Quantum Mechanics and the Geometry of Spacetime

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100th anniversary of General Relativity session

GR produced two stunning predictions

- Black holes
- Expanding universe

"Your math is great, but your physics is dismal" (Einstein to LeMaitre)

Both involve drastic stretching of space and/or time

Incorporating Quantum Mechanics

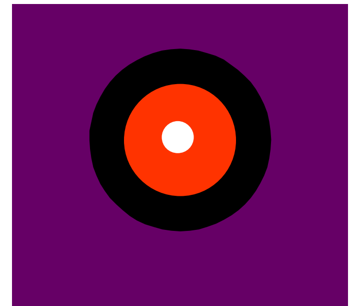
A simple approach

- General relativity → is a classical field theory
- We should quantize it
- It is hard to change the shape of spacetime
- For most situations → quantum fields in a fixed geometry is a good approximation
- General relativity as an effective field theory → systematic low energy approximation.

- Even this simple approximation gives surprising predictions.

Two surprising predictions

- Black holes have a temperature



We can have white ``black holes''

$$T \sim \frac{\hbar}{r_H}$$

Hawking

- An accelerating universe also has a temperature

Chernikov, Tagirov,
Figari, Hoegh-Krohn, Nappi,
Gibbons, Hawking,
Bunch, Davies, ...

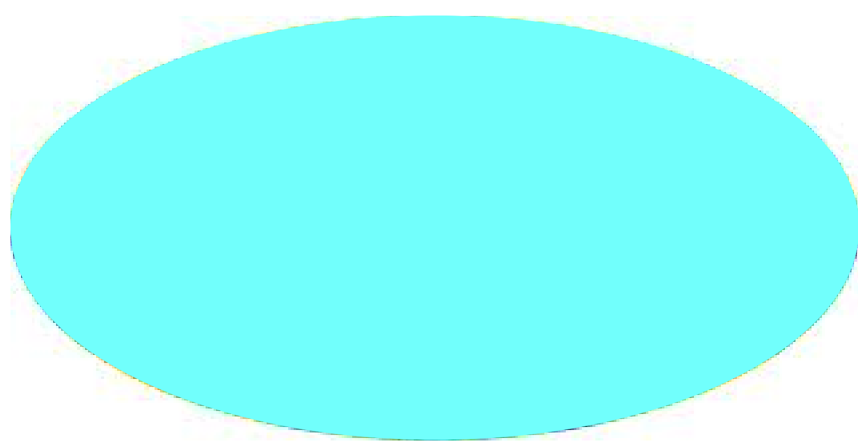
$$T \sim \hbar H = \frac{\hbar}{R_H}$$

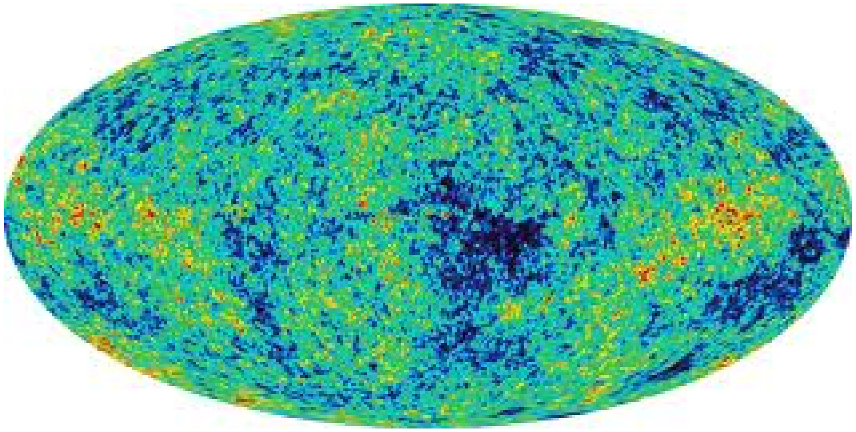
Very relevant for us!

Inflation

Starobinski, Mukhanov
Guth, Linde,
Albrecht, Steinhardt, ...

- Period of expansion with almost constant acceleration.
- Produces a large homogeneous universe



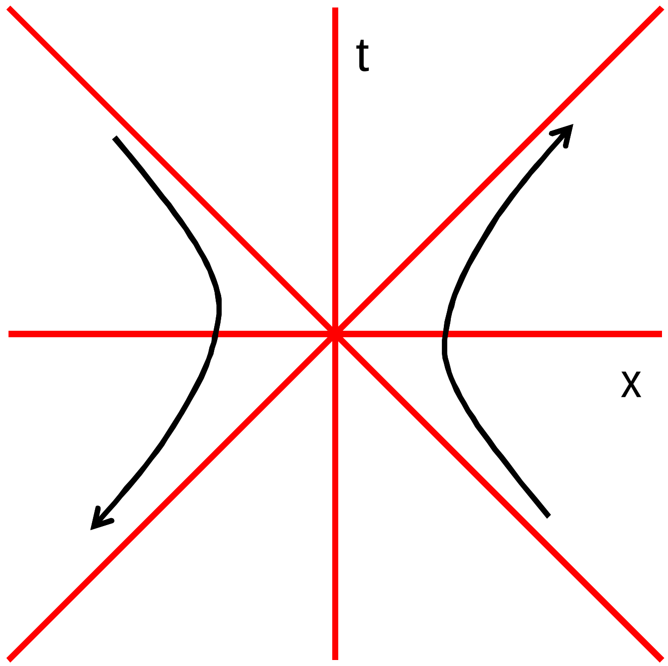


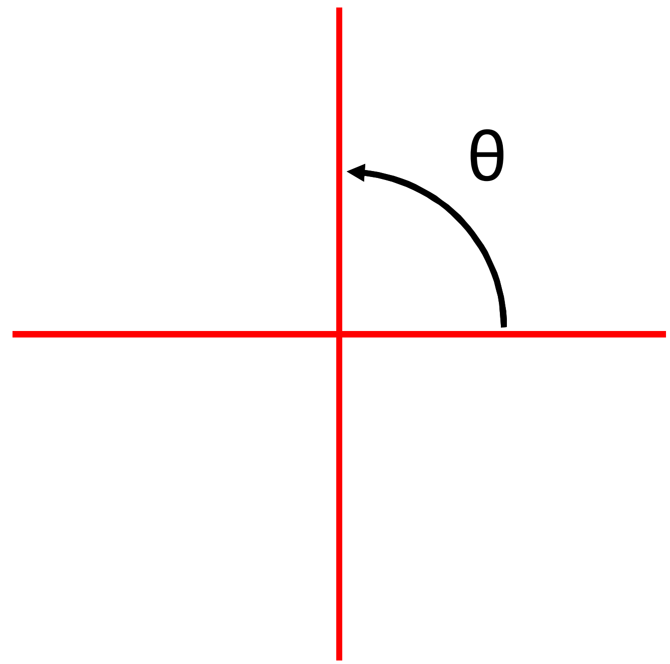
Quantum mechanics is crucial for understanding the large scale geometry of the universe.

Slide 7

JM2 Juan Maldacena, 4/24/2011

Why a temperature ?



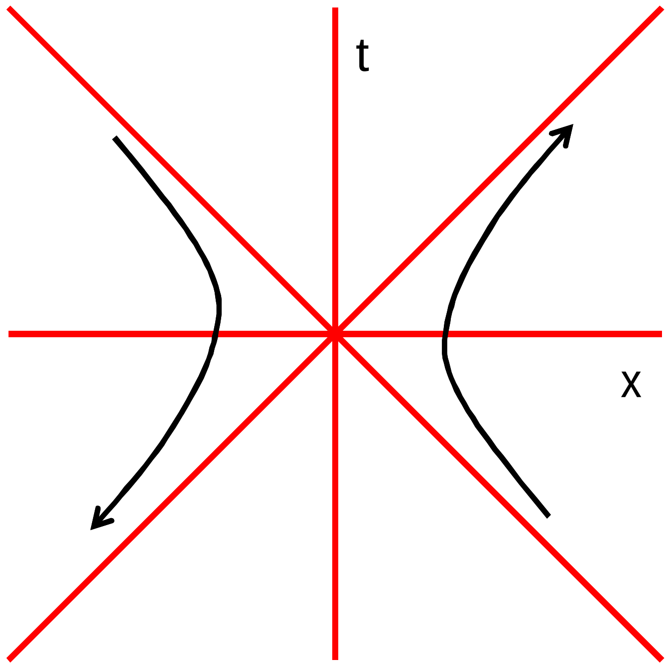


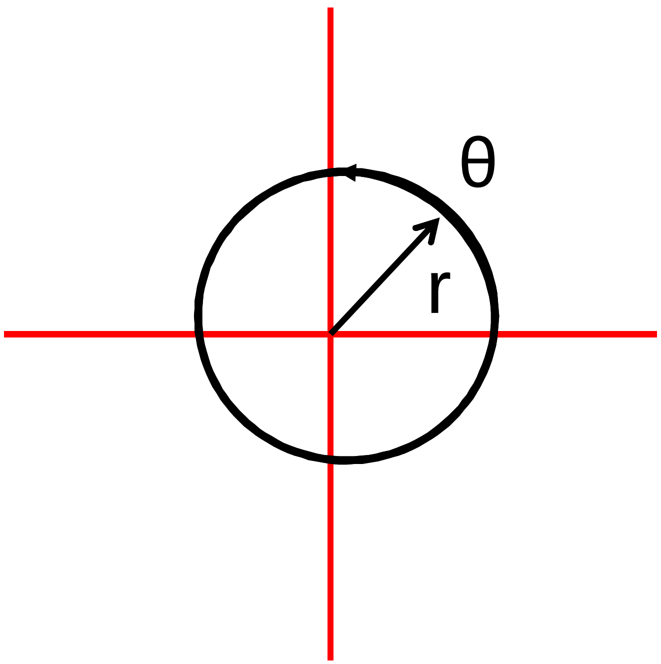
Special relativity + quantum mechanics

Accelerated observer → energy = boost generator.

Continue to Euclidean space → boost becomes rotation.

Why a temperature ?





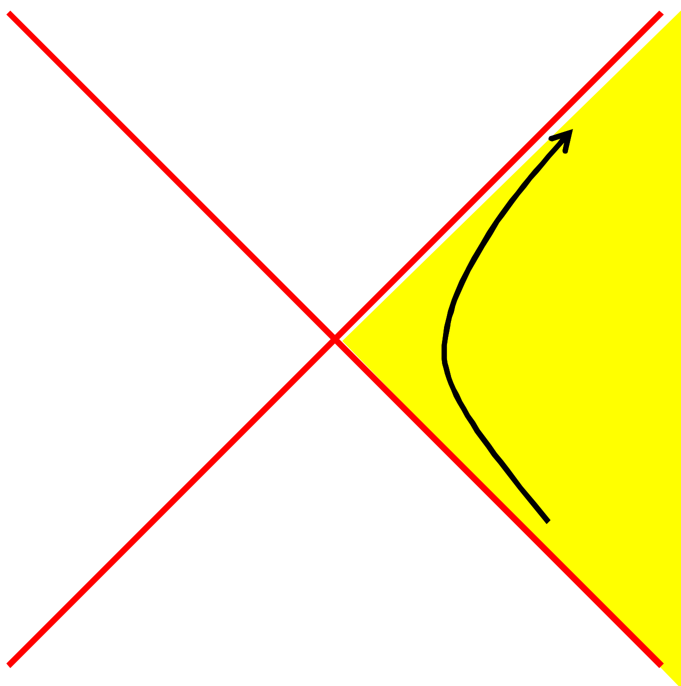
Continue to Euclidean space → boost becomes rotation.

Angle is periodic → temperature

$$\beta = \frac{1}{T} = 2\pi r$$

Bisognano Weichman, Unruh

Entanglement & temperature

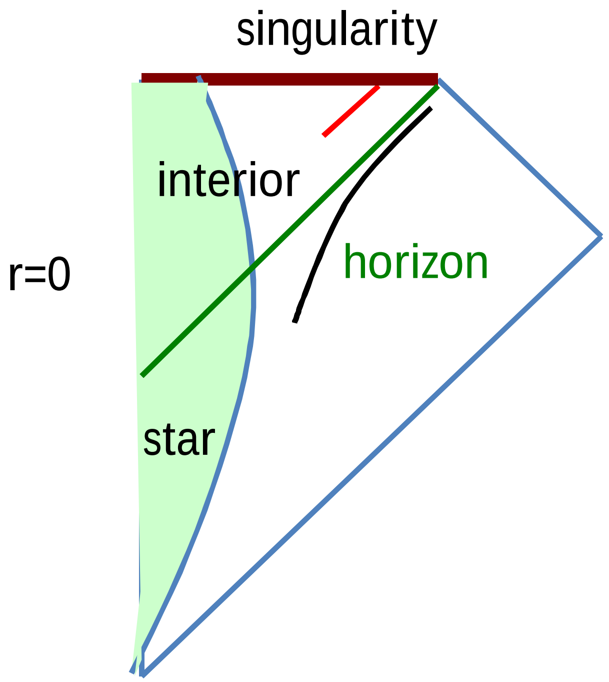


Accelerated observer only has access to the right wedge.

If we only make observations on the right wedge → do not see the whole system → get a mixed state.

Vacuum is highly entangled !

Black hole case



Black hole from collapse

We only see the region outside the horizon, if we stay outside.

Black hole entropy

$$T \sim \frac{\hbar}{r_H}$$

Special relativity near the black hole horizon

$$r_H \leftrightarrow M$$

Einstein equations

$$dM = TdS$$

1st Law of thermodynamics

$$S = \frac{(\text{Area})}{4\hbar G_N}$$

Black hole entropy

Bekenstein, Hawking

2nd Law → area increase from Einstein equations and positive null energy condition.

Hawking

General relativity and thermodynamics

- Viewing the black hole from outside, this suggests that that general relativity is giving us a thermodynamic (approximate) description of the system if we stay outside.
- Quantum mechanics suggests that there should be an exact description where entropy does not increase. (As viewed from outside). And where Hawking radiation is not mixed.
- 2nd law suggests that information is not lost (if information were lost, why should the 2nd law be valid ?). View entropy as the information that we could in principle have but we don’t.

Unitarity from outside ?

- Identify the degrees of freedom that give rise to black hole entropy.
- Black hole entropy depends only on gravity → fundamental degrees of freedom of quantum gravity.
- Should reveal the quantum structure of spacetime.
- Understand their dynamics.

- This is something that requires going beyond perturbation theory, beyond gravity as an effective theory.

String theory

- String theory started out defined as a perturbative expansion.
- For the black hole problem→ we need to go beyond perturbation theory.
- String theory contains interesting solitons: D-branes.
- D-branes inspired some non-perturbative definitions of the theory in some cases.

Witten's talk

Polchinski

Matrix theory: Banks, Fischler, Shenker, Susskind
Gauge/gravity duality: JM, Gubser, Klebanov, Polyakov, Witten

Gauge/Gravity Duality

(or gauge/string duality, AdS/CFT, holography)

Quantum Field Theory

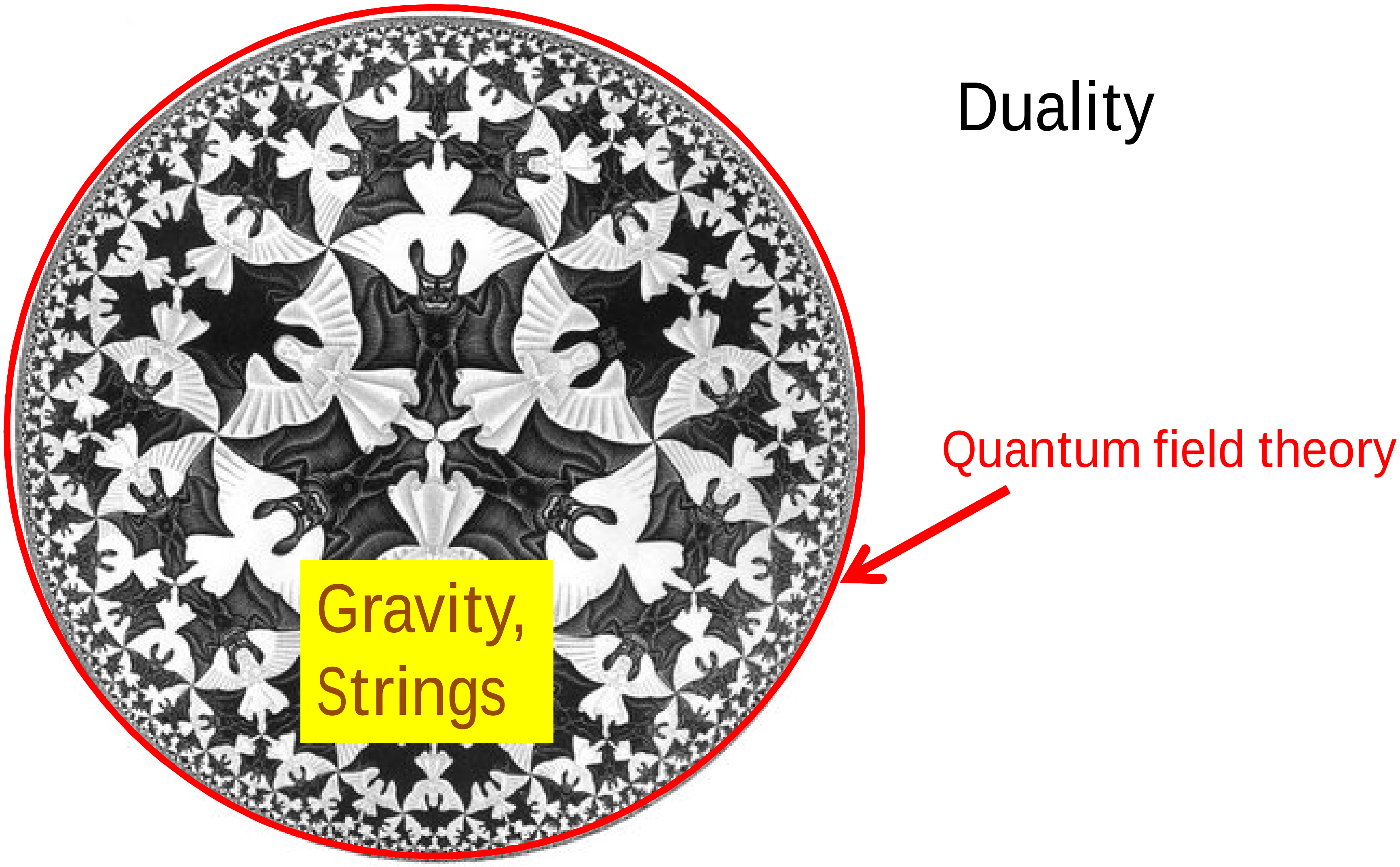
Theories of quantum interacting particles

Dynamical Space-time (General relativity) string theory

Gravity in asymptotically Anti de Sitter Space

Anti de Sitter = hyperbolic space
with a time-like direction

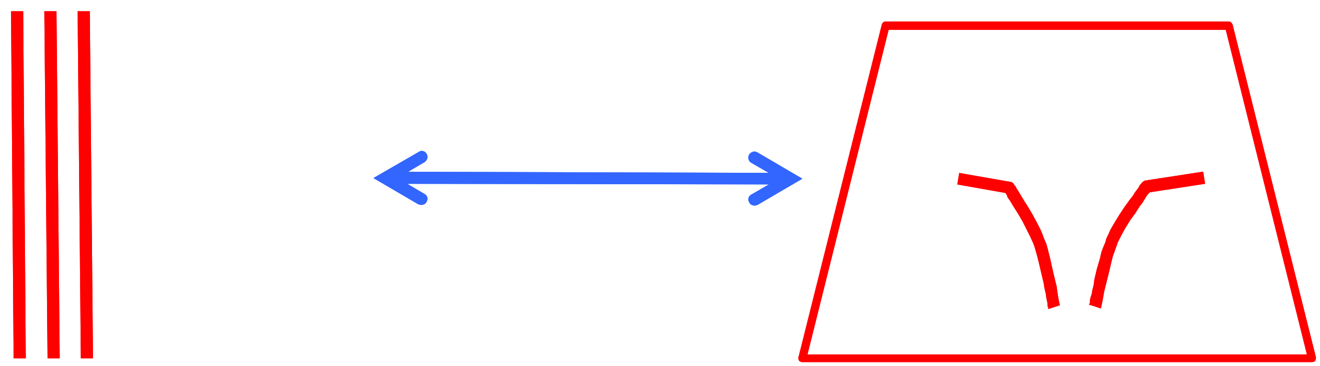
Gravity in asymptotically Anti de Sitter Space



Brane argument

JM 1997

Polchinski



Horowitz
Strominger

Collection of N 3-branes

Geometry of a black 3-brane

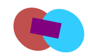
Low energies

SU(N) Super Quantum Chromodynamics in four dimensions

=

string theory on $AdS_5 \times S^5$

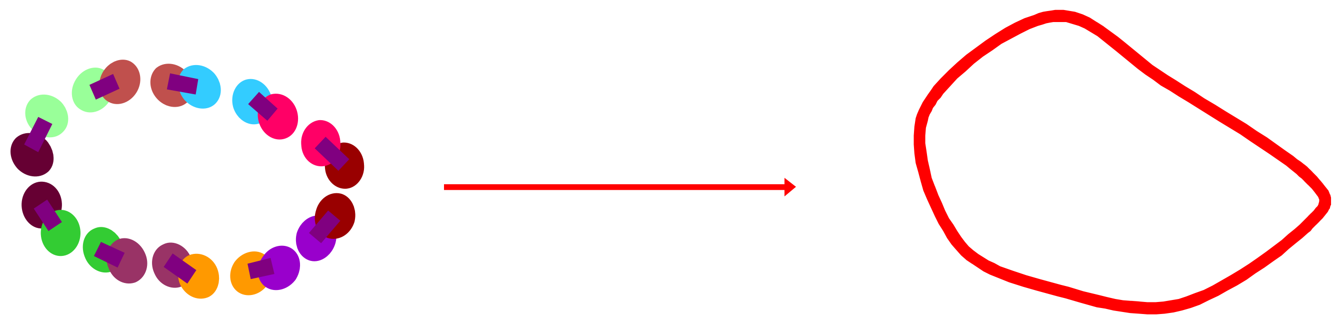
Large N gauge theories and strings

Gluon: color and anti-color 

Take N colors instead of 3, SU(N)

t' Hooft '74

Large N limit

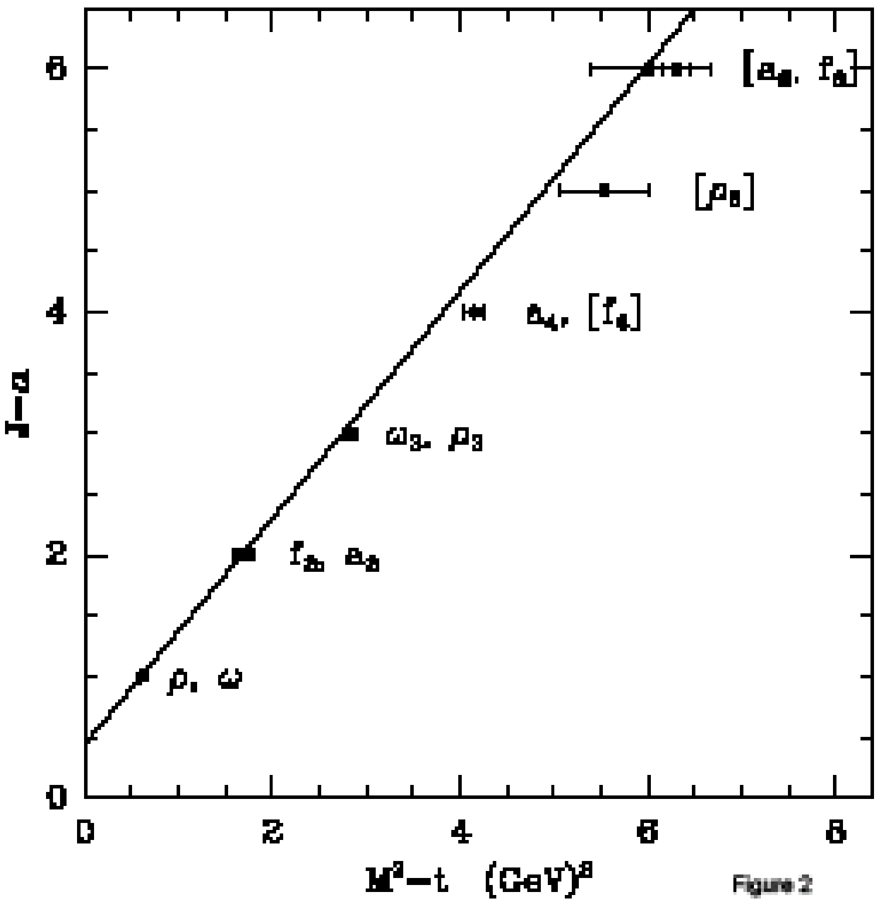


g^2N = effective interaction strength.
Keep it fixed when N → infinity

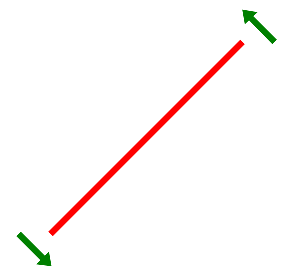
Closed strings → glueballs

String coupling ~ 1/N

Experimental evidence for strings in chromodynamics



From E. Klempt **hep-ex/0101031**

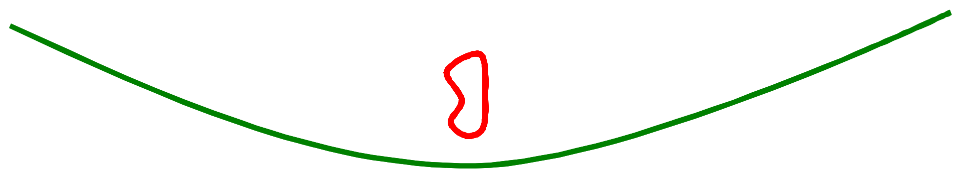


Rotating String model

$$m^2 \sim TJ_{\max} + const$$

Gravity from strings

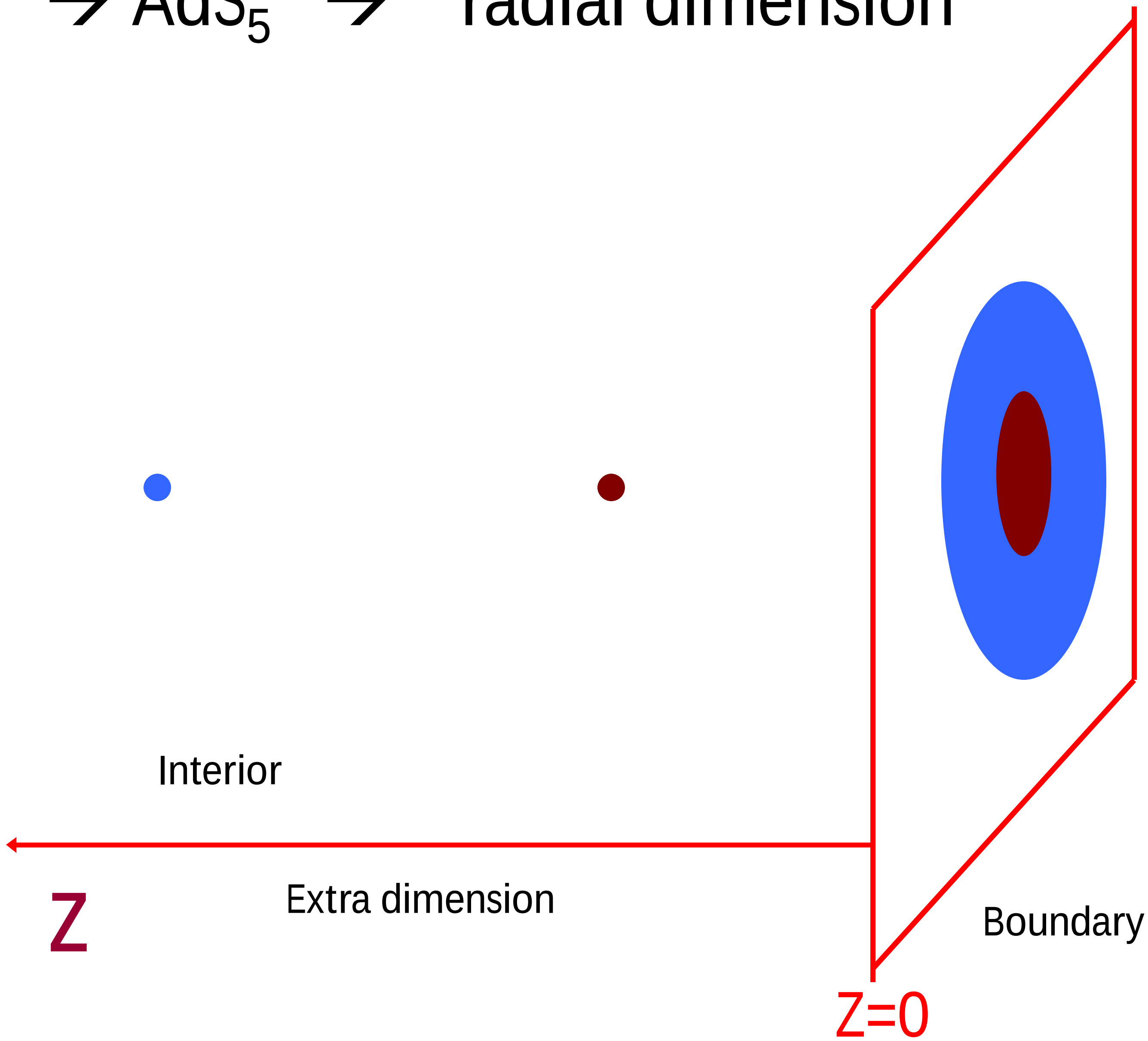
$$\frac{\text{Radius of curvature}}{\text{size of string}} \sim (\text{effective field theory coupling})^{\text{positive}}$$

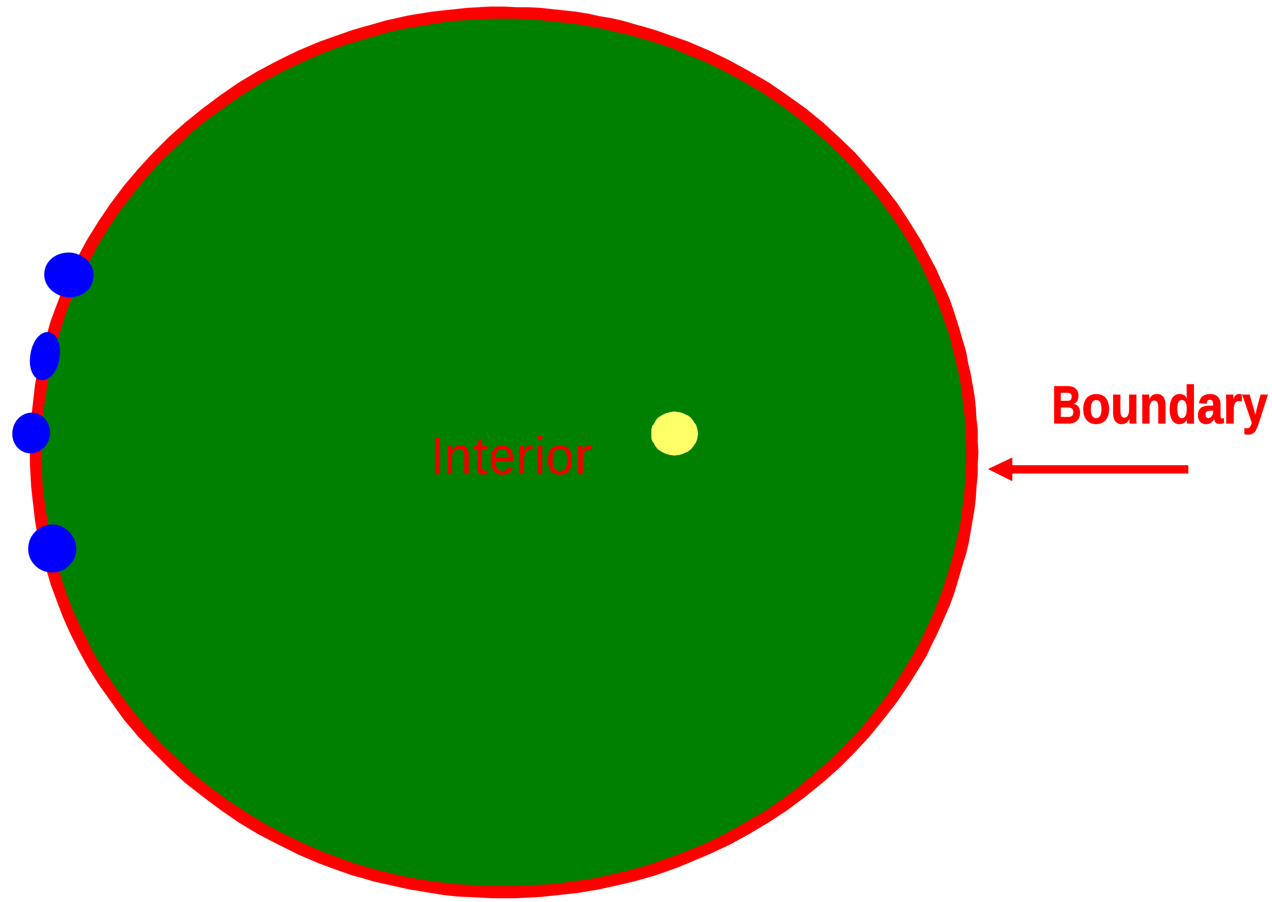


$$\text{string coupling} \sim G_N \propto \frac{1}{N^2}$$

Einstein gravity → We need large N and strong coupling.

3+1 → AdS_5 → radial dimension





Einstein Gravity in the interior → Described by very strongly interacting particles on the boundary.

BLACK HOLES = High energy, thermalized states on the boundary

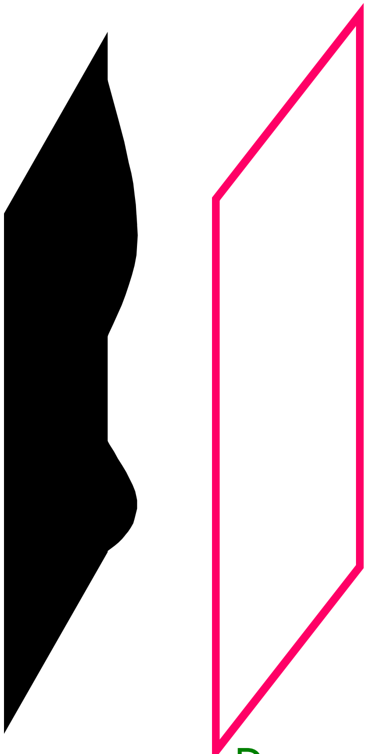
- Entropy = Area of the horizon = Number of states in the boundary theory.

Strominger, Vafa,...

- Falling into the black hole = thermalization of a perturbation in the boundary theory.

Black holes and hydrodynamics

- Field theory at finite temperature = black brane in Anti-de-Sitter space



Ripples on the black brane = hydrodynamic modes

Absorption into the black hole = dissipation, viscosity.

Transport coefficients → Solving wave equations in this background.

Einstein equations → hydrodynamics (Navier Stokes equations)

Discovery of the role of anomalies in hydrodynamics

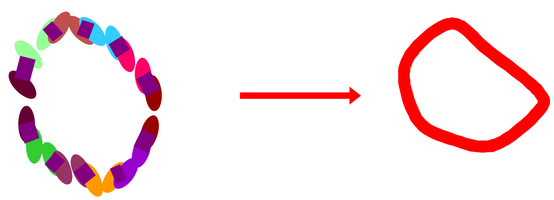
Damour, Herzog, Son, Kovtun, Starinets, Bhattacharyya, Hubeny, Loganayagam, Mandal, Minwalla, Morita, Rangamani, Reall, Bredberg, Keeler, Lysov, Strominger…

- We can form a black hole and predict what comes out by using the boundary theory.
- If you assume the duality → unitary evolution for the outside observer, no information loss.

How established is the gauge/gravity duality ?

- Lots of evidence in the simplest examples.
- Large N: Techniques of integrability → computations at any value of the effective coupling.

Minahan, Zarembo,
Beisert, Eden, Staudacher
Gromov, Kazakov, Vieira
Arutynov, Frolov
Bombardeli, Fioravanti, Tateo
....



- No explicit change of variables between bulk and boundary theories (as in a Fourier transform).

In the meantime...

Black holes as a source of information

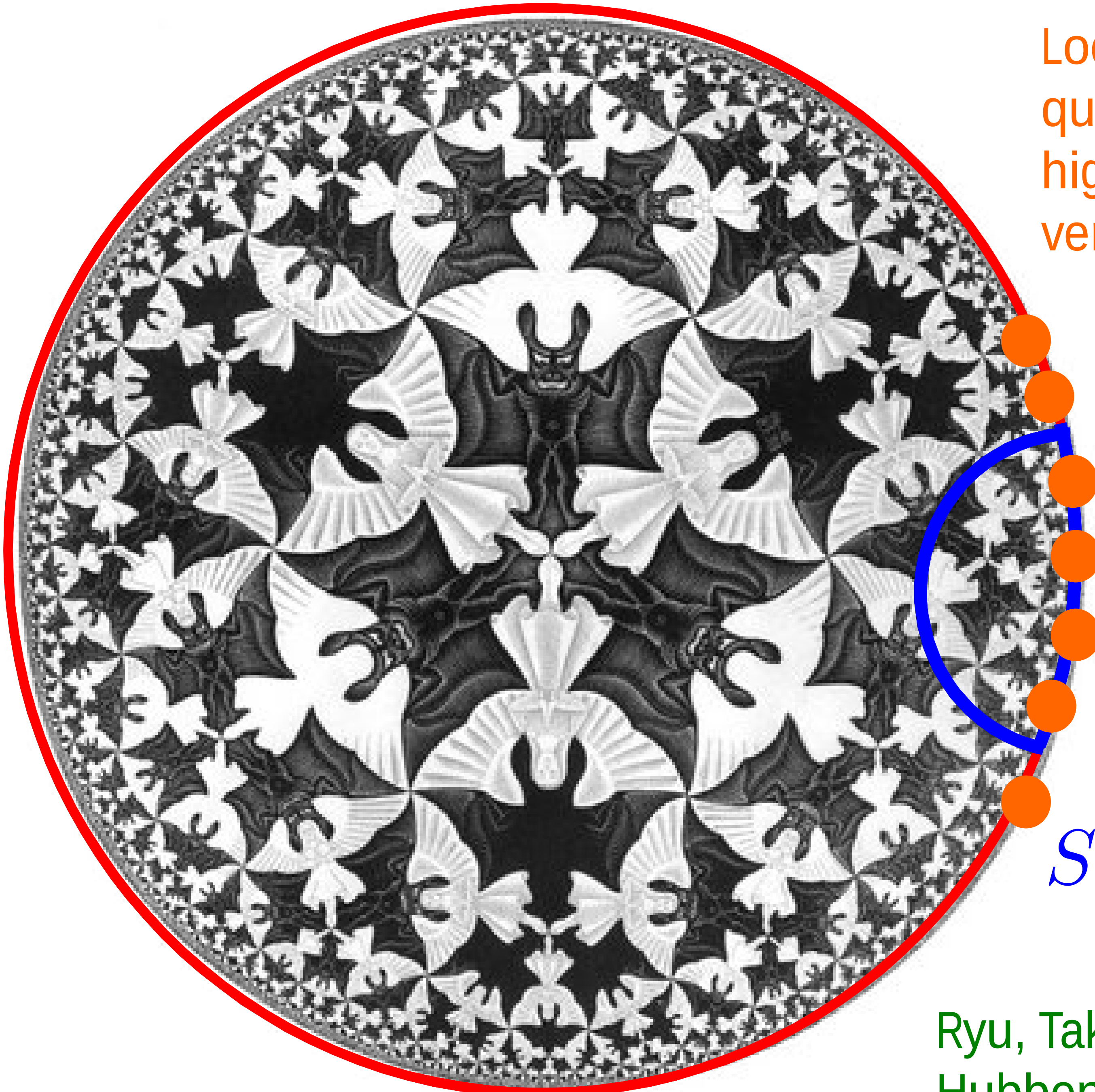
- Strongly coupled field theory problems → Simple gravity problems.

  Heavy ion collisions, high temperature superconductors, etc..

- Geometrization of physics !
- Why could strong coupling simplify the problem?

Ex: Gas of particles → Hydrodynamics
Need some strong enough interactions (zero interactions → Infinite viscosity)

Gravity is the "hydrodynamics of entanglement"



Local boundary quantum bits are highly interacting and very entangled

$$S(R) = \frac{A_{\min}}{4G_N}$$

Ryu, Takayanagi, Hubbeny, Rangamani

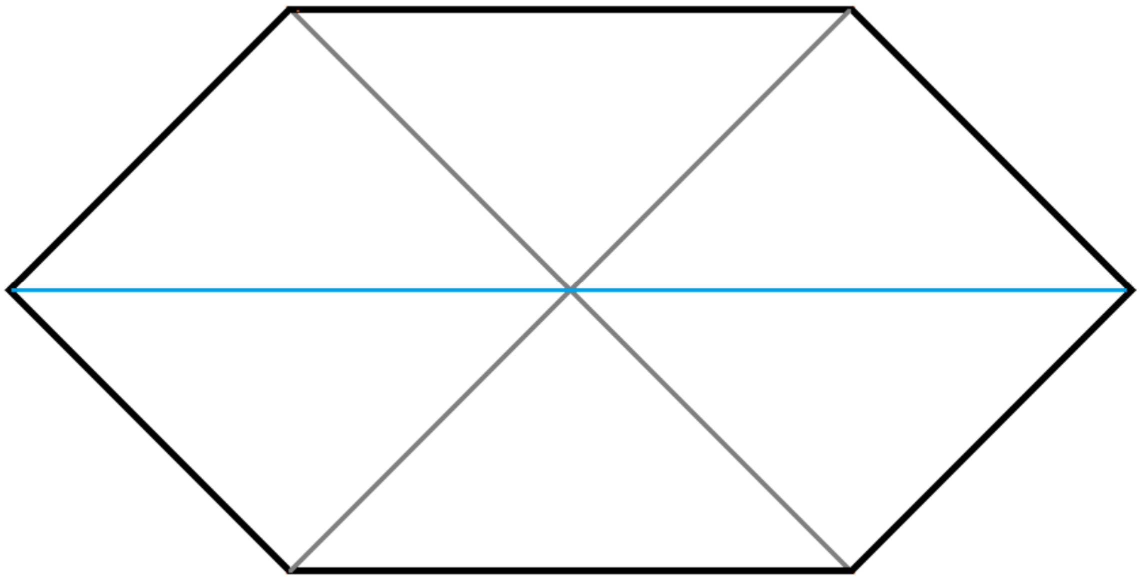
Entanglement and geometry

- The entanglement pattern present in the state of the boundary theory can translate into geometrical features of the interior.

- Spacetime is closely connected to the entanglement properties of the fundamental degrees of freedom.

- Slogan: Entanglement is the glue that holds spacetime together…

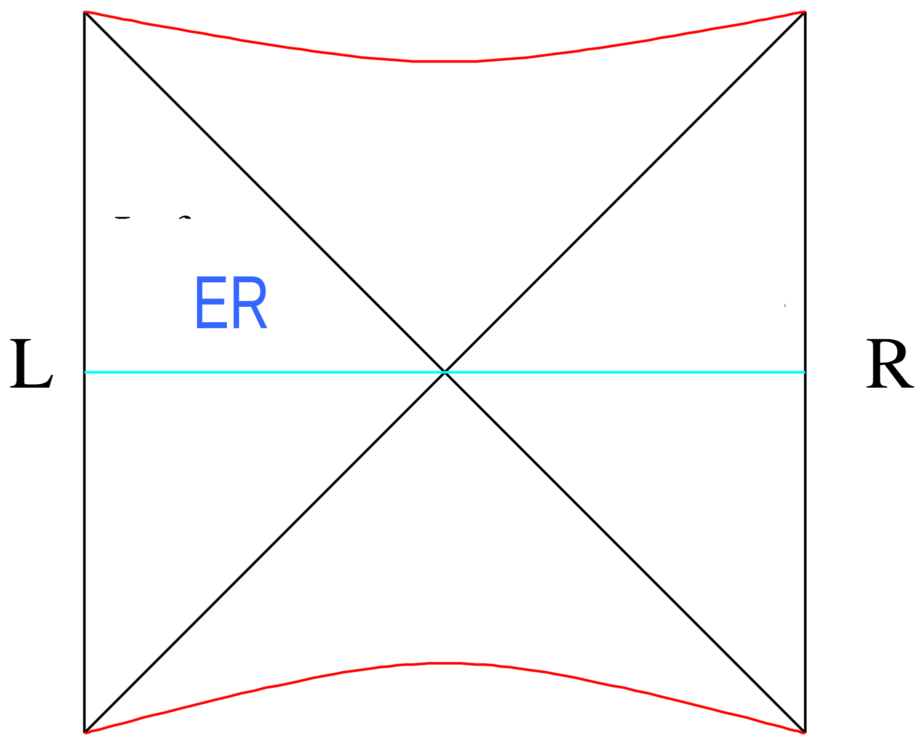
Two sided Schwarzschild solution



Eddington, Lemaitre,
Einstein, Rosen,
Finkelstein
Kruskal

Simplest spherically symmetric solution of pure Einstein gravity
(with no matter)

Two sided AdS black hole



Entangled state in two non-interacting CFT's.

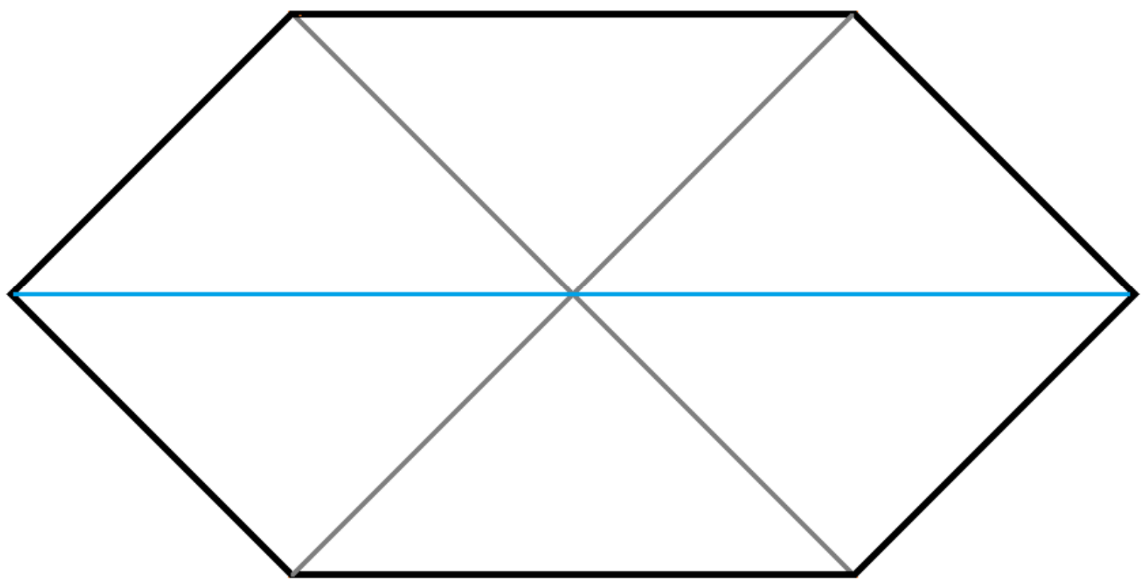
Geometric connection from entanglement

Israel
JM

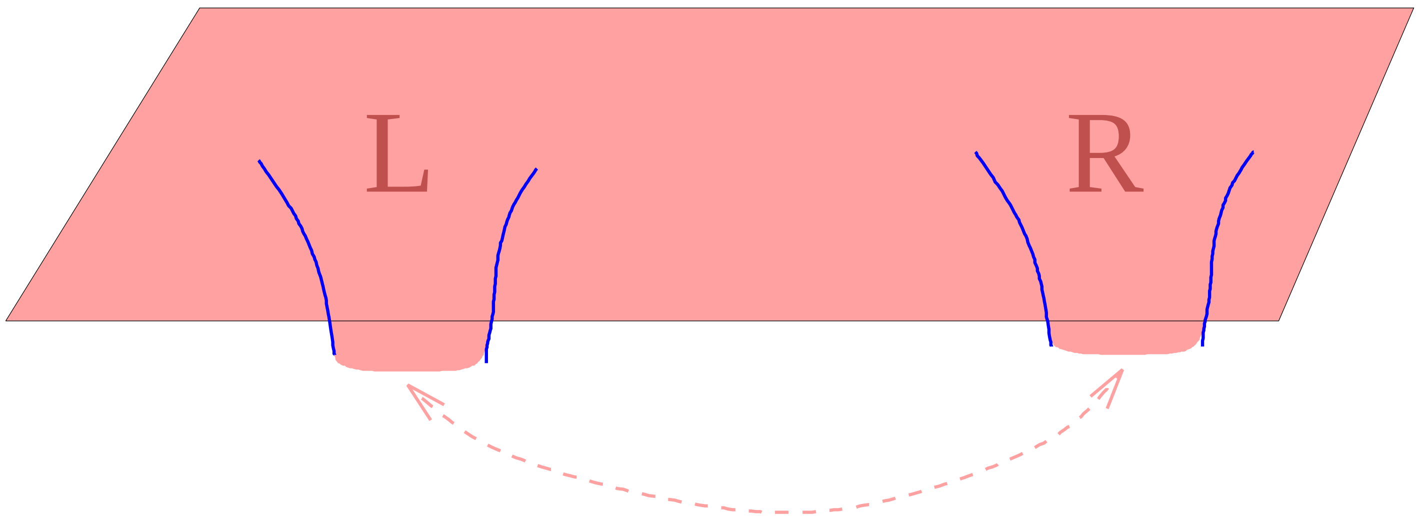
$$|\Psi\rangle = \sum_n e^{-\beta E_n/2} |\bar{E}_n\rangle_L \times |E_n\rangle_R$$

EPR

Back to the two sided Schwarzschild solution

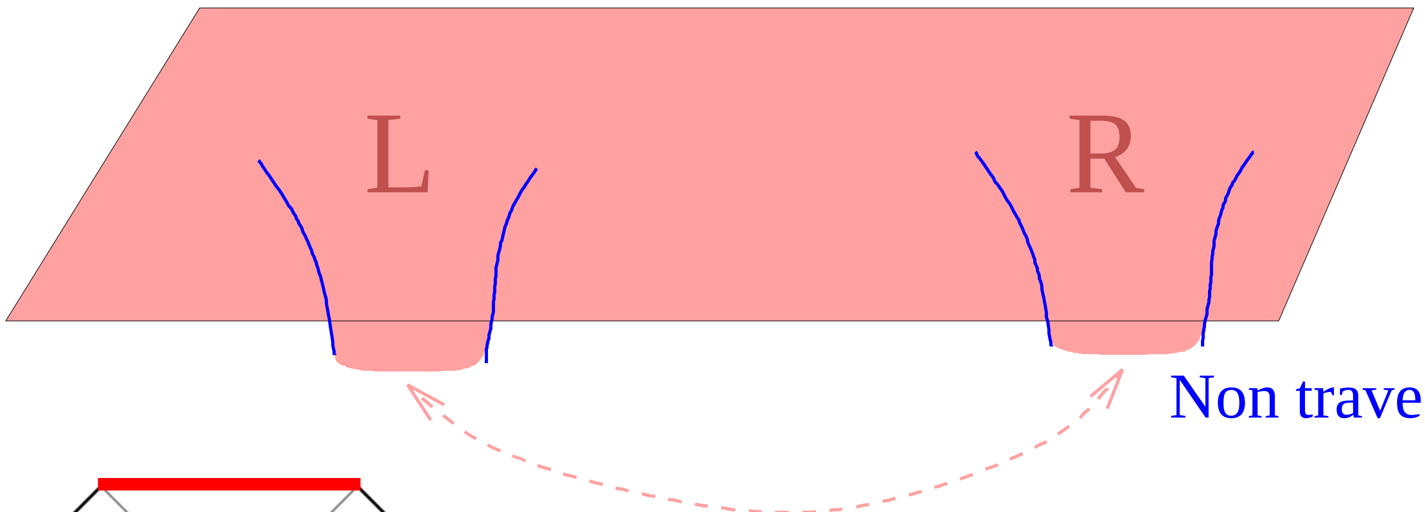


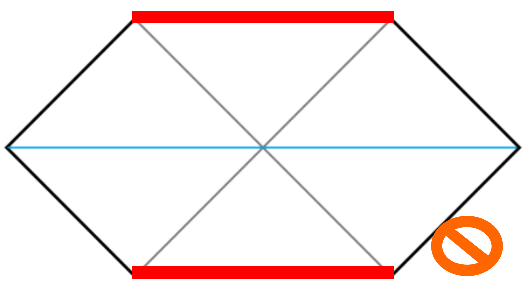
Wormhole interpretation.



Note: If you find two black holes in nature, produced by gravitational collapse, they will not be described by this geometry

No faster than light travel





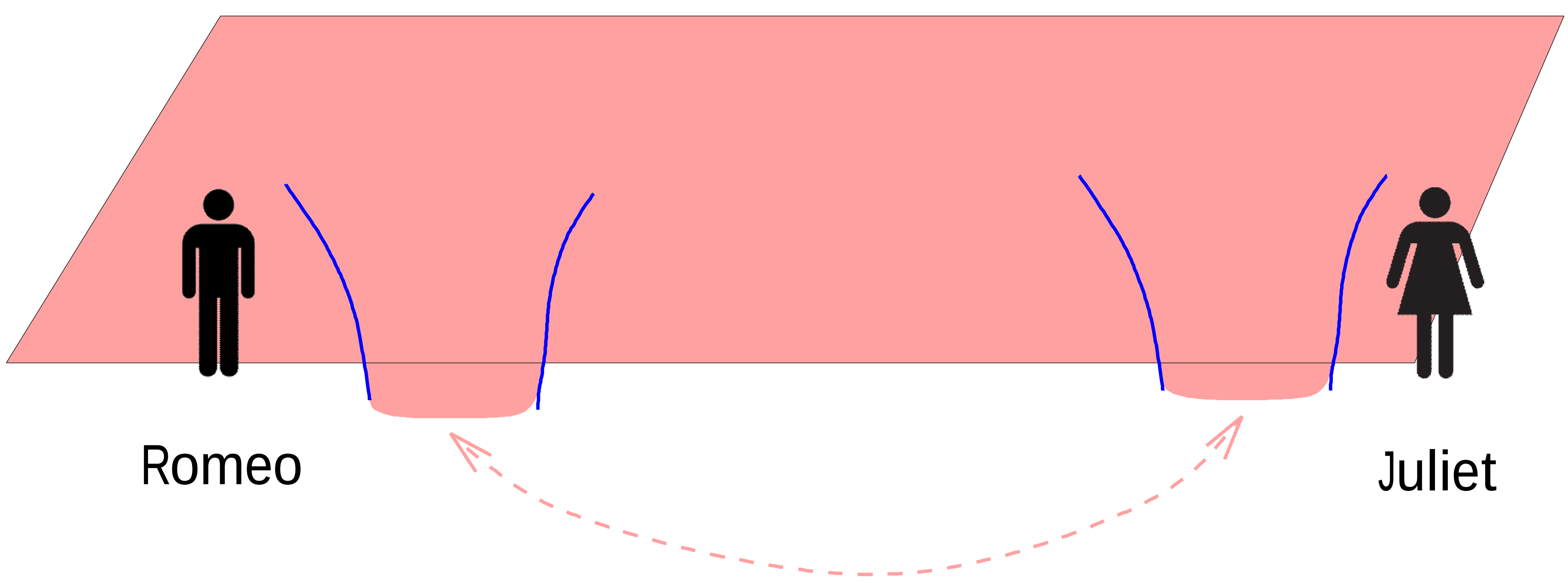
Non travesable

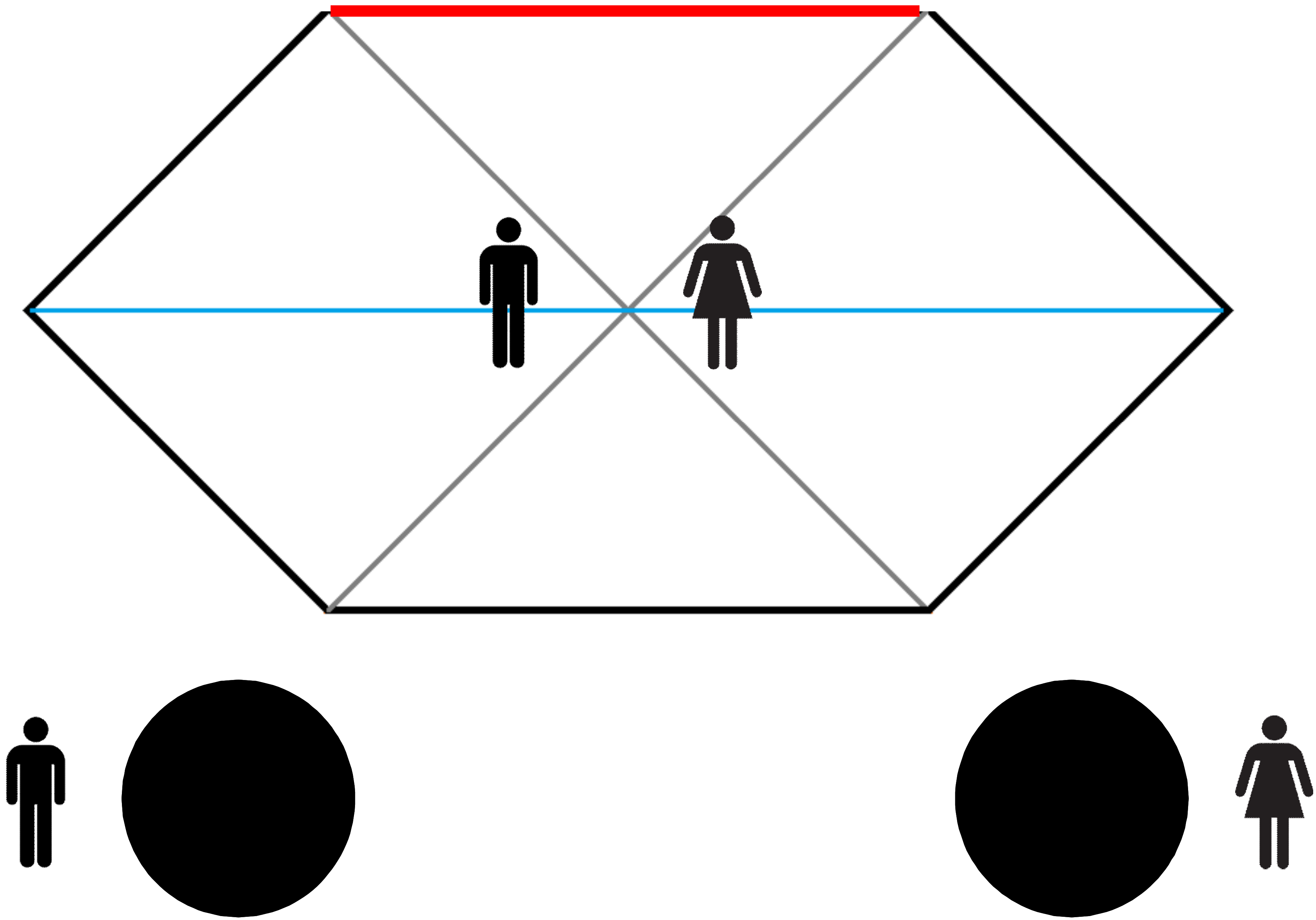
No signals

No causality violation

Fuller, Wheeler, Friedman, Schleich, Witt, Galloway, Wooglar

A forbidden meeting





ER = EPR

- Wormhole = EPR pair of two black holes in a particular entangled state:

- Large amounts of entanglement can give rise to a geometric connection.

J.M., Susskind

- We can complicate the entanglement of the two sided black hole → get longer wormhole

Stanford, Shenker, Roberts, Susskind

Black hole interior

- We do not understand how to describe it in the boundary theory.
- General relativity tells us that we have and interior but it is not clear that the exterior is unitary.
- Some paradoxes arise in some naïve constructions

Hawking,
Mathur, Almheiri, Marolf, Polchinski, Sully

- Actively explored... Under construction...

Error correcting codes

Nonlinear quantum mechanics

Entanglement

Firewalls/Fuzzballs

Non-locality

Final state projection

Conclusions

- Quantum mechanics in curved spacetime gives rise to interesting effects: Hawking radiation and primordial inflationary fluctuations.
- These effects are crucial for explaining features of our universe.
- Black hole thermodynamics poses interesting problems: Entropy, Unitarity, Information problem.

Conclusions

- Exploration of these problems lead to connections between strongly coupled quantum field theories and gravity.
- This connection has ``practical'' applications to other fields of physics. GR for superconductors.
- Patterns of entanglement are connected to geometry.
- The black hole interior continues to be a puzzling problem, whose resolution will give us new insights into the structure of spacetime.