
- Study boundary correlators of QFT in AdS
  from BCFT correlators (UV) to scattering amplitudes (IR)

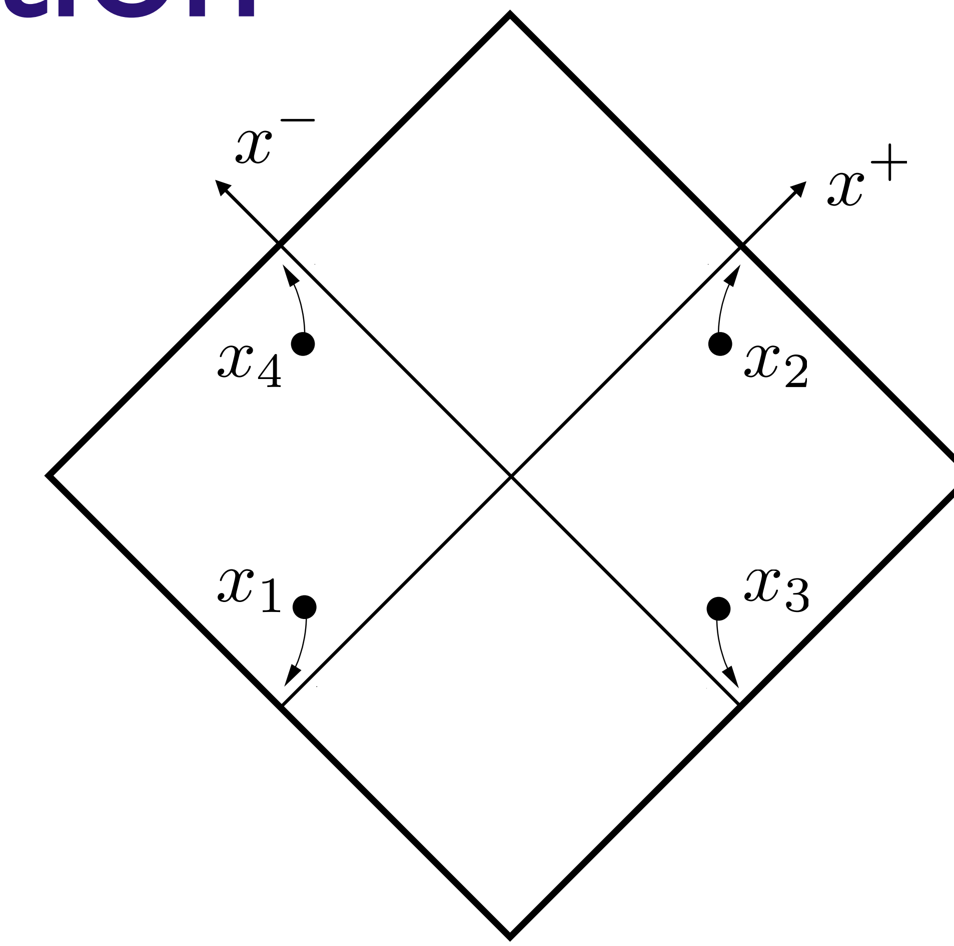
Thank you!

Extra slides

Regge limit of 4-pt function

"Maldelstam invariants"

$$s = -(p_1 + p_3)^2 = \Delta_1 + \Delta_3 - 2\gamma_{13}$$
$$t = -(p_1 + p_2)^2 = \Delta_1 + \Delta_2 - 2\gamma_{12}$$

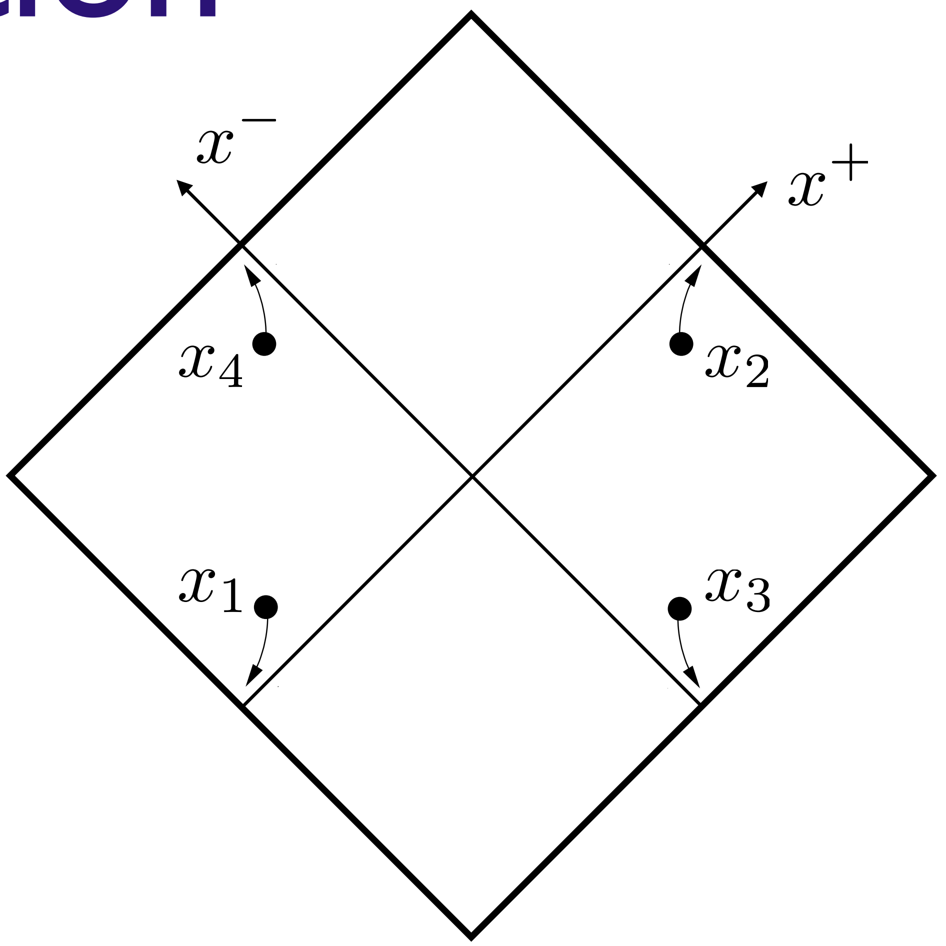


Regge limit: $s \to \infty \;, \quad t \text{ fixed}$

Regge limit of 4-pt function

"Maldelstam invariants"

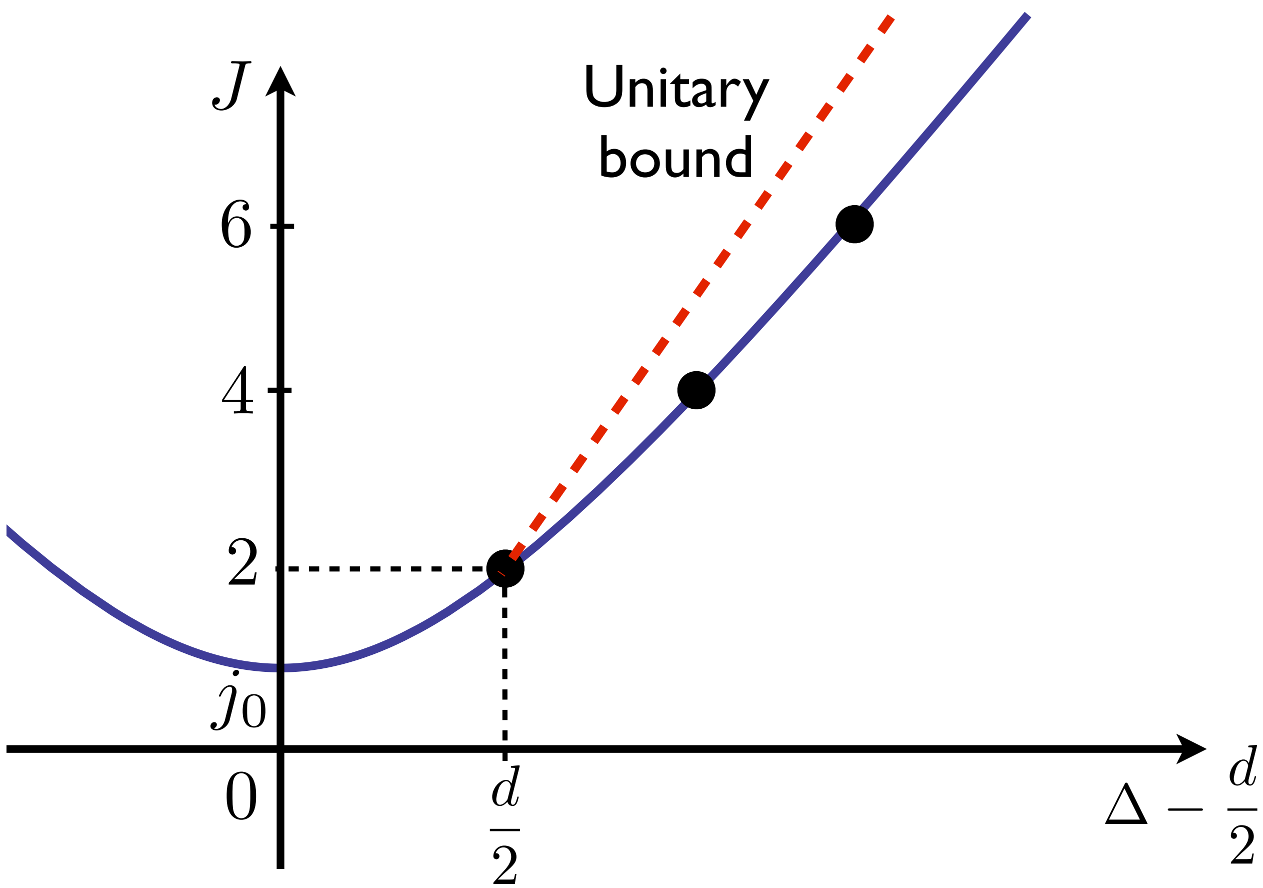
$$s = -(p_1 + p_3)^2 = \Delta_1 + \Delta_3 - 2\gamma_{13}$$
$$t = -(p_1 + p_2)^2 = \Delta_1 + \Delta_2 - 2\gamma_{12}$$



Regge limit: $s \to \infty \ , \quad t \text{ fixed}$

$$\boxed{M(s,t) \sim s^{j_0}}$$

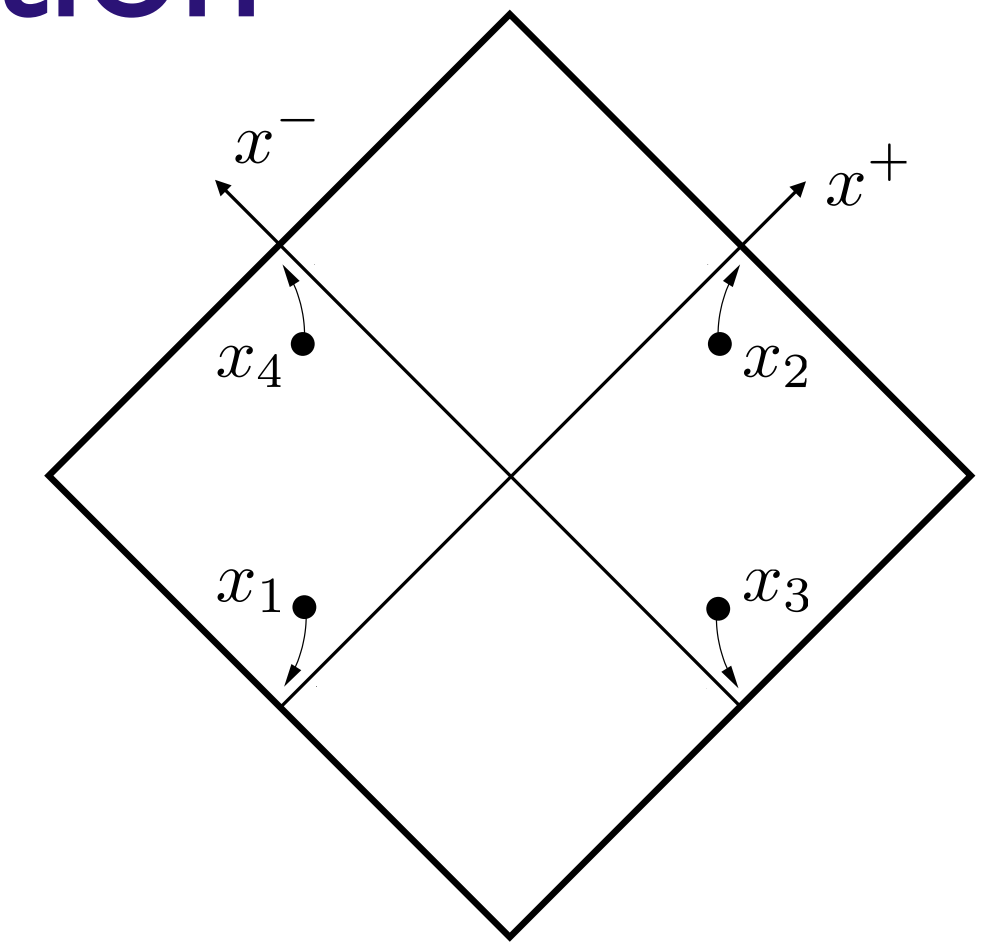
[Cornalba '07]
[Costa, Gonçalves, JP '12]



Regge limit of 4-pt function

"Maldelstam invariants"

$$s = -(p_1+p_3)^2 = \Delta_1 + \Delta_3 - 2\gamma_{13}$$
$$t = -(p_1+p_2)^2 = \Delta_1 + \Delta_2 - 2\gamma_{12}$$



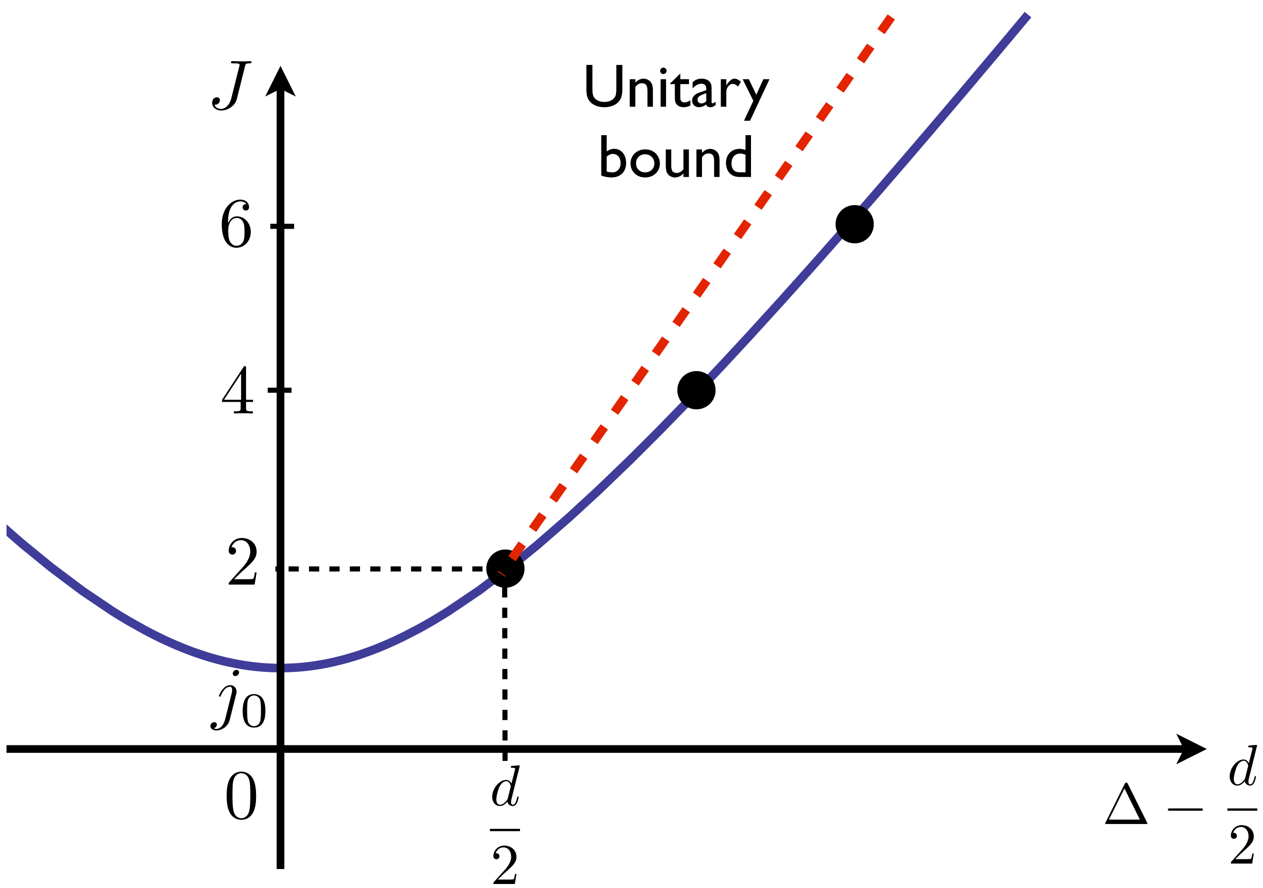
Regge limit: $s \to \infty \ , \quad t$ fixed

$$\boxed{M(s,t) \sim s^{j_0}}$$

[Cornalba '07]
[Costa, Gonçalves, JP '12]

Planar limit $\Rightarrow$ $\boxed{j_0 \le 2}$

[Maldacena, Shenker, Stanford '15]



Planar OPE?

2pt and 3pt functions of single-trace operators —?→ n-pt functions of single-trace operators

Planar OPE?

~~OPE~~

2pt and 3pt functions
of single-trace operators

—?→

n-pt functions
of single-trace operators

Planar OPE?

2pt and 3pt functions
of single-trace operators

~~OPE~~

—?→

String
Theory?

n-pt functions
of single-trace operators

Planar OPE?

2pt and 3pt functions of single-trace operators

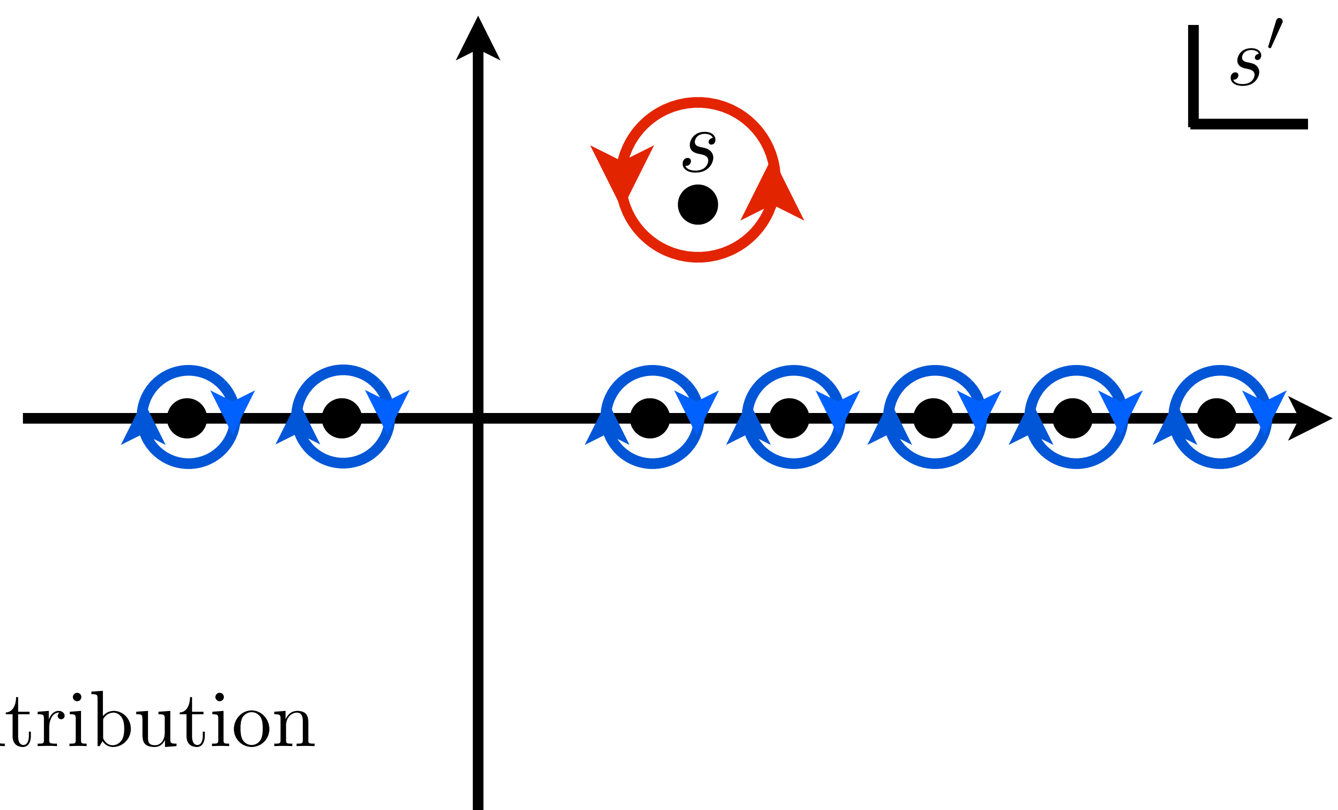
~~OPE~~

—?→

String Theory?

n-pt functions of single-trace operators

$$M(s,t) = \oint \frac{ds'}{2\pi i}\frac{M(s',t)}{s'-s}$$



$$M(s,t) \sim s^{j_0} \qquad j_0 \leq 2$$

$$M(s,t) = \sum_{s_*} \frac{\mathrm{Res}_{s=s_*} M(s,t)}{s-s_*} + \text{contribution from } s=\infty$$

Mellin Redundancy

$$\text{\# independent cross ratios} = \begin{cases} \frac{n(n-3)}{2}\,, & n \leq d+2 \\ nd - \frac{(d+2)(d+1)}{2}\,, & n \geq d+2 \end{cases}$$

For $n > d+2$ the Mellin amplitude is not unique.

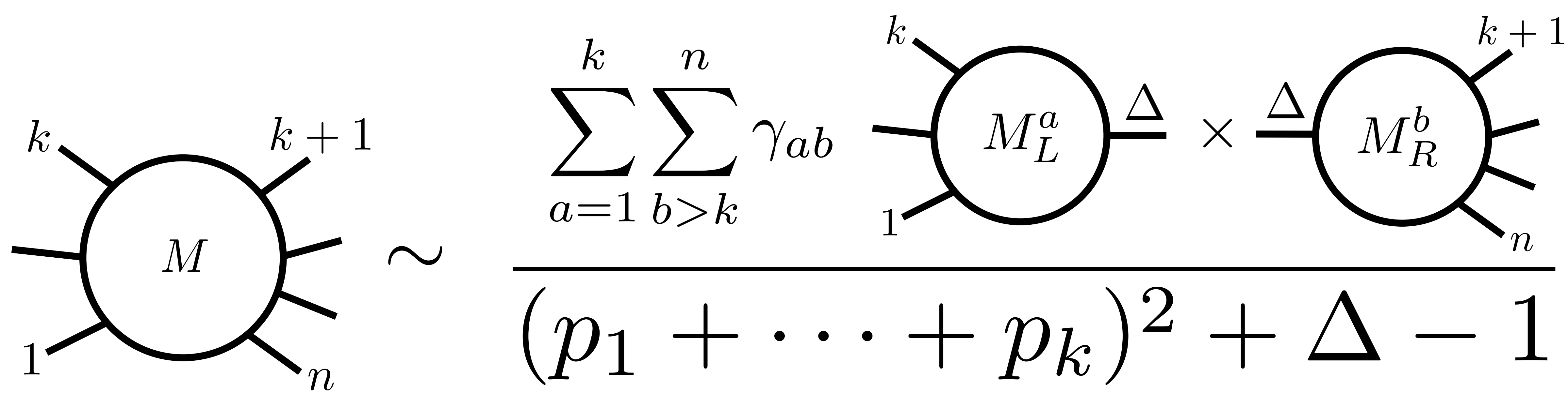
$$\det_{1\leq i,j\leq d+3} P_i \cdot P_j = 0 \qquad\qquad P_i \in \mathbb{R}^{d+2}$$

We can add to the Mellin amplitude the Mellin transform of this determinant times any function.

External Spin

$$\langle \mathcal{O}^A(P)\mathcal{O}(P_1)\dots\mathcal{O}(P_n)\rangle = \sum_{a=1}^{n} P_a^A \int [d\gamma]\, M^a(\gamma_{ij}) \prod_{i<j}^{n} \frac{\Gamma(\gamma_{ij})}{(-2P_i\cdot P_j)^{\gamma_{ij}}} \prod_{i=1}^{n} \frac{\Gamma(\gamma_i+\delta_i^a)}{(-2P_i\cdot P)^{\gamma_i+\delta_i^a}}$$

$$\gamma_i = -\sum_{j=1}^{n}\gamma_{ij}\ , \qquad \gamma_{ij}=\gamma_{ji}\ , \qquad \gamma_{ii}=-\Delta_i\ , \qquad \sum_{i,j=1}^{n}\gamma_{ij} = 1-\Delta$$



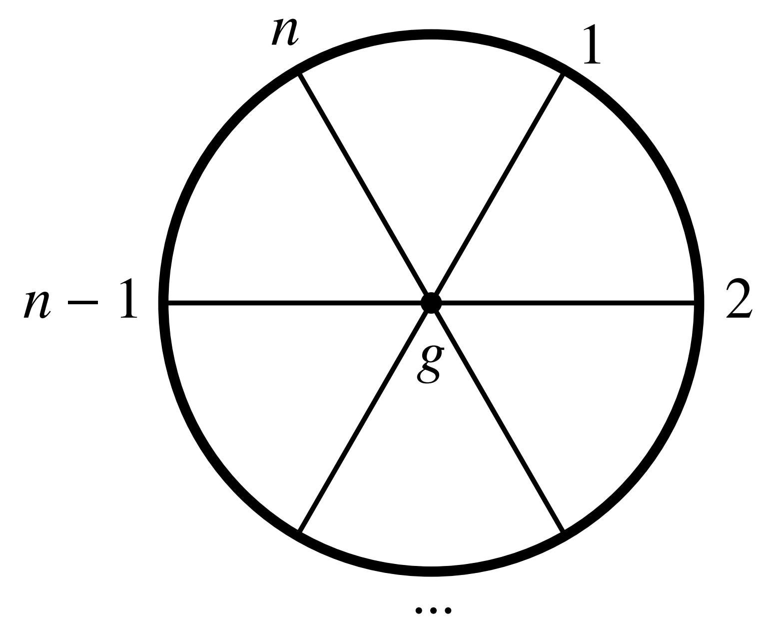
[Gonçalves, JP, Trevisani, '14]

Flat Space Limit of AdS

Evidence for $T = \lim \int \ldots M$

1) Works for an infinite set of interactions

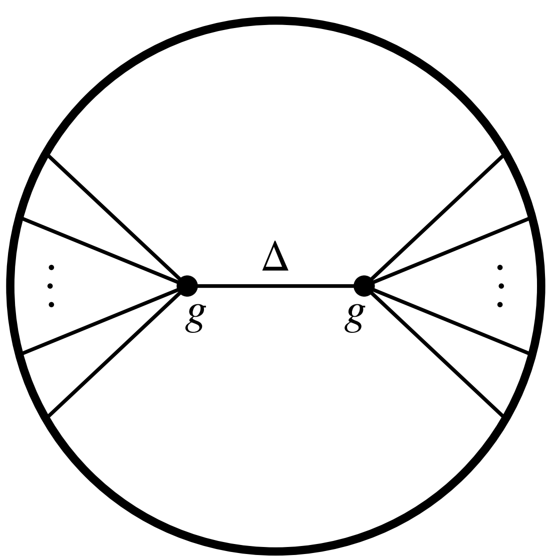
$$g\nabla \ldots \nabla\phi_1\nabla \ldots \nabla\phi_2 \ldots \nabla \ldots \nabla\phi_n$$

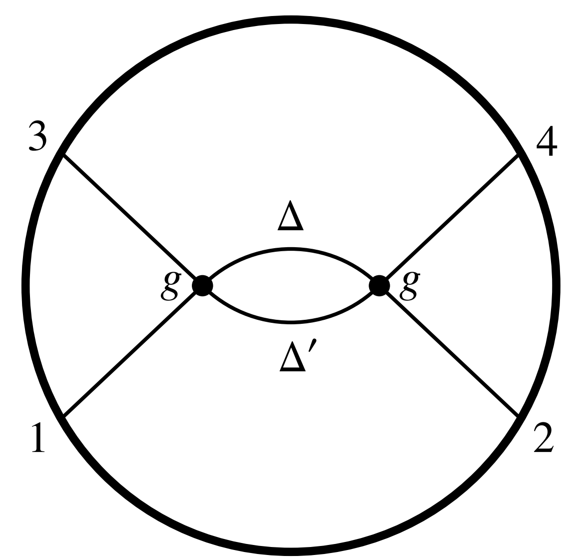


2) Derived from wave-packet scattering localized in small region of AdS

[Fitzpatrick, Kaplan '12]
[Gary, Giddings, JP '09] [Okuda, JP '10]

3) Verified in several non-trivial examples



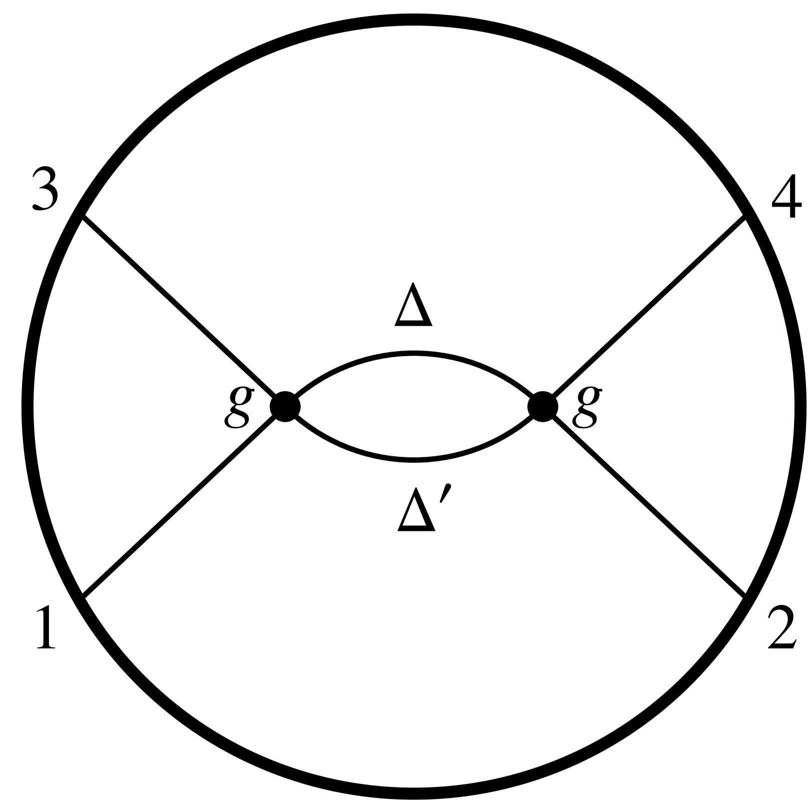


UV and IR divergences

UV divergences are the same in AdS and in flat space.

IR divergences are absent in AdS.

The Mellin amplitudes can be thought as IR regulated scattering amplitudes.



Application: from SYM to IIB strings

$\mathcal{N}=4$ SYM $\longleftrightarrow$ type IIB strings

$$g^2_{\mathrm{YM}} = 4\pi g_s$$
$$g^2_{\mathrm{YM}} N = \lambda = (R/\ell_s)^4$$

$\mathcal{O}(x) =$ Lagrangian density

$\phi =$ Dilaton

4pt function
$\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\mathcal{O}(x_3)\mathcal{O}(x_4)\rangle$

$R \to \infty$

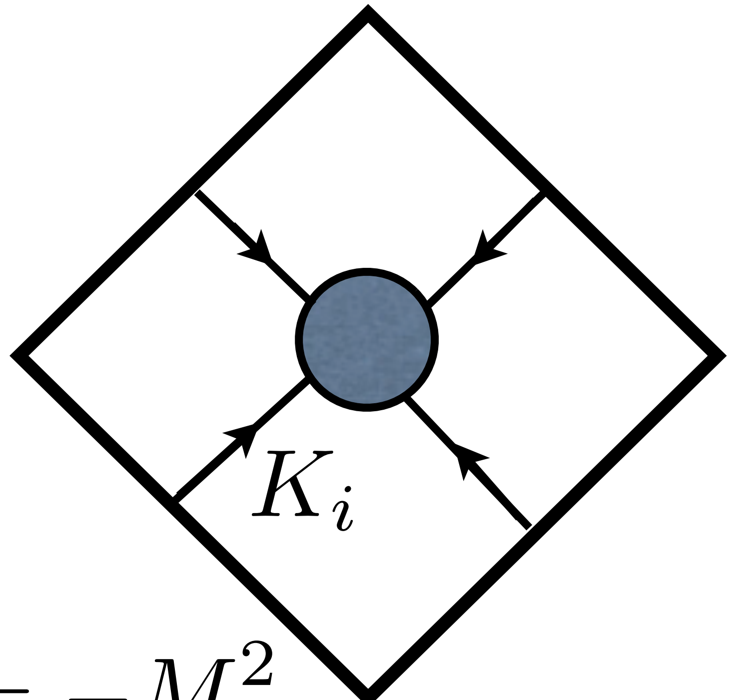
2 → 2 scattering amplitude

$$\lim_{\lambda\to\infty} \lambda^{\frac{3}{2}} M(g^2_{\mathrm{YM}}, \lambda, s_{ij} = \sqrt{\lambda}\alpha_{ij}) = \frac{1}{120\pi^3 \ell_s^6} \int_0^\infty d\beta \beta^5 e^{-\beta} T_{10}\left(g_s, \ell_s, S_{ij} = \frac{2\beta}{\ell_s^2}\alpha_{ij}\right)$$

Mellin amplitude

FSL of AdS for massive particles

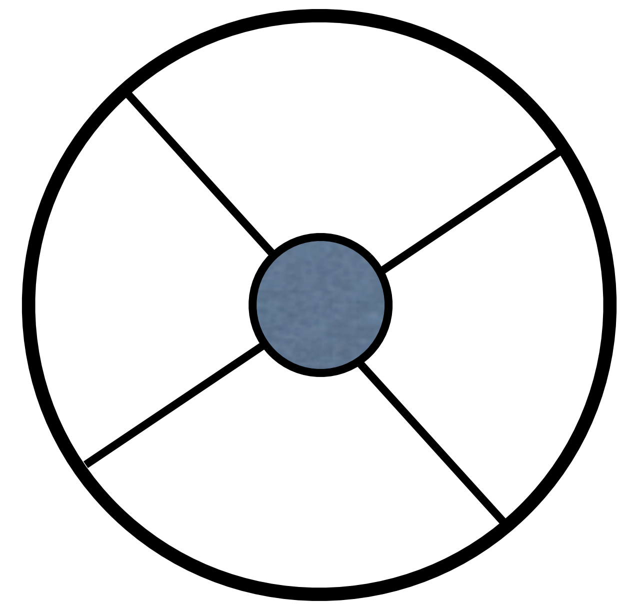
Minkowski



$K_i^2 = -M_i^2$

$R \to \infty$

Anti-de Sitter



$M_i^2 R^2 = \Delta_i(\Delta_i - d)$

[work in progress]

$$T(K_i) = \lim_{R\to\infty} \tilde{\mathcal{N}}\, M\left(\Delta_i = RM_i, \gamma_{ij} = R\frac{M_iM_j + K_i\cdot K_j}{\sum_l M_l}\right)$$

Scattering amplitude

Mellin amplitude